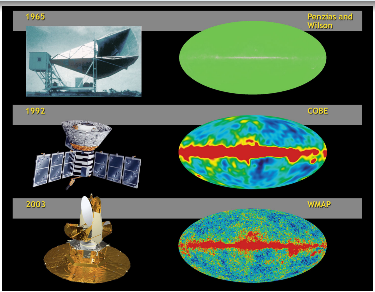
SLOOPS AND DARK MATTER ANNIHILATION AT ONE-LOOP

N. Baro, F. Boudjema, G. Chalons, Sun Hao

RELIC DENSITY OF DARK MATTER

- WMAP : $0.0997 < \Omega_{DM} h^2 < 0.1221$ (10% precision)
- PLANCK : 2% precision



PRECISION MEASUREMENTS

Must be matched by th. calculations $\Rightarrow$ One-loop

RELIC DENSITY IN THE STANDARD COSMOLOGICAL SCENARIO

$$\Omega_{DM} h^2 \simeq \frac{3 \times 10^{-27} cm^3 s^{-1}}{\langle \sigma(\chi\chi \rightarrow SM) v \rangle}$$

PRECISION

- Need for precise theoretical predictions w.r.t experimental measurements.
- Precision needed at the level of $\sigma \Rightarrow$ One-loop calculations (at least).
- If SUSY found $\Rightarrow$ Reconstruction of fundamental underlying parameters.
- Radiative corrections must be under control to be able to constrain the cosmological underlying scenario.

SOME PREVIOUS WORK AT 1-L IN SUSY

One-loop calculation EW + QCD corrections

- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \gamma\gamma, Z\gamma, gg$: Boudjema, Semenov, Temes, *Phys. Rev.* **D72**, 055024 (2005)
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to ZZ, W^+W^-$: Baro, Boudjema, Semenov, *Phys. Lett.* **B660** (2008) 550
  Baro, Boudjema, Chalons, Sun Hao, *Phys. Rev* **D81** (2008) 015005
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \tau^+\tau^-, b\bar{b}$: Baro, Boudjema, Semenov, *Phys. Lett* **B660** (2008) 550
- Co-annihilation with $\tilde{\tau}$: Baro, Boudjema, Semenov, *Phys. Lett* **B660** (2008) 550,

QCD corrections

- Co-annihilation with $\tilde{t}$ Freitas *Phys. Lett.* **B652** (2007) 280
- Annihilation into massive quarks Hermann, Klasen, Kovarik *Phys. Rev.* **D79** (2009)
  Herrmann, Klasen, *Phys. Rev.* **D76** (2007) 117704
  Herrmann, Klasen and Kovarik, *Phys. Rev.* **D80** (2009) 085025

FROM TREE TO LOOPS : NEED FOR AUTOMATION

- At tree-level we have for $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to WW$ 7 diagrams.
- Relic density predictions involve many annihilation (and coannihilation) channels.

Some efficient tree-level codes already exist for relic density calculations :

- DarkSUSY [Bergström *et al.* (2004)]
- micrOMEGAs [Bélanger, Boudjema, Pukhov, Semenov (2002)]
- Mainly $2 \to 2$ processes are taken into account in the computation.

At one-loop we have $\simeq$ 7000 diagrams

Then for an accurate and reliable relic density prediction at one-loop order we need :

$\longrightarrow$ A coherent renormalisation scheme and a choice of input parameters.

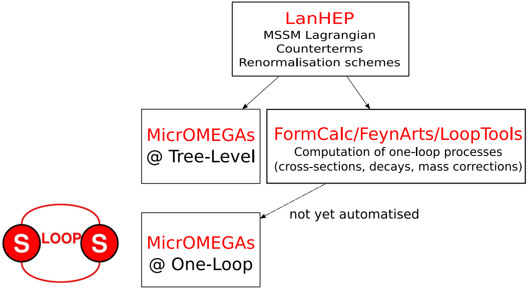
$\longrightarrow$ To generate counter-terms, for SUSY gigantic task.

$\longrightarrow$ To compute a huge amount of loop diagrams.

$\longrightarrow$ Loop Integrals library to handle Gram determinant when $v \to 0$.

$\longrightarrow$ To deal with IR and collinear divergencies $\longrightarrow$ include bremsstrahlung.

$\longrightarrow$ To evaluate many processes entering $\langle \sigma v \rangle$.

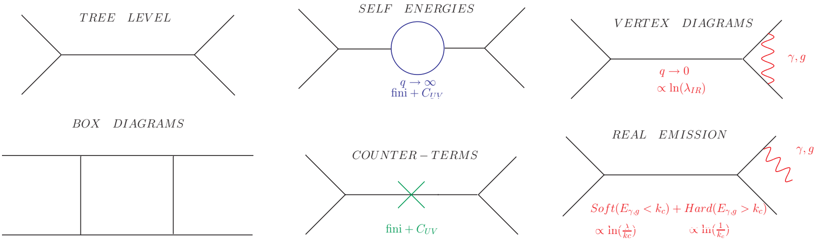


- Evaluation of one-loop diagrams including a complete and coherent renormalisation of each sector of the MSSM with an OS scheme.
- Modularity between different renormalisation schemes.
- Non-linear gauge fixing.
- Handles a large number of Feynman diagrams.
- Checks : results UV,IR finite and gauge independent.

http://code.sloops.free.fr/

DIVERGENCES

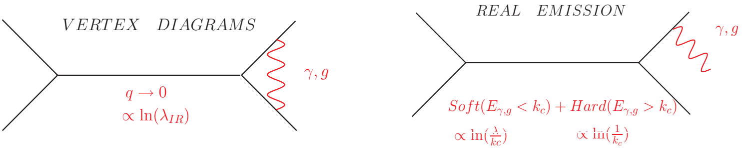
- Due to perturbative development in the coupling constant.



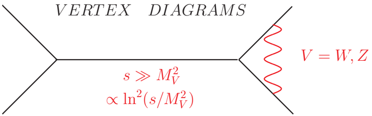
REGULARISATION

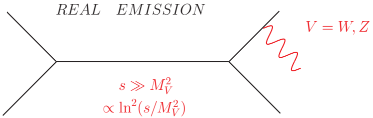
Isolate infinite parts in loops

- UV : $\ln \Lambda_{UV}$ with cut-off, $1/\epsilon_{UV}$ poles in DR.
- IR : $\ln \lambda_{IR}$ with cut-off, $1/\epsilon_{IR}$ poles in DR.



- Originate from
  - ↪ Massless gauge bosons (γ, g) coupling to on-shell external legs.
  - ↪ Soft and collinear regions of integration over boson momenta (appear as double log $\ln^2(\lambda_{IR})$ or $1/\epsilon_{IR}^2$).
- Adding real emission remove unphysical dependency in the cut-off λ_{IR} or $1/\epsilon_{IR}^2$.
- Integration over 3-particles phase space can be complicated.
- Usually for DM calculation $2 \to 2$ processes are enough, but if real corrections $\simeq$ vertex corrections, $2 \to 3$ processes should also be included.





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- Integration over 3-particles phase space can be complicated.
- Usually for DM calculation $2 \to 2$ processes are enough, but if real corrections $\simeq$ vertex corrections, $2 \to 3$ processes should also be included.
- If c.m energy $\sqrt{s} \gg M_V$, EW bosons behave like a photon $\Rightarrow$ Mass singularities in soft and collinear logs $\propto \ln^2(s/M_W^2)$

ON-SHELL RENORMALISATION OF THE MSSM SECTORS

FERMION + GAUGE SECTOR

Input parameters as in the Standard Model $m_f, \alpha(0), M_W, M_Z$

HIGGS SECTOR

Input parameters : $M_{A^0}, t_\beta = v_2/v_1$. Several definitions for δt_β :

