

Flavour violation in SUSY GUTs

$SU(5) \times A_4$ case study

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Flavour in the MSSM

Flavoured Grand Unification

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Summary and perspectives

based mainly on work in collaboration with
Jordan Bernigaud, Stephen F. King, Samuel J. Rowley
to be published — arXiv:1812.01xyz

Minimal Supersymmetric Standard Model

SM Particles		Spin		Spin	Superpartners	
Quarks	$(u_L \ d_L)$	1/2	Q	0	$(\tilde{u}_L \ \tilde{d}_L)$	Squarks
	u_R^\dagger	1/2	\bar{u}	0	\tilde{u}_R^*	
	d_R^\dagger	1/2	\bar{d}	0	\tilde{d}_R^*	
Leptons	$(\nu \ e_L)$	1/2	L	0	$(\tilde{\nu} \ \tilde{e}_L)$	Sleptons
	e_R^\dagger	1/2	\bar{e}	0	\tilde{e}_R^*	
Higgs	$(H_u^+ \ H_u^0)$	0	H_u	1/2	$(\tilde{H}_u^+ \ \tilde{H}_u^0)$	Higgsinos
	$(H_d^0 \ H_d^-)$	0	H_d	1/2	$(\tilde{H}_d^0 \ \tilde{H}_d^-)$	
W bosons	W^0, W^\pm	1		1/2	$\tilde{W}^0, \tilde{W}^\pm$	Winos
B boson	B^0	1		1/2	\tilde{B}^0	Bino
Gluon	g	1		1/2	\tilde{g}	Gluino
Graviton	G	2		3/2	\tilde{G}	Gravitino

Minimal Supersymmetric Standard Model

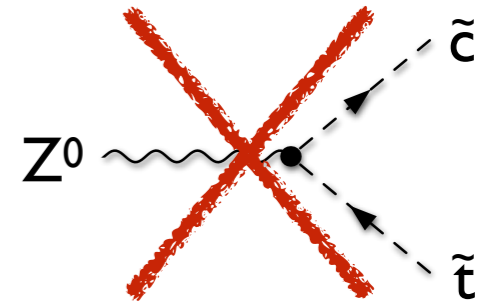
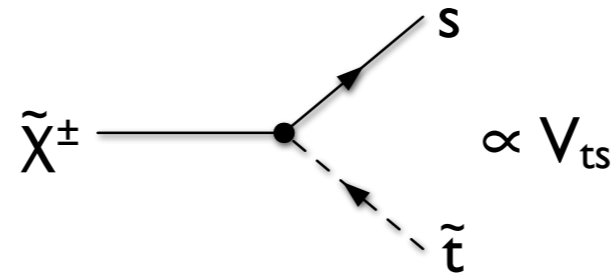
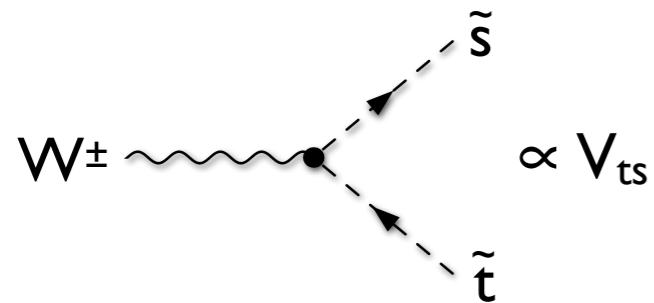
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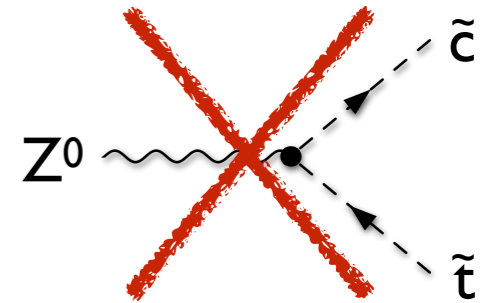
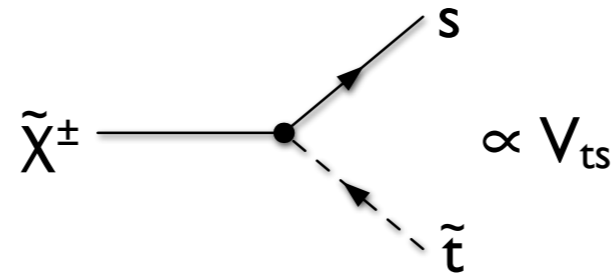
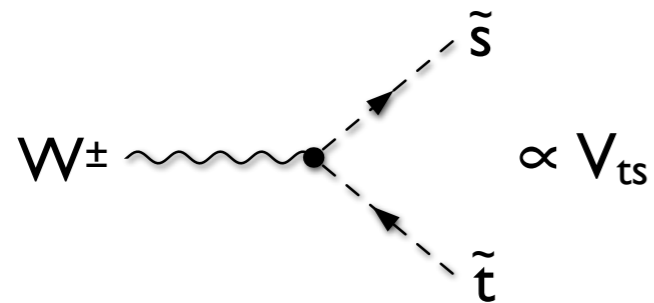
Flavour structure in the MSSM

Assume **same flavour structure** as in Standard Model: flavour-changing currents are related to CKM/PMNS-matrices — **minimal flavour violation** (MFV)

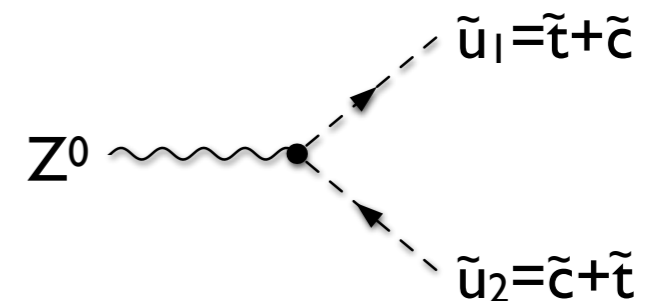
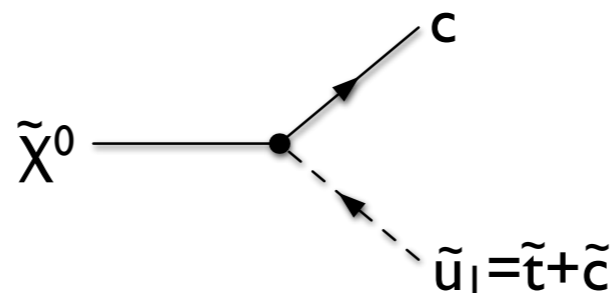
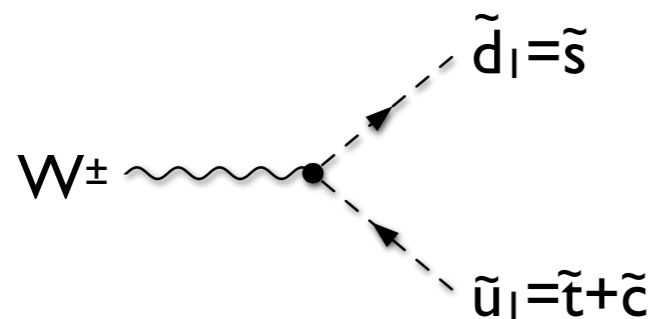


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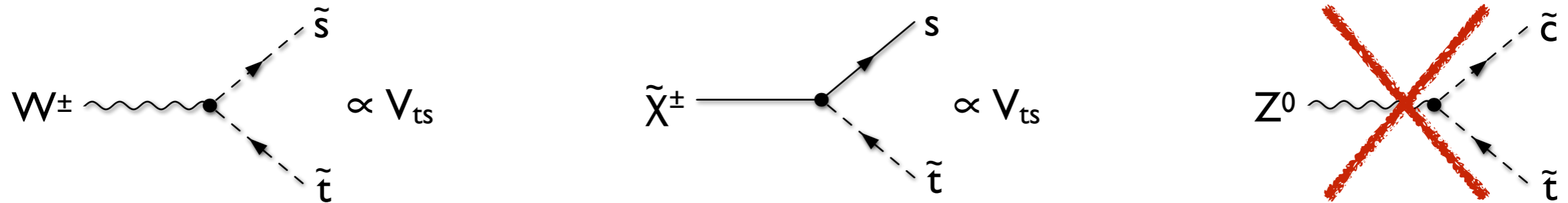


Allow for **new sources** of flavour violation: corresponding interactions not related to CKM/PMNS-matrices any more — **non-minimal flavour violation** (NMFV)



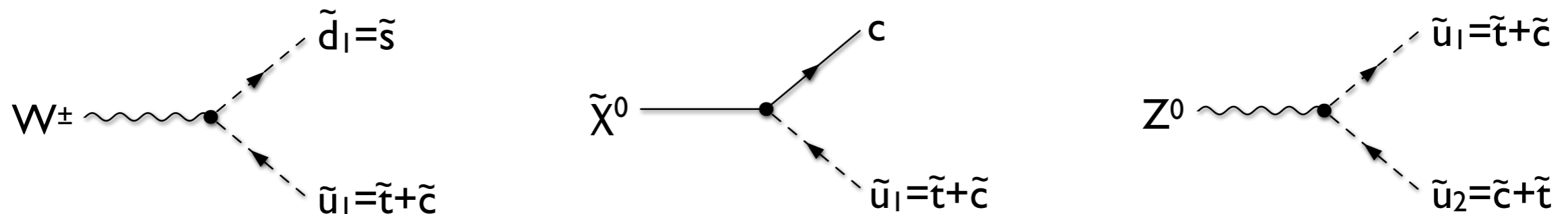
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MFV vs. NMFV...?

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Sfermion sector in the MSSM

In the **super-CKM basis**, the sfermion sector is parametrized by **four mass matrices**:

$$\mathcal{M}_{\tilde{u}}^2 = \begin{pmatrix} V_{\text{CKM}} M_{\tilde{Q}}^2 V_{\text{CKM}}^\dagger + m_u^2 + D_{\tilde{u},L} & \frac{v_u}{\sqrt{2}} T_u^\dagger - m_u \frac{\mu}{\tan \beta} \\ \frac{v_u}{\sqrt{2}} T_u - m_u \frac{\mu^*}{\tan \beta} & M_{\tilde{U}}^2 + m_u^2 + D_{\tilde{u},R} \end{pmatrix}$$

$$\mathcal{M}_{\tilde{d}}^2 = \begin{pmatrix} M_{\tilde{Q}}^2 + m_d^2 + D_{\tilde{d},L} & \frac{v_d}{\sqrt{2}} T_d^\dagger - m_d \mu \tan \beta \\ \frac{v_d}{\sqrt{2}} T_d - m_d \mu^* \tan \beta & M_{\tilde{D}}^2 + m_d^2 + D_{\tilde{d},R} \end{pmatrix}$$

$$\mathcal{M}_{\tilde{\ell}}^2 = \begin{pmatrix} M_{\tilde{L}}^2 + m_e^2 + D_{\tilde{\ell},L} & \frac{v_d}{\sqrt{2}} T_e - m_e \mu \tan \beta \\ \frac{v_d}{\sqrt{2}} T_e - m_e \mu^* \tan \beta & M_{\tilde{E}}^2 + m_e^2 + D_{\tilde{\ell},R} \end{pmatrix}$$

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5 independent mass matrices and 3 trilinear coupling matrices

$$M_{\tilde{Q}}^2, M_{\tilde{U}}^2, M_{\tilde{D}}^2, M_{\tilde{L}}^2, M_{\tilde{E}}^2 \quad T_u, T_d, T_e$$

(3x3 matrices in flavour space — 48 independent parameters)

Sfermion sector in the MSSM

Non-minimally flavour-violating terms manifest as **non-diagonal entries** in the soft mass matrices $(M_{\tilde{Q}}^2, M_{\tilde{U}}^2, M_{\tilde{D}}^2, M_{\tilde{L}}^2, M_{\tilde{E}}^2)$ and the trilinear coupling matrices (T_u, T_d, T_e)

— **dimensionless and scenario-independent parametrization:**

$$\begin{aligned} (\delta_{LL}^Q)_{ij} &= \frac{(M_{\tilde{Q}}^2)_{ij}}{(M_{\tilde{Q}})_{ii}(M_{\tilde{Q}})_{jj}} & (\delta_{RR}^U)_{ij} &= \frac{(M_{\tilde{U}}^2)_{ij}}{(M_{\tilde{U}})_{ii}(M_{\tilde{U}})_{jj}} & (\delta_{RL}^U)_{ij} &= \frac{v_u}{\sqrt{2}} \frac{(T_u)_{ij}}{(M_{\tilde{Q}})_{ii}(M_{\tilde{U}})_{jj}} \\ (\delta_{RR}^D)_{ij} &= \frac{(M_{\tilde{D}}^2)_{ij}}{(M_{\tilde{D}})_{ii}(M_{\tilde{D}})_{jj}} & (\delta_{RL}^D)_{ij} &= \frac{v_d}{\sqrt{2}} \frac{(T_d)_{ij}}{(M_{\tilde{Q}})_{ii}(M_{\tilde{D}})_{jj}} \\ (\delta_{RR}^L)_{ij} &= \frac{(M_{\tilde{L}}^2)_{ij}}{(M_{\tilde{L}})_{ii}(M_{\tilde{L}})_{jj}} & (\delta_{RR}^E)_{ij} &= \frac{(M_{\tilde{E}}^2)_{ij}}{(M_{\tilde{E}})_{ii}(M_{\tilde{E}})_{jj}} & (\delta_{RL}^E)_{ij} &= \frac{v_d}{\sqrt{2}} \frac{(T_e)_{ij}}{(M_{\tilde{L}})_{ii}(M_{\tilde{E}})_{jj}} \end{aligned}$$

