Partie 1 : Théorème de Wick pour trois champs fermioniques

$$\psi_i = \psi_i^+ + \psi_i^- \quad (i=1,2,3)$$

$$\psi_1\psi_2 = (\psi_1^+ + \psi_1^-)(\psi_2^+ + \psi_2^-) = \psi_1^+\psi_2^+ + \psi_1^+\psi_2^- + \psi_1^-\psi_2^+ + \psi_1^-\psi_2^-$$

$$:\psi_1\psi_2: = \psi_1^+\psi_2^+ - \psi_2^-\psi_1^+ + \psi_1^-\psi_2^+ + \psi_1^-\psi_2^-$$

$$\Rightarrow \quad \psi_1\psi_2 - :\psi_1\psi_2: = \psi_1^+\psi_2^- + \psi_2^-\psi_1^+ = \{\psi_1^+, \psi_2^-\} = \underbrace{\varphi_1\varphi_2}$$

$$\Rightarrow \quad \boxed{\psi_1\psi_2 = :\psi_1\psi_2: + \underbrace{\psi_1\psi_2}}$$

$$\psi_1\psi_2\psi_3 = \left(:\psi_1\psi_2: + \underbrace{\psi_1\psi_2}\right)\psi_3 = :\psi_1\psi_2\psi_3^+: + :\psi_1\psi_2:\psi_3^- + \underbrace{\psi_1\psi_2}\psi_3$$

$$:\psi_1\psi_2:\psi_3^- = \psi_1^+\psi_2^+\psi_3^- - \psi_2^-\psi_1^+\psi_3^- + \psi_1^-\psi_2^+\psi_3^- + \psi_1^-\psi_2^-\psi_3^-$$

$$\psi_1^+\psi_2^+\psi_3^- = \psi_1^+\underbrace{\varphi_2\varphi_3} - \underbrace{\varphi_1\varphi_3}\varphi_2^+ + \varphi_3^-\varphi_1^+\varphi_2^+$$

$$-\psi_2^-\psi_1^+\psi_3^- = -\underbrace{\psi_1\psi_3}\psi_2^- - \psi_3^-\psi_2^-\psi_1^+$$

$$\psi_1^-\psi_2^+\psi_3^- = \underbrace{\psi_2\psi_3}\psi_2^- - \psi_3^-\psi_2^-\psi_1^+$$

$$\psi_1^-\psi_2^-\psi_3^- = \psi_3^-\psi_1^-\psi_2^-$$

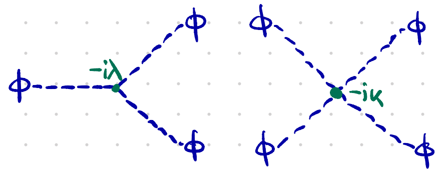
$$\Rightarrow \quad \boxed{\psi_1\psi_2\psi_3 = :\psi_1\psi_2\psi_3: + \underbrace{\psi_2\psi_3}\psi_1 - \underbrace{\psi_1\psi_3}\psi_2 + \underbrace{\psi_1\psi_2}\psi_3}$$

$$\Rightarrow \quad \boxed{T\{\psi_1\psi_2\} = :\psi_1\psi_2: + \overbrace{\psi_1\psi_2}}$$

$$\boxed{T\{\psi_1\psi_2\psi_3\} = :\psi_1\psi_2\psi_3: + \overbrace{\psi_2\psi_3}\psi_1 - \overbrace{\psi_1\psi_3}\psi_2 + \overbrace{\psi_1\psi_2}\psi_3}$$

Partie 2: Interactions scalaires

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)}_{\text{cinétique}} - \underbrace{\tfrac{1}{2}m^2\phi^2}_{\text{masse}} \underbrace{- \lambda\phi^3 - \kappa\phi^4}_{\text{interactions}}$$



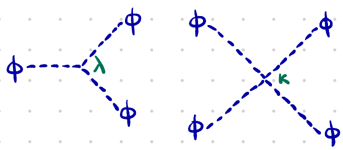
$$S = T\left\{\exp\left[i\int d^4x\, \mathcal{L}_{int}(x)\right]\right\}$$

$$= T\left\{\exp\left[-i\int d^4x\, :\lambda\phi^3(x) + \kappa\phi^4(x):\right]\right\}$$

$$= 1 - i\int d^4x\, T\{:\lambda\phi^3(x) + \kappa\phi^4(x):\} + i^2\int d^4x\int d^4y\, T\{:\lambda\phi^3(x) + \kappa\phi^4(x)::\lambda\phi^3(y) + \kappa\phi^4(y):\} + \dots$$

Ordre 0: pas d'interaction

Ordre 1: $S \sim \lambda\phi^3(x) + \kappa\phi^4(x)$



Ordre 2: $S \sim T\{:\lambda\phi^3(x) + \kappa\phi^4(x)::\lambda\phi^3(y) + \kappa\phi^4(y):\}$

$$\sim \lambda^2 :\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y)$$

$$+ \lambda^2 :\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y)$$

$$+ \lambda^2 :\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y)$$

$$+ \lambda\kappa :\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y):$$

$$+ \lambda\kappa :\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y):$$

$$+ \lambda\kappa :\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y):$$

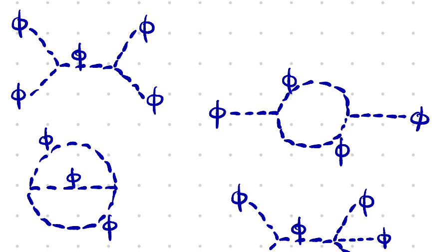
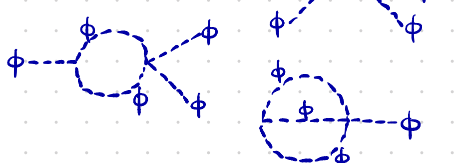
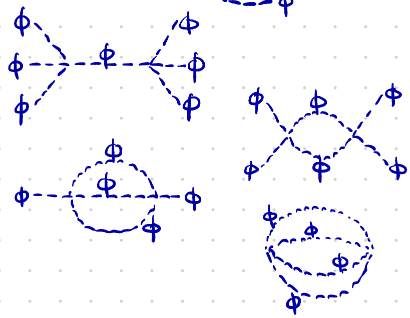
$$+ \{x \leftrightarrow y\}$$

$$+ \kappa^2 :\phi(x)\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y)\phi(y):$$

$$+ \kappa^2 :\phi(x)\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y)\phi(y):$$

$$+ \kappa^2 :\phi(x)\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y)\phi(y):$$

$$+ \kappa^2 :\phi(x)\phi(x)\phi(x)\phi(x)\,\phi(y)\phi(y)\phi(y)\phi(y):$$

Contributions à $\phi\phi \to \phi\phi$: