

# Radiative Corrections for Higgs Study at the ILC

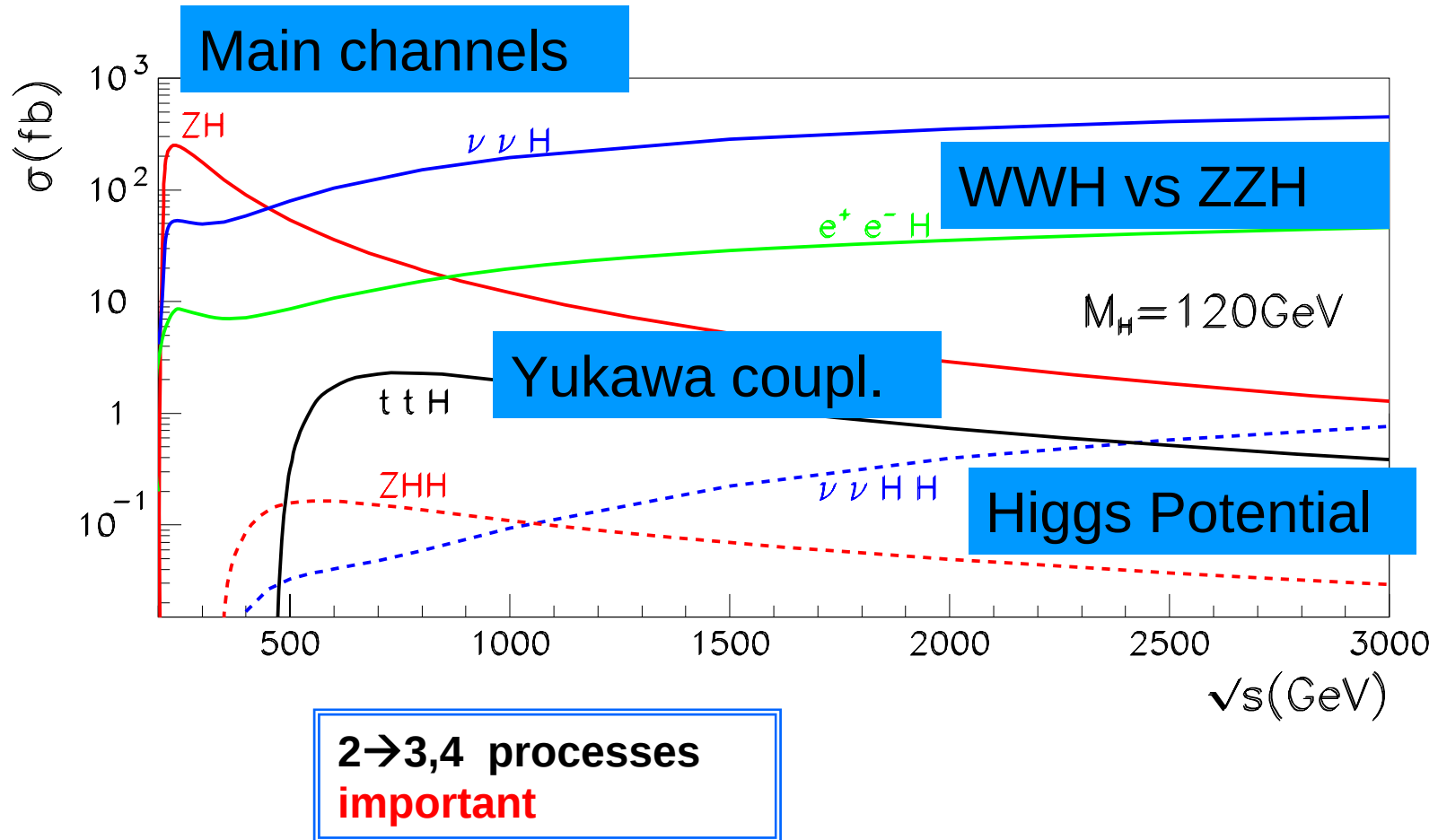
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and ACPP(Automated Calculations in Particle Physics)  
EPS05, Lisbon  
2005.07.23



# Higgs@ILC

- Most important target :  
LHC : discovery of Higgs  
ILC : detailed physics for Higgs  
→ gauge structure, Yukawa coupling  
Higgs potential
- Final states : 2-, 3-, 4-bodies  
→ large scale calculation
- High statistics experiments  
→ higher order calculation required