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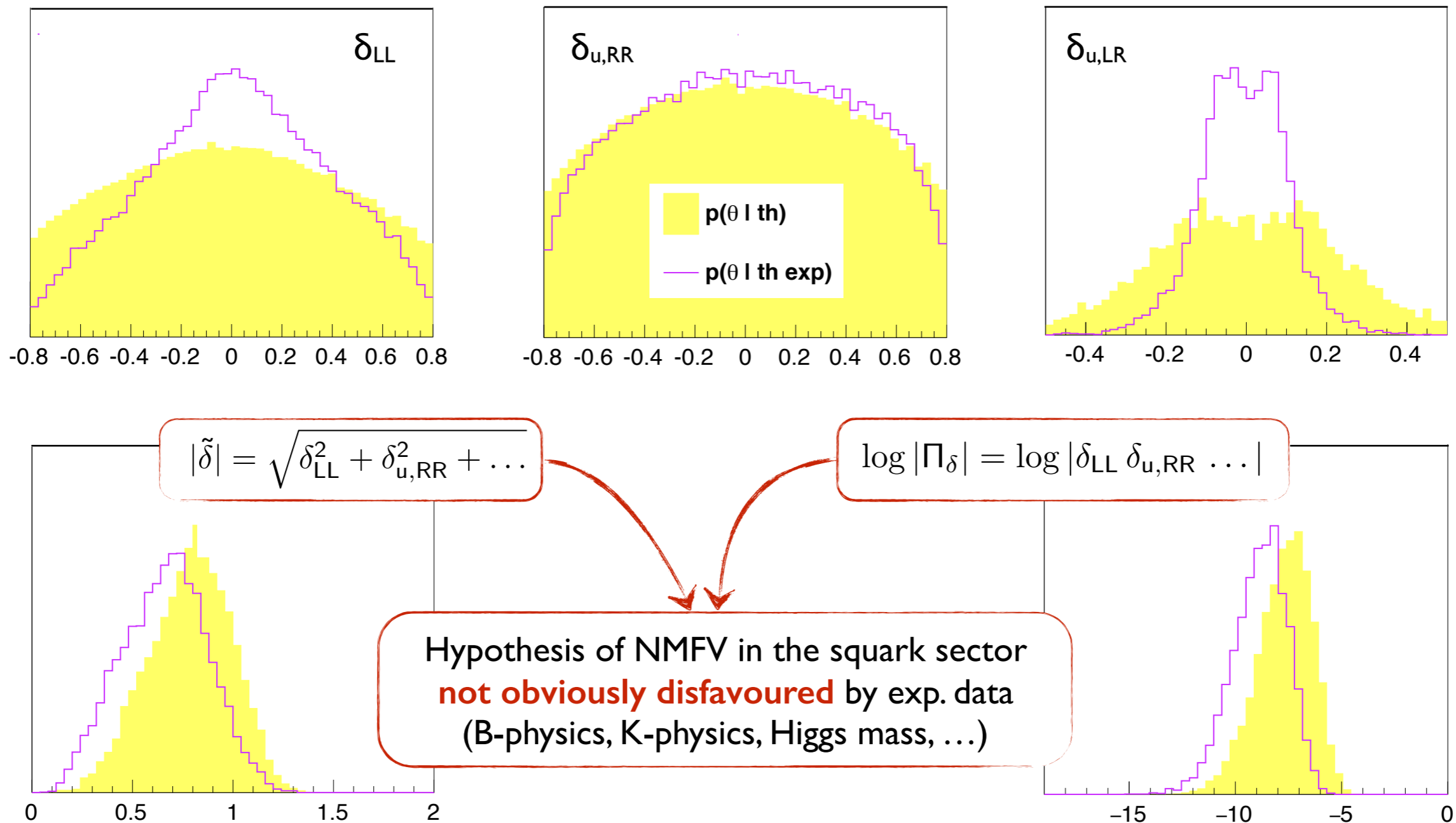
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 \end{aligned}$$

Mass eigenstates are obtained via 6x6 rotation matrices (generalized “mixing angles”):

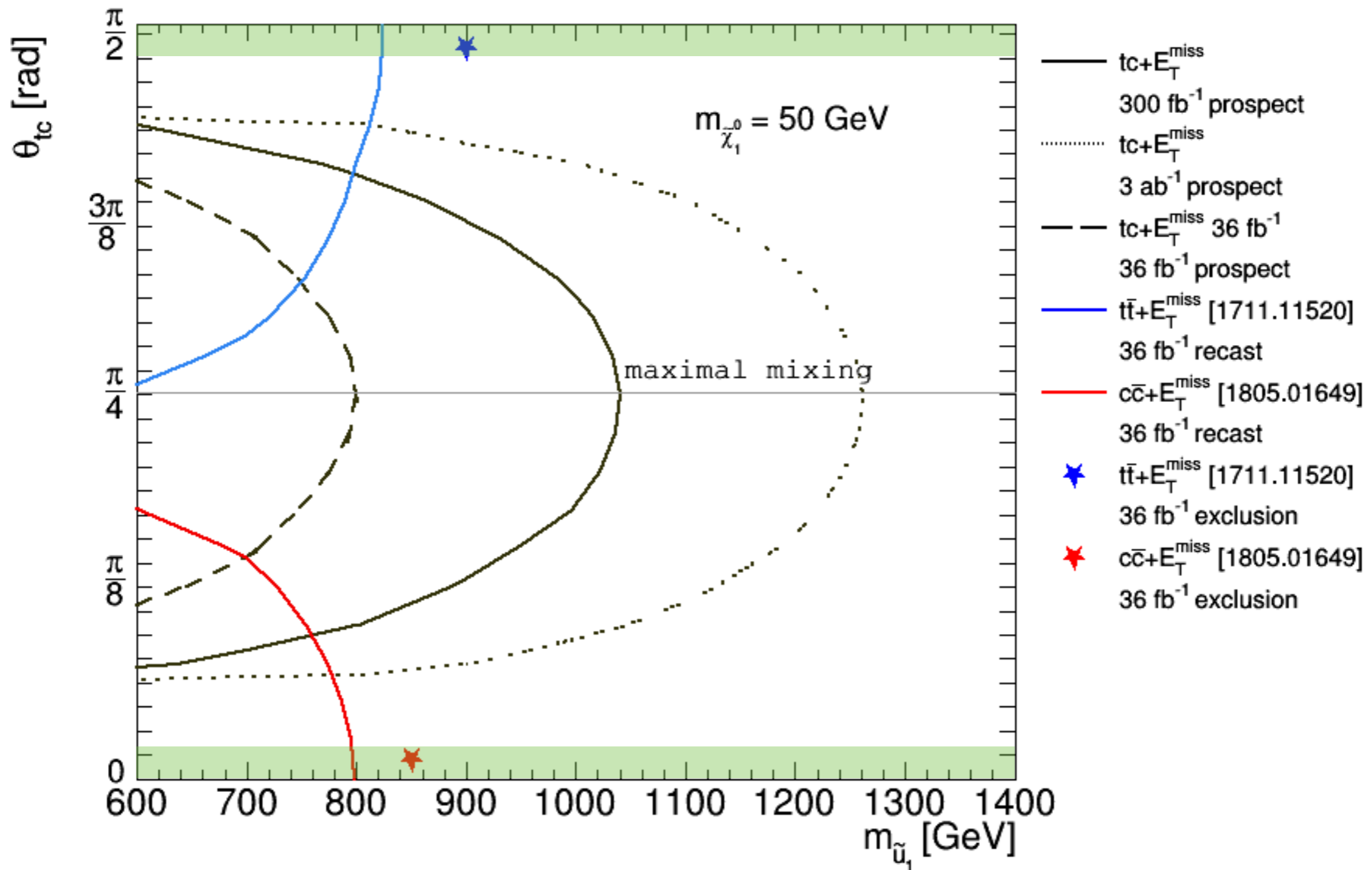
$$\begin{aligned}
 \text{diag}(m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2, \dots, m_{\tilde{u}_6}^2) &= \mathcal{R}_{\tilde{u}} \mathcal{M}_{\tilde{u}}^2 \mathcal{R}_{\tilde{u}}^\dagger & \text{diag}(m_{\tilde{d}_1}^2, m_{\tilde{d}_2}^2, \dots, m_{\tilde{d}_6}^2) &= \mathcal{R}_{\tilde{d}} \mathcal{M}_{\tilde{d}}^2 \mathcal{R}_{\tilde{d}}^\dagger \\
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 \end{aligned}$$

TeV scale MSSM — flavour-violating parameters

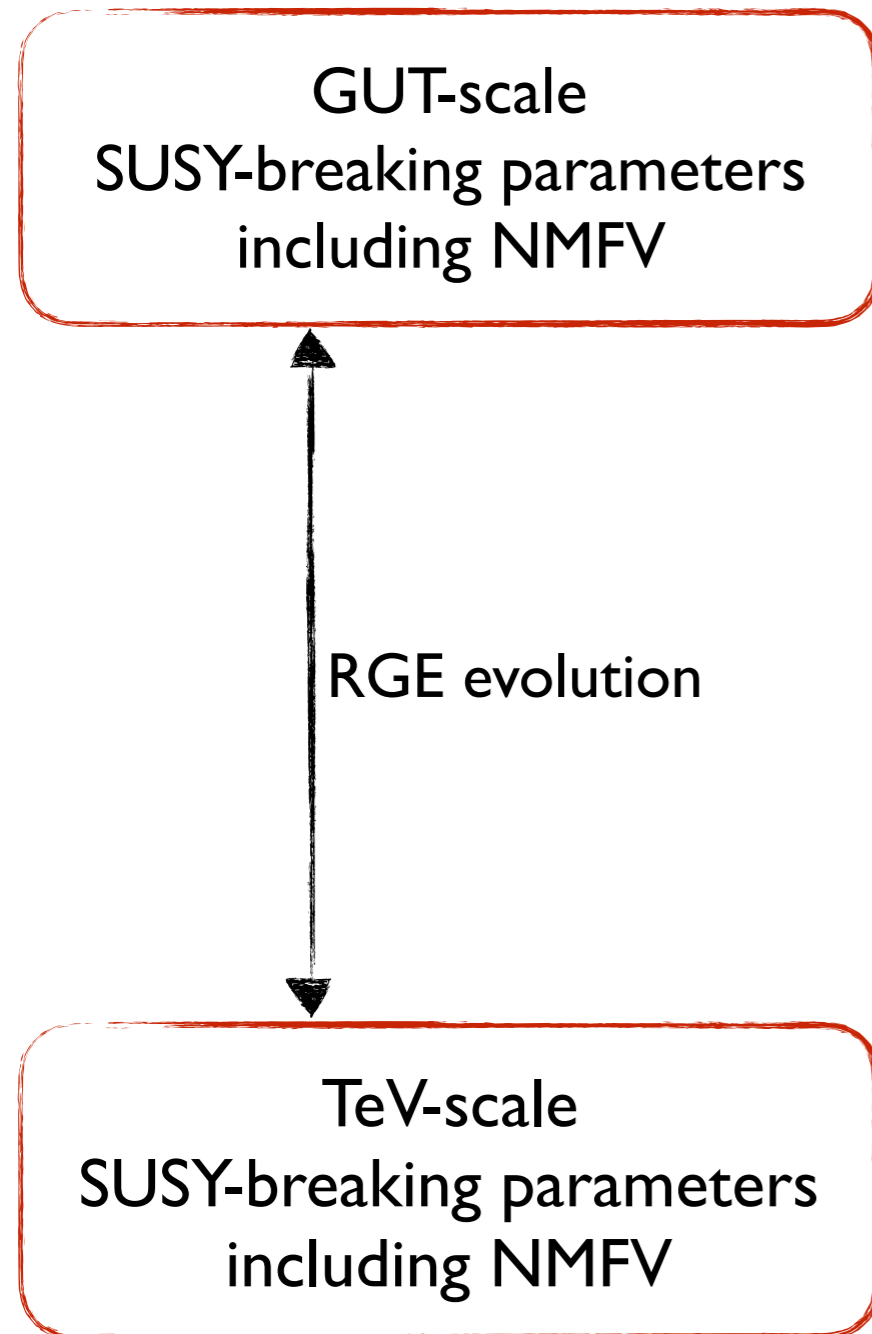
Extensive analysis of the MSSM with squark NMFV featuring 22 parameters at the TeV scale
— **Markov Chain Monte Carlo (MCMC) study**



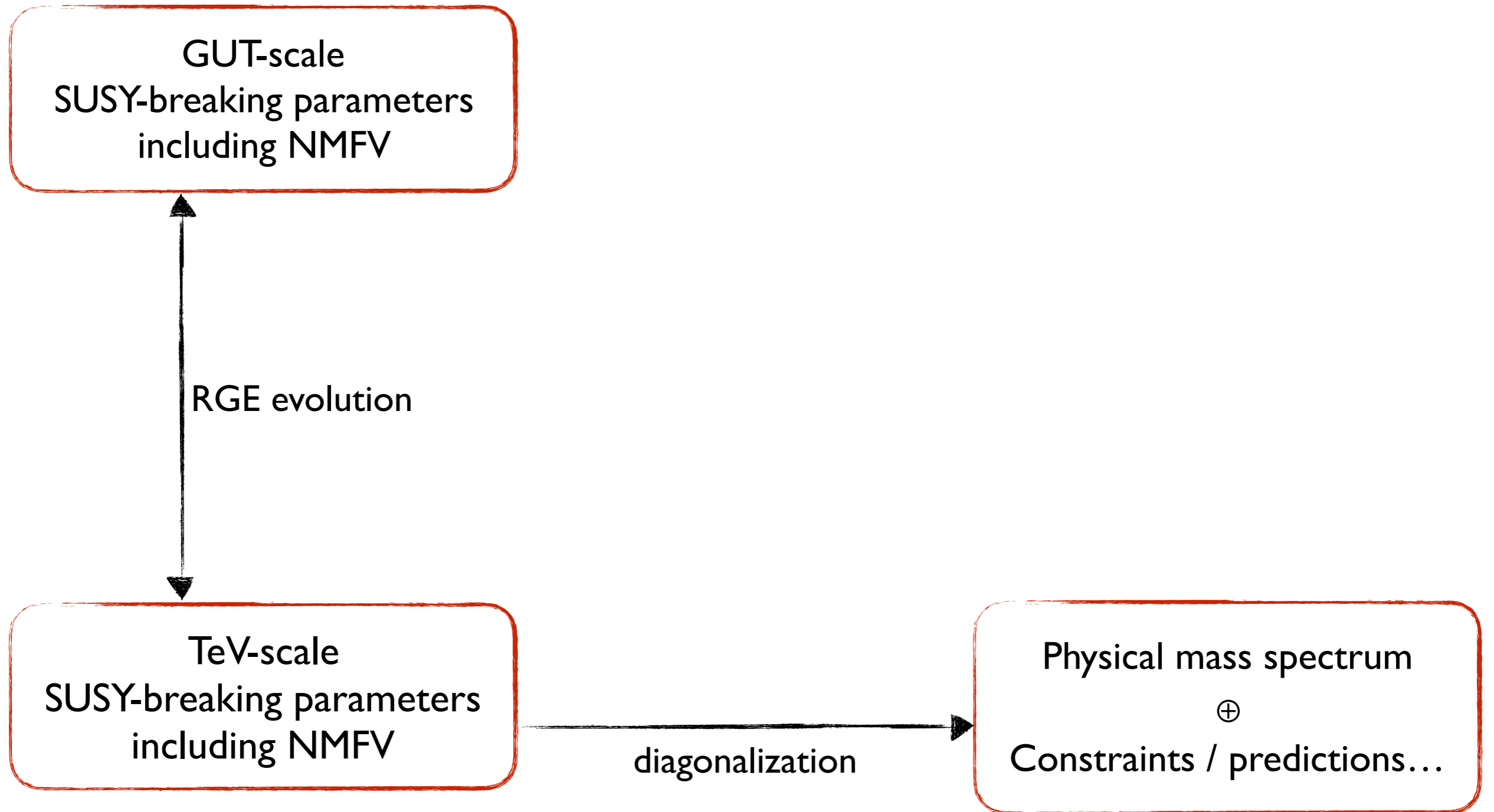
LHC squark mass limits and search proposal



