Tools and Monte-Carlos

for the

New Physics

Fawzi BOUDJEMA LAPTH-Annecy, France

- What's a tool and what it takes to make one
- New Physics vs the Standard Model Physics
- Modular Structures and interfaces

OUTLINE

A. From the parton to the detector

not necessarily BSM

B. From the Lagrangian to the MEG

codes talk, interface

NLO issues

not necessarily BSM

Tools for Dark Matter

Definitely BSM

OUTLINE A and B

- What's a tool and what it takes to make one: Structure of an event
- Components of a MC EG (Monte Carlo Event Generator)
- Integration and MC techniques (probably skip)
- PS: Parton Shower in a MC
- Matrix Element vs PS
- ME generation and ME generators
- Modular structure of codes, Les Houches Accords
- Tools for the New Physics

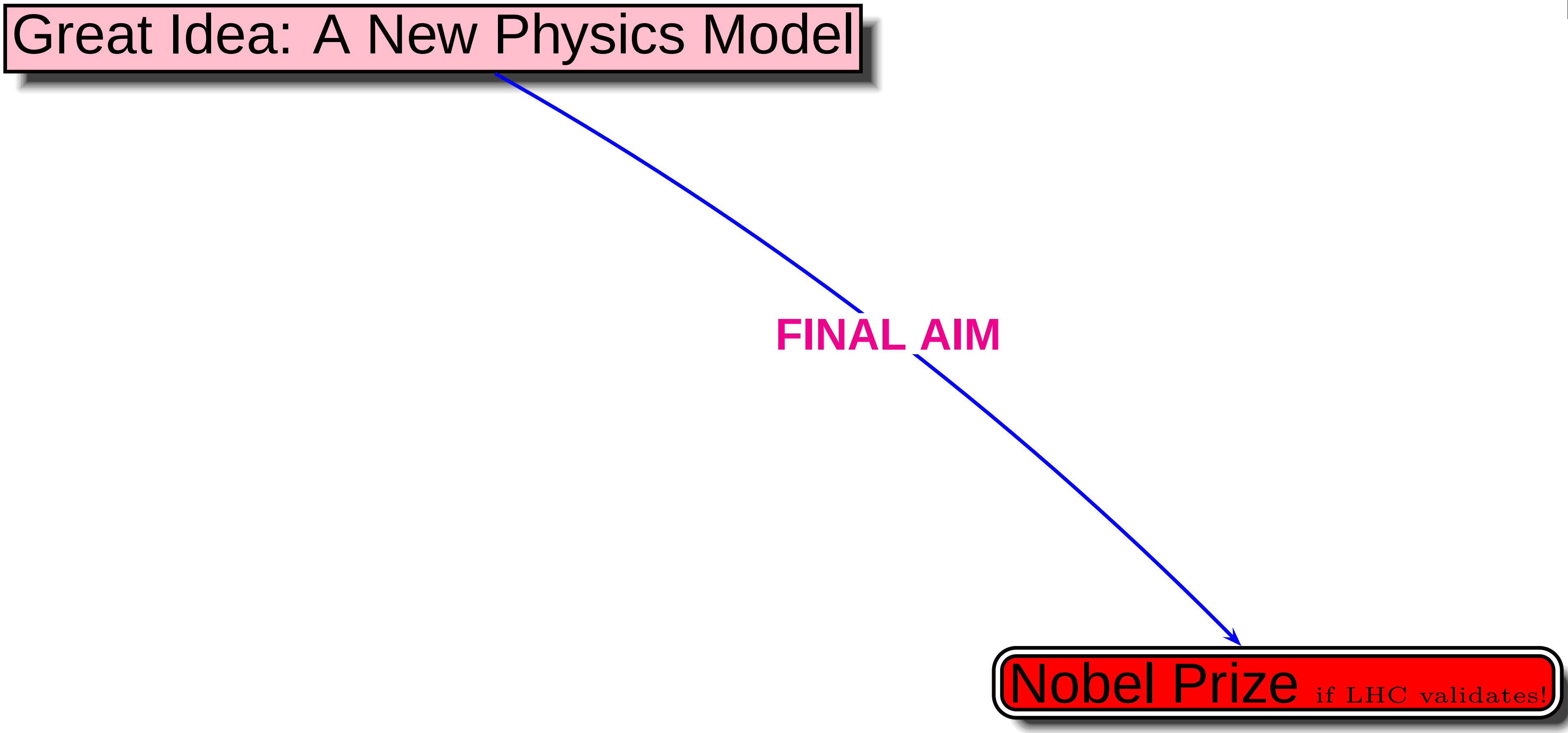
Further Reading and from where I borrowed

- Frank Krauss Bonn Lectures, 2006
  http://projects.hepforge.org/sherpa/dokuwiki/publications/presentations/index
- Fabio MALTONI HEPTOOLS School, Torino, 2008
  http://personalpages.to.infn.it/ maina/scuola08/Maltoni_Torino08.pdf
- Steve Mrenna CTEQSS, CTEQ05
- Peter Richardson CTEQ06 School, IPPP Durham, 2006
- Mike Seymour CERN Training Lectures 2003
  http://seymour.home.cern.ch/seymour/slides/CERNlecture1.ppt
- Torbjrn Sjostrand, 2006 European School of HEP, Aronsborg YETI06, IPPP Durham, see Pythia website http://www.thep.lu.se/ torbjorn
- Brian Webber 1st MCnet School, IPPP Durham 2007
- Les Houches Guidebook Les Houches Guidebook to Monte Carlo Generators for Hadron Collider Physics, hep-ph/0403045
- R.K. Ellis, W.J. Stirling and B.R. Webber QCD and Collider Physics Cambridge Monographs, Cambridge University Press

Further Reading and where you can find info on (and download) the codes

- Les Houches Guidebook Les Houches Guidebook to Monte Carlo Generators for Hadron Collider Physics, hep-ph/0403045
- BSM Tools Repository: http://www.ippp.dur.ac.uk/montecarlo/BSM/ .
- SUSY Bestiary of Public Codes: Allanach, arXiv:0805.2088 [hep-ph] .
- SUSY Tools for Dark Matter and at the Colliders. Fawzi Boudjema, Joakim Edsjo, Paolo Gondolo, arXiv:1003.4748
- My webpage
- Les Houches May 30-Jun 17, 2010 (not just a tools Workshop....)
- MC4BSM, Tools for SUSY and the New Physics Series (Google them)

Nobel Dreams



$$\mathcal{L}_{\text{New Physics Models}}$$

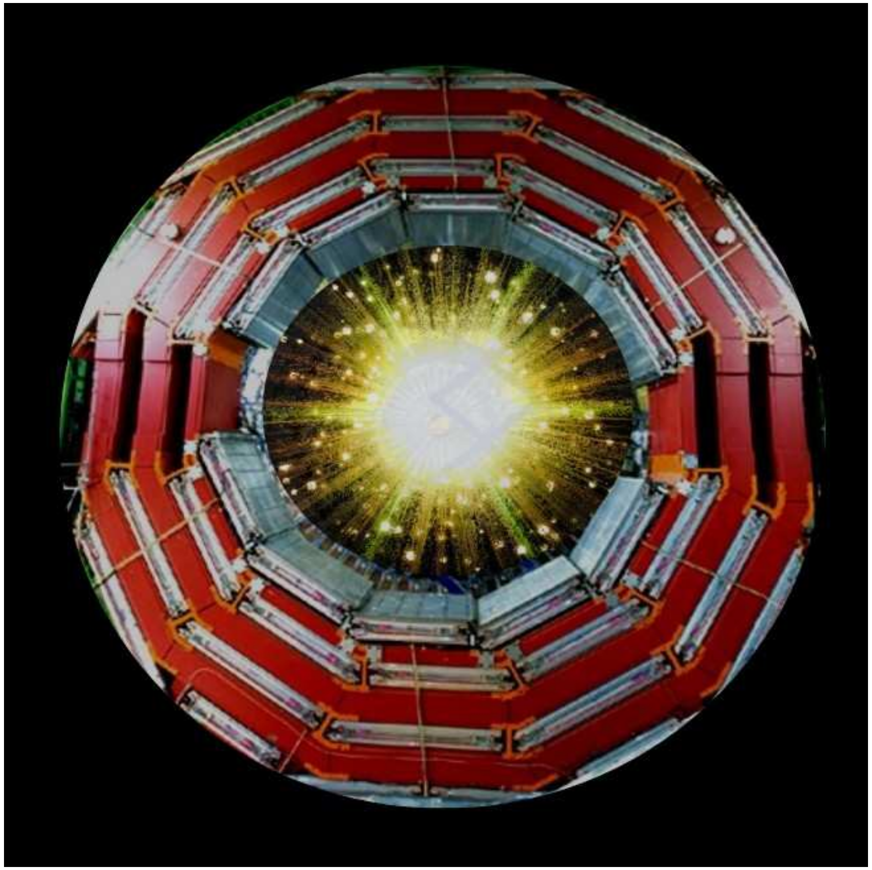
TOOLS

experimental discovery and data analyses

For a long time, Tools and MC

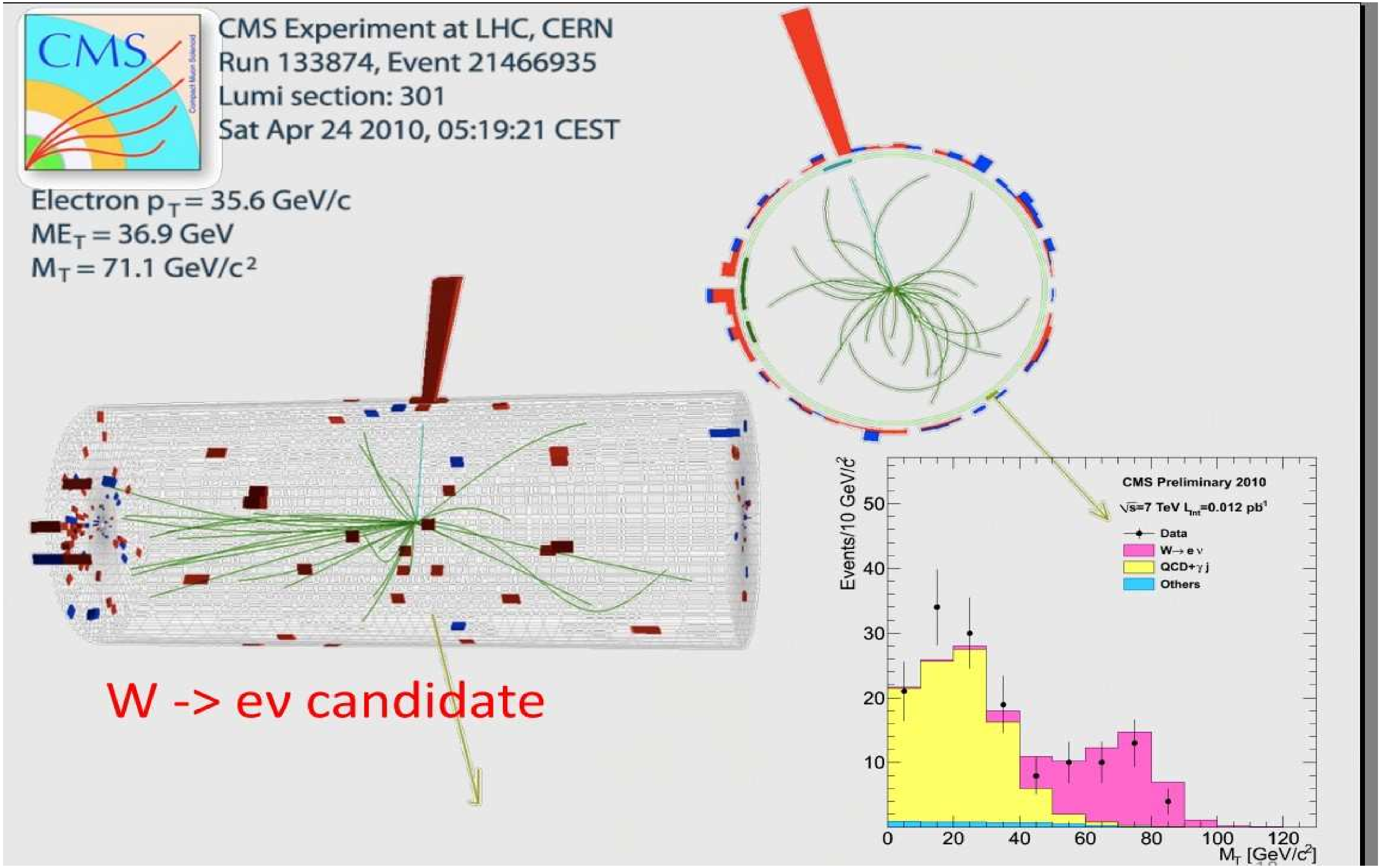
Before LHC started

For a long time, Tools and MC



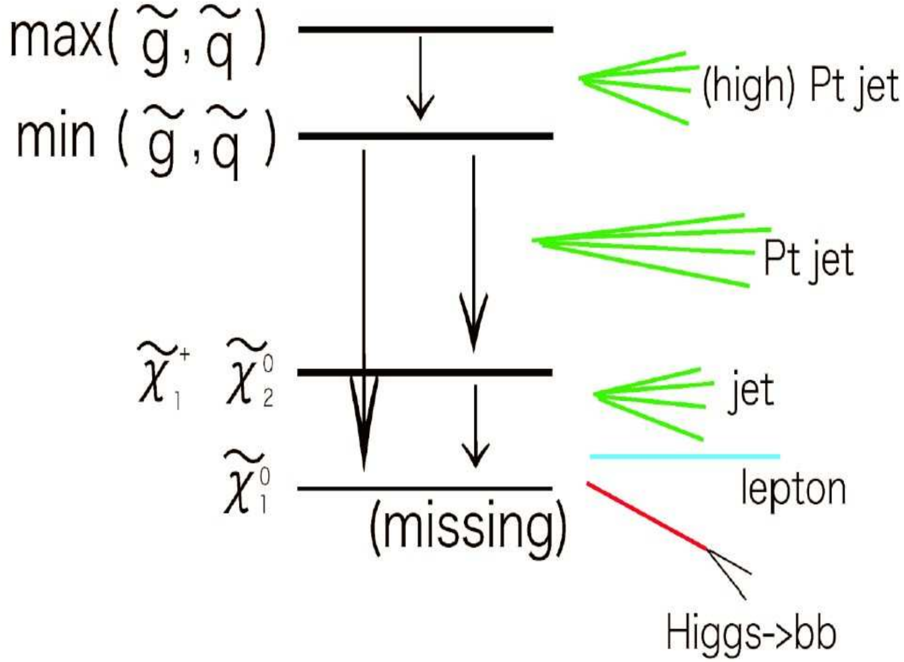
is the Higgs in there? or any other New Physics

