

Exercices

$$\begin{aligned}
 4.1] \quad T^{00} &= \dot{\varphi}^* \dot{\varphi} + \dot{\varphi}^* \dot{\varphi} - \mathcal{L} = \\
 &= 2\dot{\varphi}^* \dot{\varphi} - [\dot{\varphi}^* \dot{\varphi} - (\vec{\nabla} \varphi^*)(\vec{\nabla} \varphi) - V(\varphi)] \\
 &= \dot{\varphi}^* \dot{\varphi} + (\vec{\nabla} \varphi^*)(\vec{\nabla} \varphi) + V(\varphi)
 \end{aligned}$$

$$\begin{aligned}
 4.2] \quad [a_{\vec{k}}, a_{\vec{p}}^+] &= \frac{1}{2} [a_{\vec{k}1} + i a_{\vec{k}2}, a_{\vec{p}1}^+ - i a_{\vec{p}2}^+] = \\
 &= \frac{1}{2} ([a_{\vec{k}1}, a_{\vec{p}1}^+] + i [a_{\vec{k}2}, a_{\vec{p}1}^+] - i [a_{\vec{k}1}, a_{\vec{p}2}^+] + [a_{\vec{k}2}, a_{\vec{p}2}^+]) = \\
 &= \frac{1}{2} 2 \cdot [a_{\vec{k}1}, a_{\vec{p}1}^+] = [a_{\vec{k}1}, a_{\vec{p}1}^+] = (2\pi)^3 2\omega_{\vec{k}} \delta(\vec{k} - \vec{p})
 \end{aligned}$$

$$\begin{aligned}
 [b_{\vec{k}}, b_{\vec{p}}^+] &= \dots = (2\pi)^3 2\omega_{\vec{k}} \delta(\vec{k} - \vec{p}) \\
 [a_{\vec{k}}, b_{\vec{p}}^+] &= \dots = 0
 \end{aligned}$$

$$\begin{aligned}
 4.3] \quad \hat{q} \hat{J}^r &= iq \partial_r [\varphi^*(\partial^r \varphi) - (\partial^r \varphi^*) \varphi] = \\
 &= iq \left[(\partial_r \varphi^*)(\partial^r \varphi) + \varphi^*(\square \varphi) - (\square \varphi^*) \varphi - (\partial^r \varphi^*)(\partial_r \varphi) \right] = \\
 &= iq \left[(\square \varphi^*) \varphi - \varphi^*(\square \varphi) \right] = iq \left[-m^2 \varphi^* \varphi + m^2 \varphi^* \varphi \right] = 0
 \end{aligned}$$

↑
 Klein-Gordon

4.4.] voir cours ...