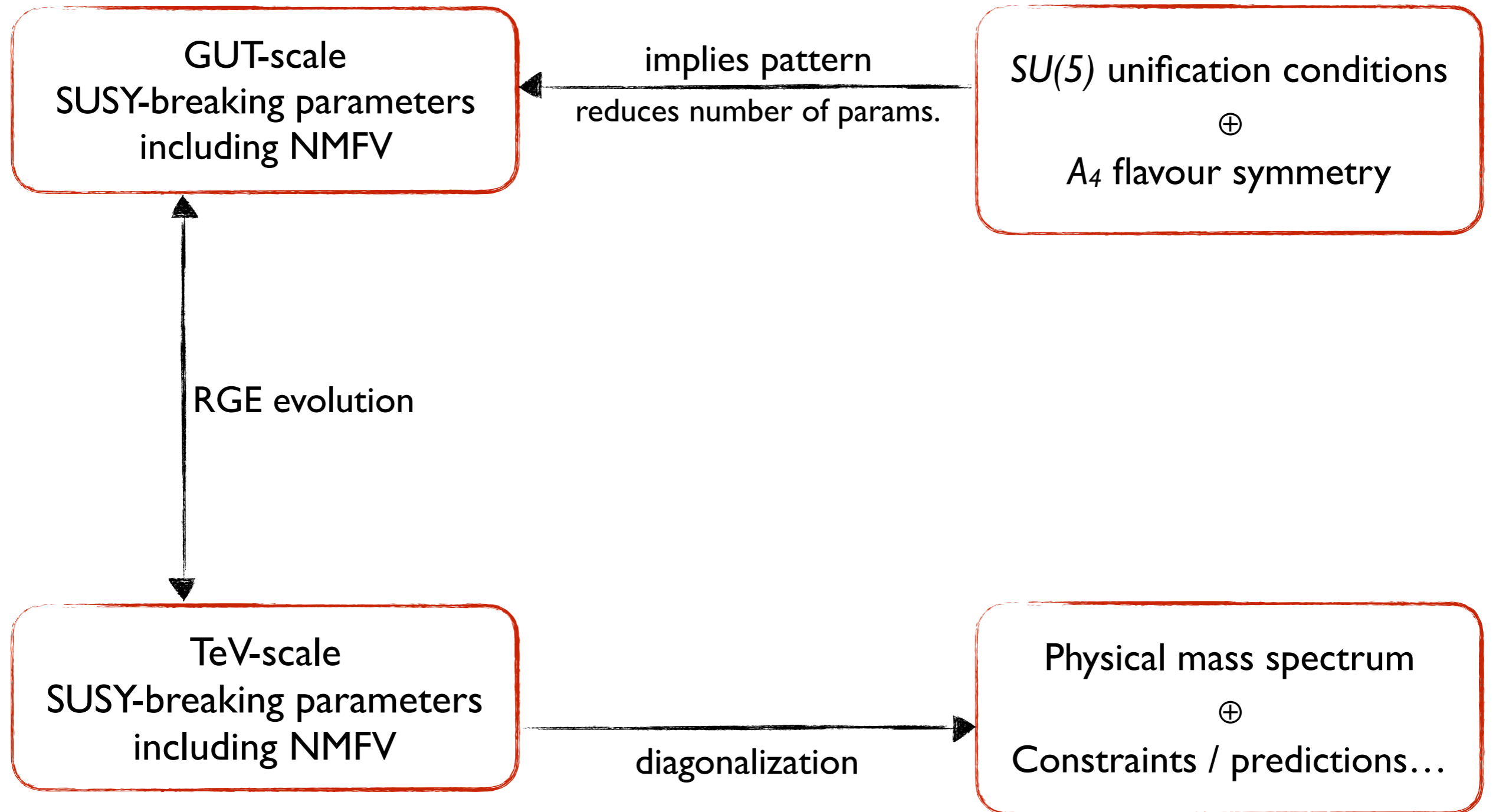
Implementation in a (flavoured) GUT framework



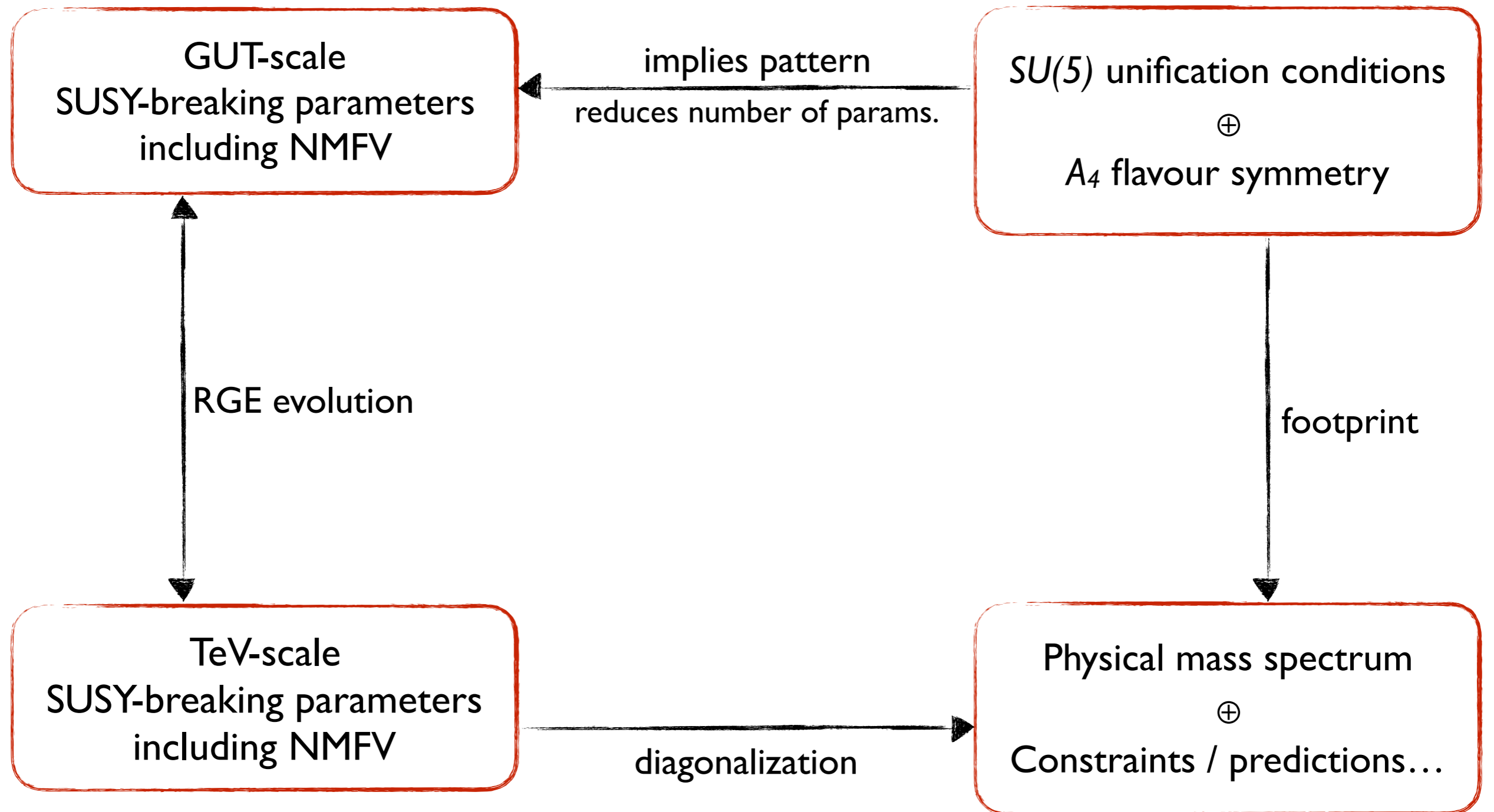
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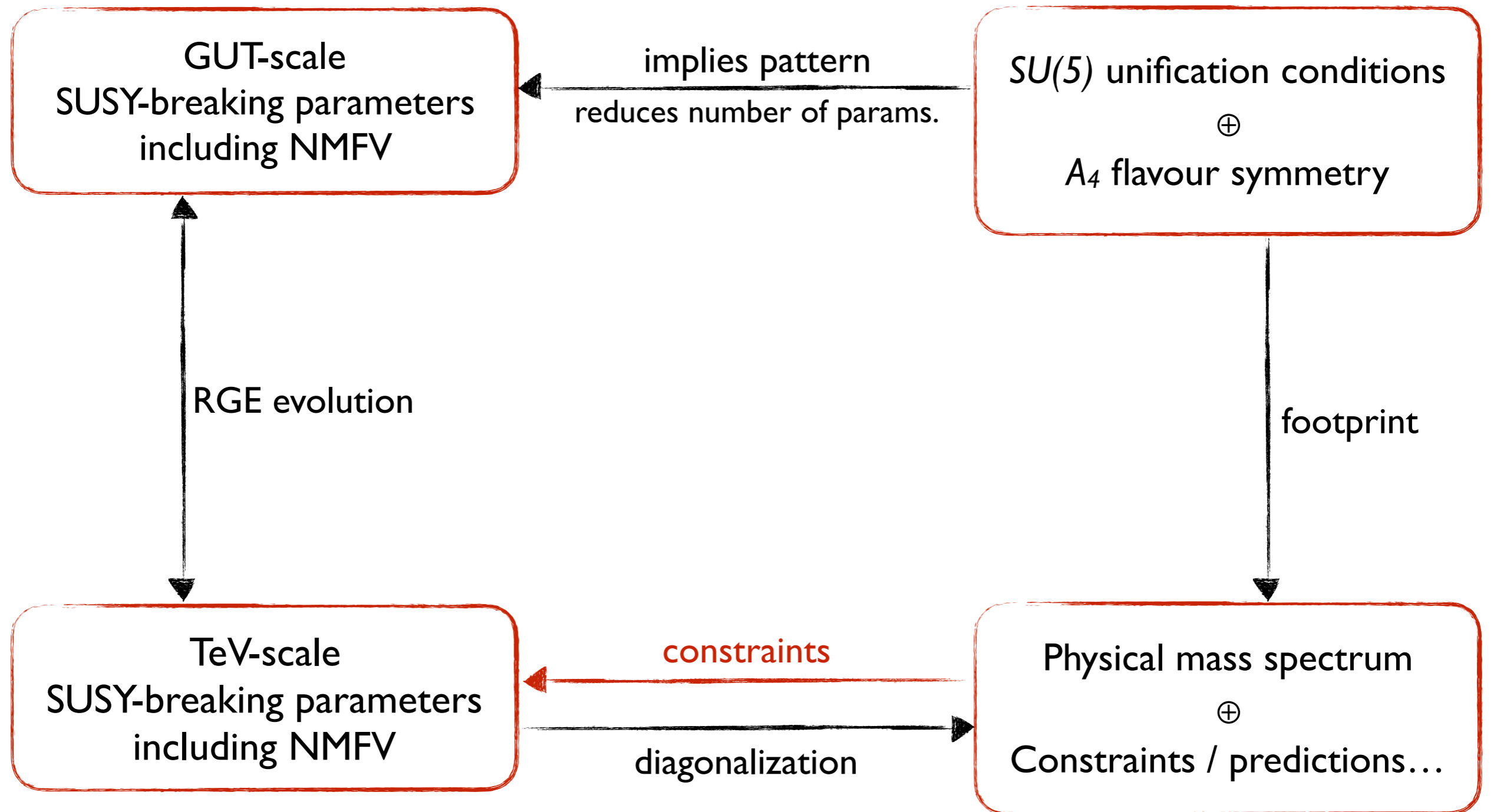
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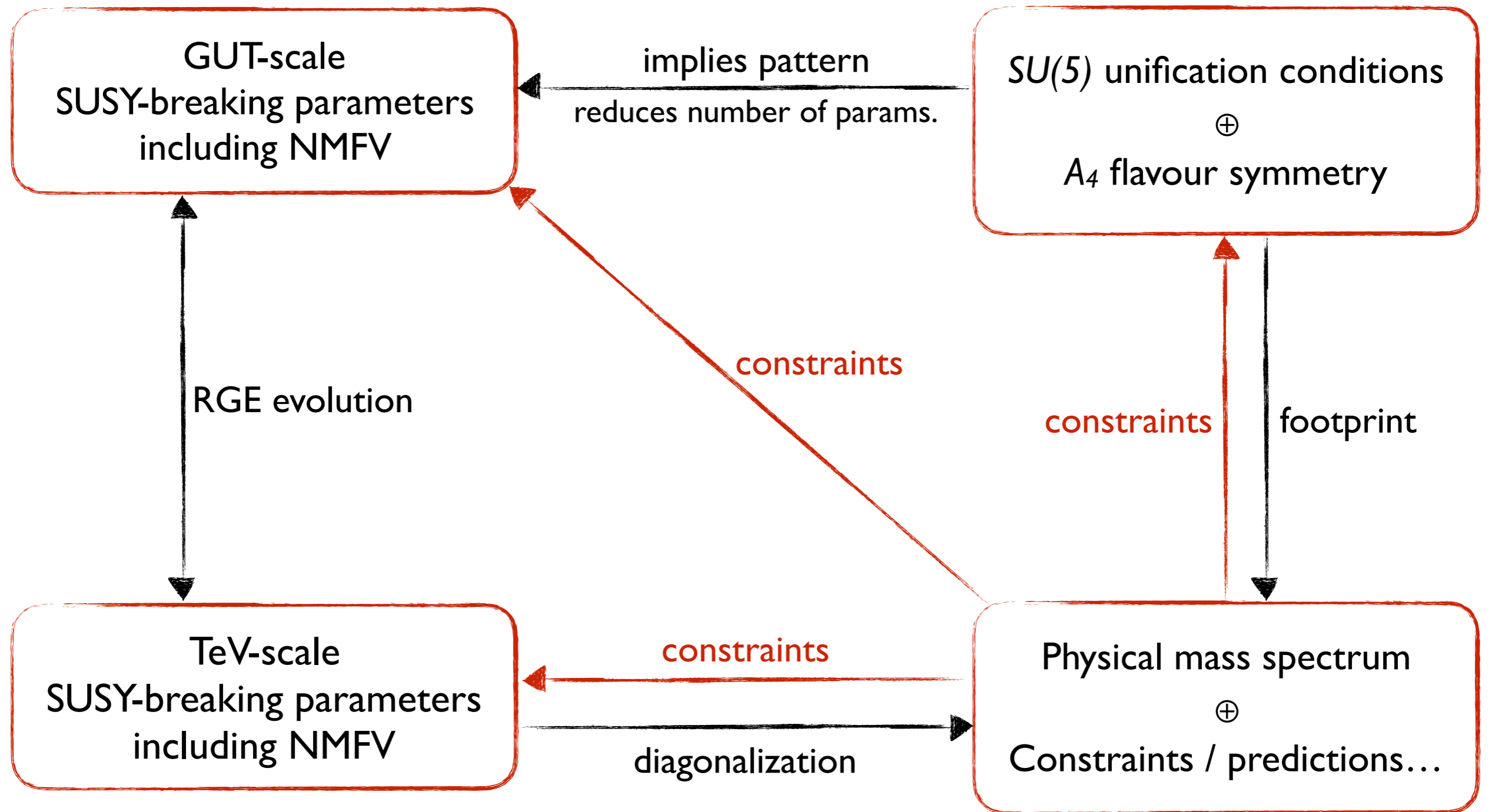
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$SU(5)$ -like Grand Unification

Standard-Model fields are neatly accommodated into the $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations of $SU(5)$

$$F = \bar{\mathbf{5}} = \begin{pmatrix} d_r^c \\ d_b^c \\ d_g^c \\ e^- \\ -\nu_e \end{pmatrix}_L \quad T = \mathbf{10} = \begin{pmatrix} 0 & u_g^c & -u_b^c & u_r & d_r \\ \cdot & 0 & u_r^c & u_b & d_b \\ \cdot & \cdot & 0 & u_g & d_g \\ \cdot & \cdot & \cdot & 0 & e^c \\ \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix}_L$$

The $SU(5)$ gauge group may be broken into the Standard-Model gauge group according to

$$SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y.$$

$$\bar{\mathbf{5}} = d^c(\bar{\mathbf{3}}, \mathbf{1}, 1/3) \oplus L(\mathbf{1}, \bar{\mathbf{2}}, -1/2),$$

$$\mathbf{10} = u^c(\bar{\mathbf{3}}, \mathbf{1}, -2/3) \oplus Q(\mathbf{3}, \mathbf{2}, 1/6) \oplus e^c(\mathbf{1}, \mathbf{1}, 1)$$

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Extending to Supersymmetry, $SU(5)$ symmetry provides the following relationships between soft terms at the Grand Unification scale:

$$\begin{aligned} M_{\tilde{D}}^2 &= M_{\tilde{L}}^2 \equiv M_F^2 & A_d &= A_e^t \equiv A_{FT} \\ M_{\tilde{Q}}^2 &= M_{\tilde{U}}^2 = M_{\tilde{E}}^2 \equiv M_T^2 & A_u &\equiv A_{TT} \end{aligned}$$

Adding the A_4 flavour symmetry

Unify three families of $\bar{\mathbf{5}} = F = (d^c, L)$ into the triplet of A_4

while the three $\mathbf{10}_i = T_i = (Q, u^c, e^c)_i$ representations are singlets of A_4

$$M_F^2 = \begin{pmatrix} m_F^2 & 0 & 0 \\ 0 & m_F^2 & 0 \\ 0 & 0 & m_F^2 \end{pmatrix} \quad M_T^2 = \begin{pmatrix} m_{T_1}^2 & 0 & 0 \\ 0 & m_{T_2}^2 & 0 \\ 0 & 0 & m_{T_3}^2 \end{pmatrix}$$

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Generally, non-minimal flavour violation is expected in this type of setup
(presence of flavons related to the breaking of A_4 ...)