- $\overline{DR}$: δt_β is a pure divergence
- MH : δt_β is defined from the measurement of the mass m_H
- $A^0\tau\tau$: δt_β is defined from the decay $A^0 \to \tau^+\tau^-$($vertex \propto m_\tau t_\beta$)

SFERMIONS SECTOR

Input parameters : 3 sfermions masses $m_{\tilde{d}_1}, m_{\tilde{d}_2}, m_{\tilde{u}_1}$ and 2 conditions for $A_{u,d}$

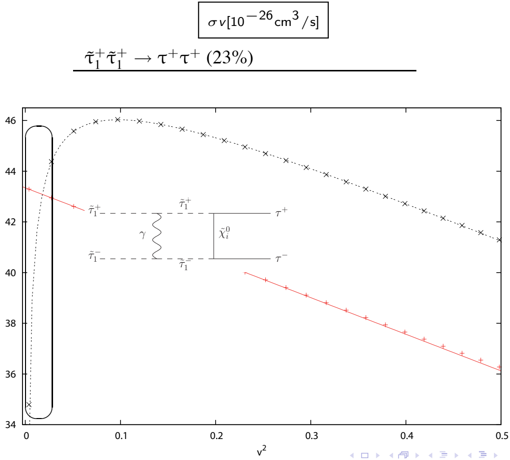
NEUTRALINOS/CHARGINOS SECTOR

Input parameters : 2 charginos $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}$ and 1 neutralino $\tilde{\chi}_1^0$

ANNIHILATION INTO FERMIONS

- Baro, Boudjema, Semenov, *Phys. Lett* **B660** (2008) 550
- annihilation cross section $\propto m_f \Rightarrow$ mainly into heavy ones (τ, b, t).
- bulk region in mSUGRA paradigm
- coannihilation with staus

- Example : bino-scenario : coannihilation with $\tilde{\tau}$
- Chirality suppression mechanism no more at play at 1L
- Peculiar feature when $v \longrightarrow 0$ for $\tilde{\tau}_1^+ \tilde{\tau}_1^+ \longrightarrow \tau^+ \tau^+$
- $\Rightarrow$ Coulomb effect
- $\dfrac{\sigma_1^{Coul}}{\sigma_0} = -\dfrac{\pi\alpha}{v}$
- No sensitive effect on $\Omega_\chi h^2$



Second example : annihilation into $b\bar{b}$

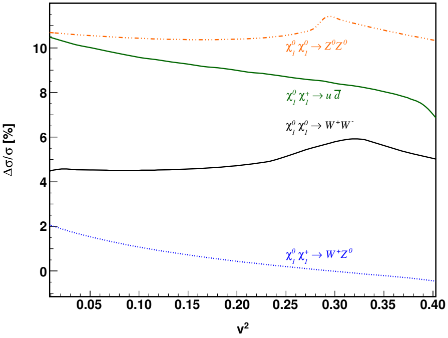
- Mixed case : bino-higgsino
- Important A^0-exchange in s-channel
  $m_{\tilde{\chi}_1^0} = 106$ GeV, $m_{A^0} = 300 GeV$
- $\sigma v = a + bv^2$
- δt_β scheme dependency
- Important corrections to $A^0 \to b\bar{b}$ vertex
  $\to$ anomalous dimension, Δm_b
- Modified Yukawa coupling implemented in `micrOMEGAs`

$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to b\bar{b}$	$A_{\tau\tau}$	$\overline{\mathrm{DR}}$	MH
$\delta a/a$ EW	-1%	+3%	+31%
$\delta a/a$ QCD	-26%	-26%	-26%
$\delta b/b$ EW	-1%	+3%	+29%
$\delta b/b$ QCD	-30%	-30%	-30%

- Baro, Boudjema, Chalons, Sun Hao, *Phys. Rev* **D81** (2008) 015005
- Most difficult channels : Gauge invariance plays a prominent role.