# Higgs@LC:tree cross sections



*Full 1-loop RC available*

$$e^+e^- \rightarrow ZH$$

A.Denner, et al., ZPC56(1992) 261;  
B.A.Kniehl, ZPC55(1992) 605

$$e^+e^- \rightarrow \nu\bar{\nu}H$$

GRACE, PLB559(2003)252  
Denner et al., NPB660(2003)289

$$e^+e^- \rightarrow t\bar{t}H$$

GRACE, PLB571(2003)163  
You et al., PLB571(2003)85  
Denner et al., PLB575(2003)290

$$e^+e^- \rightarrow e^+e^-H$$

GRACE, PLB600(2004)65

$$e^+e^- \rightarrow ZHH$$

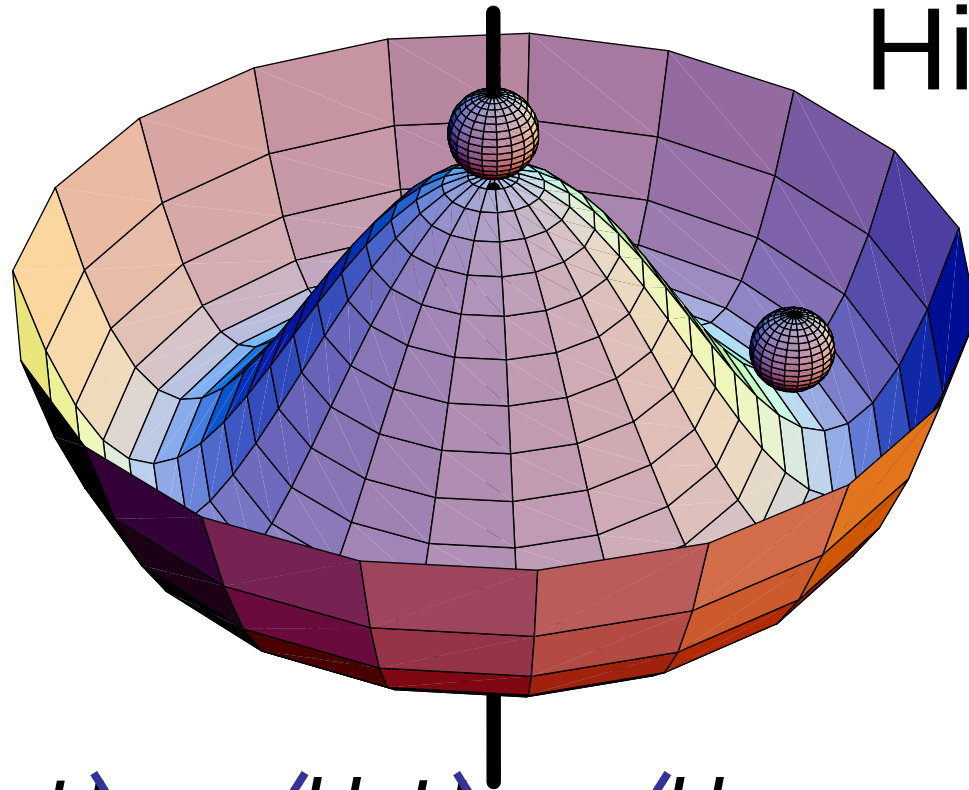
GRACE, PLB576(2003)152  
Zhang et al., PLB578(2004)349

$$e^+e^- \rightarrow \nu\bar{\nu}HH$$

GRACE, at ECFAC (Sep.2004),  
LCWS05 (Mar.2005)

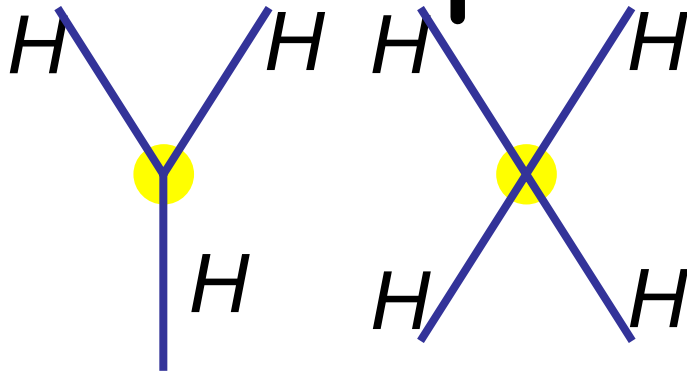
**This talk.**

# Higgs Potential



*Something required to  
provide mass  
Key for the structure of  
standard model*