S. Antusch, S. F. King, M. Spinrath — Phys. Rev. D 87 (2013) 096018 — arXiv:1301.6764 [hep-ph]
M. Dimou, S. F. King, C. Luhn — JHEP 1602 (2016) 118 — arXiv:1511.07886 [hep-ph]
M. Dimou, S. F. King, C. Luhn — Phys. Rev. D 93 (2016) 075026 — arXiv:1512.09063 [hep-ph]

**Muon $g-2$ and dark matter suggest nonuniversal gaugino masses:
 $SU(5) \times A_4$ case study at the LHC**Alexander S. Belyaev,^{1,2,*} Steve F. King,^{1,†} and Patrick B. Schaefers^{1,‡}

arXiv:1801.00514 [hep-ph]

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$$\Omega_{\text{CDM}} h^2 \lesssim 0.1224$$

$$\sigma_{\text{DD}}^{\text{SI}} \lesssim 7.64 \cdot 10^{-11} \text{ pb}$$

$$m_h = (125.09 \pm 1.50) \text{ GeV}$$

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all other states heavy ($>3 \text{ TeV}$)

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What about NMFV effects in this setup...?

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$$\mathcal{M}_{\tilde{\ell}}^2 = \begin{pmatrix} M_{\tilde{L}}^2 + m_e^2 + D_{\tilde{\ell},L} & \frac{v_d}{\sqrt{2}} T_e - m_e \mu \tan \beta \\ \frac{v_d}{\sqrt{2}} T_e - m_e \mu^* \tan \beta & M_{\tilde{E}}^2 + m_e^2 + D_{\tilde{\ell},R} \end{pmatrix}$$



5 independent mass matrices and 3 trilinear coupling matrices

$$M_{\tilde{Q}}^2, M_{\tilde{U}}^2, M_{\tilde{D}}^2, M_{\tilde{L}}^2, M_{\tilde{E}}^2 \quad T_u, T_d, T_e$$

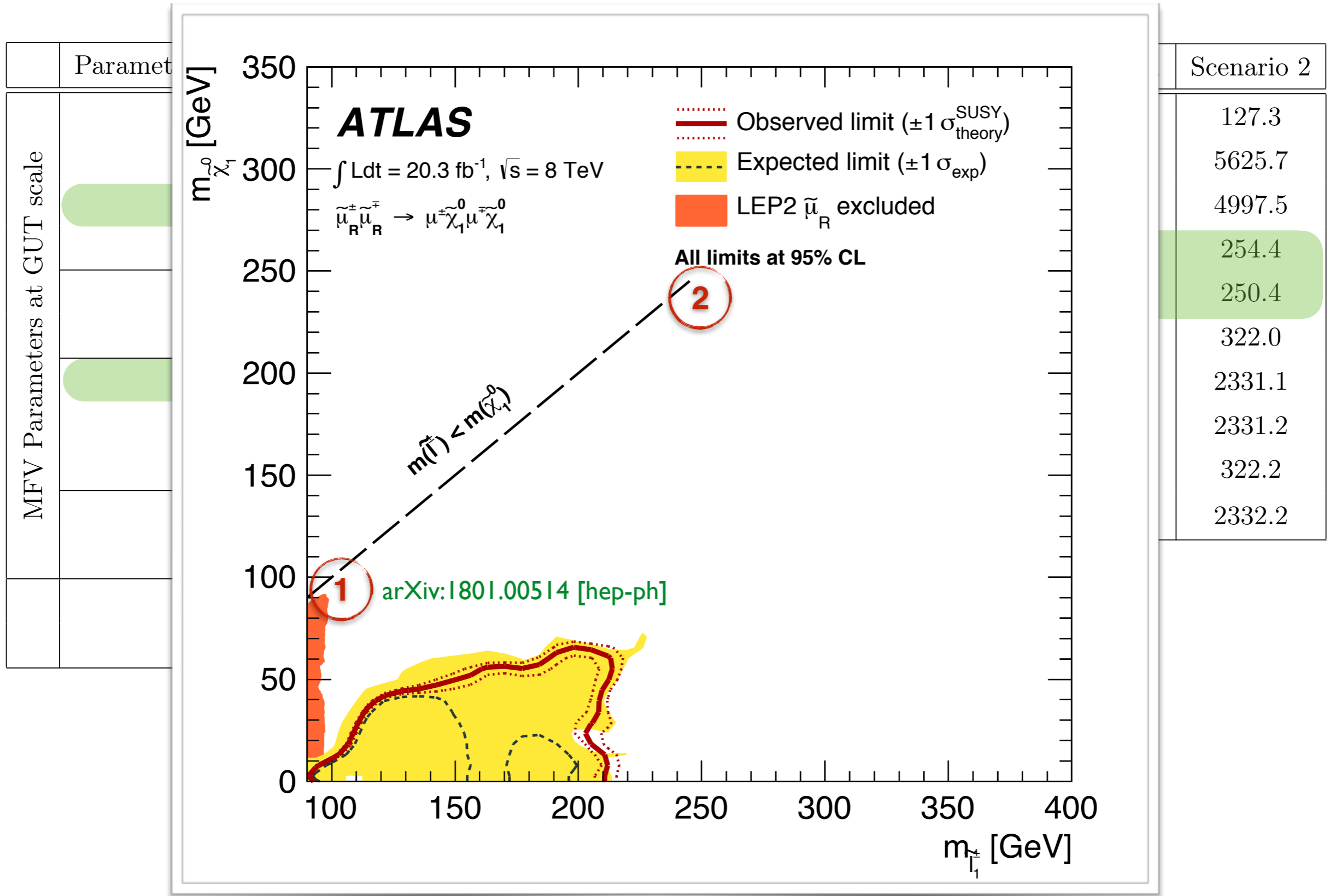
(3x3 matrices in flavour space — 48 independent parameters)

MFV Reference points

	Parameter/Observable	Scenario 1	Scenario 2
MFV Parameters at GUT scale	m_F	5000	5000
	m_{T_1}	5000	5000
	m_{T_2}	200	233.2
	m_{T_3}	2995	2995
	a_{33}^{TT}	-940	-940
	a_{33}^{FT}	-1966	-1966
	M_1	250.0	600.0
	M_2	415.2	415.2
	M_3	2551.6	2551.6
	M_{H_u}	4242.6	4242.6
	M_{H_d}	4242.6	4242.6
		$\tan \beta$	30
	μ	-2163.1	-2246.8

	Parameter/Observable	Scenario 1	Scenario 2
Physical masses	m_h	126.7	127.3
	$m_{\tilde{g}}$	5570.5	5625.7
	$m_{\tilde{\mu}_L}$	4996.7	4997.5
	$m_{\tilde{\mu}_R}$	102.1	254.4
	$m_{\tilde{\chi}_1^0}$	94.6	250.4
	$m_{\tilde{\chi}_2^0}$	323.6	322.0
	$m_{\tilde{\chi}_3^0}$	2248.8	2331.1
	$m_{\tilde{\chi}_4^0}$	2248.8	2331.2
	$m_{\tilde{\chi}_1^\pm}$	323.8	322.2
	$m_{\tilde{\chi}_2^\pm}$	2249.8	2332.2

MFV Reference points



NMFV parameter study

Parameters	Scenario 1	Scenario 2
$(\delta^T)_{12}$	$[-2.00, 2.00] \times 10^{-2}$	$[-5.57, 5.15] \times 10^{-2}$
$(\delta^T)_{13}$	$[-8.01, 8.01] \times 10^{-2}$	$[-0.267, 0.301]$
$(\delta^T)_{23}$	0.0	$[-5.73, 5.73] \times 10^{-2}$
$(\delta^F)_{12}$	$[-8.00, 8.00] \times 10^{-3}$	$[-8.00, 8.00] \times 10^{-3}$
$(\delta^F)_{13}$	$[-1.00, 1.00] \times 10^{-2}$	$[-8.00, 8.00] \times 10^{-2}$
$(\delta^F)_{23}$	$[-1.60, 1.60] \times 10^{-2}$	$[-8.00, 8.00] \times 10^{-2}$
$(\delta^{TT})_{12}$	$[-8.69, 10.43] \times 10^{-4}$	$[-7.46, 8.95] \times 10^{-4}$
$(\delta^{TT})_{13}$	$[-1.74, 1.74] \times 10^{-3}$	$[-3.48, 1.74] \times 10^{-3}$
$(\delta^{TT})_{23}$	$[-0.0174, 0.145]$	$[-0.0871, 0.124]$
$(\delta^{FT})_{12}$	$[-4.64, 4.64] \times 10^{-5}$	$[-5.47, 5.47] \times 10^{-5}$
$(\delta^{FT})_{13}$	$[-7.74, 7.74] \times 10^{-5}$	$[-3.87, 3.87] \times 10^{-4}$
$(\delta^{FT})_{21}$	0.0	$[-1.04, 1.04] \times 10^{-4}$
$(\delta^{FT})_{23}$	$[-1.16, 1.16] \times 10^{-4}$	$[-2.32, 2.32] \times 10^{-4}$
$(\delta^{FT})_{31}$	$[-1.39, 1.39] \times 10^{-5}$	$[-8.81, 8.81] \times 10^{-5}$
$(\delta^{FT})_{32}$	0.0	$[-1.49, 1.49] \times 10^{-4}$

parameters at GUT scale

$$(\delta^T)_{ij} = \frac{(M_T^2)_{ij}}{(M_T)_{ii}(M_T)_{jj}}$$

$$(\delta^F)_{ij} = \frac{(M_F^2)_{ij}}{(M_F)_{ii}(M_F)_{jj}}$$

$$(\delta^{TT})_{ij} = \frac{v_u}{\sqrt{2}} \frac{(T_u)_{ij}}{(M_T)_{ii}(M_T)_{jj}}$$

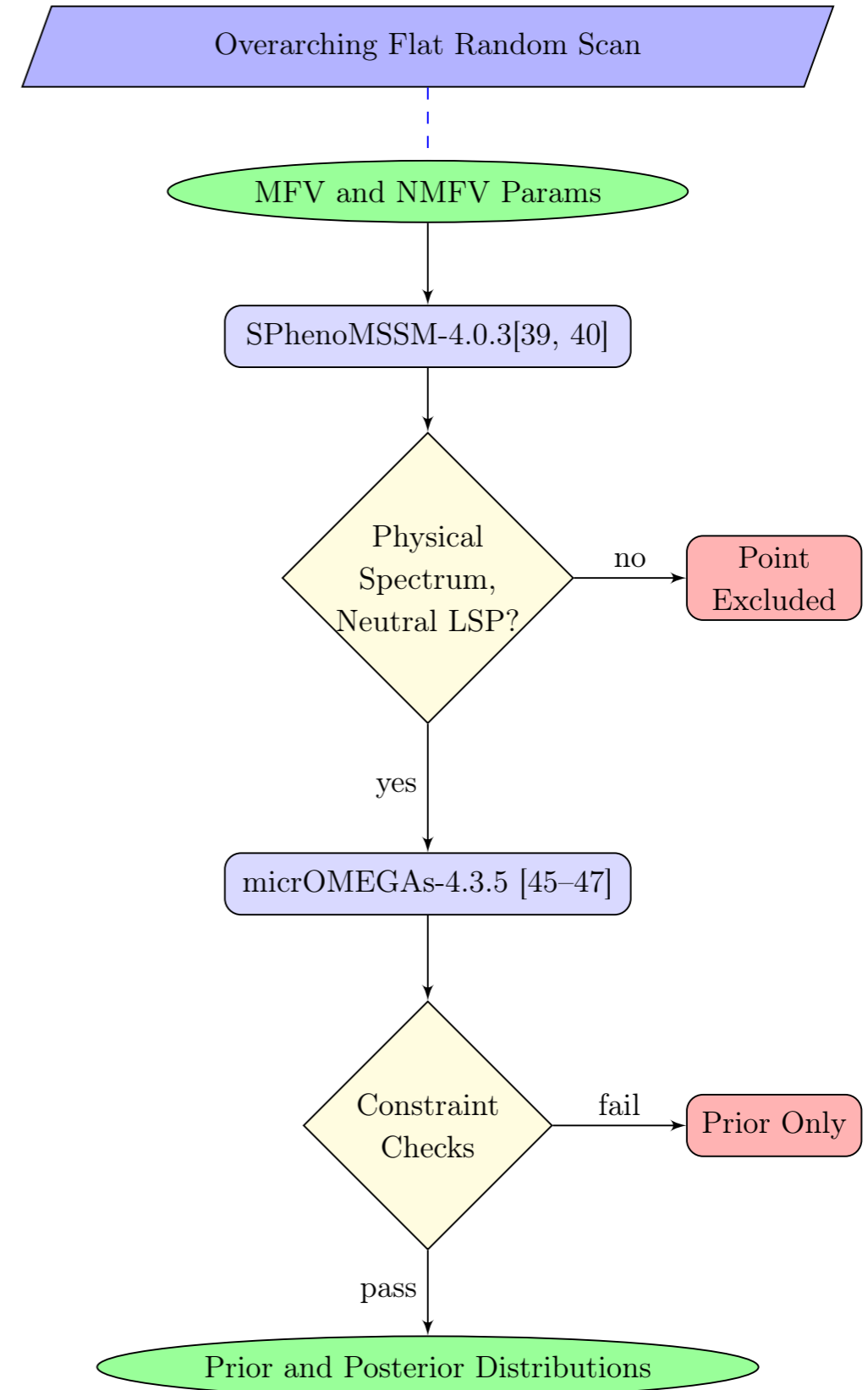
$$(\delta^{FT})_{ij} = \frac{v_u}{\sqrt{2}} \frac{(T_d)_{ij}}{(M_T)_{ii}(M_F)_{jj}}$$