Data from LHC, after some cleaning

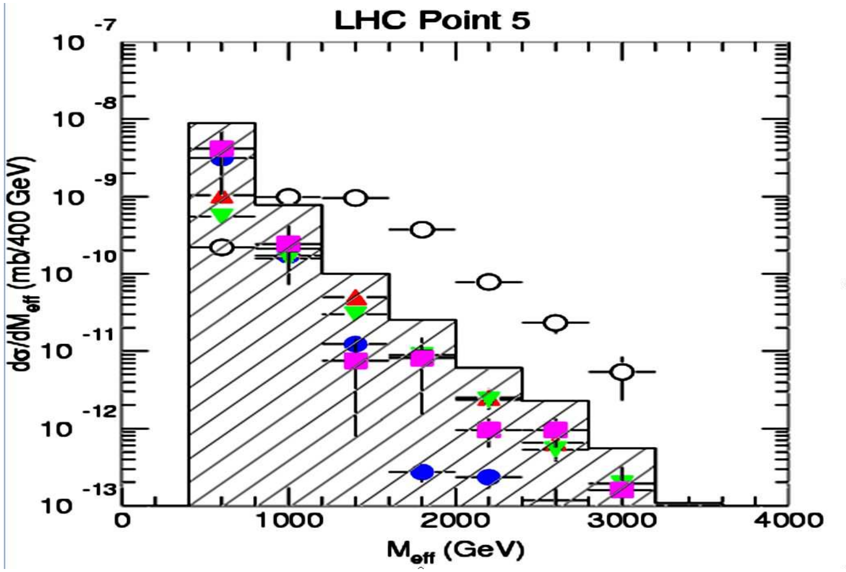


Turn on the machine!

in 1998 we were told to expect an early SUSY discovery

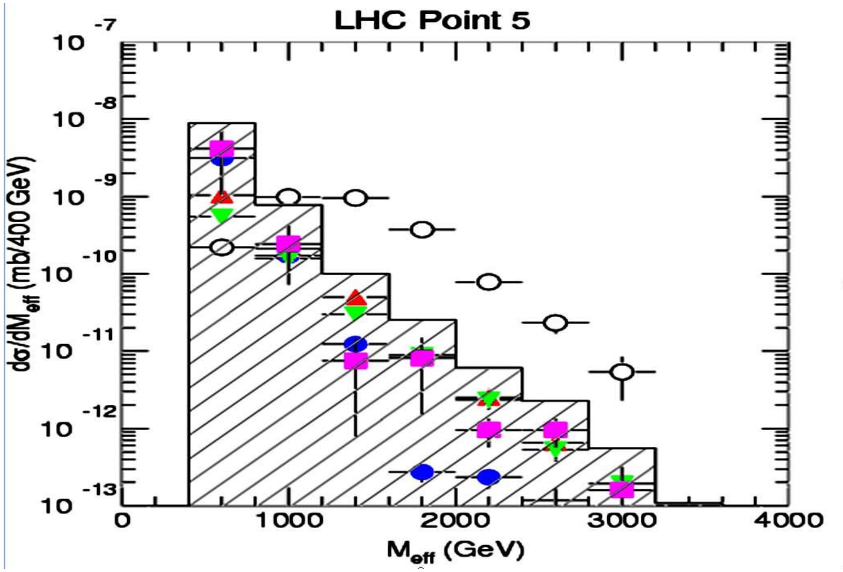


ATLAS TDR (same with CMS)



ATLAS TDR 98
(mSUGRA point, PreWMAP)

ATLAS TDR (same with CMS)



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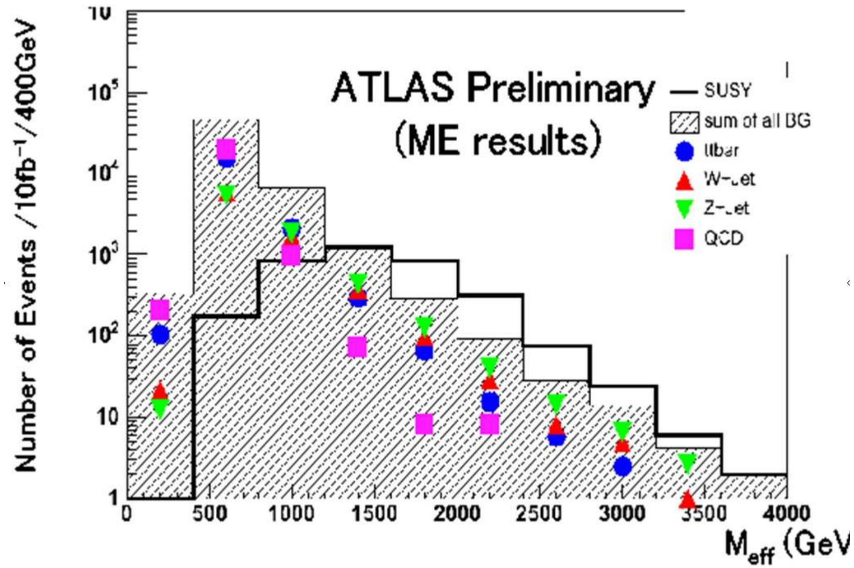
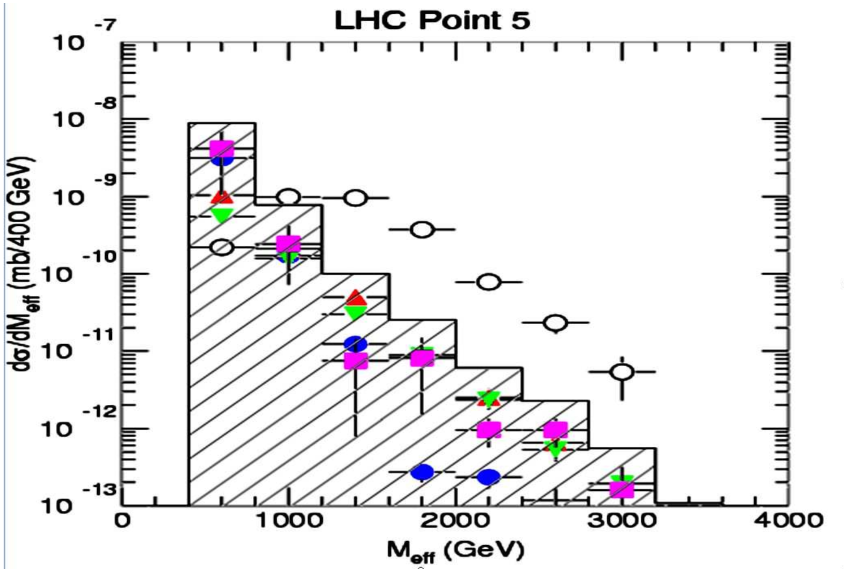


ATLAS 2006

ATLAS TDR (same with CMS)



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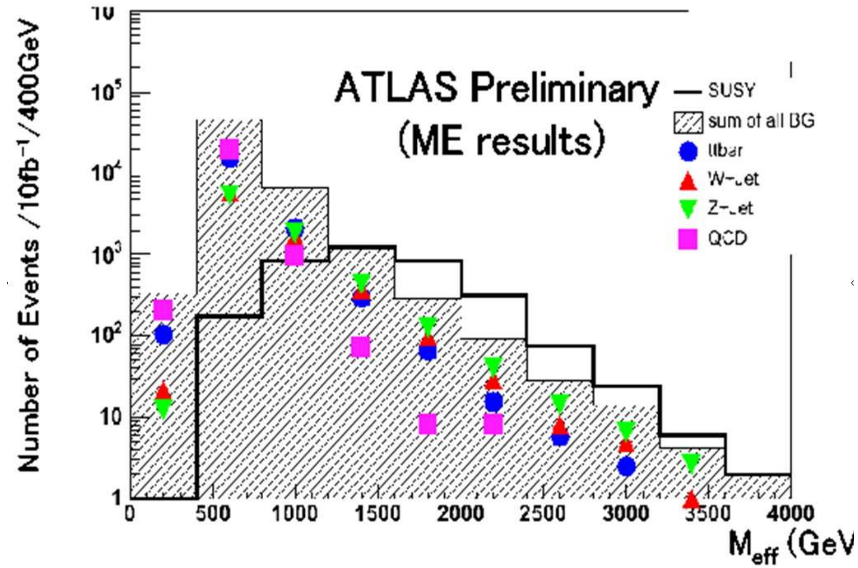


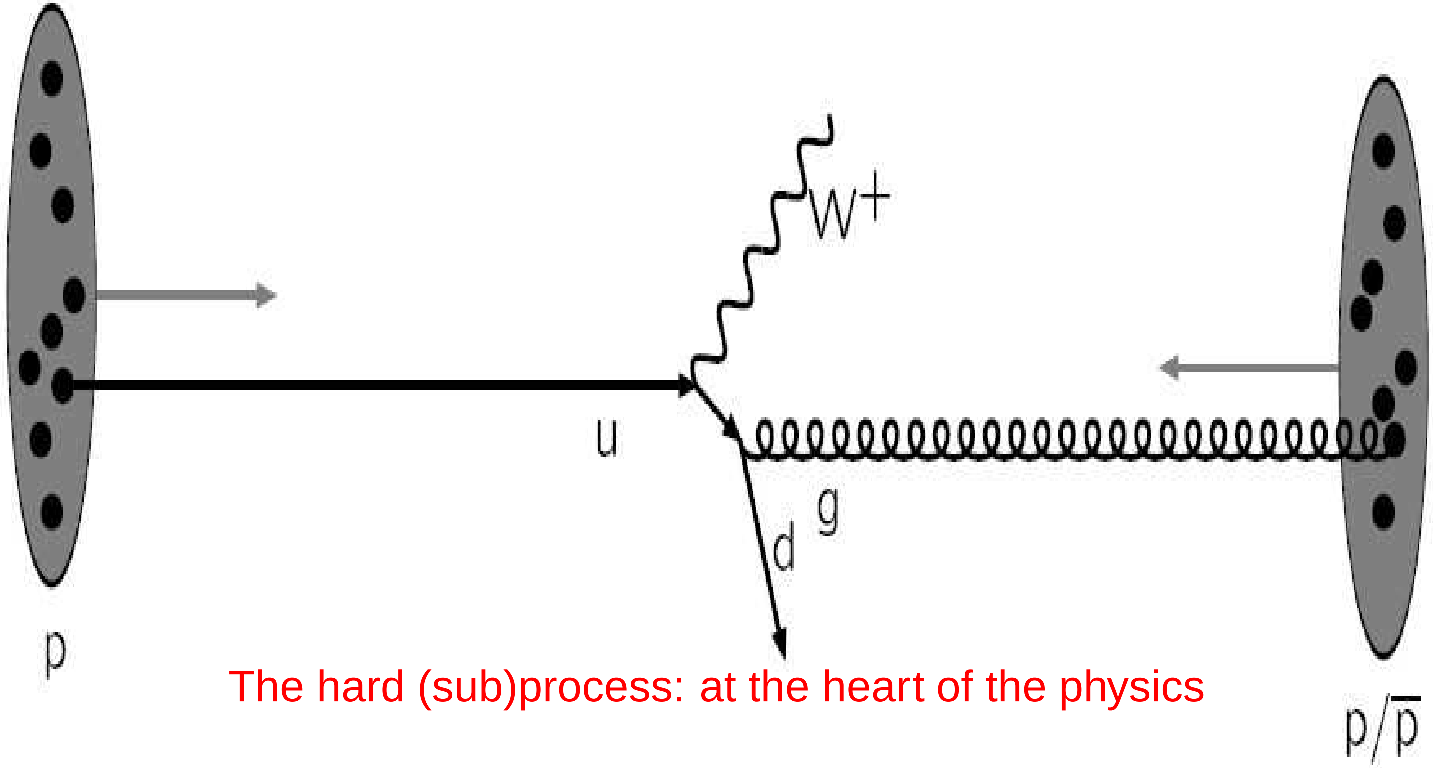
ATLAS 2006

What happened?

What we hope for!

Let's turn the machine again, slow motion!