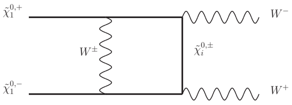
Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value	400	350	-250	4	1000	650	0	800

$$\tilde{\chi}_1^0 = 0.11\tilde{B} - 0.31\tilde{W} - 0.70\tilde{H}_1^0 - 0.63\tilde{H}_2^0$$



	$A_{\tau\tau}$	$\overline{\mathrm{DR}}$	MH
$\delta\Omega h^2/\Omega h^2$	-2.4%	-2.5%	-3.3%

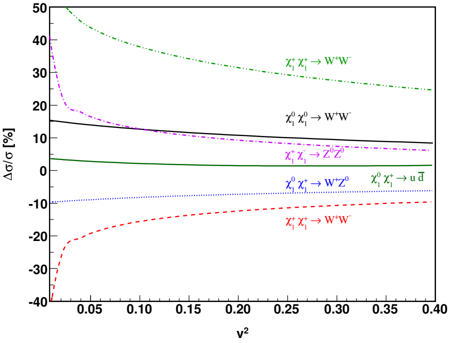
- Bulk of corrections to the s-wave coefficient
- Small δt_β scheme dependence
- QCD corrections to $u\bar{d} \simeq 3$ %
- Bump = $\tilde{\chi}_1^\pm$ threshold in boxes, not present at Tree-Level
- $a + bv^2$ expansion doesn't work anymore at 1-L



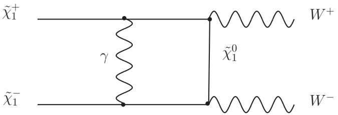
ANNIHILATION INTO GAUGE BOSONS : LIGHT-WINO

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{u}_L}$	$M_{\tilde{e}_L}$	$M_{\tilde{u}_R,\tilde{e}_R}$	A_i	M_{A^0}
Value	550	210	-600	30	1200	387	360	800	0	700

$$\tilde{\chi}_1^0 = 0.005\tilde{B} - 0.99\tilde{W} - 0.15\tilde{H}_1^0 - 0.05\tilde{H}_2^0$$



- At 1-L new feature appear for $v \longrightarrow 0$: Coulomb effect
- Possible to capture its one-loop manifestation
- Degeneracy lifted between processes
- Large corrections
- Almost no δt_β scheme dependence
- Strong cancellations between QCD/EW corrections



	$A_{\tau\tau}$	$\overline{\text{DR}}$	MH
$\delta\Omega h^2/\Omega h^2$	-1.9%	-1.9%	-1.9%

ANNIHILATION INTO GAUGE BOSONS : HEAVY-WINO

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value(GeV)	3500	1800	4500	15	5000	5000	0	5000

$$\tilde{\chi}_1^0 = 0.000\tilde{B} - 0.999\tilde{W} + 0.004\tilde{H}_1^0 + 0.032\tilde{H}_2^0$$

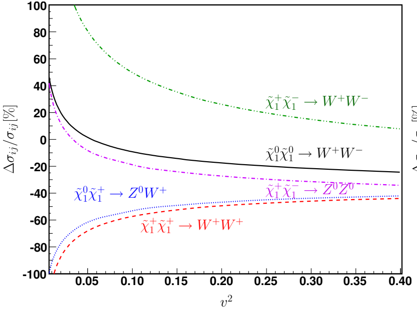
		Tree
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ [10%]	a	$+2.43$
	b	$+0.52$
$\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$ [10%]	a	$+1.22$
	b	$+0.26$
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to Z^0W^+$ [9%]	a	$+0.51$
	b	$+0.12$
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to t\bar{b}$ [9%]	a	$+0.54$
	b	-0.23
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to u\bar{d}$ [9%]	a	$+0.54$
	b	-0.23
$\tilde{\chi}_1^+\tilde{\chi}_1^- \to Z^0Z^0$ [6%]	a	$+0.73$
	b	$+0.16$
$\tilde{\chi}_1^+\tilde{\chi}_1^- \to W^+W^-$ [6%]	a	$+0.65$
	b	$+0.17$
$\Omega_\chi h^2$		0.0997

