Multi-legs calculations with GRACE/LOOP system

— Toward Radiative Corrections to $e^+e^- \to \mu\bar{\nu}u\bar{d}$ —

Junpei Fujimoto

KEK, Japan

for the collaboration with

F. Boudjema, T. Ishikawa, T. Kaneko,

K. Kato, Y. Kurihara and Y. Shimizu.

Loop & Legs 2004

Zinnowitz

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Contents

1. Introduction

At L&L/RADCOR 2002, the numerical results of $O(\alpha)$ corrections to $e^+e^- \to \nu\bar{\nu}H$ was presented by GRACE group.

Since then, several radiative corrections to important $2 \to 3$ processes were performed by several authors:

- for $e^+e^- \to \nu\bar{\nu}H$;
  - G. Belanger et. al. PLB 559(2003)252.
  - A. Denner et. al. PLB 560(2003)196.
  - A. Denner et. al. NPB 660(2003)289.
- for $e^+e^- \to t\bar{t}H$;
  - Y. You et. al. PLB 571(2003)85.
  - G. Belanger et. al. PLB 571(2003)163.
  - A. Denner et. al. PLB 575(2003)290.
  - A. Denner et. al. NPB 680(2004)85.
- for $e^+e^- \to ZHH$;
  - G. Belanger et. al. PLB 576(2003)152.
  - R. Zhang et. al. PLB 578(2004)349.
- for $\gamma\gamma \to t\bar{t}H$;

– H. Chen et. al. NPB 683(2004)196.

- for $e^+e^- \to e^+e^-H$;
  – F. Boudjema et. al. contribution to ACAT03(Dec. 2003); hep-ph/0404098.
- for $e^+e^- \to \nu\bar{\nu}\gamma$;
  – F. Boudjema et. al. contribution to ACAT03(Dec. 2003); hep-ph/0404098.

Full EW 1-loop calculations are well under control for $2 \to 3$ processes in SM.

Now the time to attack $2 \to 4$ **!!**

(A. Vicini, "2nd ECFA/DESY Linear Collider Workshop"(Oct. 1999))

We take a typical LEP-2 process

$$e^+ + e^- \to \mu + \bar{\nu} + u + \bar{d}$$

for the first trial.

2. Motivation

At LEP2 experiments, Double Pole Approximation(DPA) or the fermion loop scheme was used to predict the cross sections of $e^+e^- \to$ 4-fermions. Features are:

- Gauge invariance is guaranteed.
- It was sensible to split the diagrams into W-pair production(CC03) and others.

But the energy region at the future linear-collider, non-CC03 diagrams are not negligible.

For example, at $\sqrt{s} = 500$ GeV, the tree level cross sections of CC03 and CC10 are

CC03	CC10	1-CC03/CC10
213.56 fb	222.39 fb	4%

The size of the radiative corrections to (CC10-CC03) should be carefully estimated at the TeV energy region.

We need exact $O(\alpha)$ corrections to 4-fermion processes.

3. Steps of calculation

- 1-loop calculation consists of the following three parts:
  - the numerators
    $\Longrightarrow$ using Symbolic manipulation system
    $\Longrightarrow$ to shorten the size of matrix elements, m_e, m_μ, m_u, m_d are neglected.
  - the denominators
    $\Longrightarrow$ 5- and 6-point integrals are reduced to 4-point.
    $\Longrightarrow$ FF and other analytic formulas
    $\Longrightarrow$ all the masses are kept for the mass singularity.
  - Kinematics $\Longrightarrow$ keep masses exactly

- **The reduction algorithm of $N(\geq 5)$ point functions:**

$$T^{(5)} \sim \int \frac{d^n l}{(2\pi)^n i} \frac{l_\mu l_\nu \cdots l_\rho}{D_0 D_1 \cdots D_4},$$

where $D_0 = l^2 - m_0^2$ and $D_i = (l + s_i)^2 - m_i^2, i = 1, \cdots, 4$.