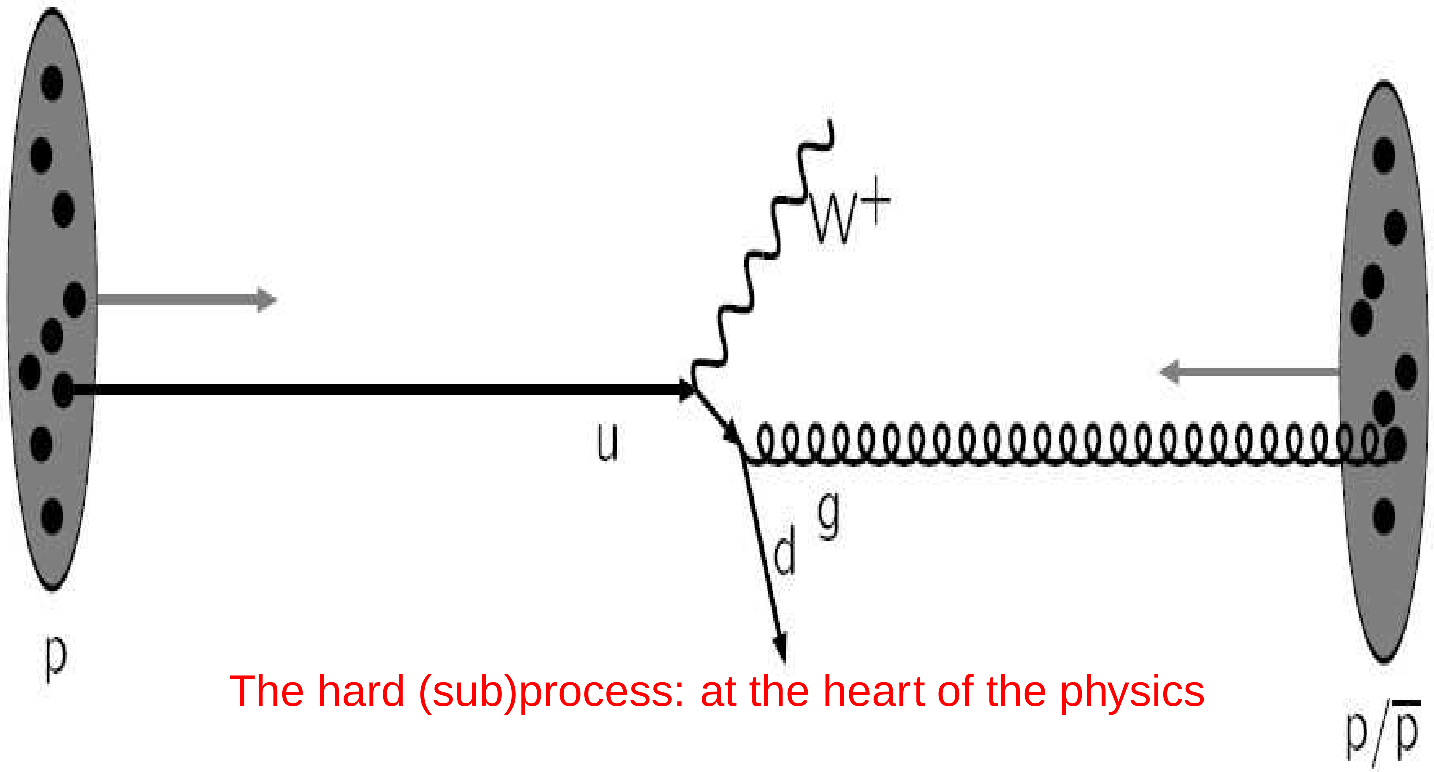
Incoming beams: partons densities



The hard (sub)process: at the heart of the physics

- Hard process is well understood and well described: relies on a firm perturbative framework.

Movie: The structure of an event



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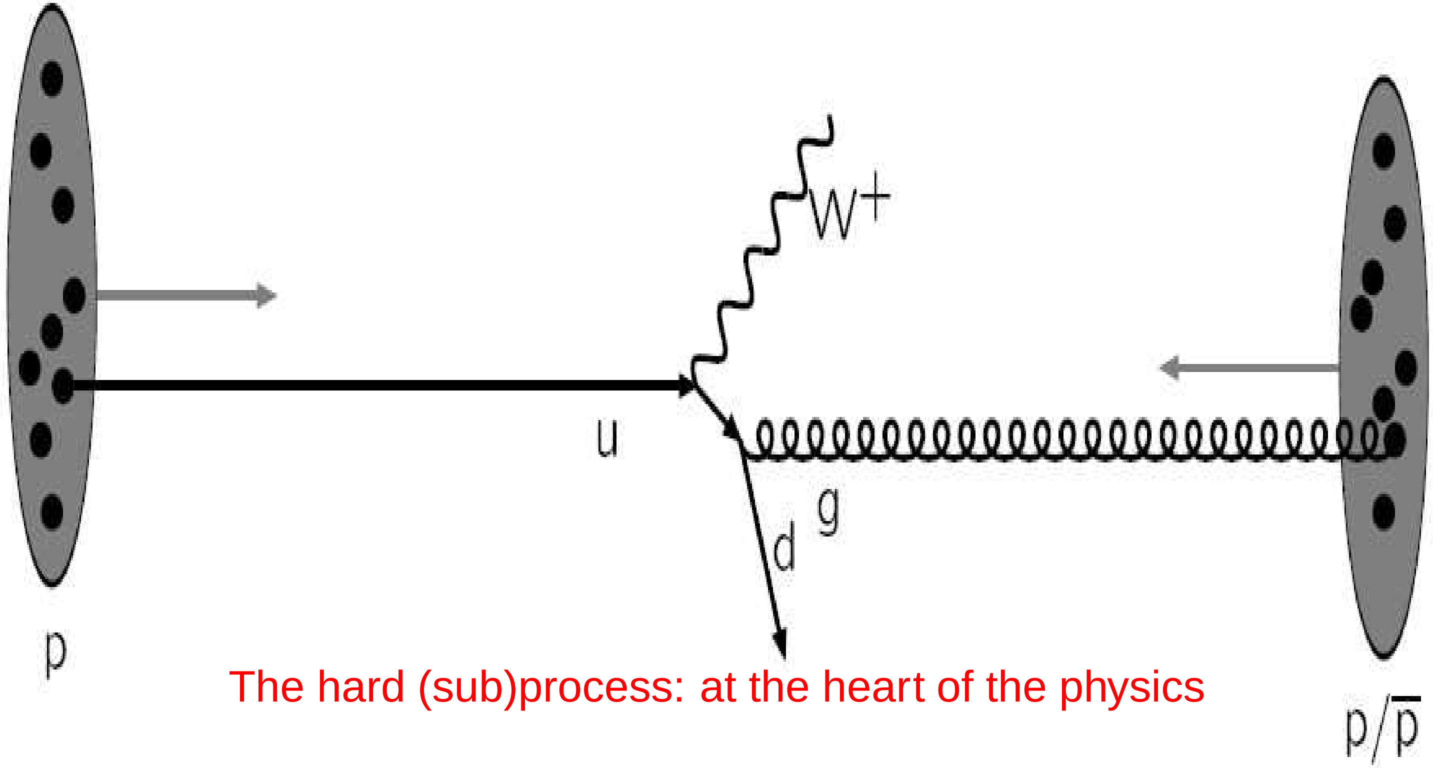
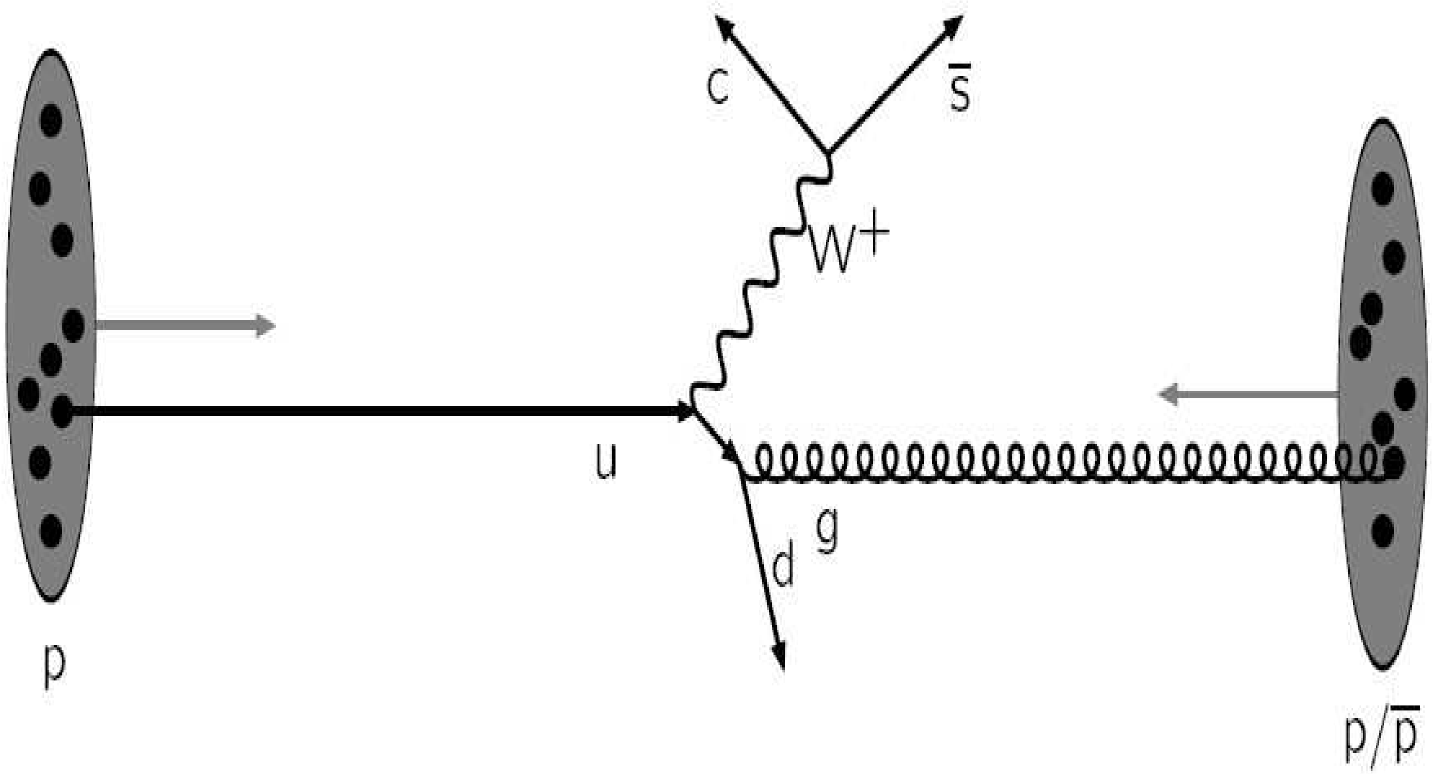
- described by Matrix Elements (ME)
  This does not mean that it is very well calculated



The hard (sub)process: at the heart of the physics

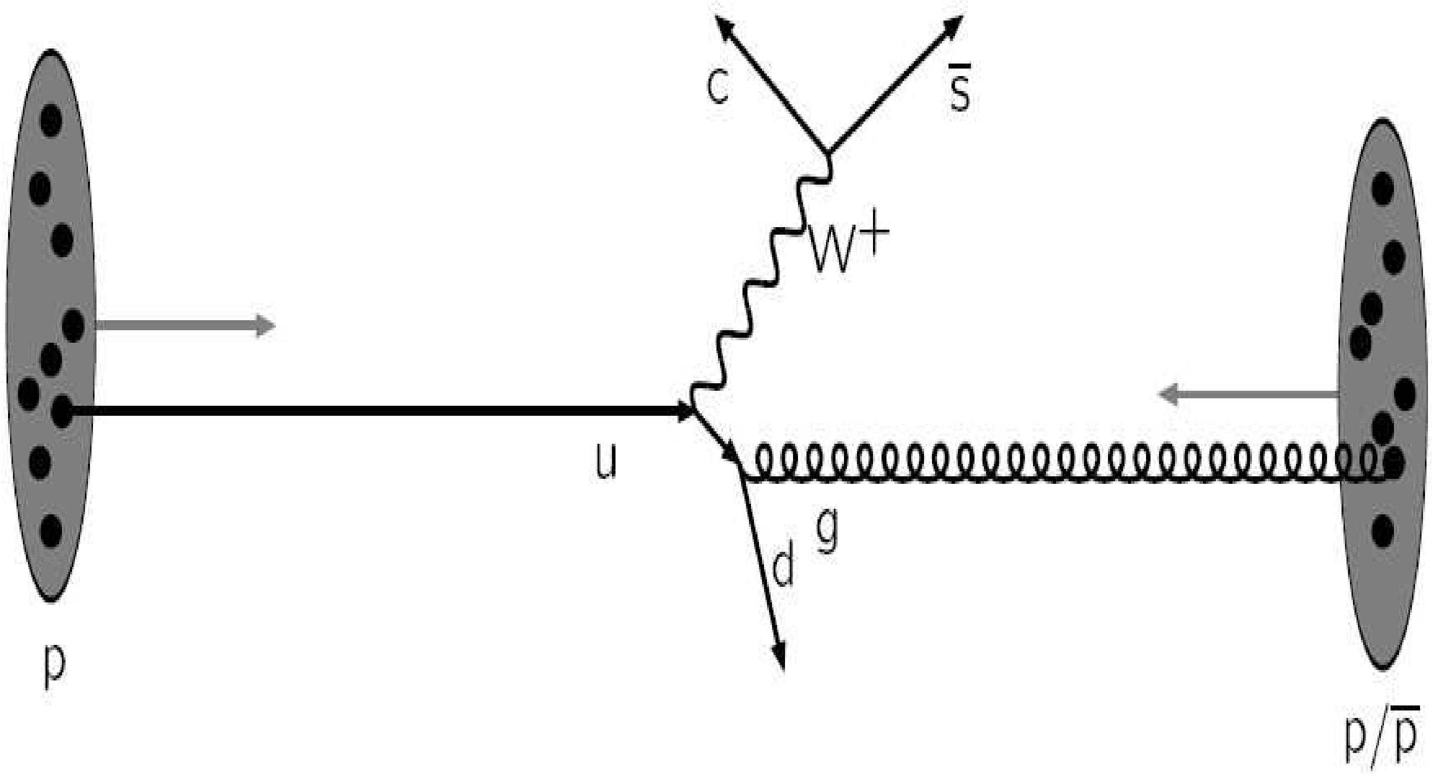
- Hard process is well understood and well described: relies on a firm perturbative framework.
- described by Matrix Elements (ME)
  This does not mean that it is very well calculated
- issue of higher order (NLO), most calculations only LO say



Decays of resonances: correlated with hard process

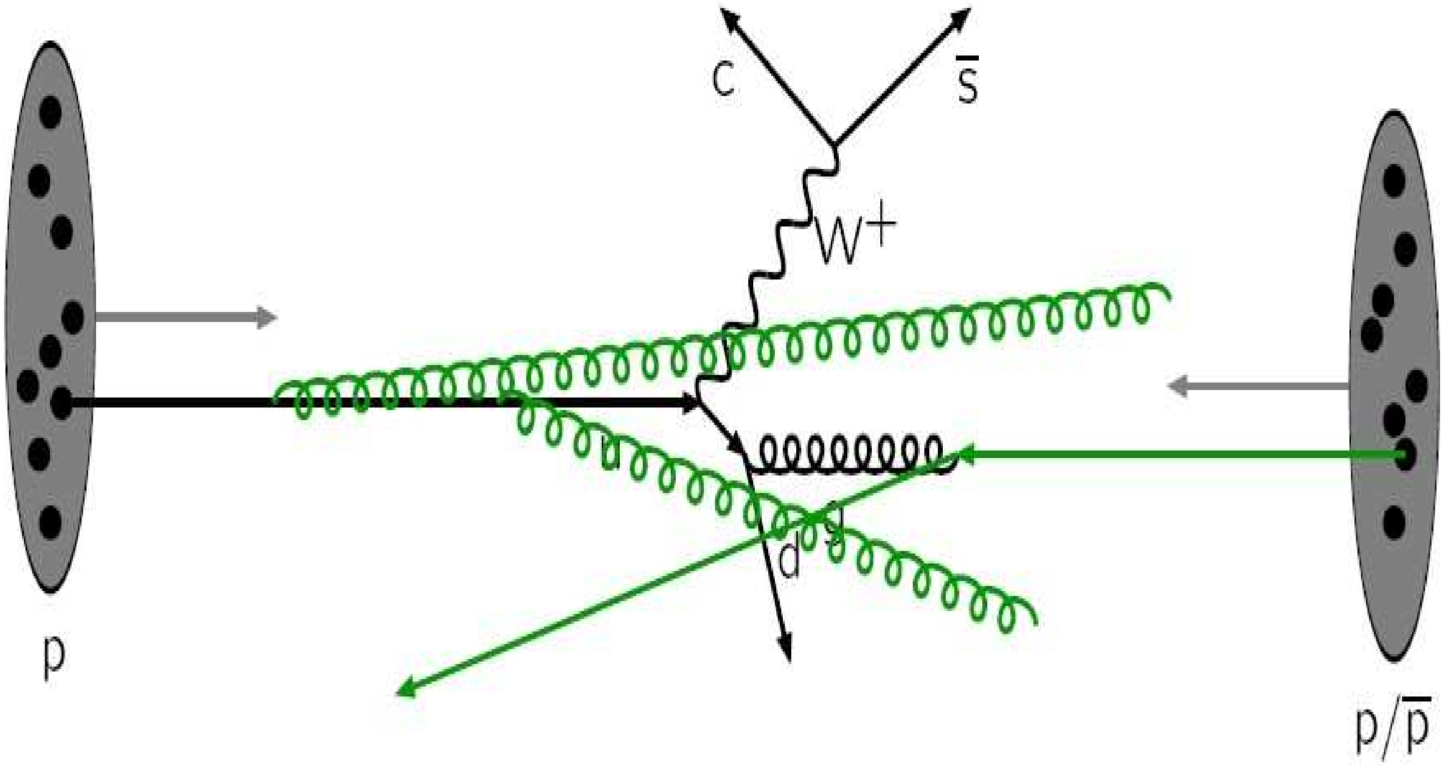
- Approximation: W on-shell

Movie: The structure of an event



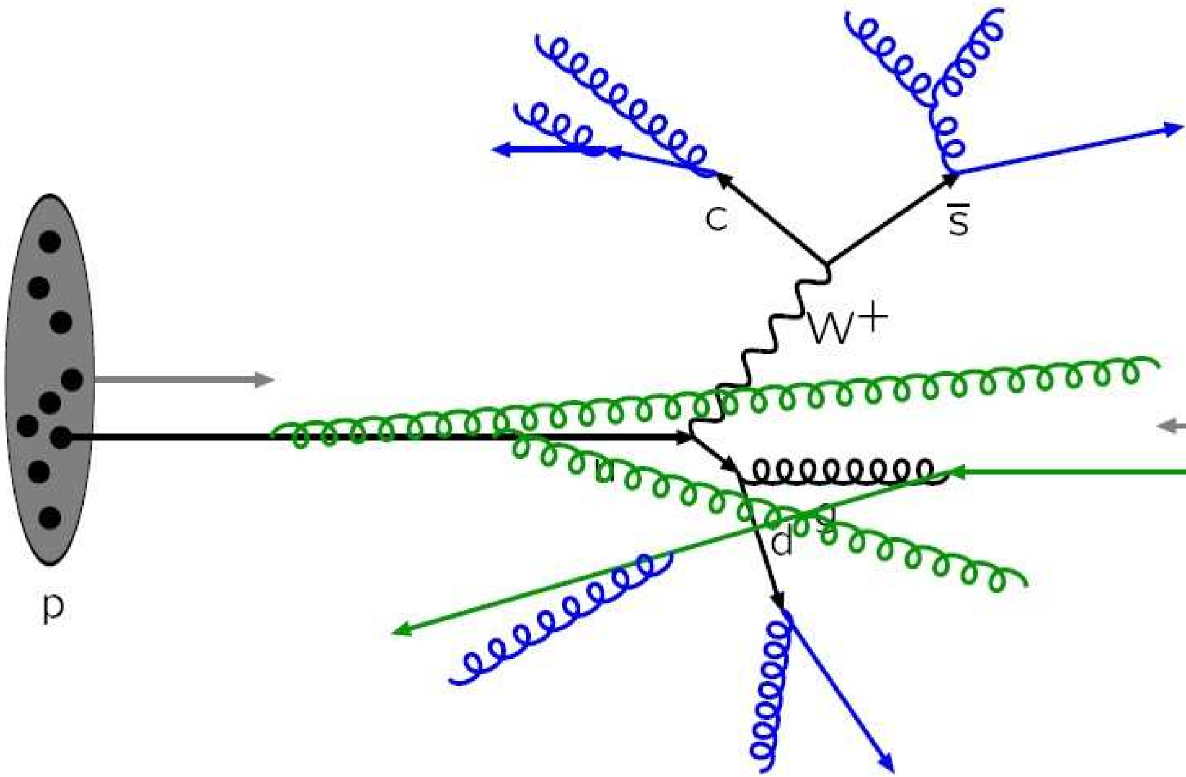
Decays of resonances: correlated with hard process

- Approximation: W on-shell
- Spin effect in decays?



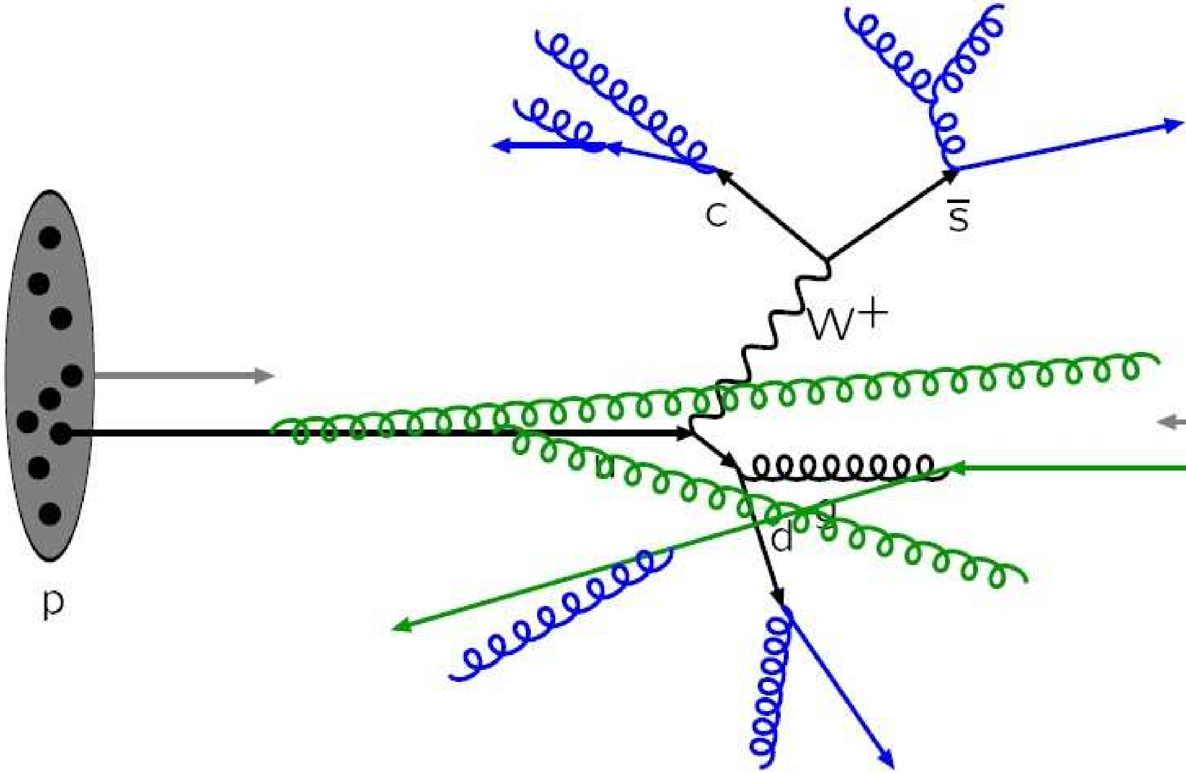
ISR: Initial State Radiation

Space-like parton showers (PS)



FSR: Final State Radiation

time-like parton showers (PS)



FSR: Final State Radiation

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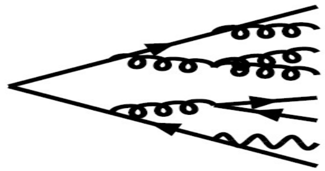
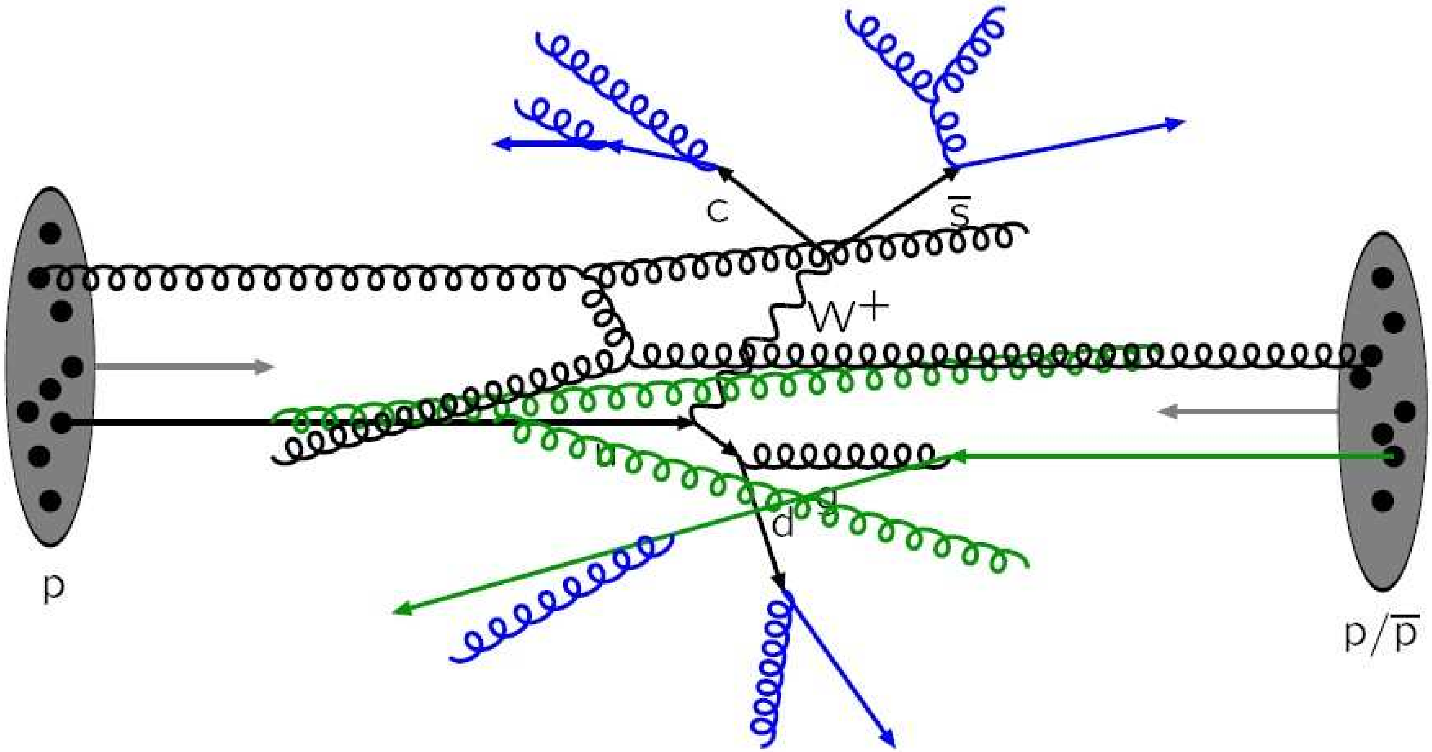


$q \longrightarrow qg$

$g \longrightarrow gg$

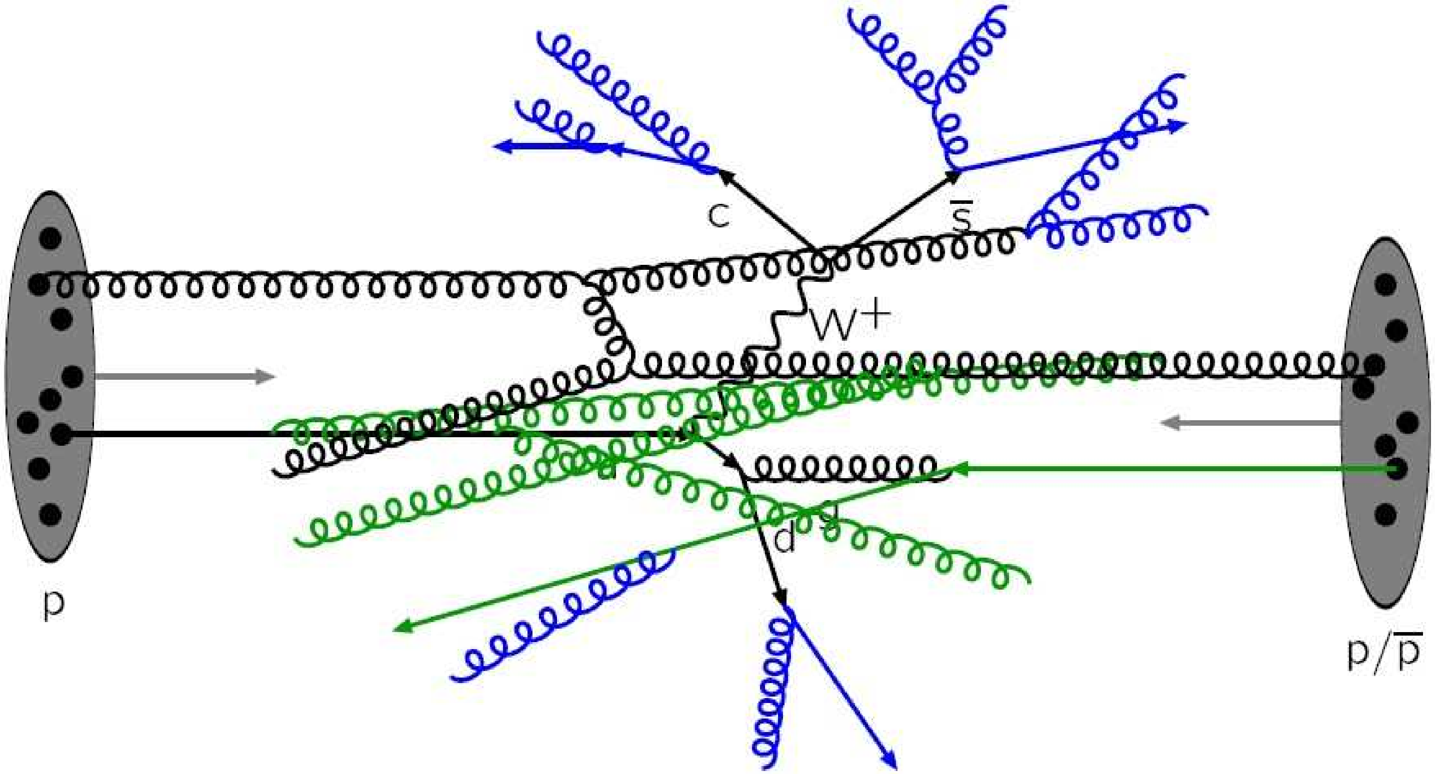
$g \longrightarrow q\bar{q}$

$q \longrightarrow q\gamma$



Multiple parton-parton interactions (MPI)

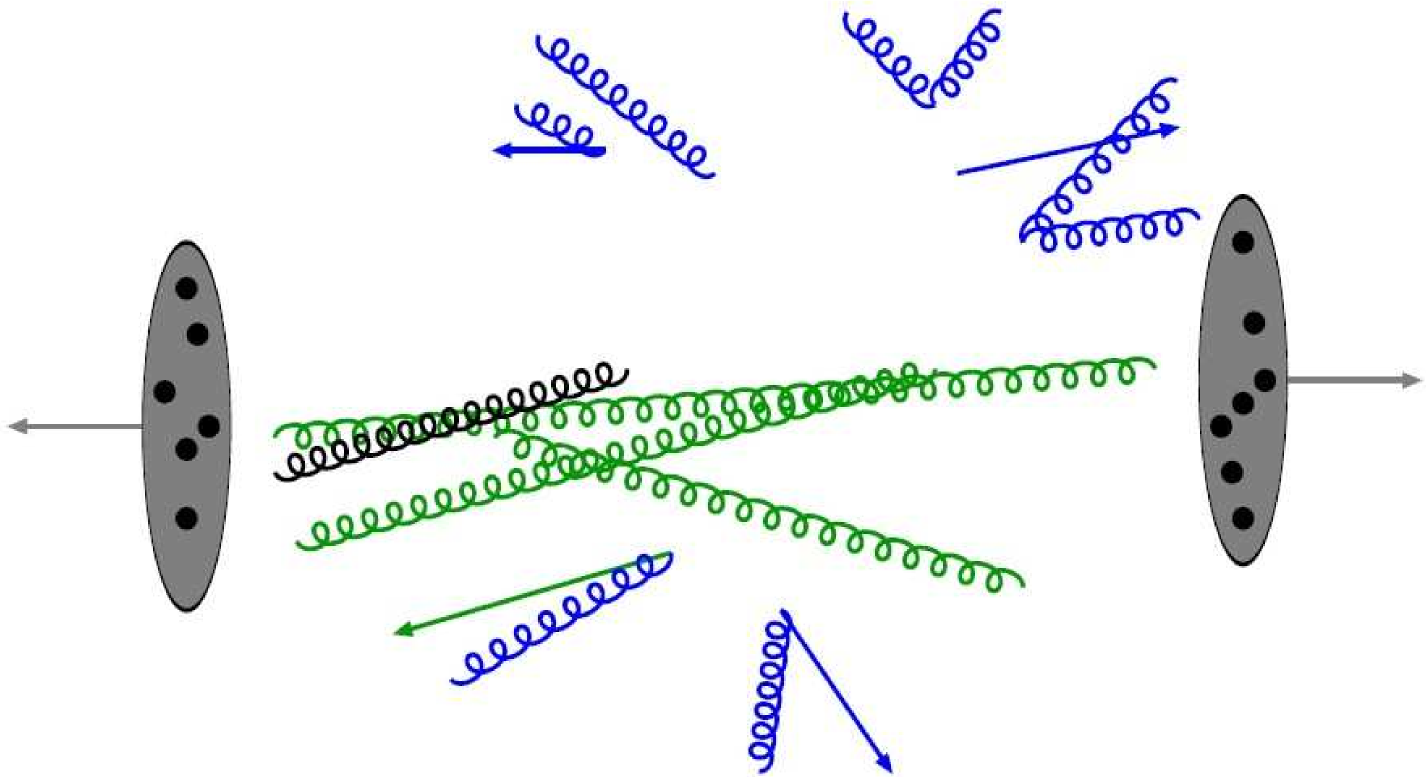
The muck



MPI with ISR and FSR !

The muck

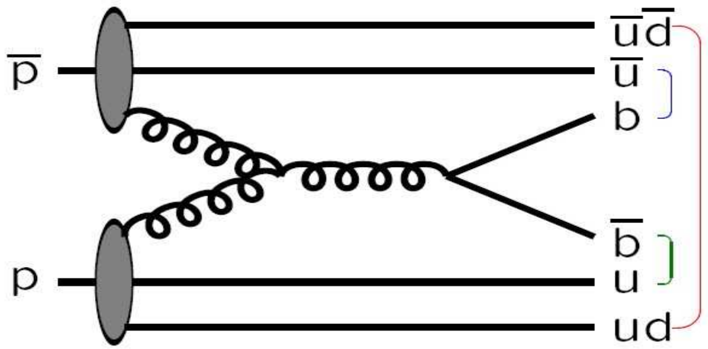
Movie: The structure of an event



Beam remnants and other outgoing partons !

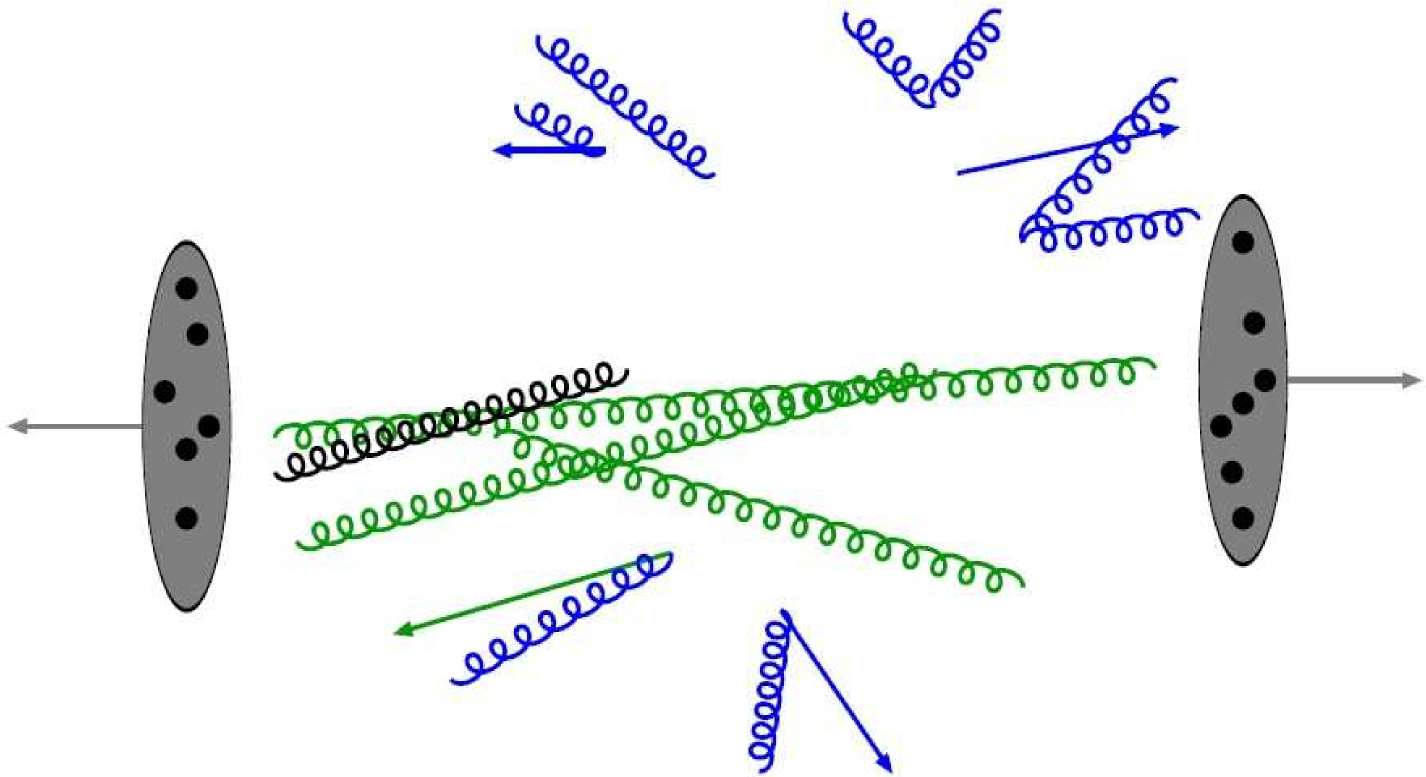
The muck

Movie: The structure of an event



Beam remnants: coloured remains of the proton not taking part in the hard process, but they are colour connected to the hard process.

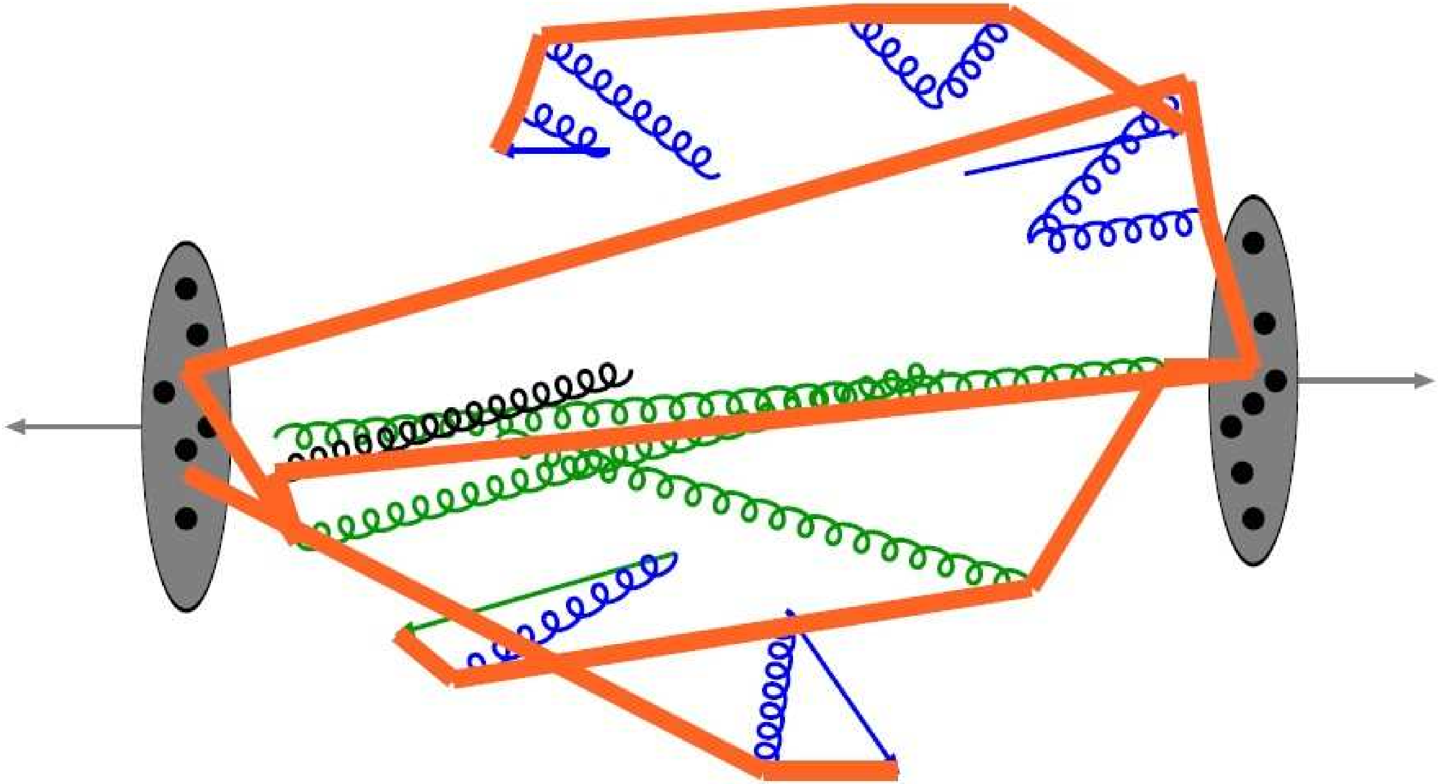
Movie: The structure of an event



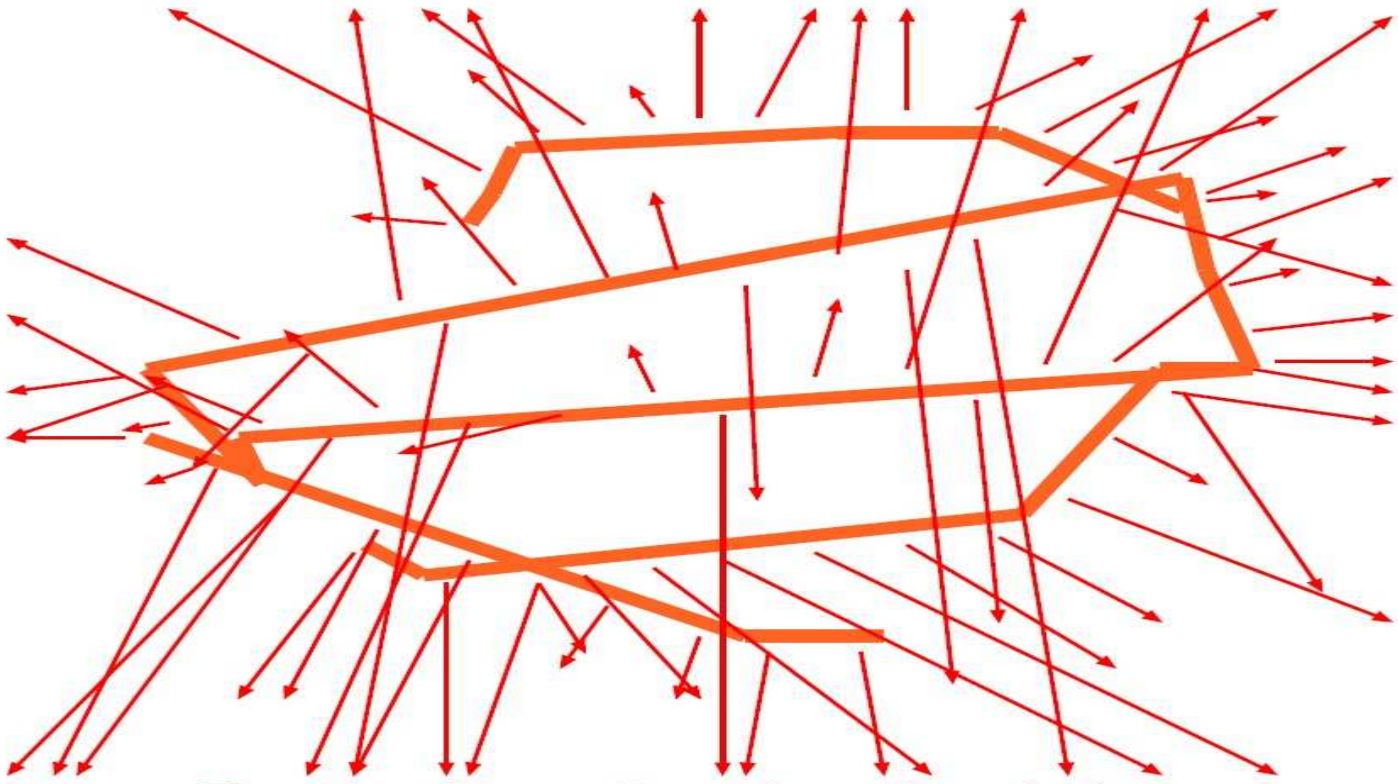
Beam remnants and other outgoing partons !

The muck: UE is not understood. It is modelled through some data that do not give strong constraints and hence extrapolation can be doubted.

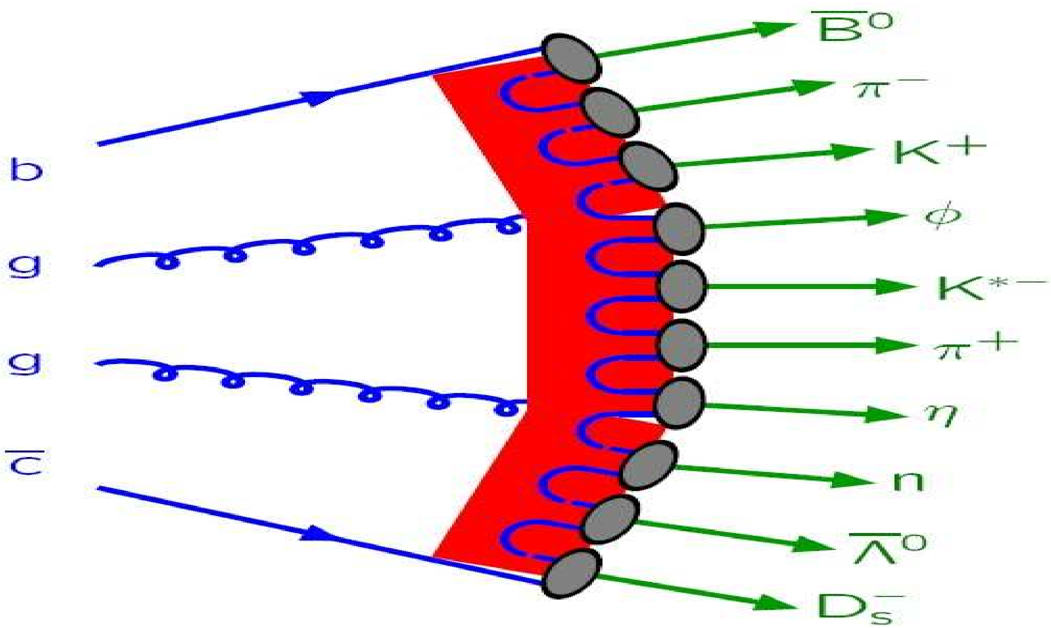
Movie: The structure of an event



Everything is connected by colour confinement (here strings)



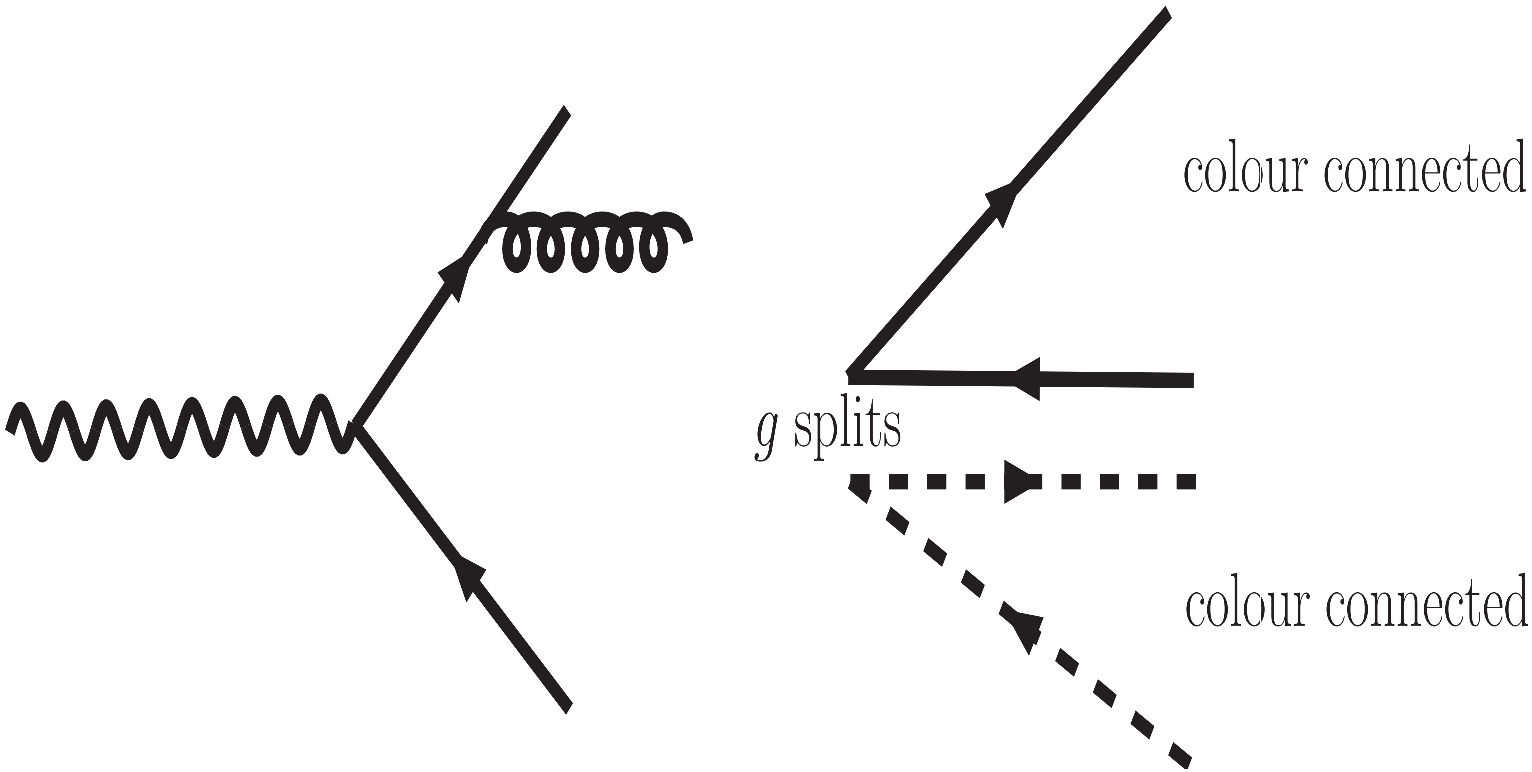
The strings fragment to produce hadrons



Hadronisation: Clusters to produce hadrons (Cluster Model, HERWIG)

Hadronisation and colour connection assumes standard QCD. In some exotic models (Rp violation) nor so trivial to perform this step.

