

Partie 1: Théorème de Wick pour trois champs fermioniques

$$\psi_i = \psi_i^+ + \psi_i^- \quad (i=1,2,3)$$

$$\psi_1 \psi_2 = (\psi_1^+ + \psi_1^-)(\psi_2^+ + \psi_2^-) = \psi_1^+ \psi_2^+ + \psi_1^+ \psi_2^- + \psi_1^- \psi_2^+ + \psi_1^- \psi_2^-$$

$$:\psi_1 \psi_2: = \psi_1^+ \psi_2^- - \psi_2^- \psi_1^+ + \psi_1^- \psi_2^+ + \psi_1^- \psi_2^-$$

$$\Rightarrow \psi_1 \psi_2 - :\psi_1 \psi_2: = \psi_1^+ \psi_2^- + \psi_2^- \psi_1^+ = \{\psi_1^+, \psi_2^-\} = \underbrace{\varphi_1 \varphi_2}$$

$$\Rightarrow \boxed{\psi_1 \psi_2 = :\psi_1 \psi_2: + \underbrace{\psi_1 \psi_2}}$$

$$\psi_1 \psi_2 \psi_3 = \left(:\psi_1 \psi_2: + \underbrace{\psi_1 \psi_2} \right) \psi_3 = :\psi_1 \psi_2 \psi_3: + :\psi_1 \psi_2: \psi_3^- + \underbrace{\psi_1 \psi_2} \psi_3$$

$$:\psi_1 \psi_2: \psi_3^- = \psi_1^+ \psi_2^+ \psi_3^- - \psi_2^- \psi_1^+ \psi_3^- + \psi_1^- \psi_2^+ \psi_3^- + \psi_1^- \psi_2^- \psi_3^-$$

$$\psi_1^+ \psi_2^+ \psi_3^- = \psi_1^+ \underbrace{\varphi_2 \varphi_3} - \underbrace{\varphi_1 \varphi_3} \psi_2^+ + \varphi_3 \psi_1^+ \varphi_2^+$$

$$-\psi_2^- \psi_1^+ \psi_3^- = -\underbrace{\psi_1 \psi_3} \psi_2^- - \psi_3^- \psi_2^- \psi_1^+$$

$$\psi_1^- \psi_2^+ \psi_3^- = \underbrace{\psi_2 \psi_3} \psi_1^- - \psi_3^- \psi_2^- \psi_1^+$$

$$\psi_1^- \psi_2^- \psi_3^- = \psi_3^- \psi_1^- \psi_2^-$$

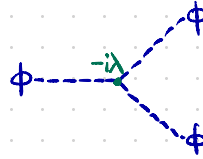
$$\Rightarrow \boxed{\psi_1 \psi_2 \psi_3 = :\psi_1 \psi_2 \psi_3: + \underbrace{\psi_2 \psi_3} \psi_1 - \underbrace{\psi_1 \psi_3} \psi_2 + \underbrace{\psi_1 \psi_2} \psi_3}$$

$$\Rightarrow \boxed{T\{\psi_1 \psi_2\} = :\psi_1 \psi_2: + \underbrace{\psi_1 \psi_2}}$$

$$\boxed{T\{\psi_1 \psi_2 \psi_3\} = :\psi_1 \psi_2 \psi_3: + \underbrace{\psi_2 \psi_3} \psi_1 - \underbrace{\psi_1 \psi_3} \psi_2 + \underbrace{\psi_1 \psi_2} \psi_3}$$

Partie 2: Interactions scalaires

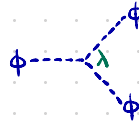
$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)(\partial^\dagger\phi)}_{\text{cinétique}} - \underbrace{\frac{1}{2}m^2\phi^2}_{\text{masse}} - \underbrace{\lambda\phi^3}_{\text{interactions}}$$



$$\begin{aligned} S &= T\left\{\exp\left[i\int d^4x \mathcal{L}_{\text{int}}(x)\right]\right\} \\ &= T\left\{\exp\left[-i\int d^4x : \lambda\phi^3(x) : \right]\right\} \\ &= 1 - i\int d^4x T\{:\lambda\phi^3(x): \} + i^2\int d^4x\int d^4y T\{:\lambda\phi^3(x)::\lambda\phi^3(y): \} + \dots \end{aligned}$$

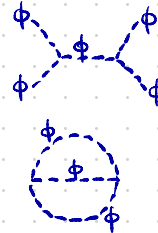
Ordre 0: pas d'interaction

Ordre 1: $S \sim \lambda\phi^3(x)$



Ordre 2: $S \sim T\{:\lambda\phi^3(x) + \kappa\phi^4(x)::\lambda\phi^3(y) + \kappa\phi^4(y): \}$

$$\begin{aligned} &\sim \lambda^2 : \phi(x)\phi(x)\phi(x)\phi(y)\phi(y)\phi(y) : \\ &+ \lambda^2 : \phi(x)\phi(x)\phi(x)\phi(y)\phi(y)\phi(y) : \\ &+ \lambda^2 : \phi(x)\phi(x)\phi(x)\phi(y)\phi(y)\phi(y) : \end{aligned}$$



Contributions à $\phi\phi \rightarrow \phi\phi$:

