Exercices

4.4) voir cours...

4.5) $$\langle 0|\, T\{\varphi(x)\varphi^\dagger(y)\}\,|0\rangle = \Theta(x^0-y^0)\underbrace{\langle 0|\varphi(x)\varphi^\dagger(y)|0\rangle}_{(i)} + \Theta(y^0-x^0)\underbrace{\langle 0|\varphi^\dagger(y)\varphi(x)|0\rangle}_{(ii)}$$

(i) $$\langle 0|\varphi(x)\varphi^\dagger(y)|0\rangle = \langle 0|\int d\tilde{k}\Big[\underset{\substack{\uparrow\\ \text{création en } x}}{a_{\vec{k}}}\, e^{-ikx} + \underbrace{b^\dagger_{\vec{k}}}_{\to 0} e^{ikx}\Big]\int d\tilde{p}\Big[\underset{\substack{\uparrow\\ \text{création en } y}}{a^\dagger_{\vec{p}}}\, e^{ipx} + \underbrace{b_{\vec{p}}}_{\to 0} e^{-ipx}\Big]|0\rangle$$

$$\langle 0|\varphi(x)\varphi^\dagger(y)|0\rangle = \int d\tilde{k}\, d\tilde{p}\; e^{-ikx}e^{ipy}\underbrace{\langle 0|a_{\vec{k}}a^\dagger_{\vec{p}}|0\rangle}_{=\tilde{\delta}_{kp}} = \int d\tilde{k}\; e^{-ik(x-y)}$$

(ii) $$\langle 0|\varphi^\dagger(y)\varphi(x)|0\rangle = \ldots = \int d\tilde{k}\; e^{ik(x-y)}$$

$$\Longrightarrow \langle 0|T\{\varphi(x)\varphi^\dagger(y)\}|0\rangle = \int d\tilde{k}\Big[\Theta(x^0-y^0)e^{-ik(x-y)} + \Theta(y^0-x^0)e^{ik(x-y)}\Big]$$
$$= -i\, G_F(x-y)$$

4.3) $$\partial_\mu j^\mu = iq\,\partial_\mu\big[\varphi^*(\partial^\mu\varphi) - (\partial^\mu\varphi^*)\varphi\big] =$$
$$= iq\big[(\partial_\mu\varphi^*)(\partial^\mu\varphi) + \varphi^*(\Box\varphi) - (\Box\varphi^*)\varphi - (\partial^\mu\varphi^*)(\partial_\mu\varphi)\big] =$$
$$= iq\big[(\Box\varphi^*)\varphi - \varphi^*(\Box\varphi)\big] \underset{\substack{\uparrow\\ \text{Klein-Gordon}}}{=} iq\big[-m^2\varphi^*\varphi + m^2\varphi^*\varphi\big] = 0$$

4.2)
$$[a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = \frac{1}{2}[a_{\vec{k}1} + i a_{\vec{k}2}, a^{\dagger}_{\vec{p}1} - i a^{\dagger}_{\vec{p}2}] =$$
$$= \frac{1}{2}\left([a_{\vec{k}1}, a^{\dagger}_{\vec{p}1}] + i[a_{\vec{k}2}, a^{\dagger}_{\vec{p}1}] - i[a_{\vec{k}1}, a^{\dagger}_{\vec{p}2}] + [a_{\vec{k}2}, a^{\dagger}_{\vec{p}2}]\right) =$$
$$= \frac{1}{2} 2\cdot [a_{\vec{k}1}, a^{\dagger}_{\vec{p}1}] = [a_{\vec{k}1}, a^{\dagger}_{\vec{p}1}] = (2\pi)^3 2\omega_{\vec{k}} \delta(\vec{k}-\vec{p})$$
$$[b_{\vec{k}}, b^{\dagger}_{\vec{p}}] = \dots = (2\pi)^3 2\omega_{\vec{k}} \delta(\vec{k}-\vec{p})$$
$$[a_{\vec{k}}, b^{\dagger}_{\vec{p}}] = \dots = 0$$

4.1)
$$T^{00} = \dot{\varphi}^{*}\dot{\varphi} + \dot{\varphi}^{*}\dot{\varphi} - \mathcal{L} =$$
$$= 2\dot{\varphi}^{*}\dot{\varphi} - [\dot{\varphi}^{*}\dot{\varphi} - (\vec{\nabla}\varphi^{*})(\vec{\nabla}\varphi) - V(\varphi)]$$
$$= \dot{\varphi}$$