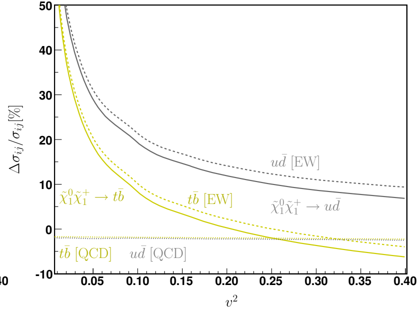
- $m_{\tilde{\chi}_1^0} = 1799.1$ GeV
- $\delta(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}) = 0.0003$ GeV

- $\sigma_0 v = a + bv^2$
- $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^\pm}$ almost degenerate
- Coannihilation very important
- Degeneracy between processes $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ and $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$
- A lot of processes contribute

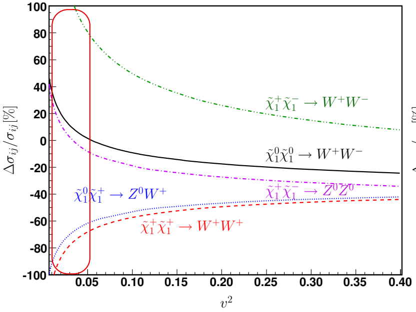
Virtual corrections + γ bremsstrahlung

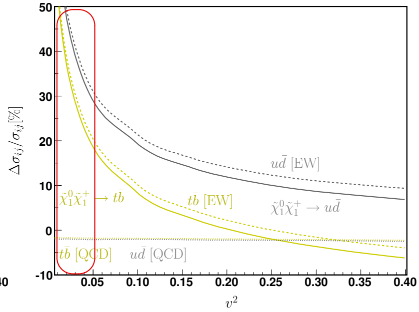
Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value(GeV)	3500	1800	4500	15	5000	5000	0	5000



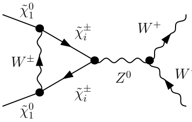


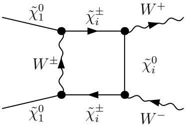
Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value(GeV)	3500	1800	4500	15	5000	5000	0	5000



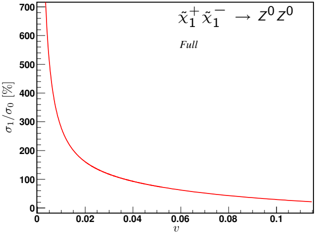


- $M_W/m_{\tilde{\chi}_1^0} = 0.045 \Rightarrow W^{\pm}, Z^0$ bosons almost considered as massless.
- $v \longrightarrow 0$: Large Sommerfeld (QED+EW) enhancement.





- The EW Sommerfeld effect is expected to be cut-off, as opposed to the QED one.
- To extract it, remove the QED Coulomb effect first.

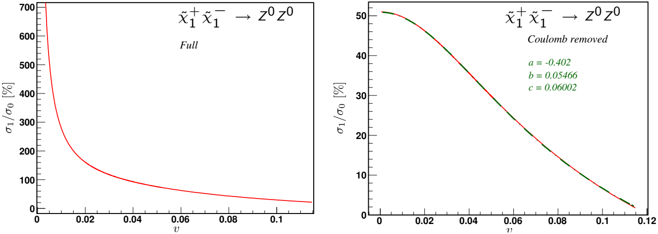


- $S_{nr} = X_{nr}/(1 - e^{-X_{nr}}) \quad X_{nr} = 2\pi\alpha Q_i Q_j / v$
- $S_{1L} = \frac{\pi\alpha}{v} \times \sigma_0 Q_i Q_j$

- The EW Sommerfeld effect is expected to be cut-off, as opposed to the QED one.
- To extract it, remove the QED Coulomb effect first.
- Then, as behavior expected to be cut-off, fit with,