Window to New  
Physics



Double (triple) Higgs  
production Process

$$e^+ e^- \rightarrow \nu \bar{\nu} H H$$

“1-loop” includes counter terms

non-linear gauge fixing model

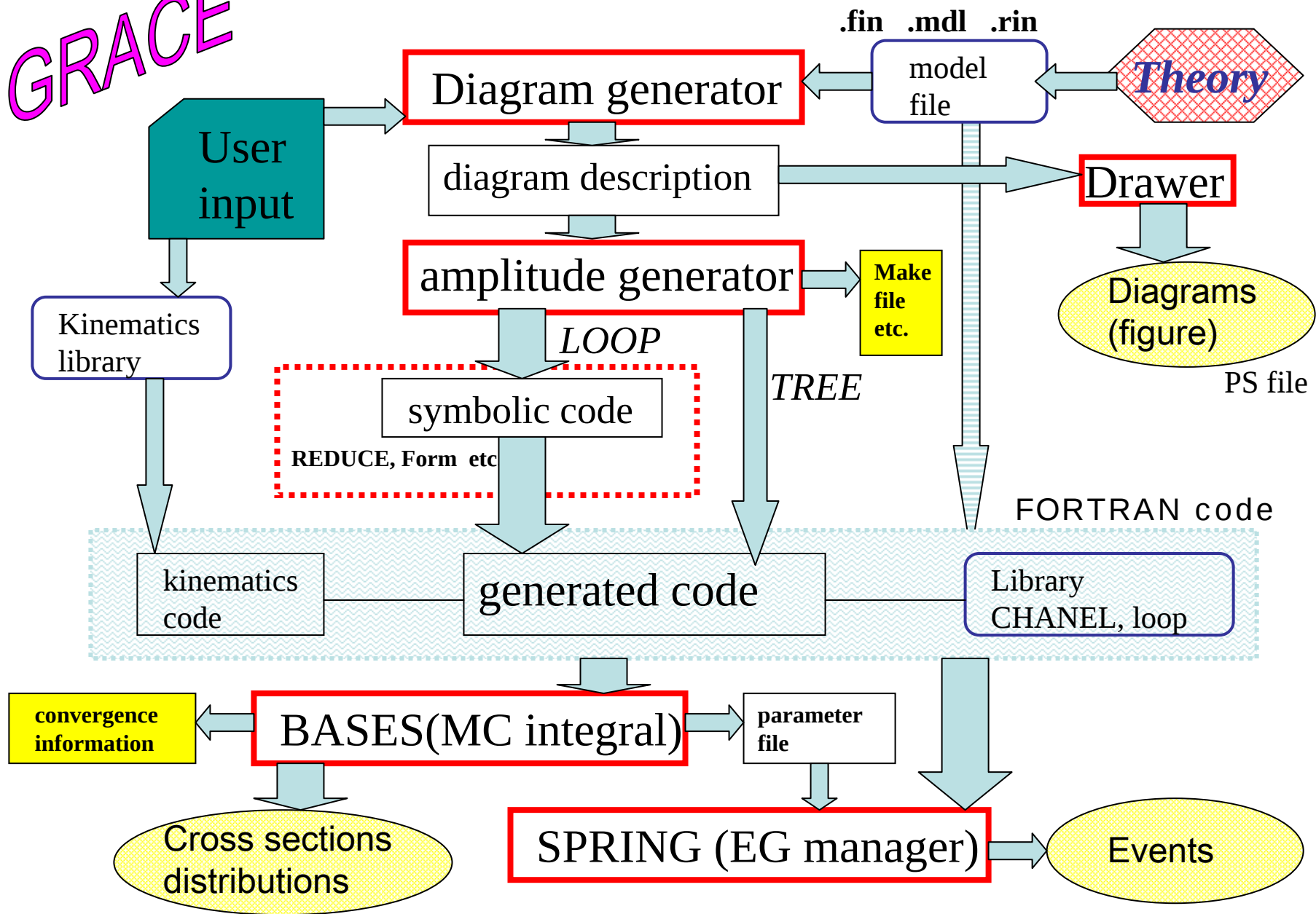
Gauge inv. check at a few p.s. points

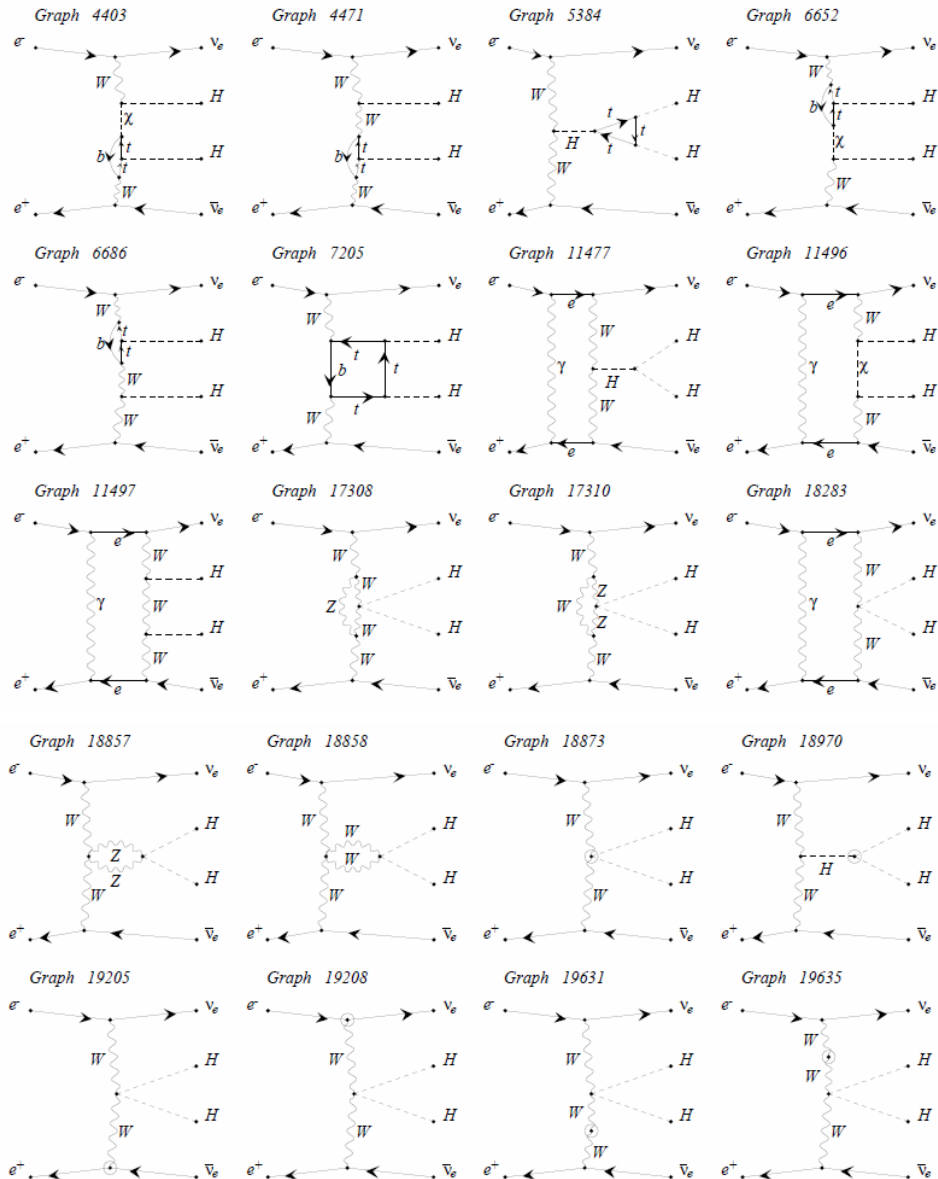
no e-scalar, integration

| Number of diagrams | Full sets |        | Production sets |        |
|--------------------|-----------|--------|-----------------|--------|
|                    | tree      | 1-loop | tree            | 1-loop |
| $\nu_\mu$          | 27        | 8292   | 6               | 1754   |
| $\nu_e$            | 81        | 19638  | 12              | 3416   |

