4.5] $\langle 0| T\{\varphi(x)\varphi^\dagger(y)\}|0\rangle = \theta(x^0-y^0)\underbrace{\langle 0|\varphi(x)\varphi^\dagger(y)|0\rangle}_{(i)} + \theta(y^0-x^0)\underbrace{\langle 0|\varphi^\dagger(y)\varphi(x)|0\rangle}_{(ii)}$

(i) $\langle 0|\varphi(x)\varphi^\dagger(y)|0\rangle = \langle 0|\int d\tilde{k}\left[\underset{\substack{\uparrow \\ \text{création en } x}}{a_{\vec{k}}} e^{-ikx} + \underbrace{b^\dagger_{\vec{k}}}_{\to 0} e^{ikx}\right]\int d\tilde{p}\left[\underset{\substack{\uparrow \\ \text{création en } y}}{a^\dagger_{\vec{p}}} e^{ipx} + \underbrace{b_{\vec{p}}}_{\to 0} e^{-ipx}\right]|0\rangle$

$$\langle 0|\varphi(x)\varphi^\dagger(y)|0\rangle = \int d\tilde{k}\, d\tilde{p}\, e^{-ikx} e^{ipy} \underbrace{\langle 0|a_{\vec{k}} a^\dagger_{\vec{p}}|0\rangle}_{=\tilde{\delta}_{kp}} = \int d\tilde{k}\, e^{-ik(x-y)}$$

(ii) $\langle 0|\varphi^\dagger(y)\varphi(x)|0\rangle = \ldots = \int d\tilde{k}\, e^{ik(x-y)}$

$$\Longrightarrow \langle 0|T\{\varphi(x)\varphi^\dagger(y)\}|0\rangle = \int d\tilde{k}\left[\theta(x^0-y^0)e^{-ik(x-y)} + \theta(y^0-x^0)e^{ik(x-y)}\right]$$
$$= -i\, G_F(x-y)$$