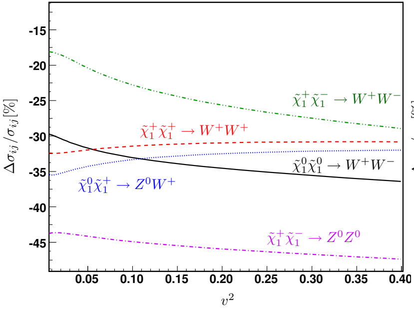
$$\sigma_1/\sigma_0 = a + \frac{b}{\sqrt{v^2 + c^2}}$$

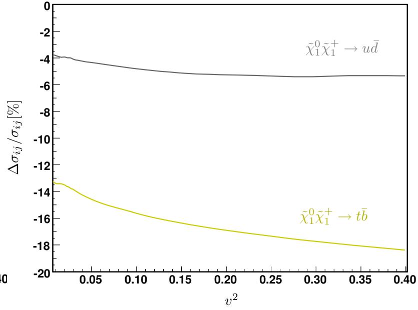
where c is supposed to be the cut-off, of order $M_W/m_{\tilde{\chi}_1^0}$.



- Large corrections but $<$ QED Sommerfeld for $v \rightarrow 0$.

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value(GeV)	3500	1800	4500	15	5000	5000	0	5000





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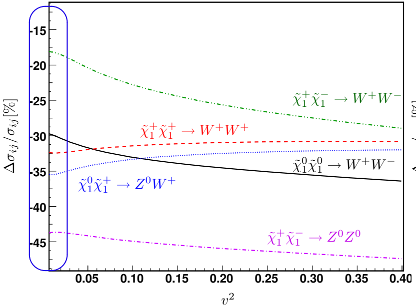
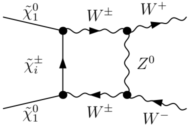
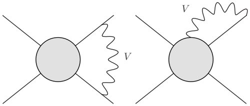


- Even after substraction of Sommerfeld effect, still large corrections of Sudakov type ($m_{\tilde{\chi}_1^0} \gg M_W, M_Z$) for individual processes.



- Originate from vertex and box diagrams involving virtual bosons.
- General form of one-loop Sudakov corrections

$$\alpha\left[C_2 \underbrace{\ln^2\left(\frac{s}{M_V^2}\right)}_{\text{LL}} + C_1 \underbrace{\ln^1\left(\frac{s}{M_V^2}\right)}_{\text{NLL}} + C_0\right] + \mathcal{O}\left(\frac{M_V^2}{s}\right) \quad V = \gamma, W^{\pm}, Z^0$$



- The $\ln(s/M_V^2)$ represents mass singularities and originate from soft and collinear regions.
- For QED corrections always present ($M_\gamma \to 0$), for EW ones when $s \gg M_{W,Z}^2$.

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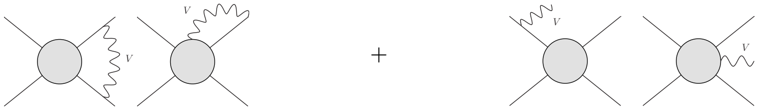
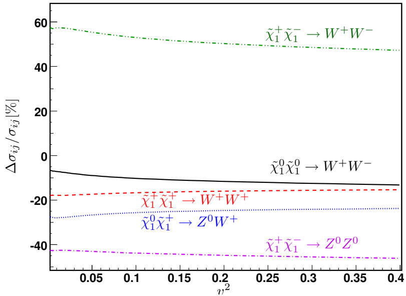
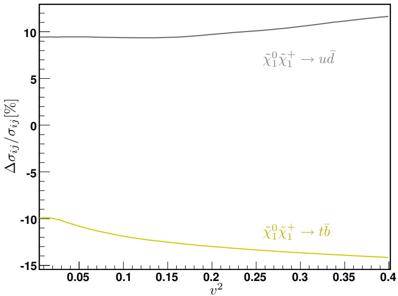


- The $\ln(s/M_V^2)$ represents mass singularities and originate from soft and collinear regions.
- For QED corrections always present ($M_\gamma \to 0$), for EW ones when $s \gg M_{W,Z}^2$.
- Dependency on M_γ unphysical $\Rightarrow$ removed by adding real emission as stated by the Bloch-Nordsieck theorem [Bloch,Nordsieck(1937)].
- For EW corrections, $M_{W,Z}$ physical and retained in the calculation.
- Adding real emission can counterbalance virtual corrections.

