Exercises I – Probability distributions[a]

Exercise 1

An examination consists of 20 questions to each of which the students have to answer "true" or "false". Each question counts one point if the answer is correct (no negative points for wrong answers). A certain student is not well prepared for the examination and answers randomly to all questions. Which probability distribution can be used to describe this situation? Find the probability that this student…

- obtains 20 points?
- obtains 0 points?
- passes the exam (if at least 12 points are required to pass)?

What is the expectation value[b] for the mark of the student?

Exercise 2

The length L of 5000 metallic pieces in normally distributed with mean[c] 151 cm and standard deviation[d] 15 cm. Find how many pieces should satisfy the following criteria:

$L < 120$ cm, $L > 130$ cm, 140 cm $< L < 180$ cm, 125 cm $< L < 145$ cm.

Exercise 3

For a standard normal[e] distribution $\mathcal{N}(0,1)$, find the parameter a such that:

$P(Z<a) = 0.9599$, $P(Z>a) = 0.01$, $P(|Z|>a) = 0.01$.

Exercise 4

A mechanism is made of three consecutive elements. The length of the first element is normally distributed with mean 2.5 cm and standard deviation 0.02 cm. The lengths of the two other elements are also normally distributed with mean 3 cm and standard deviation 0.01 cm.

What is the probability that the total length will be larger than 8.54 cm?

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[a] probability distribution *(engl.)* = loi de probabilité *(fr.)*
[b] expectation value *(engl.)* = espérance *(fr.)*
[c] mean value *(engl.)* = moyenne *(fr.)*
[d] standard deviation *(engl.)* = écart-type *(fr.)*
[e] standard normal distribution *(engl.)* = loi normal centrée réduite *(fr.)*

Exercise 5

The length of the produced pieces is measured at the end of a production line. One day's production includes 84.10% of pieces with a length larger than 30 cm, and 2.28% of pieces larger than 90 cm. Assuming that the length is normally distributed, find the mean value and the standard deviation of the length.

Exercise 6

A university professor of statistics gives a written exam composed of 100 multiple-choice questions. For each question, three answers are proposed and only one is correct. The pass mark is 40% of correct answers (no negative points for wrong answers).

1. Write down the exact formula giving the probability that a student passes the exam by entirely guessing. Do *not* evaluate this expression numerically!
2. Find an approximation for the same probability.
3. Compute numerically the probability that a student passes by entirely guessing.
4. The professor wants the probability to pass the exam by entirely guessing to be less than 1%, the success still being obtained with 40% correct answers. How many questions are necessary to fulfill this condition?

Exercise 7

The mass of a biscuit is a normal variable with mean 50 g and standard deviation 4 g. A packet contains 20 biscuits and the mass of the packing material is a normal variable with mean 100 g and standard deviation 3 g.

Find the probability that the total mass of the packet...

1. exceeds 1074 g.
2. is less than 1120 g.
3. lies between 1074 g and 1120 g.

Exercise 8

Let us consider a group of *n* persons. We will note *P(n)* the probability that at least two among the *n* persons have the same birthday (same month and day, do not consider the year, do not consider leap years[f]).

1. Before performing any calculation, guess the value of *P(k)*, where *k* is the number of persons belonging to your exercise group.
2. Calculate first *P(2)* and *P(3)*. Then find the expression for *P(n)*.
   Hint: Consider first the contrary event (i.e. evaluate the probability that the 2, 3, or *n* persons have all different birthdays).
3. Compute the value of *P(k)* and compare with your guess made in question 1.
4. How many persons are needed to have *P(n)>0.5*, *P(n)>0.9*, or *P(n)=1*?

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[f] leap year *(engl.)* = année bissextile *(fr.)*

A solution for the following homework exercises will be provided on the `Moodle` page associated to this course. Working on the homework exercises is optional and beneficial!

Exercise 9 *(homework)*

A machine produces, on average, 95% of mouldings[g] within the defined tolerance values. What is the probability that a random sample of five mouldings contains…

- no defective?
- more than one defective?

Exercise 10 *(homework)*

In a traffic survey on a motorway, the Transport Ministry Service has collected the number of cars per minute. The mean number of cars per minute is 3.

1. Justify that it is reasonable to use the Poisson distribution.
2. If the traffic flow is considered fluid for 3 or less cars per minute, on which proportion of time the traffic might be considered as fluid?

Exercise 11 *(homework)*

A university department has two photocopying machines, labeled A and B. On average per week, there are 0.8 breakdowns on machine A, and 2.0 breakdowns on machine B.

1. Which distribution can describe this situation?
2. Produce a simple graph illustrating this distribution for both A and B.
3. Find the probability of having no breakdown on machine A during two consecutive weeks.
4. Find the probability of having a total of two breakdowns in one week.

Exercise 12 *(homework)*

The number of telephone calls received by the secretary of a university department is 2 per minute around 10:00 AM. The secretary leaves the desk unattended for one minute at that time. Find the probabilities that there are

- no calls.
- one call.
- two calls.
- at least two calls.
- more than two calls.
- two, three, or four calls.

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[g] moulding *(engl.)* = moulage *(fr.)*

Exercise 13 *(homework)*

A chemical test involves measuring two independent quantities, *A* and *B*. The result of the test is $C=A-B/4$. If *A* is normally distributed with mean 10 and standard deviation 1, and *B* is also normally distributed with mean 34 and standard deviation 2, what proportion of values of *C* do you expect to be negative?

Exercise 14 *(homework)*

The weights of a large group of animals have mean 8.2 kg and variance 4.84 kg^2. What is the probability that a random selection of 80 animals will have mean weight between 8.3 and 8.4 kg?

Exercise 15 *(homework)*

A random variable is normally distributed with mean 3 and standard deviation 1. Find the probability that $(x-1)(x-3)<0$.

Exercise 16 *(homework)*

Let X be a random variable distributed normally with mean μ and standard deviation σ. Show that the random variable Z defined as

$$Z = \frac{X-\mu}{\sigma}$$

follows the standard normal distribution $\mathcal{N}(0,1)$.