Exercices

9.1)

$$\frac{1}{a\cdot b} = \frac{a-b}{ab(a-b)} = \frac{1}{a-b}\left(\frac{1}{b}-\frac{1}{a}\right) = \frac{1}{a-b}\int_b^a \frac{dz}{z^2} = \int_0^1 \frac{dx}{\left[xa+(1-x)b\right]^2}$$

$$x = \frac{z-b}{a-b} \quad\Rightarrow\quad z = x(a-b)+b = xa+(1-x)b$$

$$dx = \frac{dz}{a-b}$$

9.2)

$$\Gamma(z) = \int_0^\infty dx\, x^{z-1} e^{-x}$$

$$\Gamma(1) = \int_0^\infty dx\, e^{-x} = \left[-e^{-x}\right]_0^\infty = 1$$

$$\Gamma(z+1) = \int_0^\infty dx\, x^{(z+1)-1} e^{-x} = \int_0^\infty dx\, x^{z} e^{-x} = \underbrace{\left[-x^z e^{-x}\right]_0^\infty}_{=0} + \int_0^\infty dx\, z\, x^{z-1} e^{-x} = z\int_0^\infty dx\, x^{z-1} e^{-x} = z\,\Gamma(z)$$

$$\Rightarrow \quad \Gamma(1) = 1,\ \Gamma(2) = 1\cdot\Gamma(1) = 1,\ \Gamma(3) = 2\cdot\Gamma(2) = 2 \ \ldots\ \Gamma(n) = (n-1)!$$

$$\Gamma(1/2) = \int_0^\infty dx\, x^{\frac{1}{2}-1} e^{-x} = \int_0^\infty dx\, \frac{e^{-x}}{\sqrt{x}} = \int_0^\infty 2\, dt\, e^{-t^2} = 2\int_0^\infty dt\, e^{-t^2} = \int_{-\infty}^\infty dt\, e^{-t^2} = \sqrt{\pi}$$

$t = \sqrt{x}$

intégrale de Gauss

9.3)

$$\left.\begin{aligned} d q_E^D &= D\, q_E^{D-1}\, dq_E \\ d q_E^D &= d\left(q_E^2\right)^{D/2} = \frac{D}{2}\left(q_E^2\right)^{\frac{D}{2}-1} dq_E^2 \end{aligned}\right\} \Rightarrow\ dq_E\, q_E^{D-1} = \frac{1}{2}\left(q_E^2\right)^{\frac{D}{2}-1} dq_E^2$$

$$I_{2p} = \frac{(2\pi\mu)^{4-D}}{\pi^2}\int_0^1 dx \int d\Omega_D \int_0^\infty dq_E\, \frac{q_E^{D-1}}{\left(q_E^2+A^2\right)^2} = \frac{(2\pi\mu)^{4-D}}{\pi^2}\,\frac{2\pi^{D/2}}{\Gamma(D/2)}\int_0^1 dx \int_0^\infty dq_E^2\, \frac{1}{2}\,\frac{\left(q_E^2\right)^{\frac{D}{2}-1}}{\left(q_E^2+A^2\right)^2} =$$

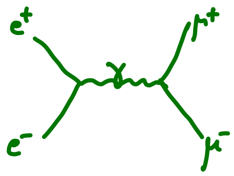
$$= \frac{(2\pi\mu)^{4-D}}{2\pi^2}\,\frac{2\pi^{D/2}}{\Gamma(D/2)}\int_0^1 dx\, \left(A^2\right)^{\frac{D}{2}-2} \underbrace{\int_0^1 dy\, (1-y)^{\frac{D}{2}-1}\, y^{2-\frac{D}{2}-1}}_{=B\left(\frac{D}{2},\,2-\frac{D}{2}\right)}$$

$$y = \frac{A^2}{q_E^2+A^2} \quad\Leftrightarrow\quad q_E^2 = \frac{A^2}{y}+A^2$$

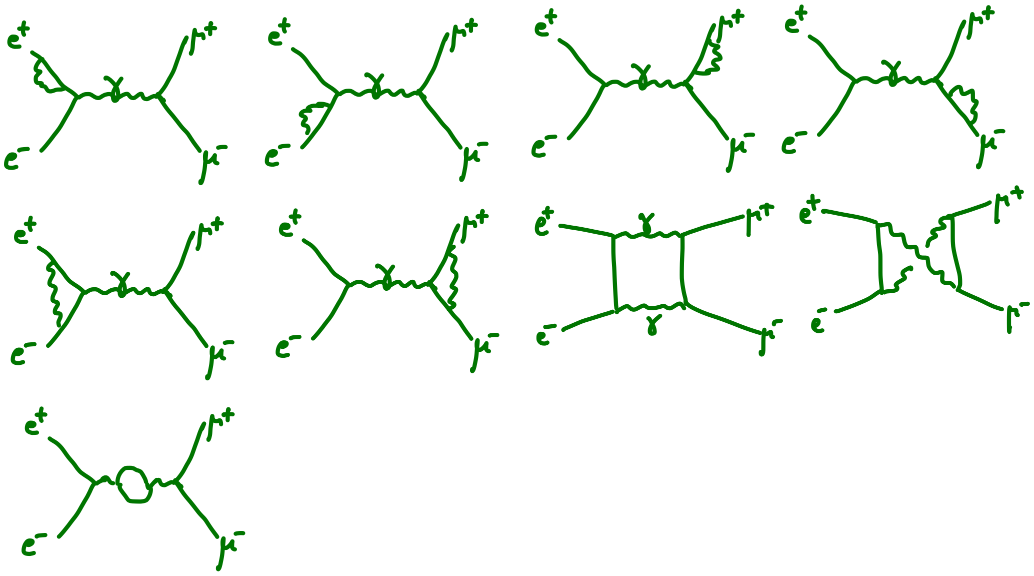
$$dq_E^2 = -\frac{A^2}{y^2}\, dy$$

9.4] $e^+e^- \to \mu^+\mu^-$

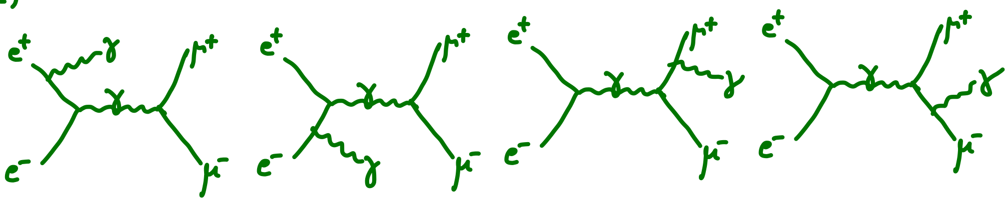
Ordre α (arbre) :



Ordre α (1 boucle) :



Ordre α (émission réelle) :



Intégrales divergentes (UV) :

Structure IR :