

CONTRÔLE CONTINU 1 - THÉORIE DE STUECKELBERG

$$\begin{aligned} \textcircled{1} \quad \mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\Pi^2}{2} A_\mu A^\mu - \frac{\lambda}{2} (\partial A)^2 - J^\mu A_\mu \\ &= -\frac{1}{2} \eta^{\mu\sigma} \eta^{\nu\sigma} \partial_\mu A_\nu (\partial_\sigma A_\mu - \partial_\sigma A_\nu) + \frac{\Pi^2}{2} \eta^{\mu\nu} A_\mu A_\nu - \frac{\lambda}{2} (\eta^{\mu\nu} \partial_\mu A_\nu)^2 - \eta^{\mu\nu} J_\mu A_\nu \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial A_\alpha} = \frac{\Pi^2}{2} \eta^{\mu\nu} \left(\underbrace{\frac{\partial A_\mu}{\partial A_\alpha}}_{\delta_\mu^\alpha} A_\nu + A_\mu \underbrace{\frac{\partial A_\nu}{\partial A_\alpha}}_{\delta_\nu^\alpha} \right) - \eta^{\mu\nu} J_\mu \underbrace{\frac{\partial A_\nu}{\partial A_\alpha}}_{\delta_\nu^\alpha} = \Pi^2 A^\alpha - J^\alpha$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial (\partial_\beta A_\alpha)} &= -\frac{1}{2} \eta^{\mu\sigma} \eta^{\nu\sigma} \frac{\partial (\partial_\mu A_\nu)}{\partial (\partial_\beta A_\alpha)} F_{\sigma\tau} - \frac{1}{2} \eta^{\mu\sigma} \eta^{\nu\sigma} \partial_\mu A_\nu \left[\frac{\partial (\partial_\sigma A_\mu)}{\partial (\partial_\beta A_\alpha)} - \frac{\partial (\partial_\sigma A_\nu)}{\partial (\partial_\beta A_\alpha)} \right] \\ &\quad - \frac{\lambda}{2} \cdot 2 (\partial A) \cdot \eta^{\mu\nu} \frac{\partial (\partial_\mu A_\nu)}{\partial (\partial_\beta A_\alpha)} = \underbrace{\delta_\sigma^\beta \delta_\tau^\alpha}_{\text{etc.}} \\ &= -F^{\beta\alpha} - \lambda \eta^{\beta\alpha} (\partial A) \end{aligned}$$

$$\begin{aligned} \partial_\beta \frac{\partial \mathcal{L}}{\partial (\partial_\beta A_\alpha)} &= -\partial_\beta F^{\beta\alpha} - \lambda \eta^{\beta\alpha} \partial_\beta (\partial A) = -\partial_\beta (\partial^\beta A^\alpha - \partial^\alpha A^\beta) - \lambda \partial^\alpha (\partial A) = \\ &= -\square A^\alpha + (1-\lambda) \partial^\alpha (\partial A) \end{aligned}$$

$$\Rightarrow \text{Euler-Lagrange: } \boxed{(\square + \Pi^2) A^\alpha = J^\alpha + (1-\lambda) \partial^\alpha (\partial A)}$$

$$\textcircled{2} \text{ pour } \lambda=0 \text{ dans le vide: } \boxed{(\square + \Pi^2) A^\alpha = \partial^\alpha (\partial A)}$$

équation de Klein-Gordon avec masse Π

$$\textcircled{3} \quad \partial_\alpha (\square + \Pi^2) A^\alpha = \underbrace{\partial_\alpha J^\alpha}_{=0} + (1-\lambda) \partial_\alpha \partial^\alpha (\partial A)$$

$$\Rightarrow (\square + \Pi^2) (\partial A) = (1-\lambda) \square (\partial A) \Rightarrow \boxed{(\lambda \square + \Pi^2) (\partial A) = 0}$$

$$\textcircled{4} \quad (\lambda \square + \Pi^2) (\partial A) = 0 \Rightarrow \left(\square + \frac{\Pi^2}{\lambda} \right) (\partial A) = 0$$

équation de Klein-Gordon avec masse m telle que $m^2 = \frac{\Pi^2}{\lambda}$

$$\textcircled{5} \quad A_S^\alpha = -\frac{1}{m^2} \partial^\alpha (\partial A)$$

$$(\square + m^2)(\partial A) = 0 \quad \Rightarrow \quad -\frac{1}{m^2} \partial^\alpha (\square + m^2)(\partial A) = 0$$

$$\Rightarrow \quad (\square + m^2) \partial^\alpha (\partial A) = 0$$

équation de Klein-Gordon avec masse m

$$\textcircled{6} \quad A_T^\alpha = A^\alpha + \frac{1}{m^2} \partial^\alpha (\partial A)$$

$$\partial_\alpha A_T^\alpha = \partial_\alpha A^\alpha + \frac{1}{m^2} \partial_\alpha \partial^\alpha (\partial A) = \left(1 + \frac{1}{m^2} \square\right) (\partial A) = \frac{1}{m^2} (m^2 + \square) (\partial A) = 0$$

$$\begin{aligned} \textcircled{7} \quad (\square + \eta^2) A_T^\alpha &= (\square + \eta^2) \left(A^\alpha + \frac{1}{m^2} \partial^\alpha (\partial A) \right) = \\ &= \square A^\alpha + \frac{1}{m^2} \square \partial^\alpha (\partial A) + \eta^2 A^\alpha + \frac{\eta^2}{m^2} \partial^\alpha (\partial A) = \\ &= (1-\lambda) \partial^\alpha (\partial A) + \frac{1}{m^2} \partial^\alpha \square (\partial A) + \lambda \partial^\alpha (\partial A) = \\ &= \frac{1}{m^2} \partial^\alpha (m^2 + \square) (\partial A) = 0 \end{aligned}$$