Exercices

4.1)

$$T^{00} = \dot{\varphi}^* \dot{\varphi} + \dot{\varphi}^* \dot{\varphi} - \mathcal{L} =$$
$$= 2\dot{\varphi}^* \dot{\varphi} - \left[\dot{\varphi}^* \dot{\varphi} - (\vec{\nabla}\varphi^*)(\vec{\nabla}\varphi) - V(\varphi) \right]$$
$$= \dot{\varphi}^* \dot{\varphi} + (\vec{\nabla}\varphi^*)(\vec{\nabla}\varphi) + V(\varphi)$$

4.2)

$$[a_{\vec{k}}, a^\dagger_{\vec{p}}] = \frac{1}{2}\left[a_{\vec{k}1} + i a_{\vec{k}2},\ a^\dagger_{\vec{p}1} - i a^\dagger_{\vec{p}2} \right] =$$
$$= \frac{1}{2}\left([a_{\vec{k}1}, a^\dagger_{\vec{p}1}] + i [a_{\vec{k}2}, a^\dagger_{\vec{p}1}] - i [a_{\vec{k}1}, a^\dagger_{\vec{p}2}] + [a_{\vec{k}2}, a^\dagger_{\vec{p}2}] \right) =$$
$$= \frac{1}{2}\, 2 \cdot [a_{\vec{k}1}, a^\dagger_{\vec{p}1}] = [a_{\vec{k}1}, a^\dagger_{\vec{p}1}] = (2\pi)^3\, 2\omega_{\vec{k}}\, \delta(\vec{k} - \vec{p})$$

$$[b_{\vec{k}}, b^\dagger_{\vec{p}}] = \dots = (2\pi)^3\, 2\omega_{\vec{k}}\, \delta(\vec{k} - \vec{p})$$

$$[a_{\vec{k}}, b^\dagger_{\vec{p}}] = \dots = 0$$

4.3)

$$\partial_\mu j^\mu = iq\, \partial_\mu \left[\varphi^* (\partial^\mu \varphi) - (\partial^\mu \varphi^*) \varphi \right] =$$
$$= iq \left[(\partial_\mu \varphi^*)(\partial^\mu \varphi) + \varphi^* (\Box \varphi) - (\Box \varphi^*) \varphi - (\partial^\mu \varphi^*)(\partial_\mu \varphi) \right] =$$
$$= iq \left[(\Box \varphi^*) \varphi - \varphi^* (\Box \varphi) \right] \underset{\text{Klein-Gordon}}{=} iq \left[-m^2 \varphi^* \varphi + m^2 \varphi^* \varphi \right] = 0$$

4.4) voir cours ...