Exercices

(1) $$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{\lambda}{4}(\phi^2-v^2)^2 = \frac{1}{2}(\partial_\mu\vec{\varphi})(\partial^\mu\vec{\varphi}) - \frac{\lambda}{4}(\vec{\varphi}^2-v^2)^2$$

$$\vec{\varphi} = \vec{\varphi}_0 + \vec{h} = \begin{pmatrix} v \\ 0 \end{pmatrix} + \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \longrightarrow \vec{\varphi}^2 = \vec{\varphi}_0^2 + 2\vec{\varphi}_0\vec{h} + \vec{h}^2 = v^2 + 2\vec{\varphi}_0\vec{h} + \vec{h}^2$$

$$\partial_\mu\vec{\varphi} = \partial_\mu\vec{h}$$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu\vec{h})(\partial^\mu\vec{h}) - \frac{\lambda}{4}(2\vec{\varphi}_0\vec{h} + \vec{h}^2) =$$

$$= \frac{1}{2}(\partial_\mu h_1)(\partial^\mu h_1) + \frac{1}{2}(\partial_\mu h_2)(\partial^\mu h_2) - \frac{\lambda}{4}(2vh_1 + h_1^2 + h_2^2)^2 =$$

$$= \frac{1}{2}(\partial_\mu h_1)(\partial^\mu h_1) + \frac{1}{2}(\partial_\mu h_2)(\partial^\mu h_2) - \lambda v h_1^2 - \lambda v h_1(h_1^2+h_2^2) - \frac{\lambda}{4}(h_1^2+h_2^2)^2$$

(2) $V(\vec{\varphi}) = V(R)$ avec $R^2 = \vec{\varphi}^2 = \sum_i \varphi_i^2$

$$\frac{\partial V}{\partial \varphi_i} = \frac{dV}{dR}\frac{\partial R}{\partial \varphi_i} = V'(R)\cdot\frac{\partial}{\partial\varphi_i}\sqrt{\sum_j \varphi_j^2} = V'(R)\cdot\frac{2\varphi_i}{2\sqrt{\sum_j\varphi_j^2}} = V'(R)\cdot\frac{\varphi_i}{R}$$

$$\frac{\partial^2 V}{\partial\varphi_i\partial\varphi_j} = \frac{\partial}{\partial\varphi_j}\left[V'(R)\cdot\frac{\varphi_i}{R}\right] = V''(R)\frac{\varphi_j}{R}\frac{\varphi_i}{R} + V'(R)\frac{\partial}{\partial\varphi_j}\left[\frac{\varphi_i}{R}\right] =$$

$$= V''(R)\frac{\varphi_j}{R}\frac{\varphi_i}{R} + V'(R)\cdot\frac{\delta_{ij}}{R} + V'(R)\,\varphi_i\underbrace{\frac{\partial}{\partial\varphi_j}\frac{1}{\sqrt{\sum\varphi_i^2}}}_{=\frac{-\varphi_j}{\sum_k\varphi_k^2}} =$$

$$= V''(R)\frac{\varphi_i\varphi_j}{R^2} + \frac{V'(R)}{R}\left[\delta_{ij} - \frac{\varphi_i\varphi_j}{R}\right]$$

(3) fait en cours.