Large scale computation required  
→ automated systems

# GRACE





$$e^+ e^- \rightarrow \nu \bar{\nu} H H$$

Relative CPU time

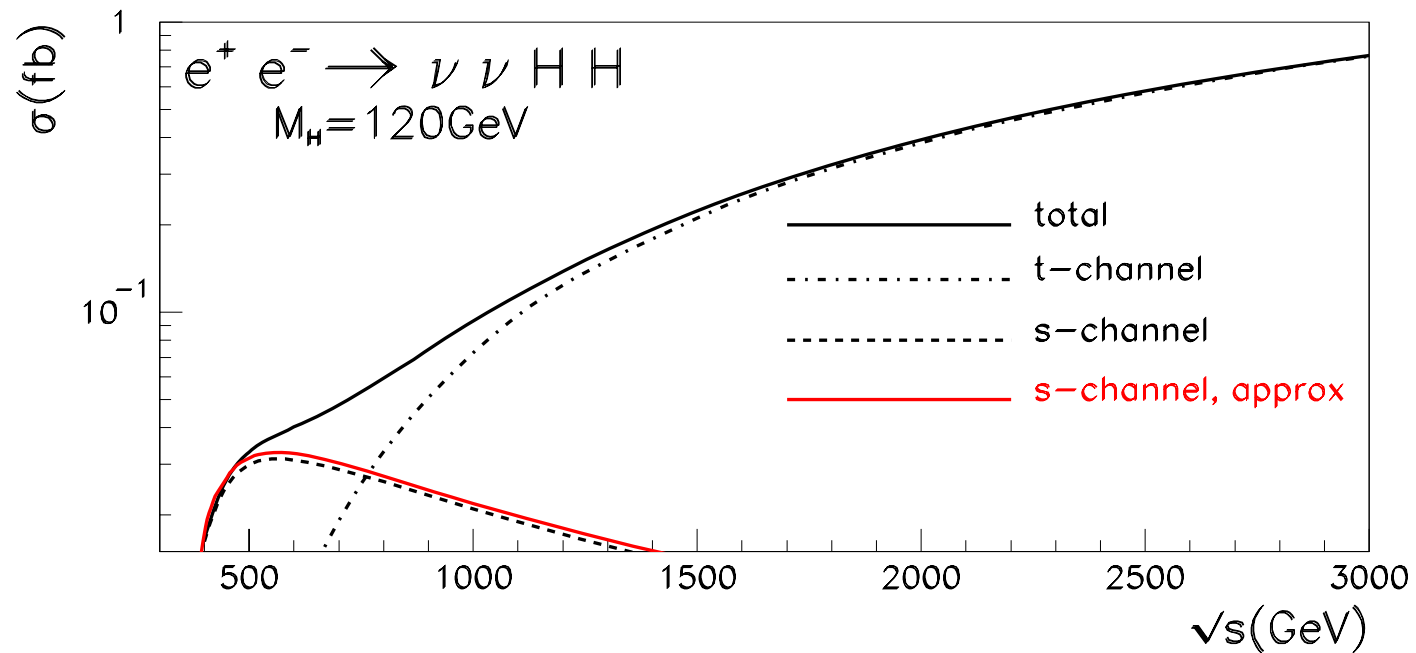
| TYPE    | #graph | CPU   |
|---------|--------|-------|
| CT,SELF | 152    | 0.7%  |
| FISH    | 434    | 0.9%  |
| VTX     | 1804   | 7.8%  |
| BOX     | 734    | 11.1% |
| PEN     | 218    | 12.5% |
| HEX     | 74     | 67.0% |
| total   | 3416   |       |

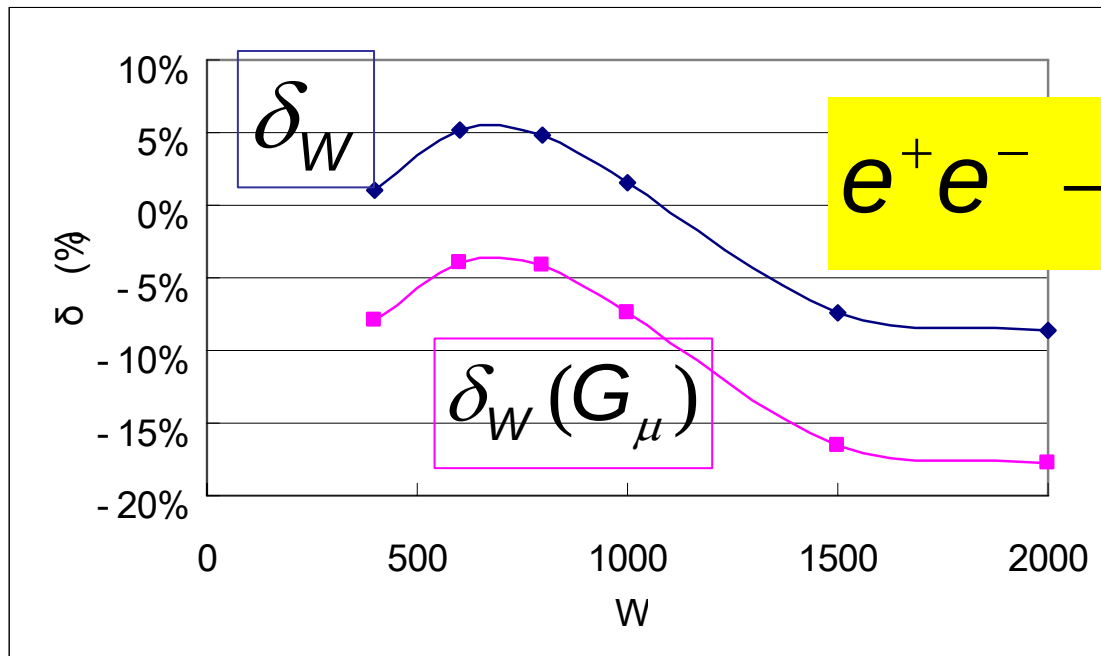
# GRACE system

- 5-, 6- point functions  $\rightarrow$  reduction  
 $\rightarrow$  sum of 4-point (and lower point) functions
- built-in check : non-linear gauge fixing
- full EW 1-loop corrections + soft – QED  
 $\rightarrow \delta_W$

$$\delta_{QED}^{e+e-} = \frac{2\alpha}{\pi} \left\{ \left( \ln \frac{s}{m_e^2} - 1 \right) \ln \frac{2k_c}{W} + \frac{3}{4} \ln \frac{s}{m_e^2} + \frac{\pi^2}{6} - 1 \right\}$$

$$e^+e^- \rightarrow \nu\nu H H$$





$$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu HH$$

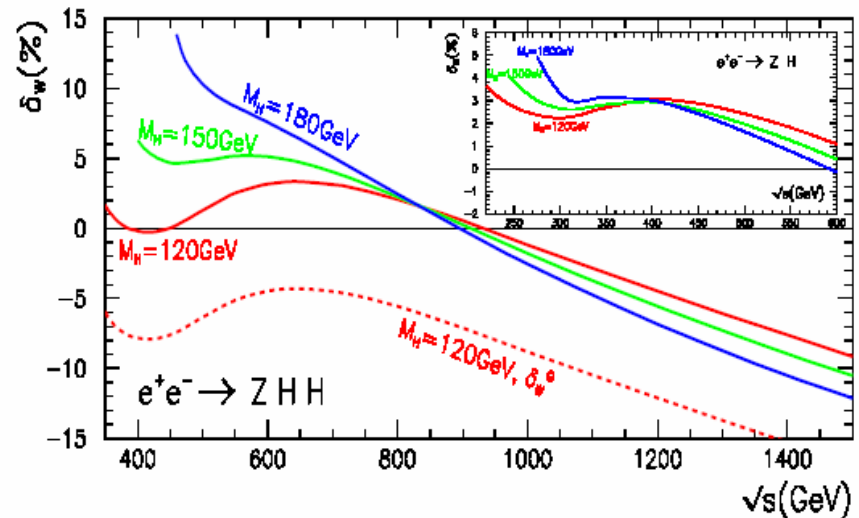
$M_H=120\text{GeV}$

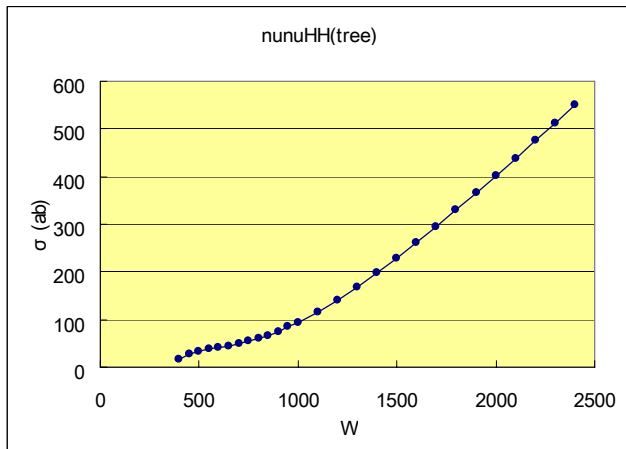
$m_t = 180\text{GeV}$

“s-channel”