- For a specific channel adding the Z^0 reduces the overall corrections.
- But result still potentially large





- Not a complete cancellation due to Bloch-Norsieck violations [Ciafaloni, Comelli (2000)]
- $W^{\pm}$ emission changes isospin → one state of a mutliplet turned into another state of the same multiplet.
- By summing/averaging over all members of the same multiplet, the cancellation should take place ⇒ Summing over all channels and processes.
- $W^{\pm}$ real emission must also be added to form an isospin singlet.

Virtual + real $2 \to 2 + \gamma, Z^0, W^{\pm}$.

Values of various cross section for $v = 0.3c$

Process	Tree-Level	1-Loop	W+Z emission	Total
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to W^+ W^-$	2.668051510	-0.232723594 (-9%)	0.608162875 ($+23\%$)	3.043490791 ($+12\%$)
$\tilde{\chi}_1^+ \tilde{\chi}_1^+ \to W^+ W^+ \times 2$	2.667542171	-1.542918600 (-58%)	0.196086090 ($+7.3\%$)	1.320709661 (-50.7%)
$\tilde{\chi}_1^+ \tilde{\chi}_1^- \to W^+ W^-$	0.713966584	0.364979903 ($+51\%$)	0.541233676 ($+76\%$)	1.620180163 ($+127\%$)
$\tilde{\chi}_1^+ \tilde{\chi}_1^- \to Z^0 Z^0$	0.805841752	-0.147277293 (-18%)	0.008798936 ($+1\%$)	0.649765523 (-17%)
$\tilde{\chi}_1^0 \tilde{\chi}_1^+ \to W^+ Z^0 \times 2$	1.127875389	-0.605309188 (-54%)	0.250488943 ($+22\%$)	0.773055144 (-32%)
$\tilde{\chi}_1^0 \tilde{\chi}_1^+ \to t\bar{b} \times 2$	1.111269832	0.099235806 ($+9\%$)	0.041938145 ($+3.8\%$)	1.252443783 ($+12.8\%$)
$\tilde{\chi}_1^0 \tilde{\chi}_1^+ \to u\bar{d} \times 2$	1.116433207	0.222165895 ($+20\%$)	0.156287258 ($+14\%$)	1.494886360 ($+34\%$)
Total	10.210980445	-1.841847071 (-18%)	1.802995923 ($+17.6\%$)	10.172129297 (-0.4%)
Total only gauge	7.983277406	-2.163248772 (-27%)	1.604770520 ($+20\%$)	7.424799154 (-7%)

- Large corrections for individual processes $\Rightarrow$ important effect for Indirect Detection [Chalons PhD Thesis, Strumia et al]
- For relic density calculation, in the thermal bath sum over all members of the isospin multiplet automatically done [Chalons PhD Thesis] $\to$ small effect expected.

CONCLUSIONS AND PERSPECTIVES

- Importance of radiative corrections in the relic density calculations, can be very large.
- Need to control them to be able to extract informations from it and to constrain the underlying cosmological scenario.
- For some cases scheme dependence.
- For a heavy neutralino scenarios taking into account $2 \to 3$ processes is necessary.
- Large corrections due to soft/collinear logs and Sommerfeld enhancement.
- Corrections can be even larger for Indirect Detection, rate and spectra for specific signatures.
- In all cases for $\Omega_\chi h^2$ @ 1-2% $\Rightarrow$ one-loop corrections mandatory.
- Study of the dependency of the results on the chargino/neutralino renormalisation scheme.
- Improve the interface with `micrOMEGAs` .