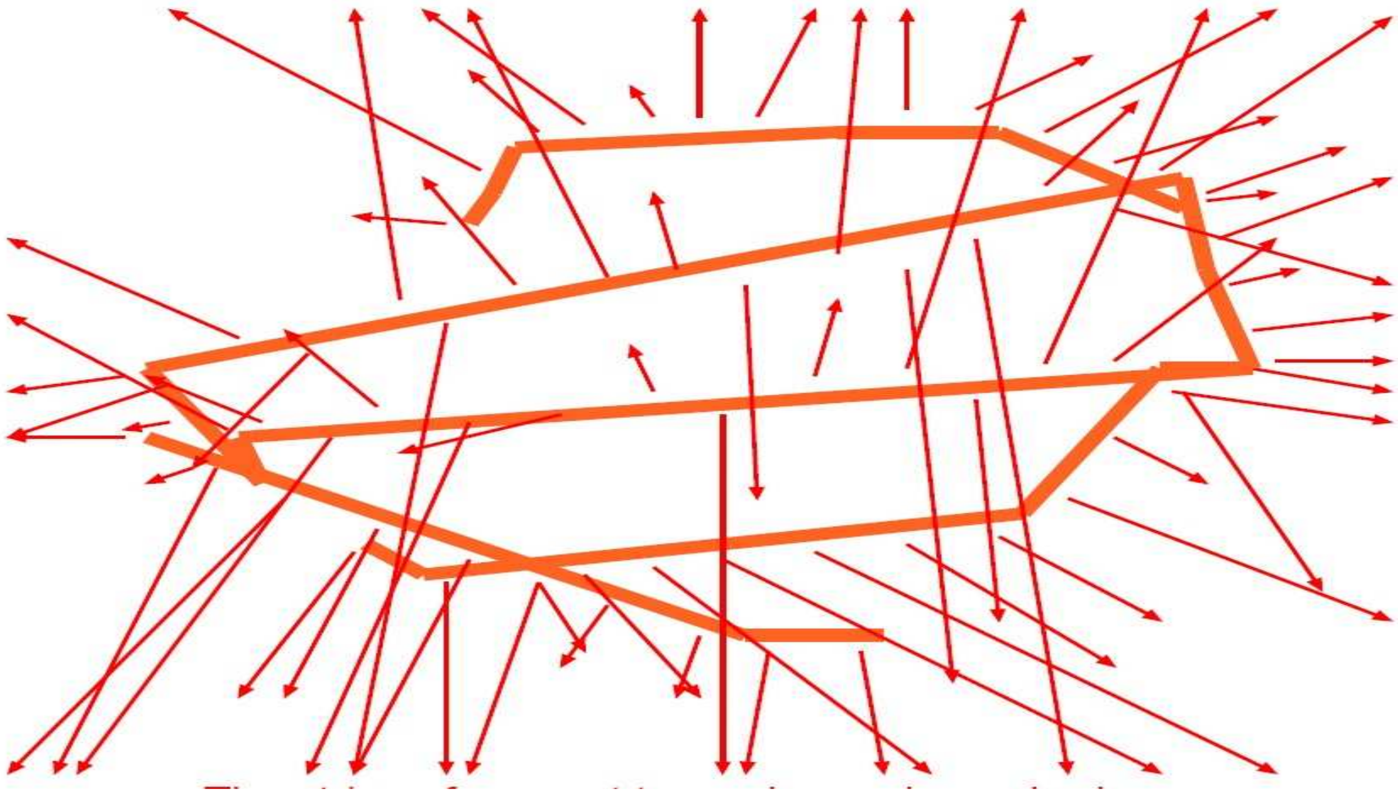
after angular ordering cluster as



$$\mathbf{W}_{R_p \mathrm{viol}} = \cdots + \frac{1}{2}\lambda''_{IJK}\epsilon^{c_1 c_2 c_3}\bar{U}^I_{c_1}\bar{D}^J_{c_2}\bar{D}^K_{c_3}\cdots$$

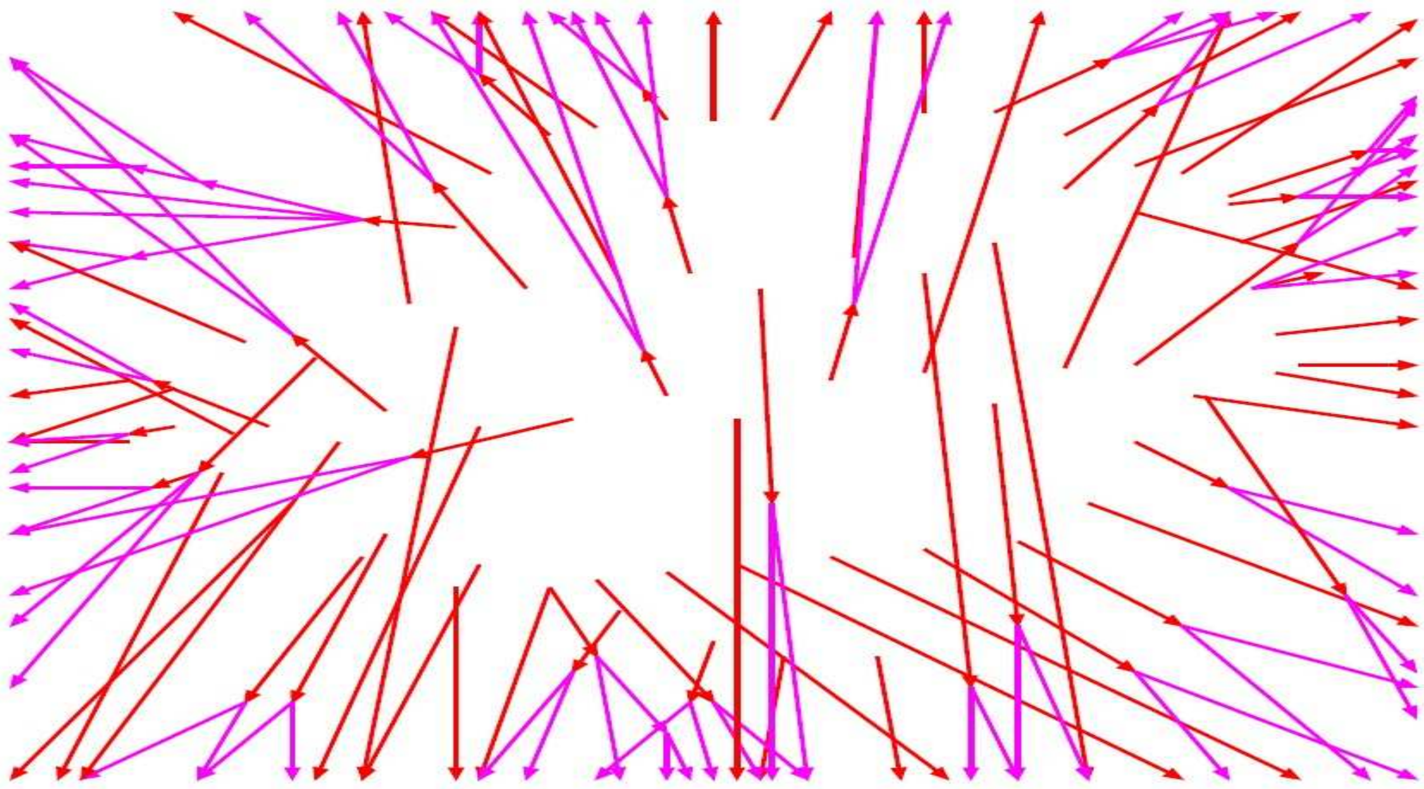
Leads to $\tilde{q} \to qq$ baryon number violation with very exotic colour flow.

You will need a PS/EVG expert!

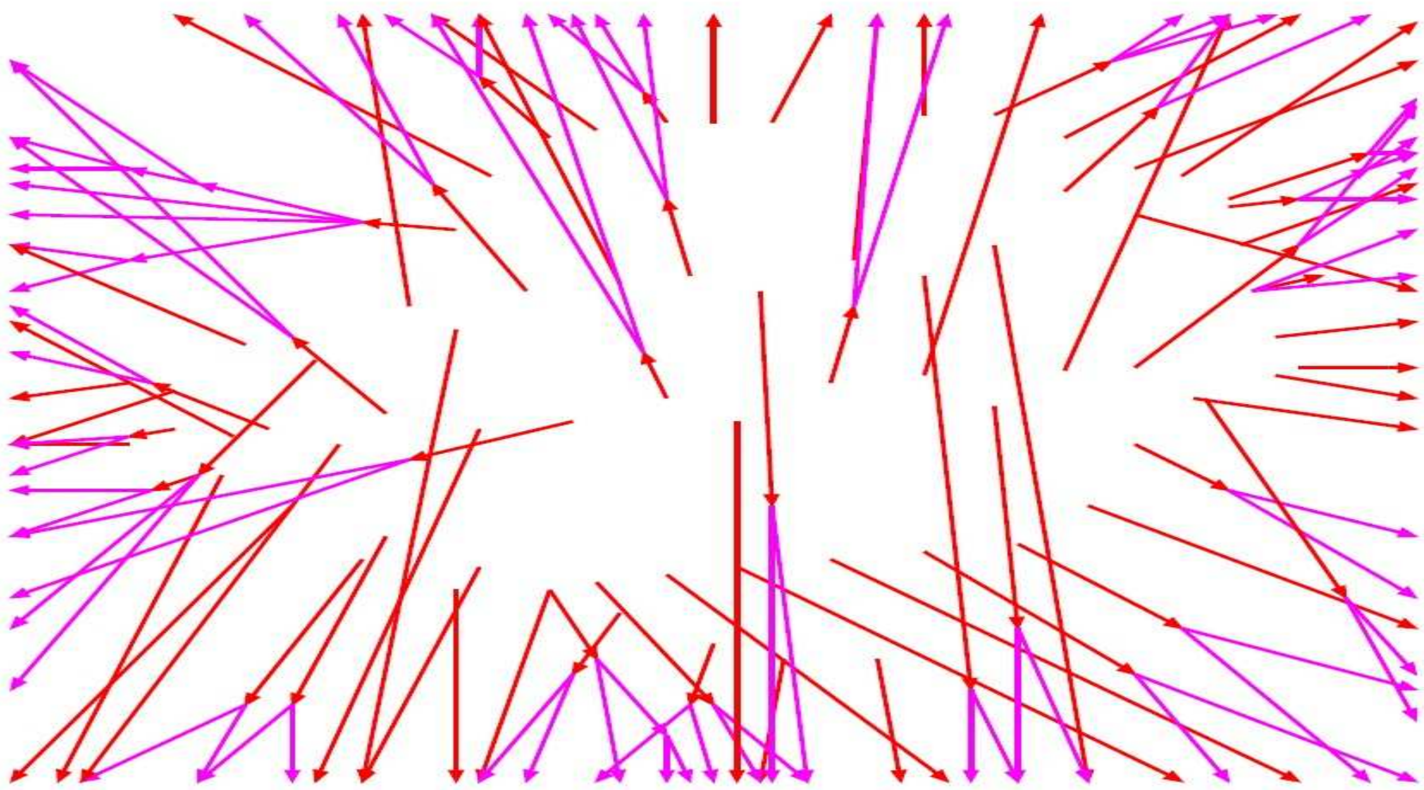


The strings fragments to produce hadrons (strings model)

Hadronisation is not really calculated from first principles, however it is modelled through various data and hence it is considered reliable



Hadrons decay



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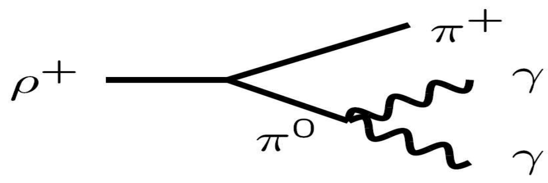
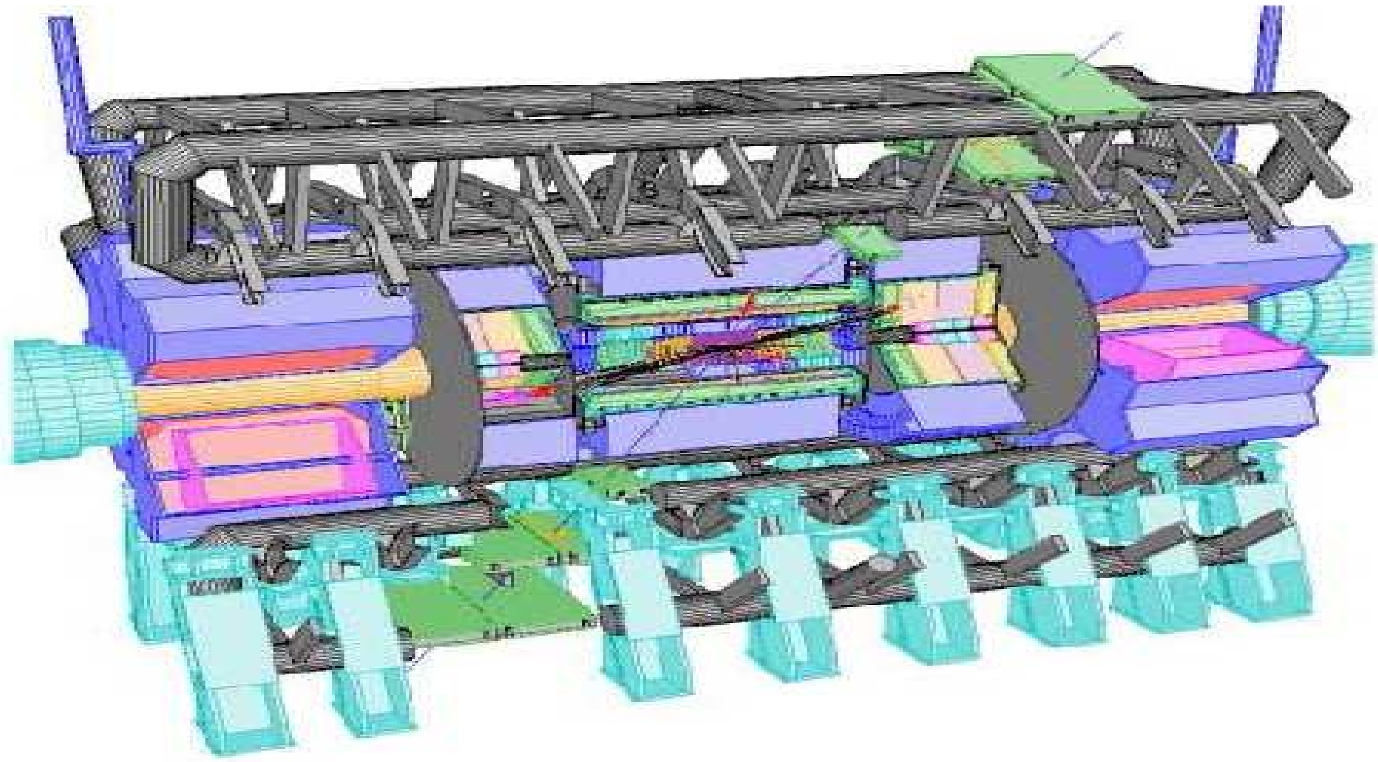