Experimental constraints

Observable	Constraint
m_h	$(125.2 \pm 2.5) \text{ GeV}$
$\text{BR}(\mu \rightarrow e\gamma)$	$< 4.2 \times 10^{-13}$
$\text{BR}(\mu \rightarrow 3e)$	$< 1.0 \times 10^{-12}$
$\text{BR}(\tau \rightarrow e\gamma)$	$< 3.3 \times 10^{-8}$
$\text{BR}(\tau \rightarrow \mu\gamma)$	$< 4.4 \times 10^{-8}$
$\text{BR}(\tau \rightarrow 3e)$	$< 2.7 \times 10^{-8}$
$\text{BR}(\tau \rightarrow 3\mu)$	$< 2.1 \times 10^{-8}$
$\text{BR}(\tau \rightarrow e^- \mu\mu)$	$< 2.7 \times 10^{-8}$
$\text{BR}(\tau \rightarrow e^+ \mu\mu)$	$< 1.7 \times 10^{-8}$
$\text{BR}(\tau \rightarrow \mu^- ee)$	$< 1.8 \times 10^{-8}$
$\text{BR}(\tau \rightarrow \mu^+ ee)$	$< 1.5 \times 10^{-8}$
$\text{BR}(B \rightarrow X_s \gamma)$	$(3.32 \pm 0.18) \times 10^{-4}$
$\text{BR}(B_s \rightarrow \mu\mu)$	$(2.7 \pm 1.2) \times 10^{-9}$
ΔM_{B_s}	$(17.757 \pm 0.042 \pm 2.7) \text{ ps}^{-1}$
ΔM_K	$(3.1 \pm 1.2) \times 10^{-15} \text{ GeV}$
ϵ_K	2.228 ± 0.29
$\Omega_{\text{CDM}} h^2$	0.1198 ± 0.0042

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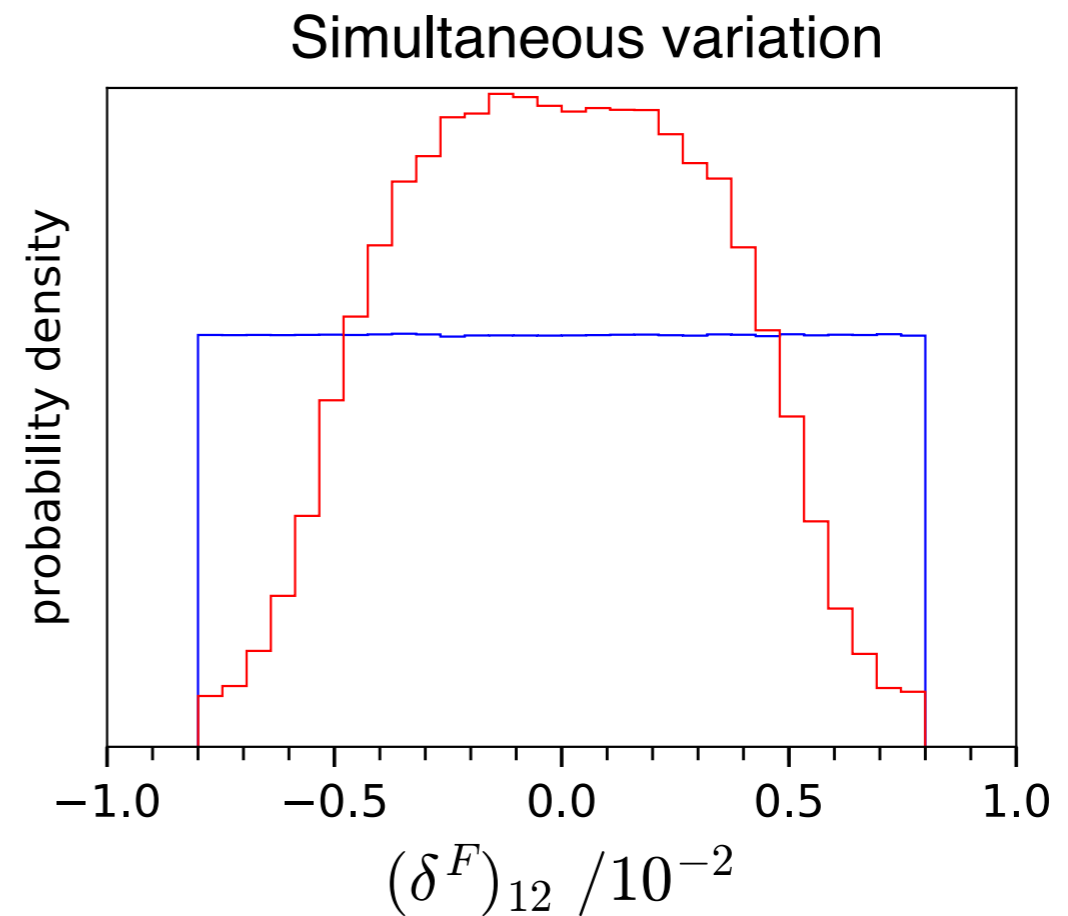
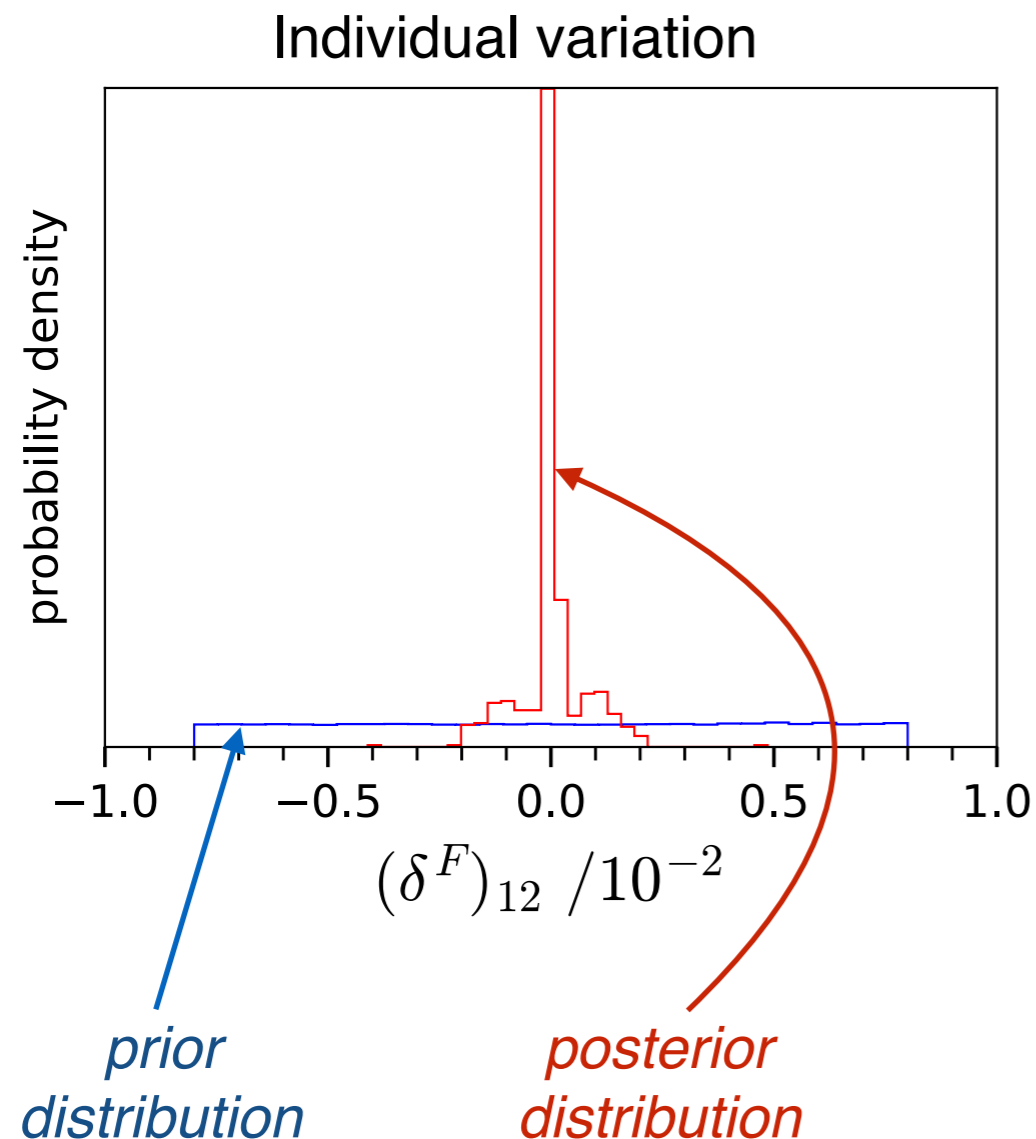
Results — Overview

Parameters	Scenario 1	Most constraining obs. 1	Scenario 2	Most constraining obs. 2
$(\delta^T)_{12}$	[-0.015, 0.015]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma, \Omega_{\tilde{\chi}_1^0} h^2$	[-0.12, 0.12] [†]	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^T)_{13}$]-0.06, 0.06[$\Omega_{\tilde{\chi}_1^0} h^2$	[-0.3, 0.3] [†]	$\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^T)_{23}$	[0, 0]*	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma$	[-0.1, 0.1] [†]	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma,$
$(\delta^F)_{12}$	[-0.008, 0.008]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$	[-0.015, 0.015] [†]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^F)_{13}$]-0.01, 0.01[$\mu \rightarrow e\gamma$	[-0.15, 0.15] [†]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^F)_{23}$]-0.015, 0.015[$\mu \rightarrow e\gamma, \Omega_{\tilde{\chi}_1^0} h^2$	[-0.15, 0.15] [†]	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma, \mu \rightarrow 3e$
$(\delta^{TT})_{12}$	$[-3, 3.5] \times 10^{-5}$	prior	$[-1, 1.5]^{\dagger} \times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{TT})_{13}$	$]-6, 7[\times 10^{-5}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$	$[-4, 2.5]^{\dagger} \times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{TT})_{23}$	$]-0.5, 4[\times 10^{-5}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$	$[-0.25, 0.2]^{\dagger}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{FT})_{12}$	[-0.0015, 0.0015]	$\Omega_{\tilde{\chi}_1^0} h^2$	$[-1.2, 1.2]^{\dagger} \times 10^{-4}$	$\mu \rightarrow 3e, \Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^{FT})_{13}$]-0.002, 0.002[$\Omega_{\tilde{\chi}_1^0} h^2$	$[-5, 5] \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^{FT})_{21}$	[0,0]*	prior	$[-1.2, 1.2]^{\dagger} \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2, \text{prior}$
$(\delta^{FT})_{23}$]-0.0022, 0.0022[$\Omega_{\tilde{\chi}_1^0} h^2$	$[-6, 6]^{\dagger} \times 10^{-4}$	$\mu \rightarrow 3e, \Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^{FT})_{31}$]-0.0004, 0.0004[$\Omega_{\tilde{\chi}_1^0} h^2$	$[-2, 2]^{\dagger} \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{FT})_{32}$	[0,0]*	prior	$[-1.5, 1.5] \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2$

