Exercices

7.1

$$[H_0, a_k^\dagger] = \int d\tilde{p}\, \omega_p [a_p^\dagger a_p, a_k^\dagger] = \int d\tilde{p}\, \omega_p \Big(a_p^\dagger \underbrace{[a_p, a_k^\dagger]}_{=\frac{1}{(2\pi)^4}\delta_{pk}} + \underbrace{[a_p^\dagger, a_k^\dagger]}_{=0} a_p \Big) = \omega_k a_k^\dagger$$

$$[H_0, a_k] = \dots = -\omega_k a_k$$

$$\frac{d}{dt}\varphi = i \frac{d}{dt} \int d\tilde{k} \Big(\underbrace{-a_k \omega_k}_{=[H_0, a_k]} e^{-ikx} + \underbrace{a_k^\dagger \omega_k}_{=[H_0, a_k^\dagger]} e^{ikx} \Big) = i\,[H_0, \varphi]$$

7.3

$$H_S = \underbrace{\frac{p^2}{2m} + \frac{k}{2}X^2}_{H_{OS}} + \underbrace{\varepsilon X^4}_{H_{IS}} \longrightarrow H_I = e^{iH_{os}(t-t_0)} H_{IS}\, e^{-iH_{os}(t-t_0)} =$$

$$= \varepsilon \cdot e^{iH_{os}(t-t_0)} X^4 e^{-iH_{os}(t-t_0)} = \varepsilon \cdot X^4_{free}$$

7.4

$$\mathcal{L}_I = -e :\bar{\psi} \not{A} \psi: \; = -\mathcal{H}_I$$

$$\Longrightarrow S = T\Big\{ \exp\Big(-i \int d^4x\, \mathcal{H}_I\Big)\Big\} = T\Big\{ \exp\Big(-ie \int d^4x :\bar{\psi} \not{A} \psi:\Big)\Big\}$$

7.6 Théorème de Wick pour 3 fermions

$$\psi_1\psi_2 = :\psi_1\psi_2: + \underbracket{\psi_1\psi_2}$$

On choisit : $t_3 < t_1$ et $t_3 < t_2$

$$\psi_1\psi_2\psi_3 = \left(:\psi_1\psi_2: + \underbracket{\psi_1\psi_2}\right)\psi_3 = \underbrace{:\psi_1\psi_2:\psi_3^+}_{=:\psi_1\psi_2\psi_3^+:} + :\psi_1\psi_2:\psi_3^- + \underbracket{\psi_1\psi_2}\psi_3$$

$$:\psi_1\psi_2:\psi_3^- = \underbrace{\psi_1^+\psi_2^+\psi_3^-}_{①} + \underbrace{\psi_1^-\psi_2^+\psi_3^-}_{②} - \underbrace{\psi_2^-\psi_1^+\psi_3^-}_{③} + \underbrace{\psi_1^-\psi_2^-\psi_3^-}_{=\psi_3^-\psi_1^-\psi_2^-}$$

$$① = \psi_1^+\left(-\psi_3^-\psi_2^+ + \{\psi_2^+,\psi_3^-\}\right) = -\psi_1^+\psi_3^-\psi_2^+ + \psi_1^+\{\psi_2^+,\psi_3^-\} =$$

$$= \left(\psi_3^-\psi_1^+ - \{\psi_1^+,\psi_3^-\}\right)\psi_2^+ + \psi_1^+\{\psi_2^+,\psi_3^-\} =$$

$$= \psi_3^-\psi_1^+\psi_2^+ - \underbracket{\psi_1\psi_3}\psi_2^+ + \underbracket{\psi_2\psi_3}\psi_1^+$$

$$② = \psi_1^-\left(-\psi_3^-\psi_2^+ + \{\psi_3^-,\psi_2^+\}\right) = -\psi_1^-\psi_3^-\psi_2^+ + \psi_1^-\{\psi_2^+,\psi_3^-\} =$$

$$= \psi_3^-\psi_1^-\psi_2^+ + \psi_1^-\underbracket{\psi_2\psi_3}$$

$$③ = \psi_3^-\psi_2^-\psi_1^+ - \psi_2^-\underbracket{\psi_1\psi_3}$$

$$\Rightarrow \psi_1\psi_2\psi_3 = \psi_3^-\underbrace{\left(\psi_1^+\psi_2^+ + \psi_1^-\psi_2^+ - \psi_2^-\psi_1^+ + \psi_1^-\psi_2^-\right)}_{=:\psi_1\psi_2:} + :\psi_1\psi_2:\psi_3^+$$

$$+ \underbracket{\psi_1\psi_2}\psi_3 - \underbracket{\psi_1\psi_3}\psi_2^+ - \underbracket{\psi_1\psi_3}\psi_2^- + \underbracket{\psi_2\psi_3}\psi_1^+ + \underbracket{\psi_2\psi_3}\psi_1^-$$

$$= :\psi_1\psi_2\psi_3: + \underbracket{\psi_1\psi_2}\psi_3 - \underbracket{\psi_1\psi_3}\psi_2 + \underbracket{\psi_2\psi_3}\psi_1$$

$$\Rightarrow T\{\psi_1\psi_2\psi_3\} = :\psi_1\psi_2\psi_3: + \overbracket{\psi_1\psi_2}\psi_3 - \overbracket{\psi_1\psi_3}\psi_2 + \overbracket{\psi_2\psi_3}\psi_1$$

7.7 $$\mathcal{F}(x,y) = :\bar\psi(x)\gamma^\mu\psi(x)A_\mu(x)\,\bar\psi(y)\gamma^\nu\psi(y)A_\nu(y):$$

$$+ :\bar\psi(x)\gamma^\mu\psi(x)A_\mu(x)\,\bar\psi(y)\gamma^\nu\psi(y)A_\nu(y):$$

$$+ \dots$$

$$= :\bar\psi_\alpha(x)\gamma^\mu_{\alpha\beta}\psi_\beta(x)\,\bar\psi_\eta(y)\gamma^\nu_{\eta\xi}\psi_\xi(y):\; A_\mu(x)A_\nu(y)$$

$$+ i :\bar\psi_\alpha(x)\gamma^\mu_{\alpha\beta}\,S_F(x-y)_{\beta\eta}\,\gamma^\nu_{\eta\xi}\psi_\xi(y):\; A_\mu(x)A_\nu(y)$$

$$+ \dots$$