Using the identities

$$\begin{aligned} g^{\mu\nu} &= \sum_{i,j=1}^{4} s_i^\mu (A^{-1})_{ij} s_j^\nu, \\ l^\mu &= \sum_{i,j} s_i^\mu (A^{-1})_{ij} (l \cdot s_j), \\ &= \frac{1}{2} \sum_{i,j} (A^{-1})_{ij} (D_j - D_0 - \Delta_j) s_i^\mu, \end{aligned}$$

with $A_{ij} = s_i \cdot s_j$ and $\Delta_i = s_i^2 - m_i^2$, the numerator can be reduced to

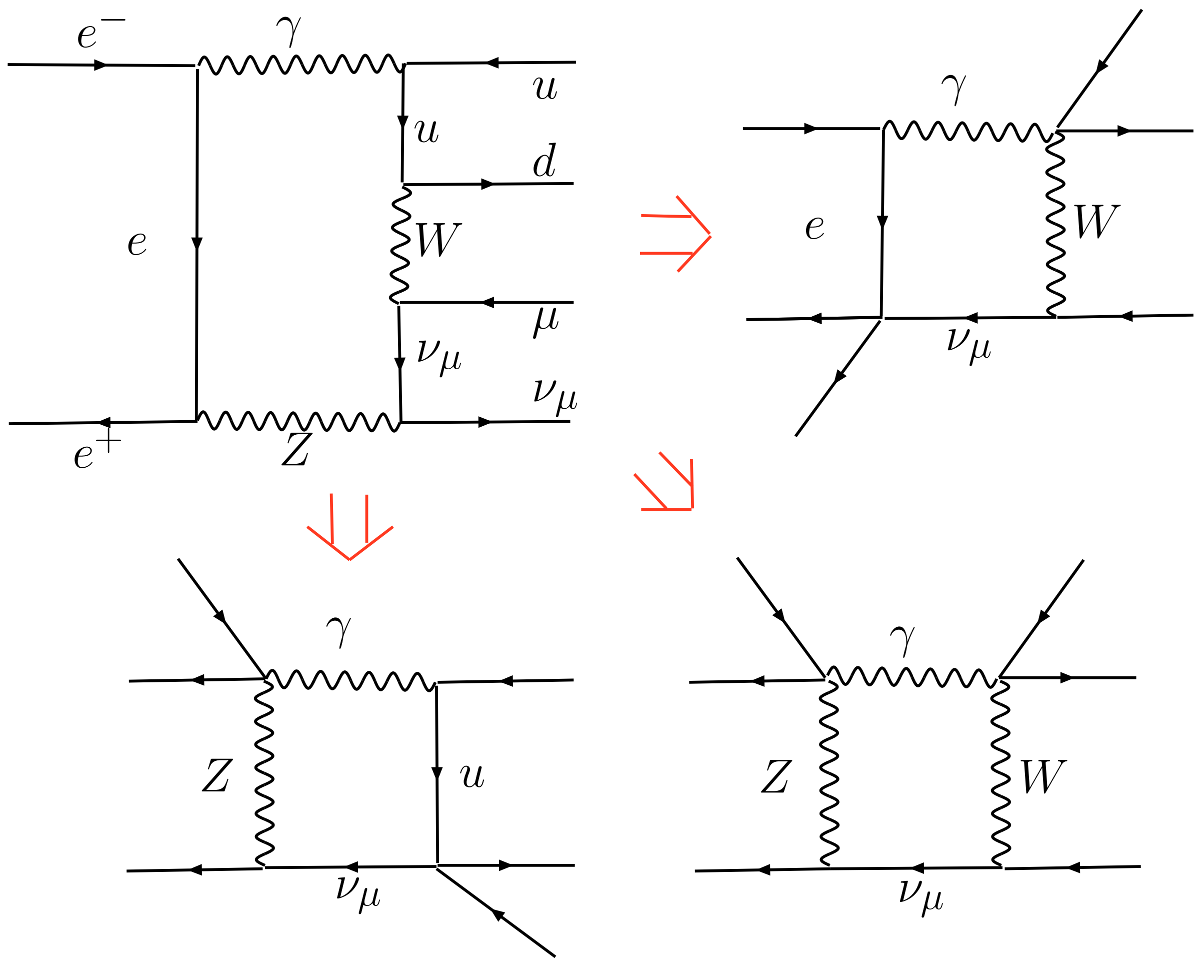
$$l_\mu l_\nu \cdots l_\rho = \sum_{i,j} (A^{-1})_{ij} (D_i - D_0 - \Delta_i) s_j^\mu l_\nu \cdots l_\rho.$$

This method also enables us to reduce a 6-point function to a sum of 4-point functions.
It is noted that the following identity is not used to reduce the numerator:

$$l^2 = \sum_{i,j} (l \cdot s_i) A_{ij} (l \cdot s_j),$$

except for the scalar integral.