$$e^+e^- \rightarrow ZHH$$

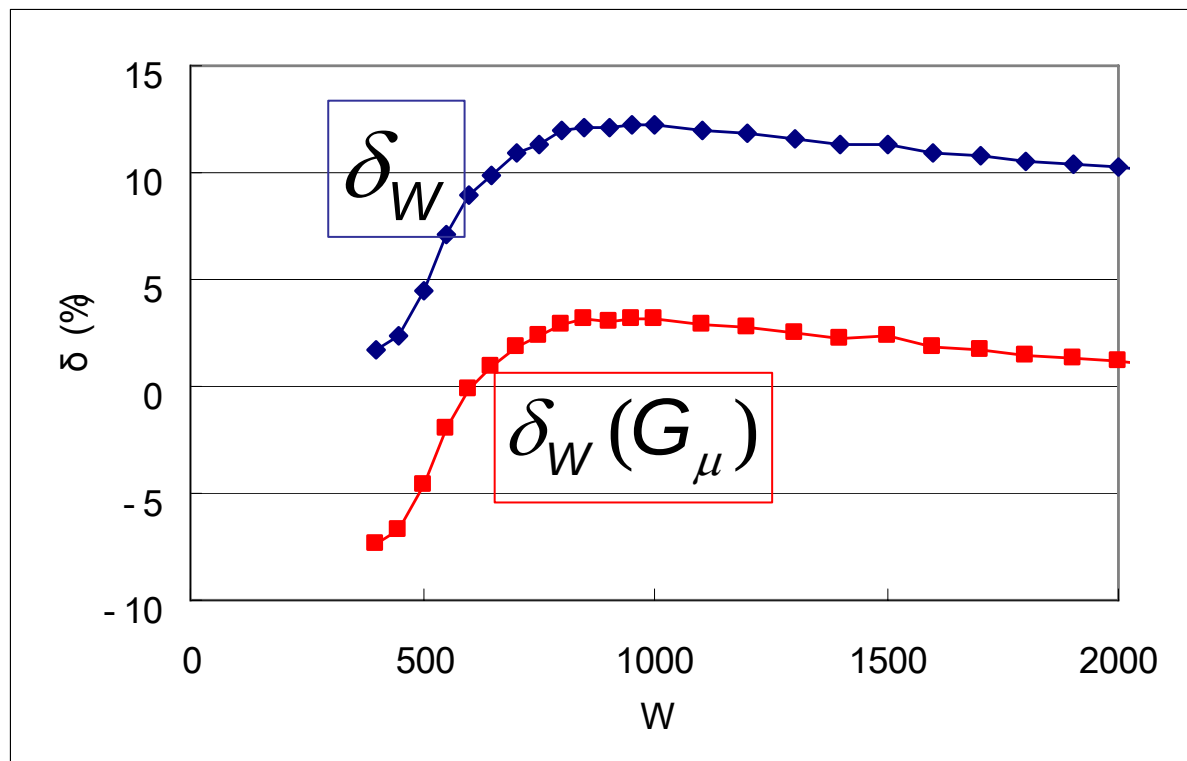
Phys.Lett.B 576(2003)152.  
 $m_t = 174\text{GeV}$