* parameter not varied

† extrapolated range

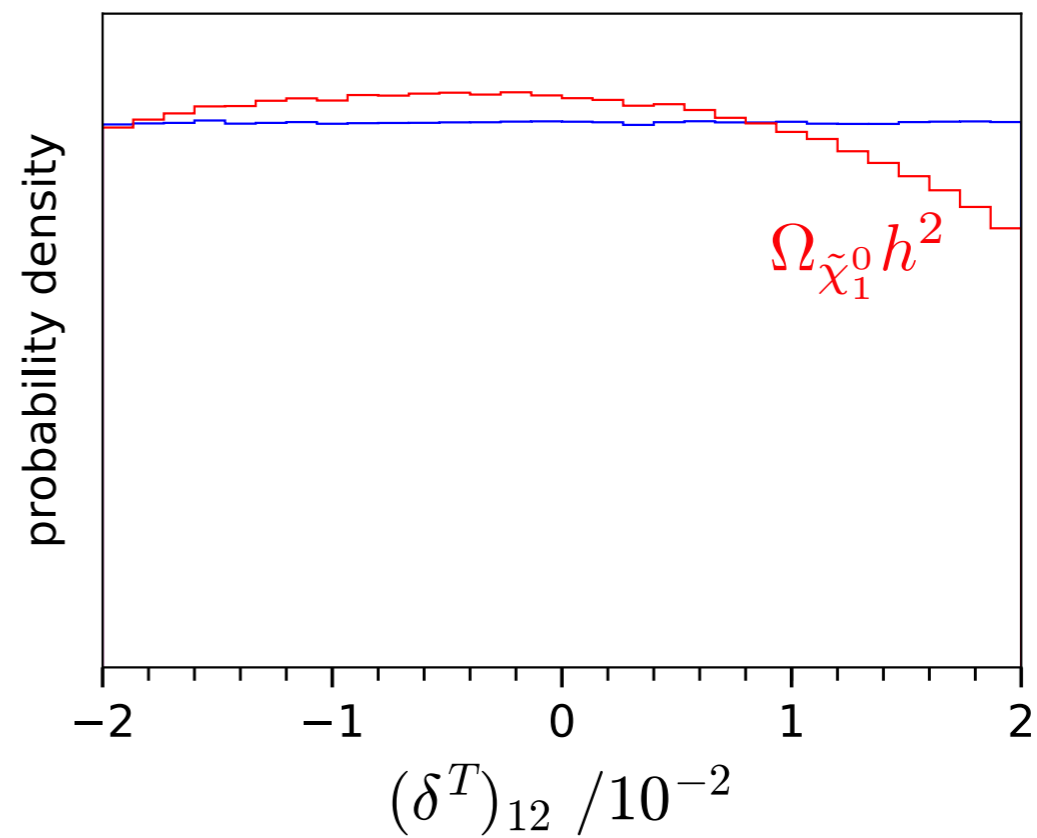
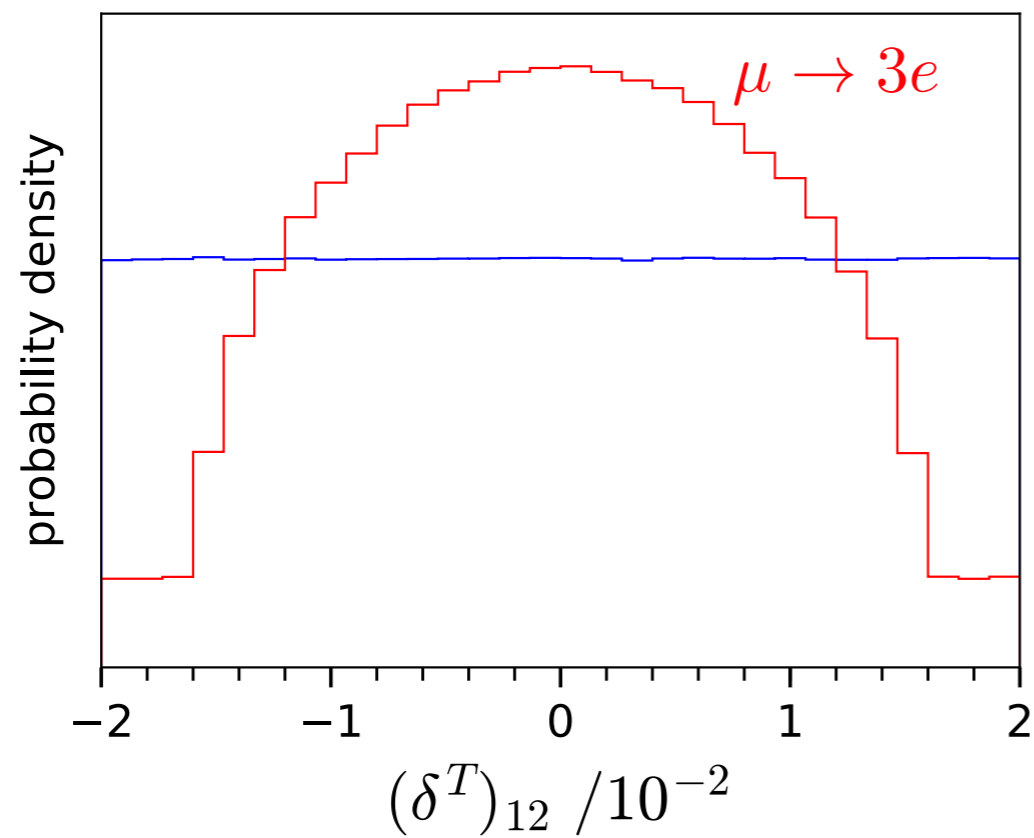
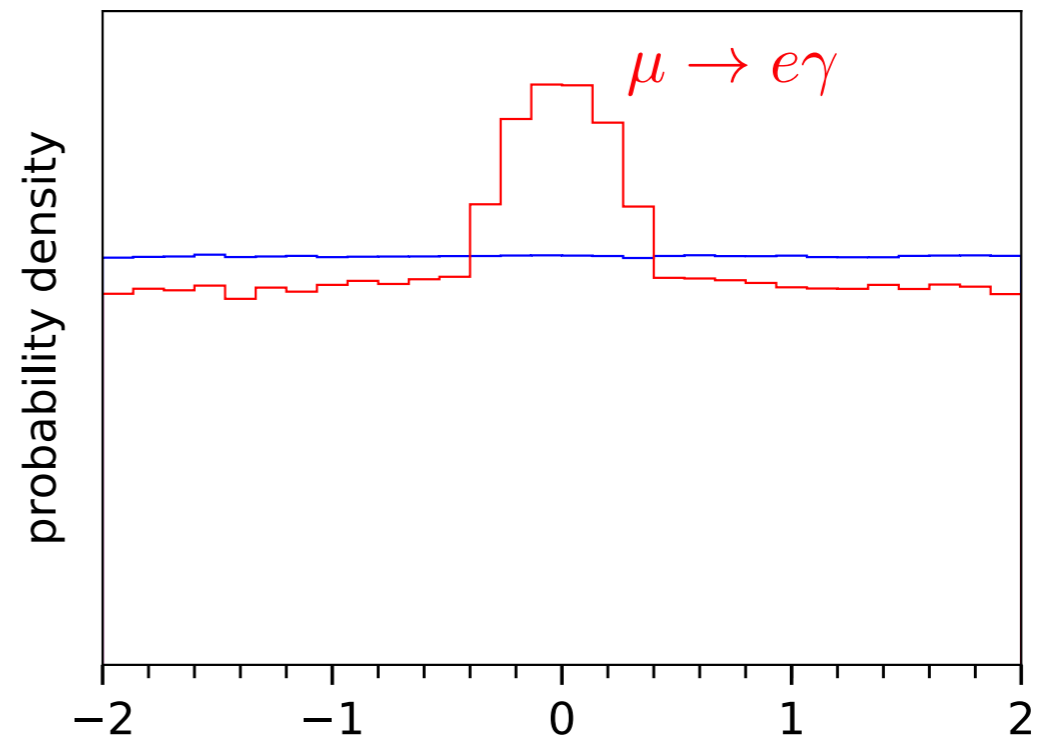
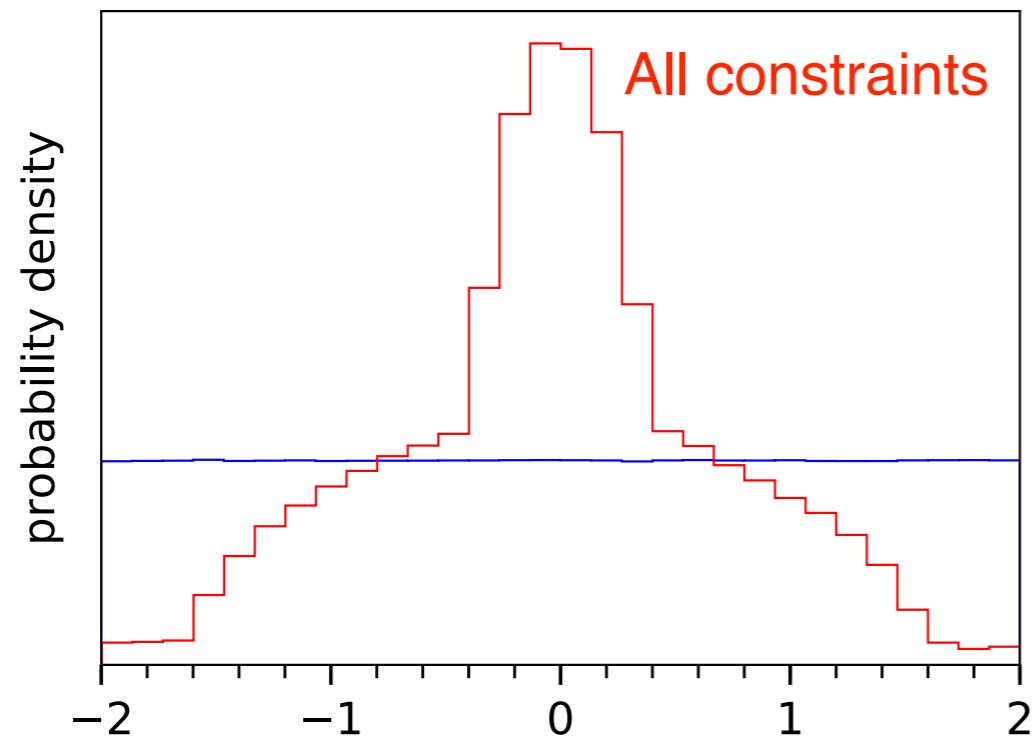
Individual vs. simultaneous variation



In a multi-dimensional parameter space, it is clearly not enough to scan each parameter individually...

→ **interference or cancellation effects in simultaneous study can be very important!**

Interplay of different constraints



$\mu \rightarrow e\gamma$ vs. $\mu \rightarrow 3e$

$$\text{BR}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$$

$$\text{BR}(\mu \rightarrow 3e) < 1.0 \times 10^{-12}$$

???

$$(\delta^F)_{12} \quad \parallel \quad \mu \rightarrow 3e, \mu \rightarrow e\gamma$$

$$(\delta^F)_{13} \quad \parallel \quad \mu \rightarrow 3e, \mu \rightarrow e\gamma$$

$\mu \rightarrow e\gamma$ vs. $\mu \rightarrow 3e$

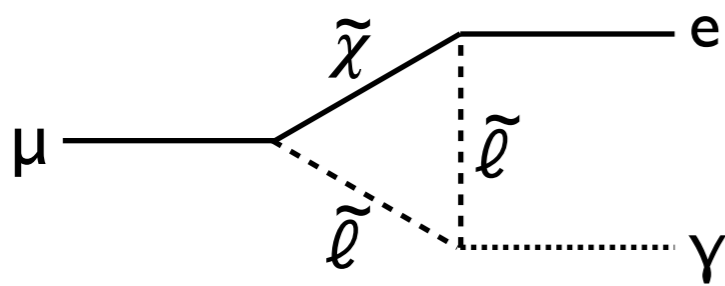
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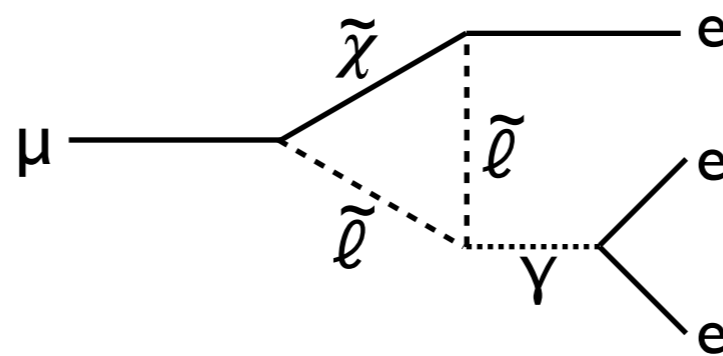
???

$$(\delta^F)_{12} \quad \parallel \quad \mu \rightarrow 3e, \mu \rightarrow e\gamma$$

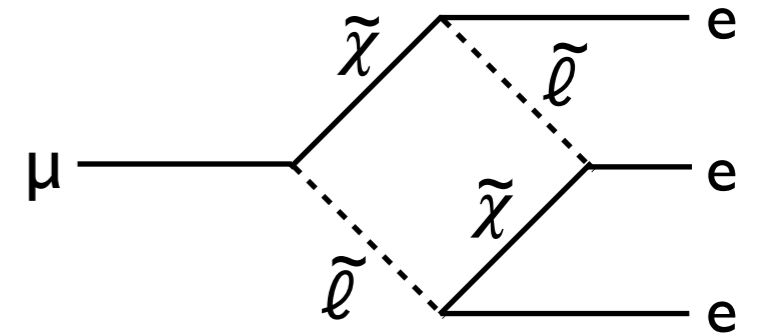
$$(\delta^F)_{13} \quad \parallel \quad \mu \rightarrow 3e, \mu \rightarrow e\gamma$$



$$\propto \frac{m_e}{m_\mu} \delta_{12} \alpha^3$$



$$\propto \frac{m_e}{m_\mu} \delta_{12} \alpha^4$$



$$\propto \delta_{12} \alpha^4$$

$\mu \rightarrow e\gamma$ vs. $\mu \rightarrow 3e$

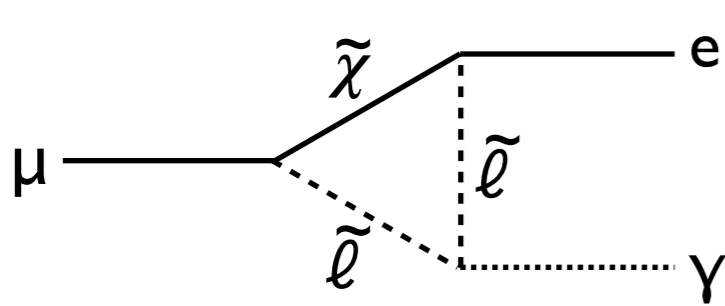
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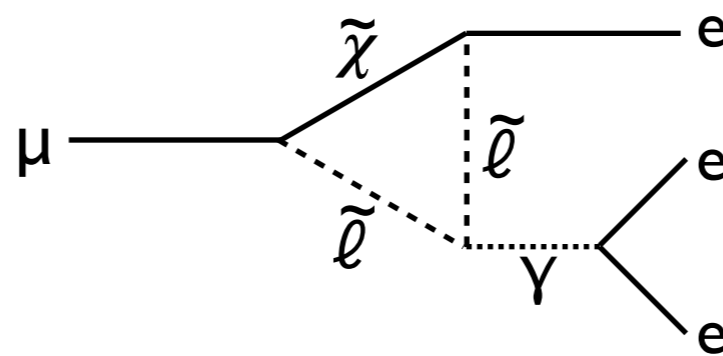
???

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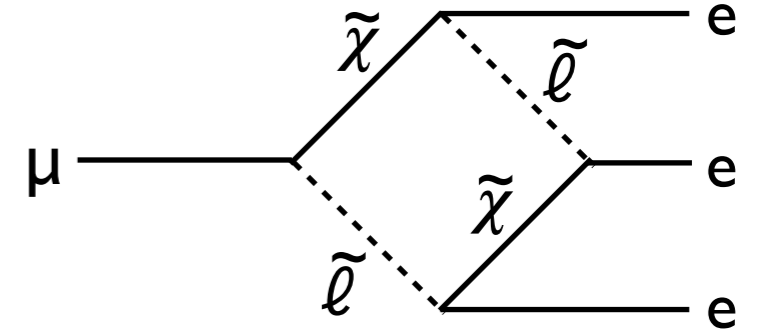
$$(\delta^F)_{13} \quad \parallel \quad \mu \rightarrow 3e, \mu \rightarrow e\gamma$$



$$\propto \frac{m_e}{m_\mu} \delta_{12} \alpha^3$$



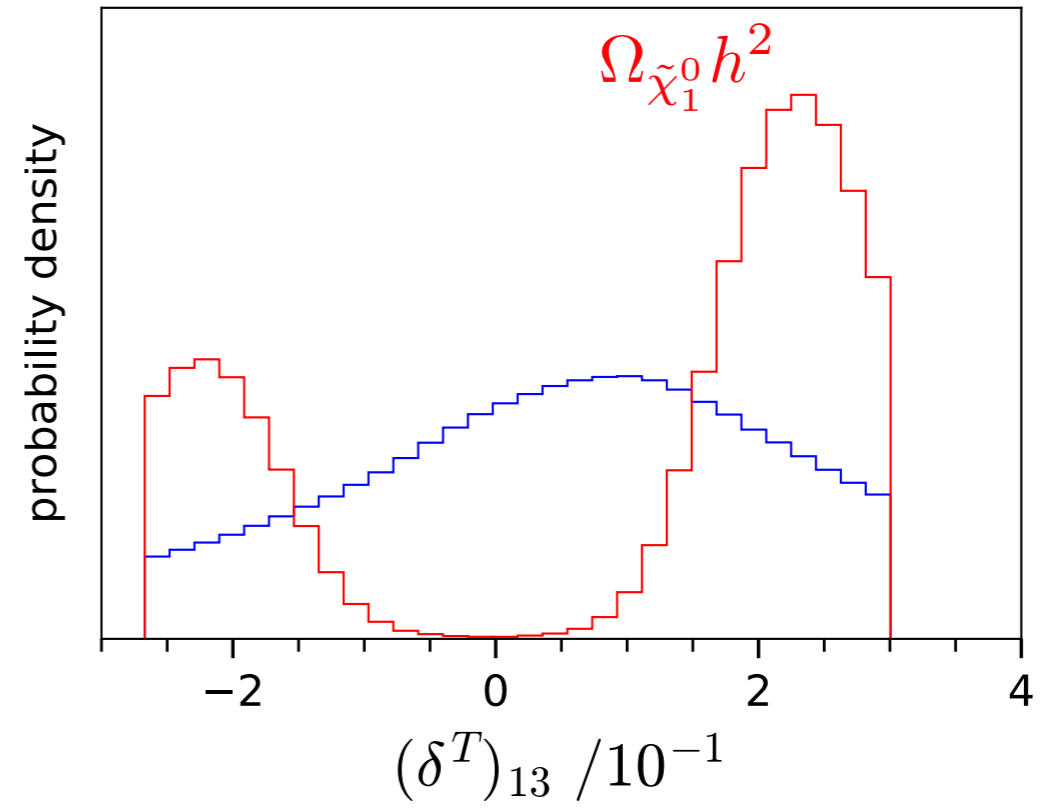
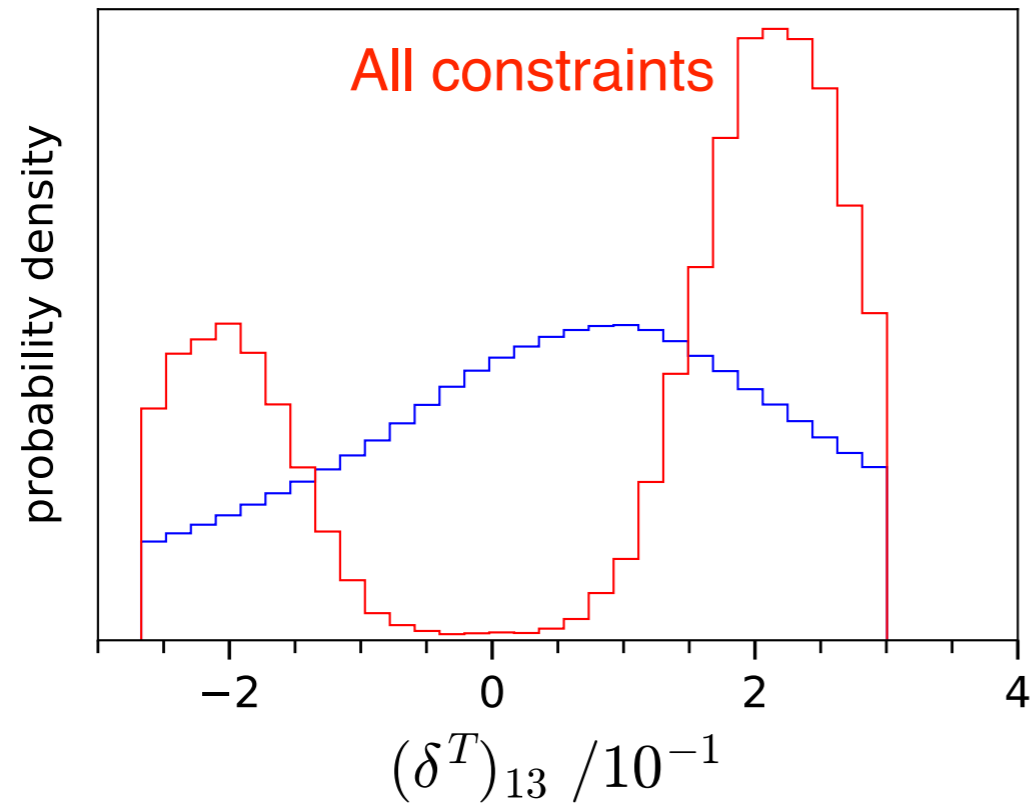
~~$$\propto \frac{m_e}{m_\mu} \delta_{12} \alpha^4$$~~



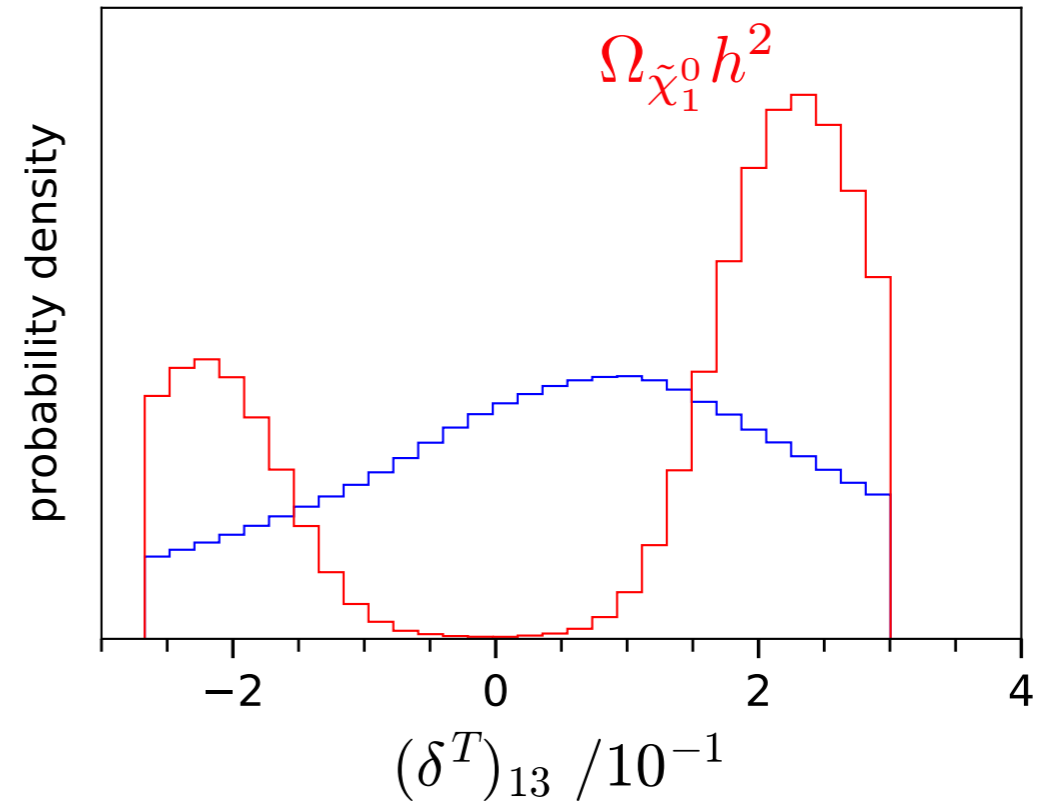
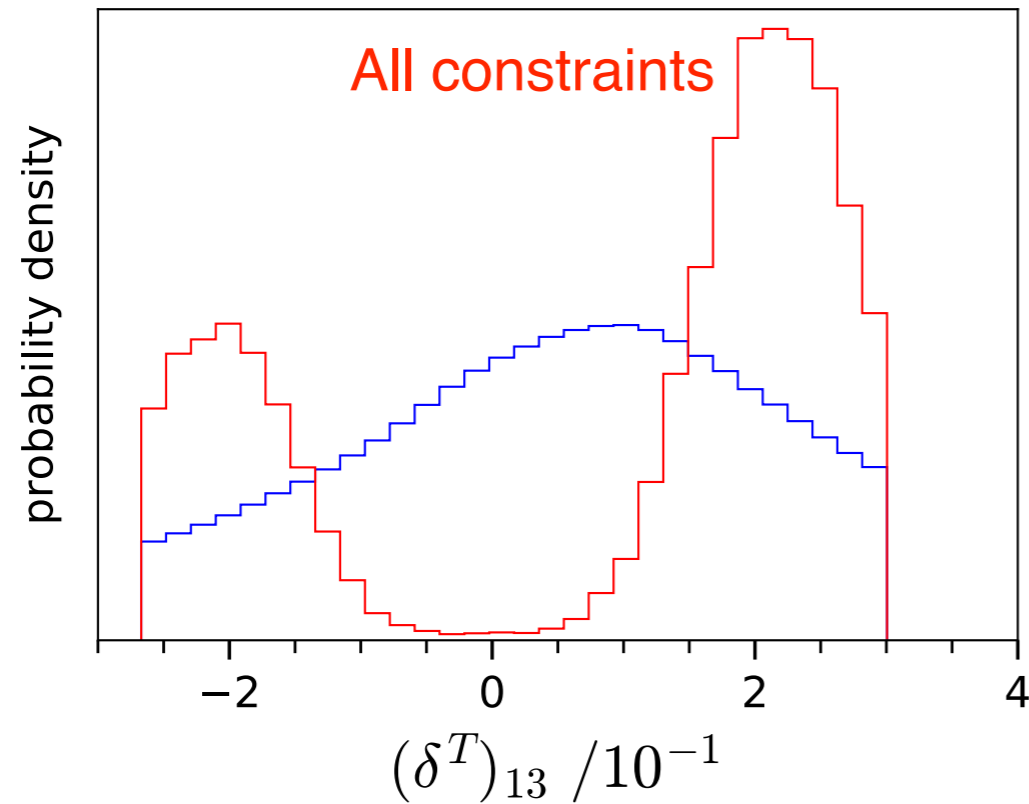
$$\propto \delta_{12} \alpha^4$$

$$A(\mu \rightarrow 3e) \sim A(\mu \rightarrow e\gamma)$$

Interplay of different constraints

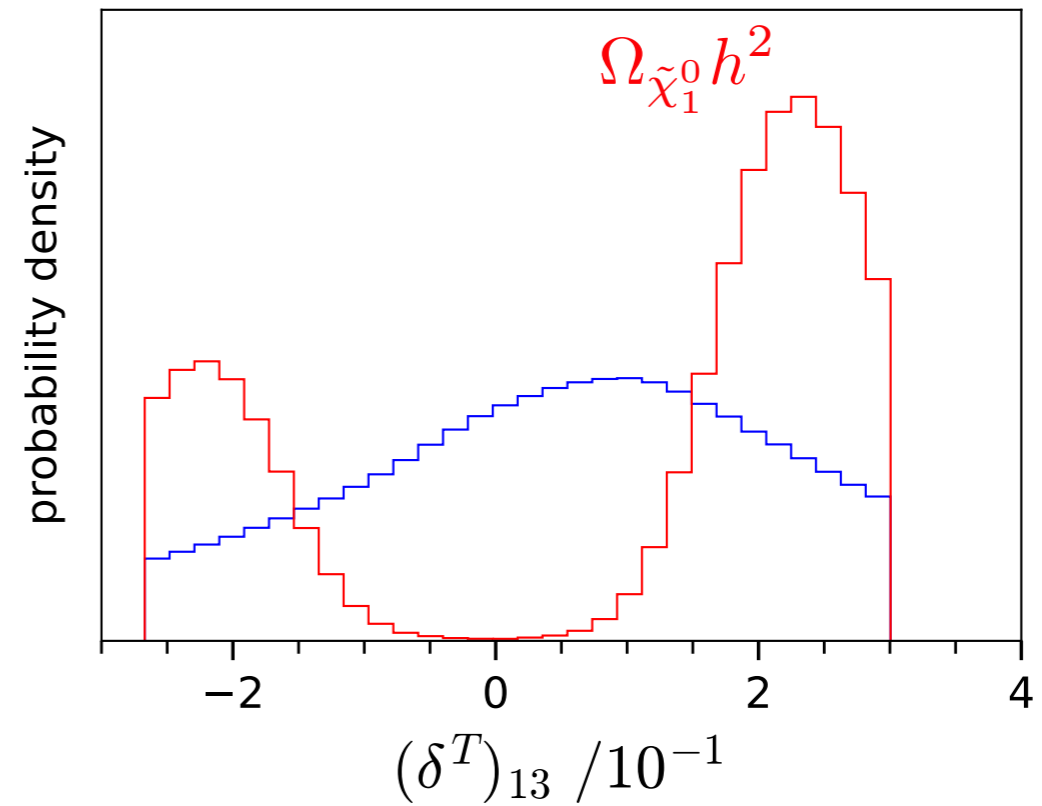
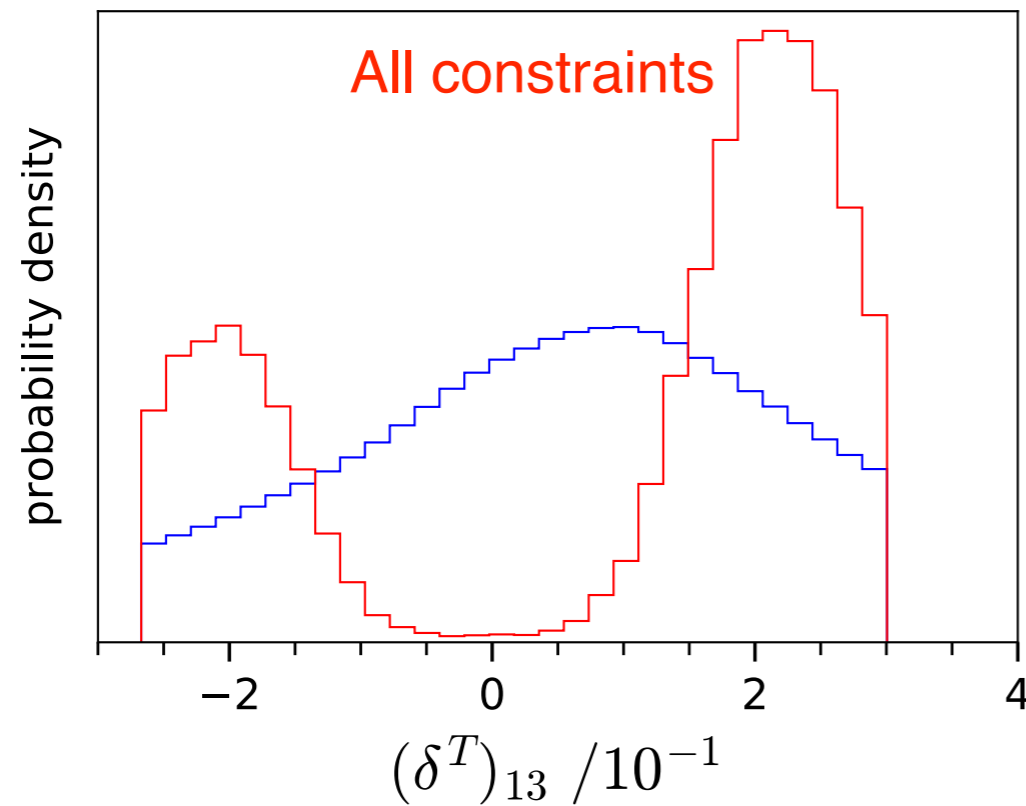


Interplay of different constraints



$(\delta^T)_{13}$ not constrained by flavour observables
other NMFV parameters driven away from zero by flavour observables,
→ **decrease in lightest smuon mass...**

Interplay of different constraints



$(\delta^T)_{13}$ not constrained by flavour observables
other NMFV parameters driven away from zero by flavour observables,
→ **decrease in lightest smuon mass...**

$(\delta^T)_{13}$ increases lightest smuon mass (due to specific pattern of mass matrix)
→ **compensation of effect of other NMFV parameters w.r.t. to the relic density**

Soft SUSY Breaking Grand Unification: Leptons vs Quarks on the Flavor Playground

M. Ciuchini,¹ A. Masiero,² P. Paradisi,^{3,4,5} L. Silvestrini,⁶ S. K. Vempati,^{7,8} and O. Vives⁴

Nucl. Phys. B 783 (2007) 112-142 — arXiv:hep-ph/0702144

Bounds on leptonic mass insertions

Type of δ_{12}^l	$\mu \rightarrow e\gamma$	$\mu \rightarrow eee$	$\mu \rightarrow e$ conversion in Ti
LL	6×10^{-4}	2×10^{-3}	2×10^{-3}
RR	-	0.09	-
LR/RL	1×10^{-5}	3.5×10^{-5}	3.5×10^{-5}

Type of δ_{13}^l	$\tau \rightarrow e\gamma$	$\tau \rightarrow eee$	$\tau \rightarrow e\mu\mu$
LL	0.15	-	-
RR	-	-	-
LR/RL	0.04	0.5	-

Type of δ_{23}^l	$\tau \rightarrow \mu\gamma$	$\tau \rightarrow \mu\mu\mu$	$\tau \rightarrow \mu ee$
LL	0.12	-	-
RR	-	-	-
LR/RL	0.03	-	0.5

Bounds on hadronic mass insertions

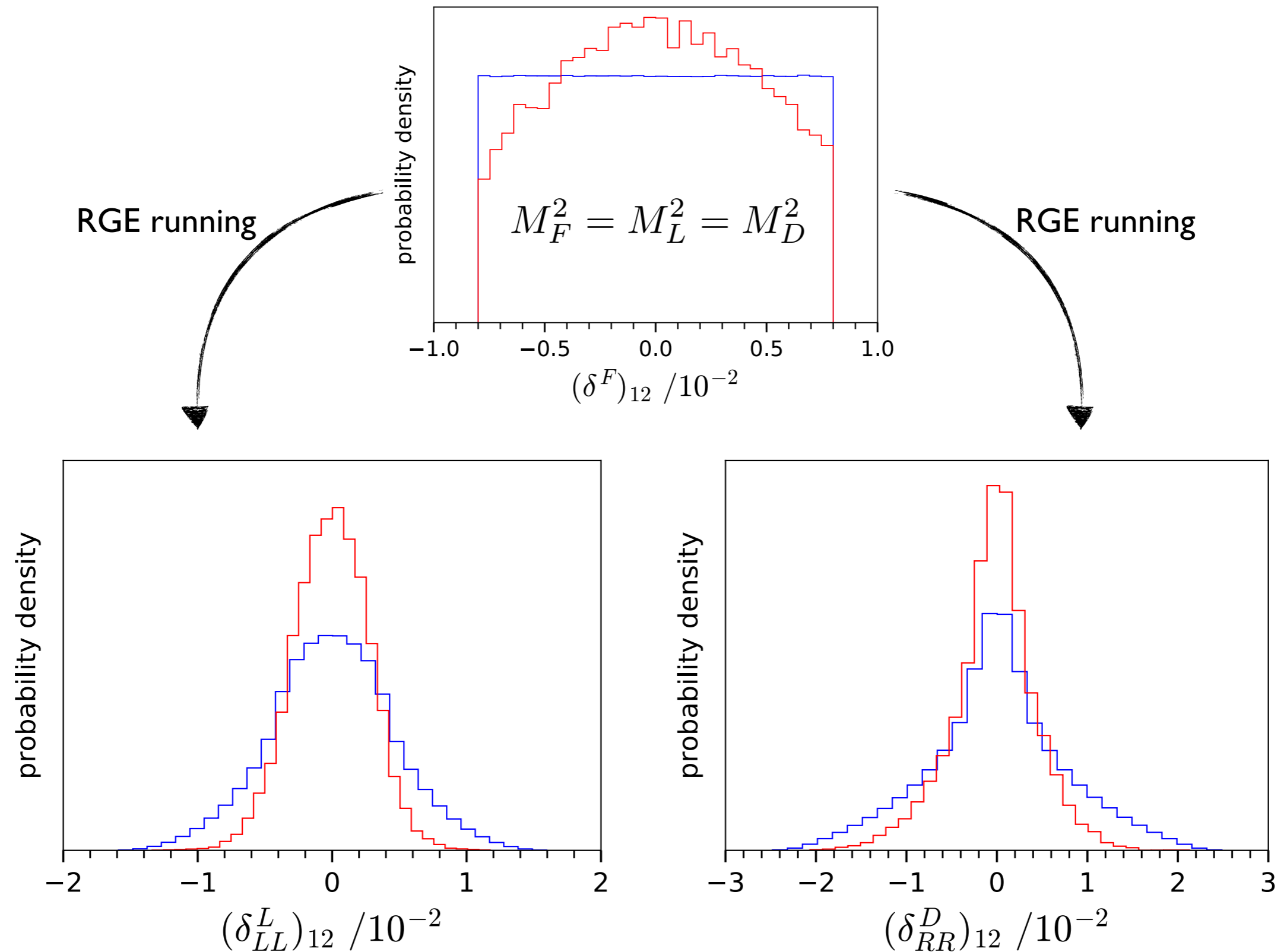
$ij \setminus AB$	LL	LR	RL	RR
12	1.4×10^{-2}	9.0×10^{-5}	9.0×10^{-5}	9.0×10^{-3}
13	9.0×10^{-2}	1.7×10^{-2}	1.7×10^{-2}	7.0×10^{-2}
23	1.6×10^{-1}	4.5×10^{-3}	6.0×10^{-3}	2.2×10^{-1}

Imposing $SU(5)$ unification conditions, hadronic mass insertions supposed to be smaller than leptonic ones,

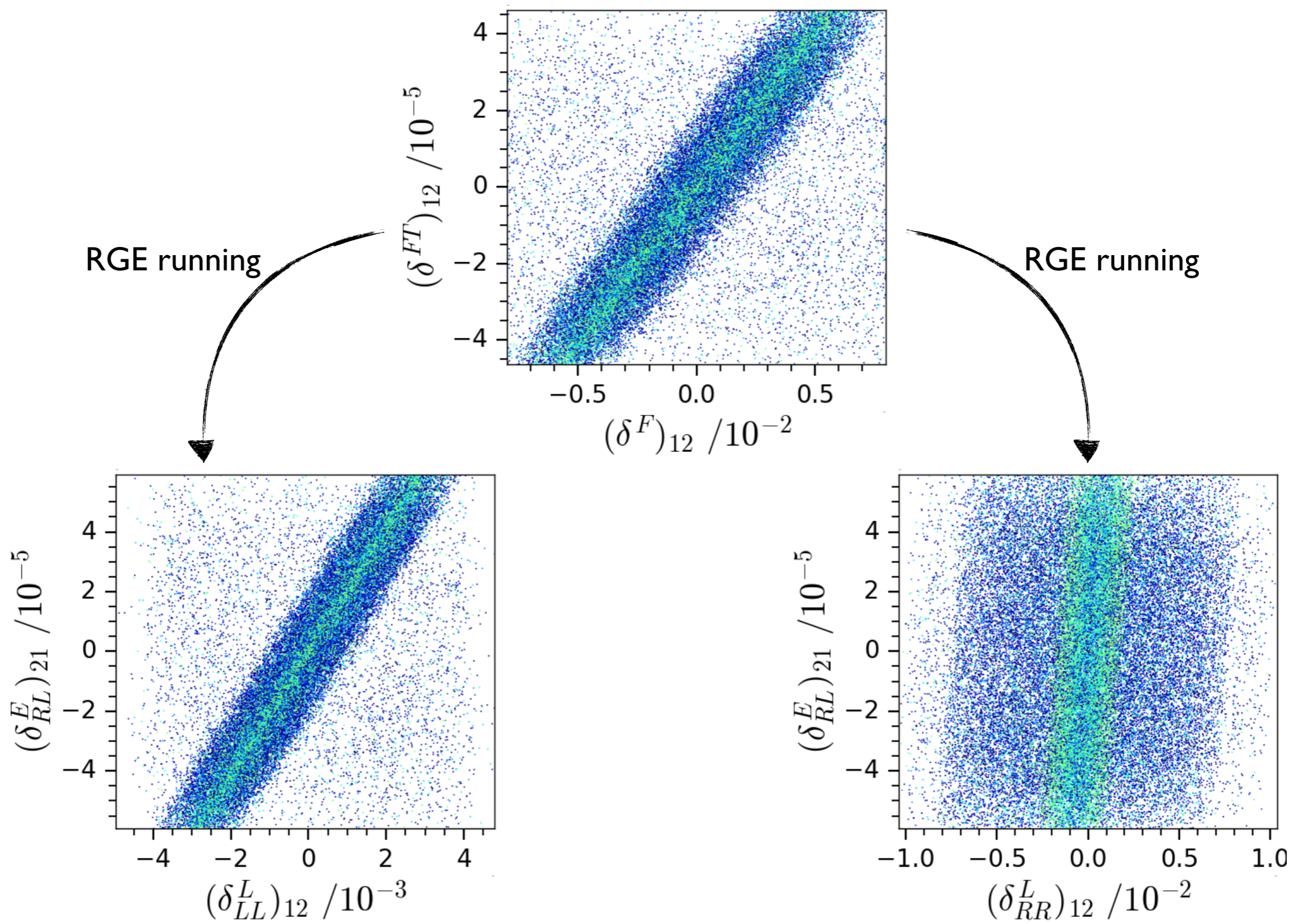
e.g.

$$|(\delta_{ij}^d)_{RR}| \leq \frac{m_L^2}{m_{dc}^2} |(\delta_{ij}^l)_{LL}|$$

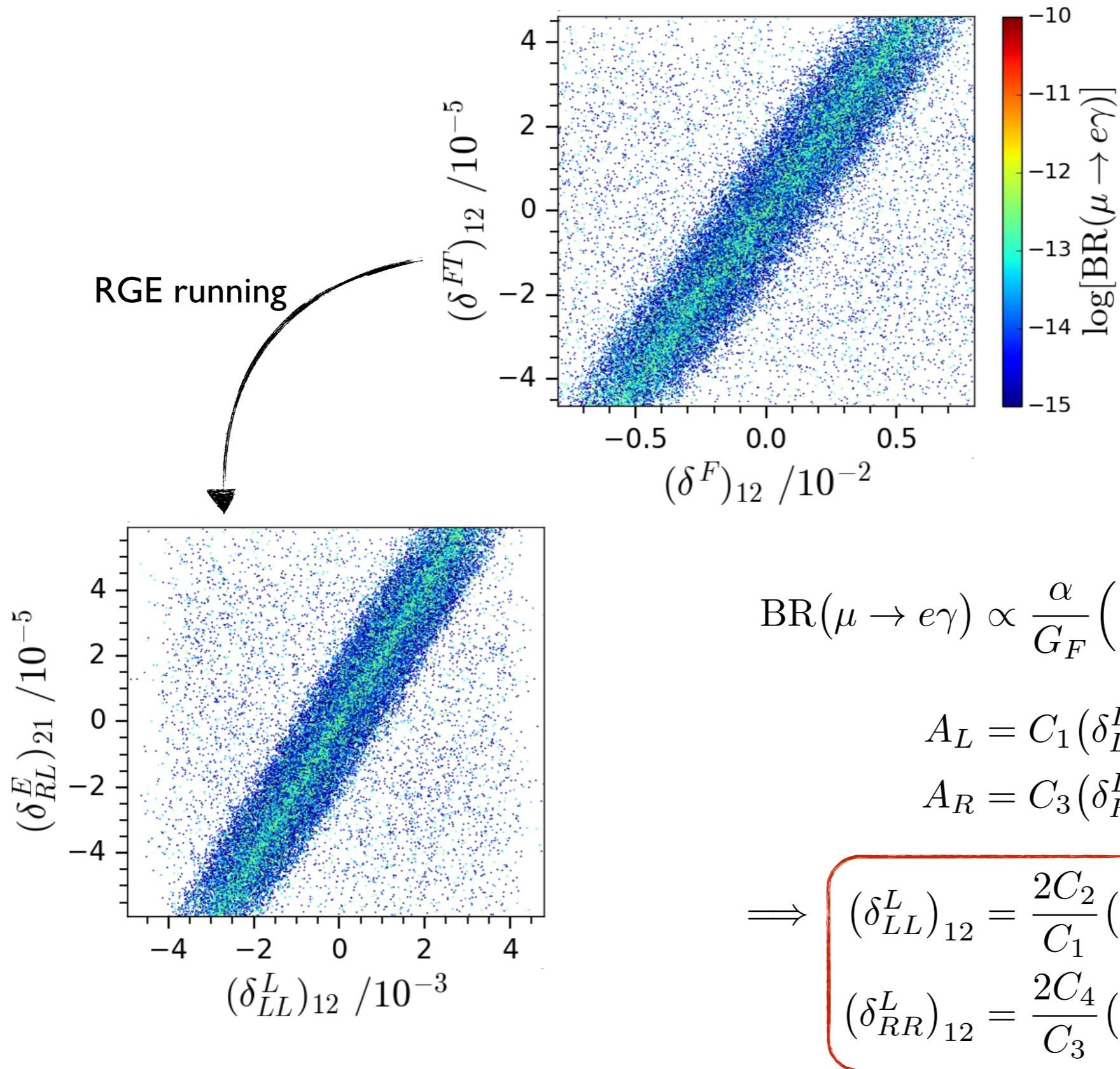
Leptonic vs. hadronic NMFV at TeV scale



Parameter correlations



Parameter correlations



Results — Summary

Parameters	Scenario 1	Most constraining obs. 1	Scenario 2	Most constraining obs. 2
$(\delta^T)_{12}$	[-0.015, 0.015]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma, \Omega_{\tilde{\chi}_1^0} h^2$	[-0.12, 0.12] [†]	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^T)_{13}$]-0.06, 0.06[$\Omega_{\tilde{\chi}_1^0} h^2$	[-0.3, 0.3] [†]	$\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^T)_{23}$	[0, 0]*	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma$	[-0.1, 0.1] [†]	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma,$
$(\delta^F)_{12}$	[-0.008, 0.008]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$	[-0.015, 0.015] [†]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^F)_{13}$]-0.01, 0.01[$\mu \rightarrow e\gamma$	[-0.15, 0.15] [†]	$\mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^F)_{23}$]-0.015, 0.015[$\mu \rightarrow e\gamma, \Omega_{\tilde{\chi}_1^0} h^2$	[-0.15, 0.15] [†]	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma, \mu \rightarrow 3e$
$(\delta^{TT})_{12}$	$[-3, 3.5] \times 10^{-5}$	prior	$[-1, 1.5]^{\dagger} \times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{TT})_{13}$	$]-6, 7[\times 10^{-5}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$	$[-4, 2.5]^{\dagger} \times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{TT})_{23}$	$]-0.5, 4[\times 10^{-5}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$	$[-0.25, 0.2]^{\dagger}$	prior, $\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{FT})_{12}$	[-0.0015, 0.0015]	$\Omega_{\tilde{\chi}_1^0} h^2$	$[-1.2, 1.2]^{\dagger} \times 10^{-4}$	$\mu \rightarrow 3e, \Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^{FT})_{13}$]-0.002, 0.002[$\Omega_{\tilde{\chi}_1^0} h^2$	$[-5, 5] \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow 3e, \mu \rightarrow e\gamma$
$(\delta^{FT})_{21}$	[0,0]*	prior	$[-1.2, 1.2]^{\dagger} \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2, \text{prior}$
$(\delta^{FT})_{23}$]-0.0022, 0.0022[$\Omega_{\tilde{\chi}_1^0} h^2$	$[-6, 6]^{\dagger} \times 10^{-4}$	$\mu \rightarrow 3e, \Omega_{\tilde{\chi}_1^0} h^2, \mu \rightarrow e\gamma$
$(\delta^{FT})_{31}$]-0.0004, 0.0004[$\Omega_{\tilde{\chi}_1^0} h^2$	$[-2, 2]^{\dagger} \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2$
$(\delta^{FT})_{32}$	[0,0]*	prior	$[-1.5, 1.5] \times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0} h^2$

* parameter not varied

† extrapolated range

Summary and outlook

Impact of non-minimal flavour violation in a flavoured GUT framework: $SU(5) \times A_4$

Limits on NMFV parameters at the GUT and TeV scales...

Interesting features already in this rather simple model...

Lepton constraints stronger than hadronic ones...

LHC phenomenology less interesting in this particular setup...

Jordan Bernigaud, B. Herrmann, Stephen F. King, Samuel J. Rowley
to be published — arXiv:1812.01xyz

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Further studies: general $SU(5)$ unified MSSM...

MSSM with $SU(5) \times S_4$ unification...

study of $(g - 2)_\ell$ in BSM...

Summary and outlook

Impact of non-minimal flavour violation in a flavoured GUT framework: $SU(5) \times A_4$

Limits on NMFV parameters at the GUT and TeV scales...

Interesting features already in this rather simple model...

Lepton constraints stronger than hadronic ones...

LHC phenomenology less interesting in this particular setup...

Jordan Bernigaud, B. Herrmann, Stephen F. King, Samuel J. Rowley
to be published — arXiv:1812.01xyz

Further studies: general $SU(5)$ unified MSSM...

MSSM with $SU(5) \times S_4$ unification...

study of $(g - 2)_\ell$ in BSM...