- When we keep the fermion masses in the numerator, the matrix elements are rather lengthy. To shorten them, those masses are neglected.
- GRACE/1-LOOP system adopts FF package for 4-point function in the case of general mass combination.
  Some non-IR 4-point functions are unstable when **internal massless** particles appear. If this case happens, we use in-house formulas.
- Introduce a decay width(constant fixed width). But it breaks the gauge invariance in $O(\alpha\Gamma_W/M_W)$.
  This causes a serious problem to IR 5-point diagrams around the W-pole. $\Longrightarrow$ See below.



e^-
γ
u
u
d
e
W
μ
ν_μ
ν_μ
e^+
Z
γ
e
W
ν_μ
γ
Z
u
ν_μ
γ
Z
W
ν_μ

5. 1-loop diagrams of $e^+e^- \to \mu\bar{\nu}u\bar{d}$

6094 diagrams for 1-loop and **44** ones for tree, when the electron-scalar couplings are kept.

$\Longrightarrow$ Approximation : ignore $m_e^2, m_\mu^2, m_u^2, m_d^2$
in the numerator of the matrix elements, which looses digits of the renormalization and infra-red independence in $O(m^2/s)$.

$\Longrightarrow$ **668** 1-loops (**30** 6-point functions, **88** 5-point functions)
and **10** trees(CC10).

$\Longrightarrow$ Figures

4. Check of codes

- Single phase space point with full-set of diagrams.
- Gauge invariance in the 1-loop level
  $\Longrightarrow$ non-linear gauge (NLG) fixing is applied in collaboration with LAPTH
  (F. Boudjema and E. Chopin, Z.Phys. C73(1996) 85.)

$\xi_W = \xi_Z = \xi_A = 1 \Longrightarrow$ Numerator $= g^{\mu\nu}$.

$$\begin{aligned}
\mathcal{L}_{\rm GF} &= -\frac{1}{\xi_W}\Big|(\partial_\mu - ie\tilde{\alpha}A_\mu - ig\cos\theta_W\tilde{\beta}Z_\mu)W^{+\mu} \\
&\qquad\qquad +\xi_W\frac{g}{2}(v+\tilde{\delta}H+i\tilde{\kappa}\chi_3)\chi^+\Big|^2 \\
&\quad -\frac{1}{2\xi_Z}\left(\partial_\mu Z^\mu + \xi_Z\frac{g}{2\cos\theta_W}(v+\tilde{\varepsilon}H)\chi_3\right)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2
\end{aligned}$$

- In total the amplitude contains 5 NLG gauge parameters:

$$\tilde{\zeta} = \tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa}$$

NLG invariance check:

$\Longrightarrow$ No width, $\Gamma_W = \Gamma_Z = 0$.

$\Longrightarrow$ the whole matrix elements has no dependence on each gauge parameters in more than 21 digits because we keep all the masses in the numerator.

	ζ^3	ζ^2	ζ^1
$\tilde{\alpha}$	29	29	21
$\tilde{\beta}$	29	28	23
$\tilde{\delta}$		30	27
$\tilde{\kappa}$		29	26
$\tilde{\epsilon}$		31	27

This table shows the number of digits canceled among coefficients of $\tilde{\zeta} = \tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa}$.

$\tilde{\alpha}$

	a^3	a^2	a^1	a^0
4V			-.1123150E-06	-.3815522E-06
6V			-.2246359E-06	-.7631044E-06
7V			.2941457E-12	-.2941457E-12
11V			.1490282E-11	-.1490282E-11
22V			.2456668E-17	-.2456668E-17
.....				
1676P			-.3506756E-07	.1672967E-06
1677P			.2598412E-19	.1957840E-17
1678P			.5263239E-19	.1010322E-17
1679P			-.7508612E-09	.7508612E-09
.....				
2812B			.1403795E-27	-.1403795E-27
2827V			-.3054008E-04	.8223128E-04
2828V	-.4346760E-07	-.6945575E-05	-.4347713E-05	.1718614E-04
2829V			-.4717141E-06	-.2049285E-05
2830V			.3500290E-06	-.3500290E-06
2831V	.9859557E-07	.7851083E-06	-.1866003E-05	.9822995E-06
.....				
4594B		.2175874E-10	.4893556E-11	-.2665229E-10
4597B			-.5193025E-21	-.1019102E-20
4602B		-.1014049E-21	.2028098E-21	-.1014049E-21
4605B			.8243866E-32	-.8243866E-32
4606B		-.1766323E-10	-.2512931E-09	.2689563E-09
.....				
5405V		.3968024E-06	.0000000E+00	-.3105600E-06
5409V		.1011671E-07	.0000000E+00	.1717128E-06
5462V		-.4721634E-06	.0000000E+00	-.2064775E-06
5498F	-.8047711E-07	-.6499034E-06	-.5010129E-06	.2058246E-05
5501F			-.1505980E-10	.0000000E+00
5502F			-.4091890E-11	.0000000E+00
5505F	.4029963E-10	-.4029963E-10	-.2497602E-10	.2497602E-10
.....				
6087		-.5141385E-25	-.1206314E-21	.2232925E-19
6088		.4693592E-25	-.3888506E-25	.3075515E-25
6089		.5834359E-36	-.4833599E-36	.3822797E-36
6090		-.5277732E-36	.1055546E-35	-.3034993E-35