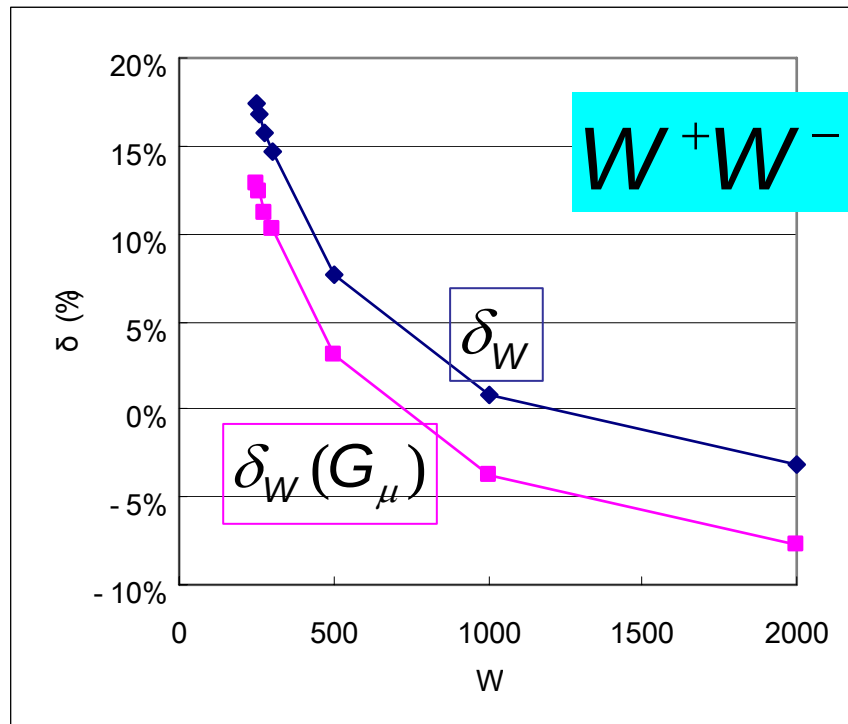




$$e^+ e^- \rightarrow \nu \bar{\nu} H H$$

$M_H = 120 \text{ GeV}$





$$W^+W^- \rightarrow HH$$

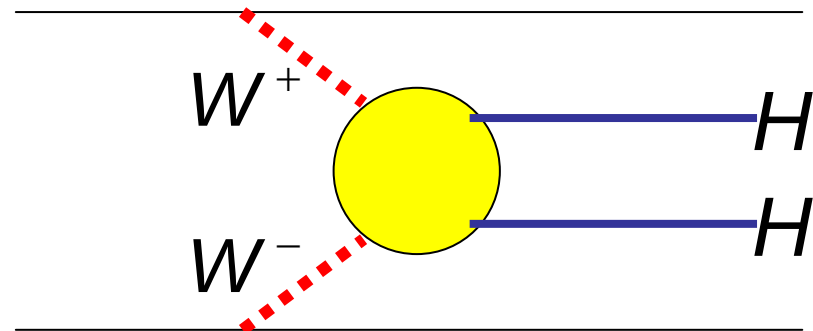
Convolution by  $M(HH)$  spectrum

...  $\delta_W$  consistent within 1% for  $W > 1\text{TeV}$

$$e^+e^- \rightarrow \nu\bar{\nu}HH$$

High  $W$

dominant mechanism (approximation)



# News for GRACE

- Two new features:  
(*ACAT05, May 05, Zeuthen*)
  - (1) Multi-precision computation/Precision control by HMLib (*with Hitachi*)  
Instability in the large scale computation, e.g.,  
Gram det.  $\sim 0$  (in “small” phase space)
  - (2) Acceleration of computation by factorization method.  
Diagram computation vs. non-diagrammatic computation(e.g., Alpha).

# HMLib: New FORTRAN Library for High-speed Multiprecision operations

- Library in FORTRAN
- Based on Integer operations  
→ fast & “lost bits” information for subtraction.

*Example: Loop amplitude, reduction of numerator*

Double precision

ReJ[1] = -1.49368718239238  
ReJ[x] = - 6.86111482424926E-0002

...  
ReJ[w\*\*3] = 93763.26727997246  Blow Up!

...  
Quadruple precision

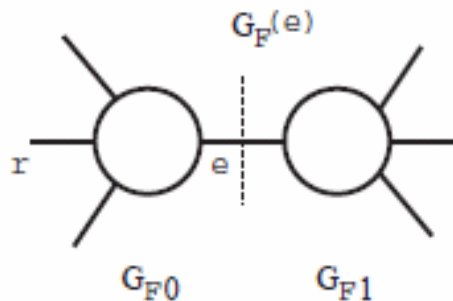
ReJ[1] = - 1.49368718238777512062307539882045  
ReJ[x] = - 6.861114708877389206553392789958382E-0002

...  
ReJ[w\*\*3] = - 1.60389378482142986480454883491878

$$e^+ e^- \rightarrow e^+ e^- \tau^+ \tau^-$$

$$\begin{aligned}\mu &= 1.6, \lambda = 10^{-20}, \\ s' &= 11.08527740873578, \\ t &= -1.46311226509359, \\ \kappa &= -2.007662965116379 \times 10^{-11} \\ \kappa' &= -3.041076446800326 \times 10^{-2}\end{aligned}$$

# Factorization method



## Basic Idea

1. Collect graphs with same propagator
2. Cut graphs at the propagator
3. Calculate amplitudes of sub-graphs
4. Multiply propagator & sub-amplitudes

## Electroweak theory (no color)

| Process                                                      | # graphs | # lines  |        | CPU-time<br>(ratio) |
|--------------------------------------------------------------|----------|----------|--------|---------------------|
|                                                              |          | GRACE    | grcft  |                     |
| $e^+e^- \Rightarrow (e^+e^-)^2$                              | 654      | 60933    | 21909  | 3.60                |
| $e^+e^- \Rightarrow (e^+e^-)^3$                              | 145128   | 16253671 | 150219 | 83.70               |
| $e^+e^- \Rightarrow e^+e^- \mu^+ \mu^- \tau^+ \tau^-$        | 12094    | 1368735  | 41648  | 15.14               |
| $e^+e^- \Rightarrow e^+e^- \mu^+ \mu^- \tau^+ \tau^- \gamma$ | 117680   | 14923326 | 101629 | 142.86              |

# Conclusion

- Most of the 1-loop **full EW RCs** for Higgs production at the ILC have been completed.
- The last one, 4-body final state process  $ee \rightarrow \nu\nu\mu\mu HH$ , for the study of Higgs, is reported. The cross section is  $O(100\text{ab})$  and  $\delta_W(G_\mu)$  is less than 5% for  $\sqrt{s} > 600\text{GeV}$ .
- The comparison with  $ee \rightarrow ZHH$ ,  $WW \rightarrow HH$  is done to understand the structure of the process.
- News for the GRACE system: Precision control and Factorization method