Exercices

③ $[D_\mu, D_\nu]\psi = ig F^a_{\mu\nu} T_a \psi =$

$= [\partial_\mu + igA_\mu, \partial_\nu + igA_\nu]\psi =$

$= \underbrace{[\partial_\mu, \partial_\nu]}_{=0}\psi + ig[A_\mu, \partial_\nu]\psi + ig[\partial_\mu, A_\nu]\psi + (ig)^2 [A_\mu, A_\nu]\psi =$

$= ig\left[\partial_\mu A_\nu - \underbrace{A_\nu \partial_\mu + A_\mu \partial_\nu}_{=0} - \partial_\nu A_\mu + ig A^a_\mu A^b_\nu \underbrace{[T_a, T_b]}_{= i C^c_{ab} T_c}\right]\psi =$

$= ig\left[\partial_\mu A^c_\nu - \partial_\nu A^c_\mu - g A^a_\mu A^b_\nu C^c_{ab}\right] T_c \psi$

$\Rightarrow \quad F^c_{\mu\nu} T_c = \left[\partial_\mu A^c_\nu - \partial_\nu A^c_\mu - g A^a_\mu A^b_\nu C^c_{ab}\right] T_c$

Pour $SO(2)$: $A^a_\mu = W^a_\mu = \{W^1_\mu, W^2_\mu, W^3_\mu\}$

$g C^c_{ab} = g \varepsilon_{abc}$

$\Rightarrow \quad F^c_{\mu\nu} = \partial_\mu W^c_\nu - \partial_\nu W^c_\mu - g W^a_\mu W^b_\nu \varepsilon_{abc}$

$\vec{F}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g \vec{W}_\mu \times \vec{W}_\nu$

④ $T_i = \frac{1}{2}\sigma^i$

$[T_a, T_b] = i C^c_{ab} T_c$

$\left[\frac{1}{2}\sigma^i, \frac{1}{2}\sigma^j\right] = i \frac{1}{4} \cdot 2 \varepsilon_{ijk} \sigma^k = i \varepsilon_{ijk} \frac{1}{2}\sigma^k$

$g_{ab} = Tr\left\{\frac{1}{2}\sigma^a \frac{1}{2}\sigma^b\right\} = \frac{1}{4} \cdot \frac{1}{2} Tr\{\sigma^a\sigma^b + \sigma^b\sigma^a\} = \frac{1}{4}\delta_{ab} Tr\{\sigma^2\} = \frac{1}{2}\delta_{ab}$