cnt	24	219	878	
sum1	.38030E-31	.25463E-31	-.28065E-23	.55657E-02
sum2	.38053E-31	.25230E-31	-.28065E-23	.55657E-02
max	.98596E-07	.91195E-05	.89766E-04	.35029E-02
s/a0	.68371E-29	.45331E-29	-.50426E-21	1.0000

C_{UV} check(Renormalization check)

Keep $C_{UV} = 1/\varepsilon - \gamma_E + \log 4\pi$ in the code and check independence of it.

IR check(Infrared divergence check)

Check independence of λ.

```
 one phase spcae point
 P ...668 Diagrams for production run,
      neglecting light fermion masses
 A ...All diagrams including all fermion masses
0  ...cuv=0
c  ...cuv=1000

lamda=10^-18
0A -0.85388129300841993023002220162825220E-03
cA -0.85388129300841993023002220166554602E-03
0P -0.85388086448063959950419248157833587E-03
cP -0.85388086452253511266116024563283106E-03
lamda=10^-21
0A -0.85388129300794622839025322460373685E-03
cA -0.85388129300794622839025322465225825E-03
0P -0.85388086990548652947683563335079978E-03
cP -0.85388086994738204263380339741156403E-03
lamda=10^-24
0A -0.85388129300747253815840381818417531E-03
cA -0.85388129300747253815840381822708744E-03
0P -0.85388087533033347105757135572379178E-03
cP -0.85388087537222898421453911977756475E-03
lamda=10^-27
0A -0.85388129300699884793860269107185114E-03
cA -0.85388129300699884793860269111500401E-03
0P -0.85388088075518041265035552905677652E-03
cP -0.85388088079707592580732329311352264E-03
```

5. Status of calculation

Input parameters:

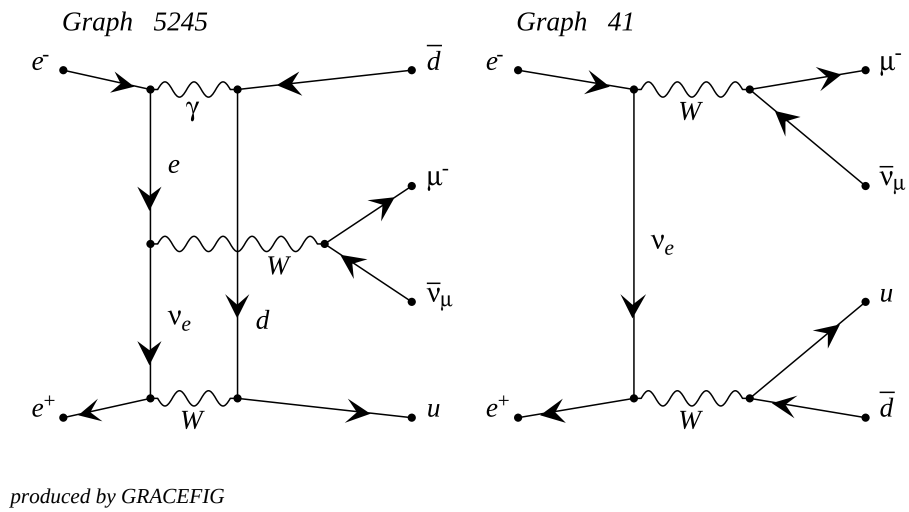
$$
\begin{aligned}
M_Z &= 91.1876 \text{ GeV}, \Gamma_Z = 2.4952 \text{ GeV},\\
M_W &= 80.4163 \text{ GeV}, \Gamma_W = 2.118 \text{ GeV},\\
M_H &= 120 \text{ GeV},\\
M_t &= 180 \text{ GeV},\\
M_u &= M_d = 63 \text{ MeV}^1,\\
M_s &= 92 \text{ MeV},\\
M_c &= 1.5 \text{ GeV},\\
M_b &= 4.7 \text{ GeV},\\
\alpha &= 1/137.0359895
\end{aligned}
$$

All the calculations were done in quadruple precision.

[1] for the denominators and vacuum polarization.

What happened with IR 5-point integrals?

Take, say loop #5245P×#41,



Write the contribution of a 5-point diagram as a sum of *five* 4-point integrals.

$$\sigma^{(5)} = I_1^{(4)IR} + I_2^{(4)IR} + I_3^{(4)} + I_4^{(4)} + I_5^{(4)},$$

which contains *two* infared divergent boxes.

Currently Γ_W, Γ_Z are retained only

1. in the IR part of box integrals

$$I_1^{(4)IR} \sim \frac{1}{D_W(q^2)} \left[\log \frac{\lambda\sqrt{\tilde{M}_W^2}}{D_W(q^2)} \log(\cdots) + \cdots \right]$$

where

$$D_W(q^2) \equiv q^2 - \tilde{M}_W^2 = q^2 - M_W^2 + i\Gamma_W M_W,$$

2. in the reduction formulas.

Put $\Gamma_Z = 0$, then the contribution of each box is summarized as

- With $\Gamma_W \neq 0$ the phase space integration does not converge.
- Without the width **3 digits cancellation** occurred at a phase space point near W-pole,

  $$(q^2 - M_W^2 + i\Gamma_W M_W)_{ud} = -169.3611 + 167.1833i \qquad (\mathrm{GeV}^2)$$

- The mechanism for this big cancellation is not fully understood, but it is highly possible that an identity including $\varepsilon_{\alpha\beta\gamma\delta}$ such as

  $$\begin{aligned} \{(l.p_1)\varepsilon(p_2,p_3,p_4,p_5) - (l.p_2)\varepsilon(p_1,p_3,p_4,p_5) \\ +(l.p_3)\varepsilon(p_1,p_2,p_4,p_5) - (l.p_4)\varepsilon(p_1,p_2,p_3,p_5) \\ +(l.p_5)\varepsilon(p_1,p_2,p_3,p_4)\} \times (l.p_6) = 0 \end{aligned}$$

  causes the trouble, by giving a relation between 5 box integrals.

- However, it seems not easy to find and remove this hidden identity in a symbolic way when it appears. General 4-point integral formulas which allows a constant width may improve the situation.

- No such phenomenon took place for 6-point diagrams nor non-IR 5-points. Also we didn't see such difficulty in $2 \to 3$ processes up to now. The number of independent fermion lines may be related with this situation.

- We are investigating the origin of this phenomena in comparing with the reduction method using only identity

  $$1 \;=\; \sum_{i=1}^{5} \left(a_i + b_{ij}(l.s_j)\right) D_j,$$

  which is derived from

  $$l^2 = \sum_{i,j} (l \cdot s_i) A_{ij} (l \cdot s_j).$$

- **Test run of integration**
  - Phase space integration is carried out by `BASES` package.
  - Those diagrams which contribute less than 0.01 fb were omitted.$\Longrightarrow$ total **361** 1-loop diagrams were taken.
  - Hard photon emission is also included.
  - In order to cure the emergency we temporarily took the following *ad hoc* replacement in the loop amplitude

    $$\sigma^{(5)} \propto [\text{IR-loop}]^{(5)} \Longrightarrow \frac{q^2 - M_W^2}{q^2 - M_W^2 + iM_W\Gamma_W}[\text{IR-loop}]^{(5)}\Big|_{\Gamma_W=0}$$

    and looked the cross section.
  - Size of the source code: 38 M Lines, 2.5 GBytes

At $\sqrt{s} = 500$ GeV: $\sigma_{tree} = 222.39 \pm 0.01$fb,

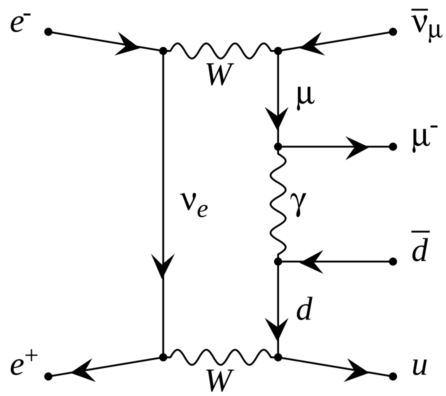
Test run

graph	original(IR)	factorized(IR)	non-IR
6-pnt	-921(16)fb	-914(6)fb	small
5-pnt	-4221(2224)	+2729(17)	-80(10)fb
4-pnt	-4041(26)	-3999(26)	+216(7)
3-pnt	+735(6)	+735(6)	-258(2)
2-pnt			-27(0.3)
self	-104(2)	-104(2)	-9(0.08)
cnt	+305(26)	+305(26)	small
soft	+990(8)		
hard			+461(0.5)
total	-7257(2223)	-258(9)	+302(9)

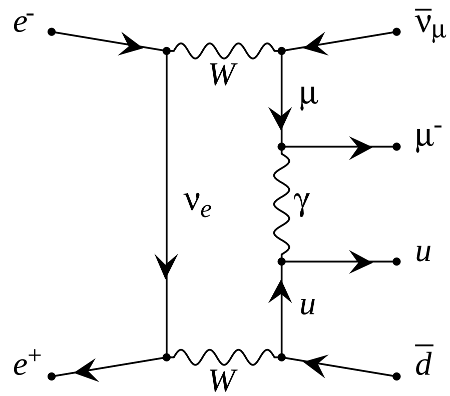
One iteration of 50,000 sampling points.

Graphs with large contributions:

Graph 4027

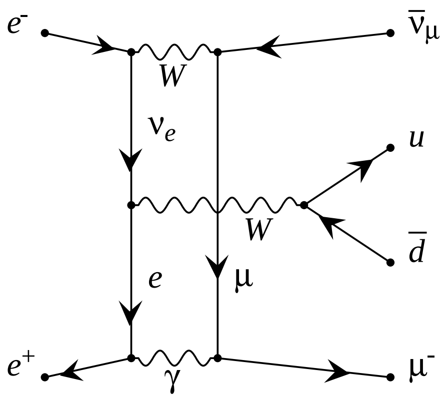


Graph 4051

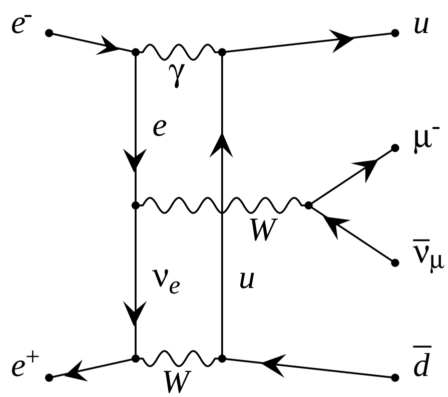


produced by GRACEFIG

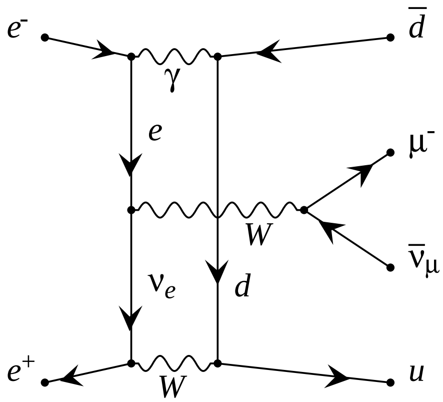
Graph 5213



Graph 5229

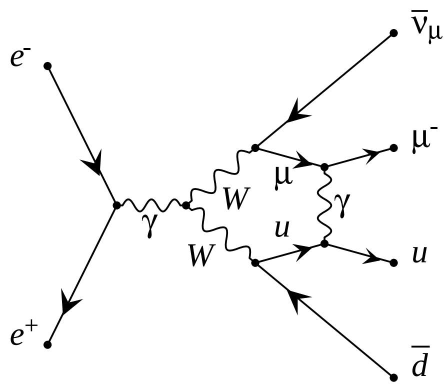


Graph 5245

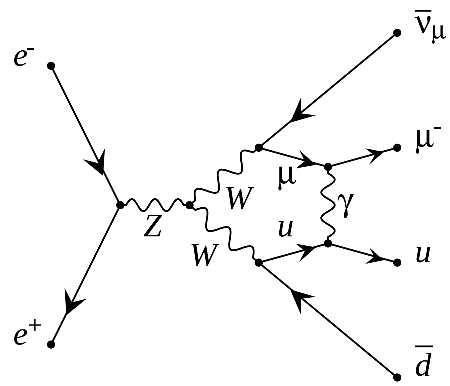


produced by GRACEFIG

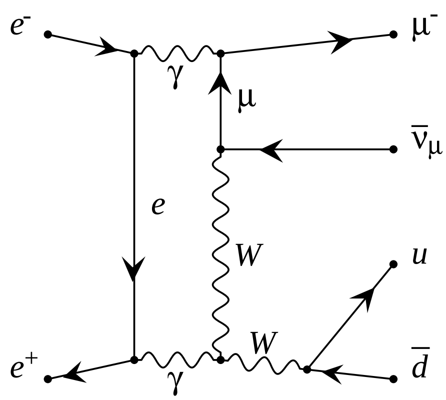
Graph 1739



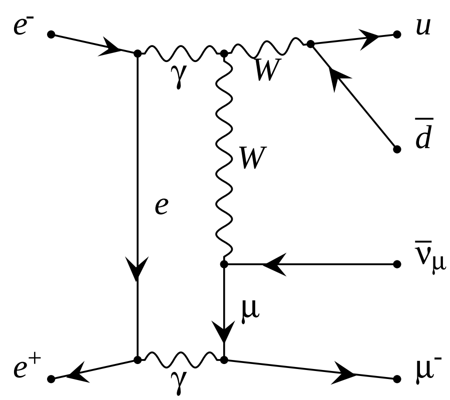
Graph 1755



Graph 3949

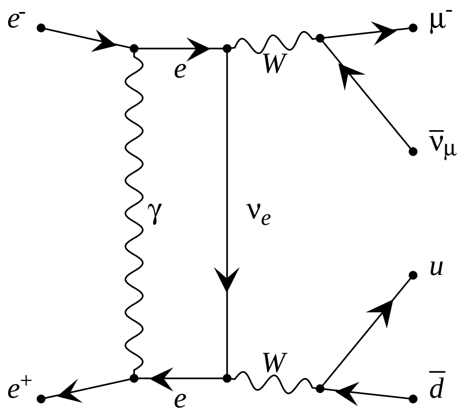


Graph 4823

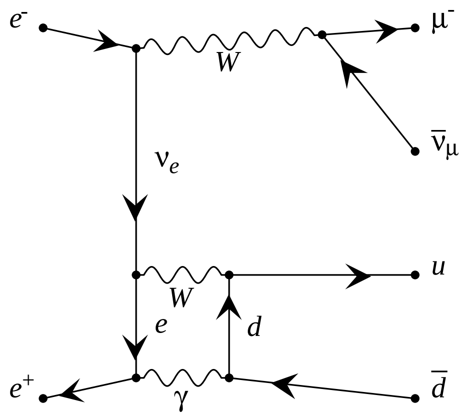


produced by GRACEFIG

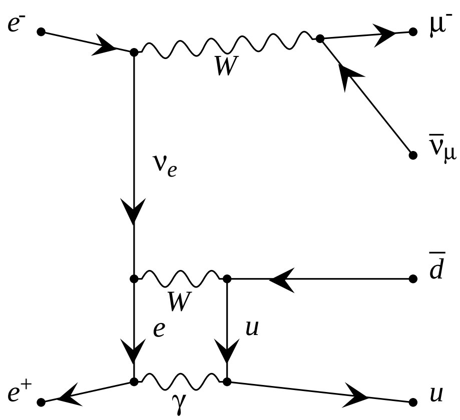
Graph 4589



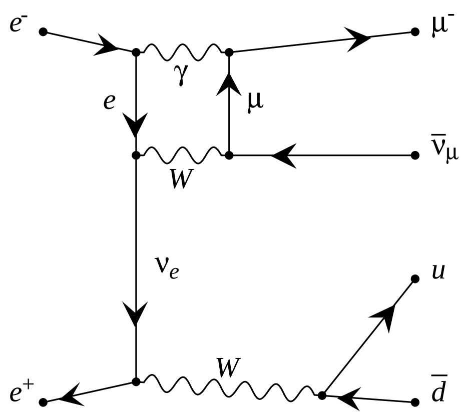
Graph 5141



Graph 5161

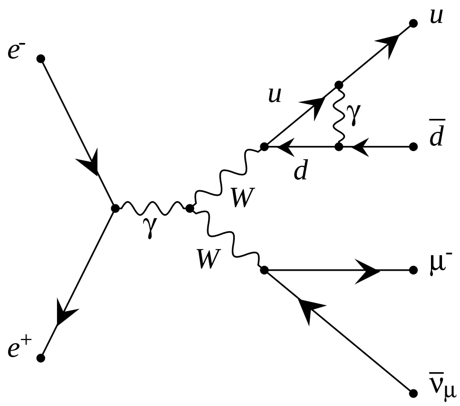


Graph 5182

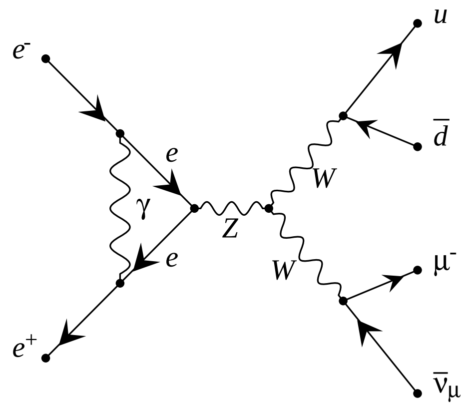


produced by GRACEFIG

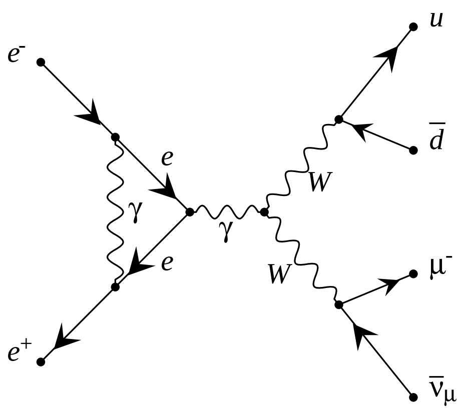
Graph 1003



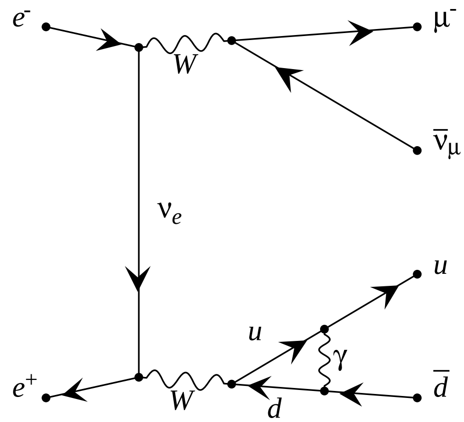
Graph 3602



Graph 3611



Graph 3767



produced by GRACEFIG

6. Summary

- One-loop amplitudes of **full 6094** diagrams for $e^+e^- \to \mu\bar{\nu}u\bar{d}$ were generated by `GRACE/1-LOOP` system.
- Non-Linear Gauge invariance has shown the consistency of the full set of amplitudes and the system itself.
- A new reduction algorithm from a 6-point function to 4-point functions works well.
- A finite decay width introduces a serious gauge invariance breaking, particularly for 5-point integrals.
- It is clearly shown that the radiative corrections to $2 \to 4$ processes are calculable, though more improvements are inevitable.

box	$\Gamma_W = 0$ (real, imag)	$\Gamma_W \neq 0$ (real, imag)
1	(-17.69217505, 66.88369265)	(-4.22744975, 2.9465712570)
2	(-929.96695924, 1372.81890328)	(-120.36833308, 39.2204483424)
3	(782.70500402, -5690.37576950)	(321.80994609, -283.271079293)
4	(410.83071602, 4557.14502347)	(-199.63435655, 275.873356743)
5	(-245.31384970, -301.36098220)	(0.65534291, -27.683645798)
sum	(0.56273604, 5.11086770)	(-1.76485038, 7.085651250)

Table 1: 5245P × 41

box	$\Gamma_W = 0$ (real, imag)	$\Gamma_W \neq 0$ (real, imag)
1	(-0.5714209451E-01,-0.2606583777E-05)	(-0.3018763824E-02,-0.2980215555E-02)
2	(-1.5656740655E+00,-1.1523430196E+00)	(-0.2746045912E-01,-0.1441615988E+00)
3	(1.7058101561E+00, 1.1486531903E+00)	(0.3021572559E-01, 0.1496464860E+00)
4	(-0.6086966576E-01, 0.2653087449E-05)	(-0.3215968725E-02,-0.3174338480E-02)
5	(-0.2212432177E-01, 0.3689906021E-02)	(-0.1361295359E-02,-0.9588860802E-03)
sum	(0.8645610115E-08, 0.1232403071E-06)	(-0.4840761440E-02,-0.1628552911E-02)

Table 2: Example