Movie: The structure of an event



These are the particles that hit the detector

- Parton Shower is well understood , perturbation theory with a few approximations
- Hadronisation is not really calculated from first principles, however it is modelled through various data and hence it is considered reliable
- UE is not understood. It is modelled through some data that do not give strong constraints and hence extrapolation can be doubted.(minimum bias??)
- Important to have a “clear” picture of the physical situation

- generate events with as much details as possible:
  W will decay.
  To τ?, τ will decay,
  there is no quark,
  only hadrons,...
  production comes with non negligible radiation
- $\sigma_{\text{final state}} = \sigma_{\text{hard process}} \, \mathcal{P}_{\text{tot}}$
- $\mathcal{P}_{\text{tot}} = \mathcal{P}_{\text{decay}} \mathcal{P}_{\text{ISR}} \mathcal{P}_{\text{FSR}} \mathcal{P}_{\text{remnants}} \mathcal{P}_{\text{hadronise}} \mathcal{P}_{\text{ord. dec.}}$
- **Divide and Conquer : each $\mathcal{P}_{\text{i}}$ handled in turn**
- an event with n particles involve about 10n random choices (flavour, mass, momentum,spin,...). At the LHC expect about 100 charged and 200 neutral particles, thus totalling a few thousand choices

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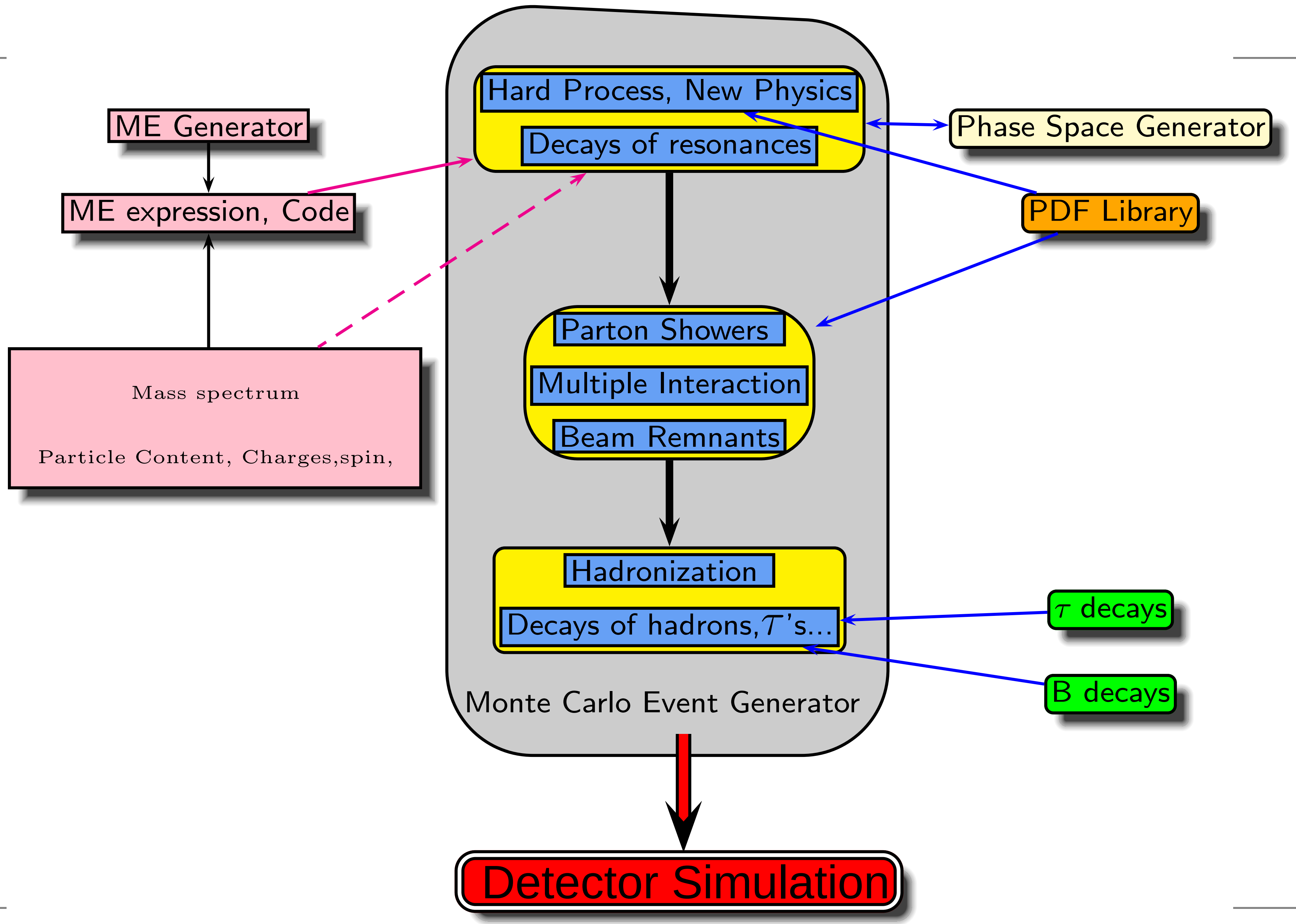
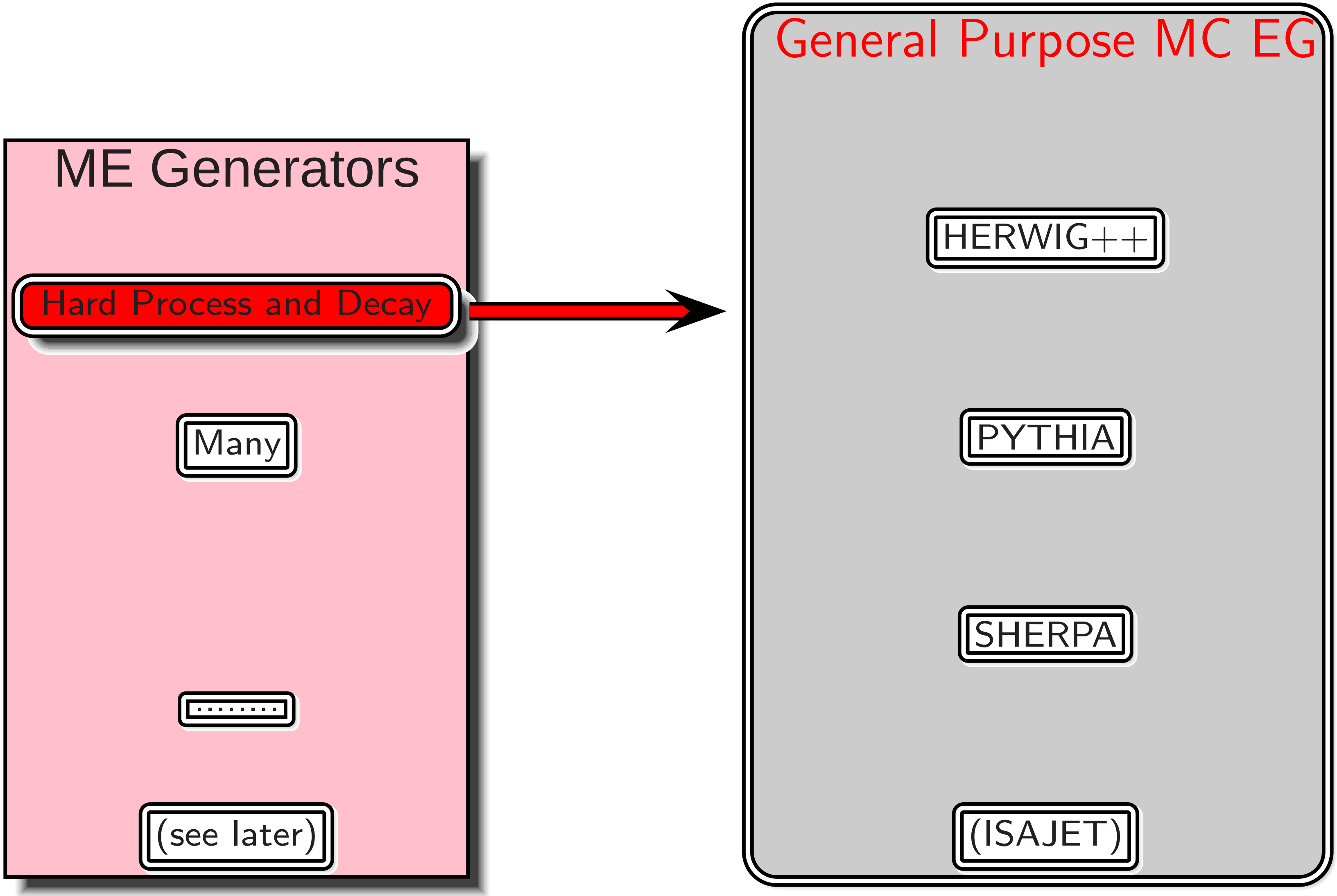
Divide and Conquer : each $\mathcal{P}_{\text{i}}$ handled in turn $\rightarrow$ Modular Structure

Putting all together

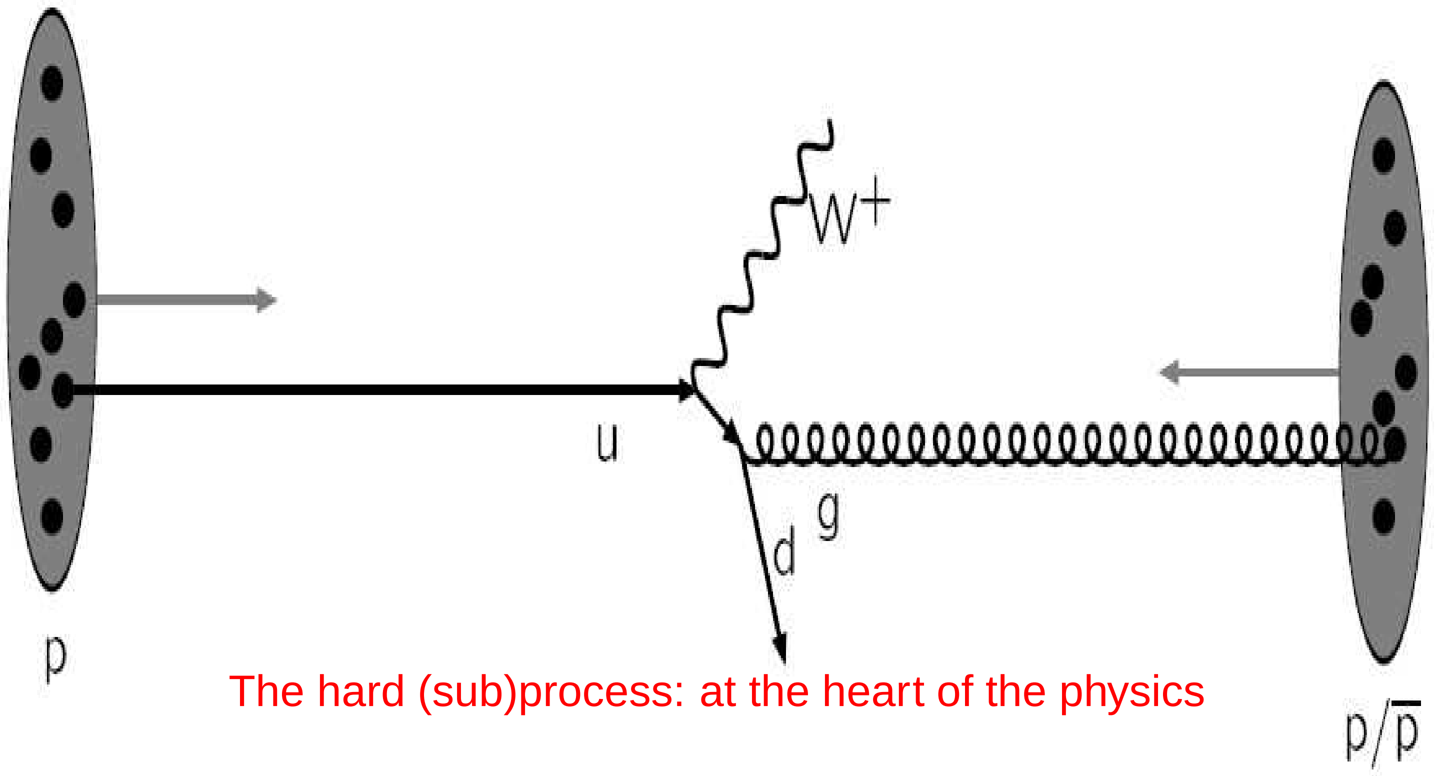


MEG vs General purpose MC EG



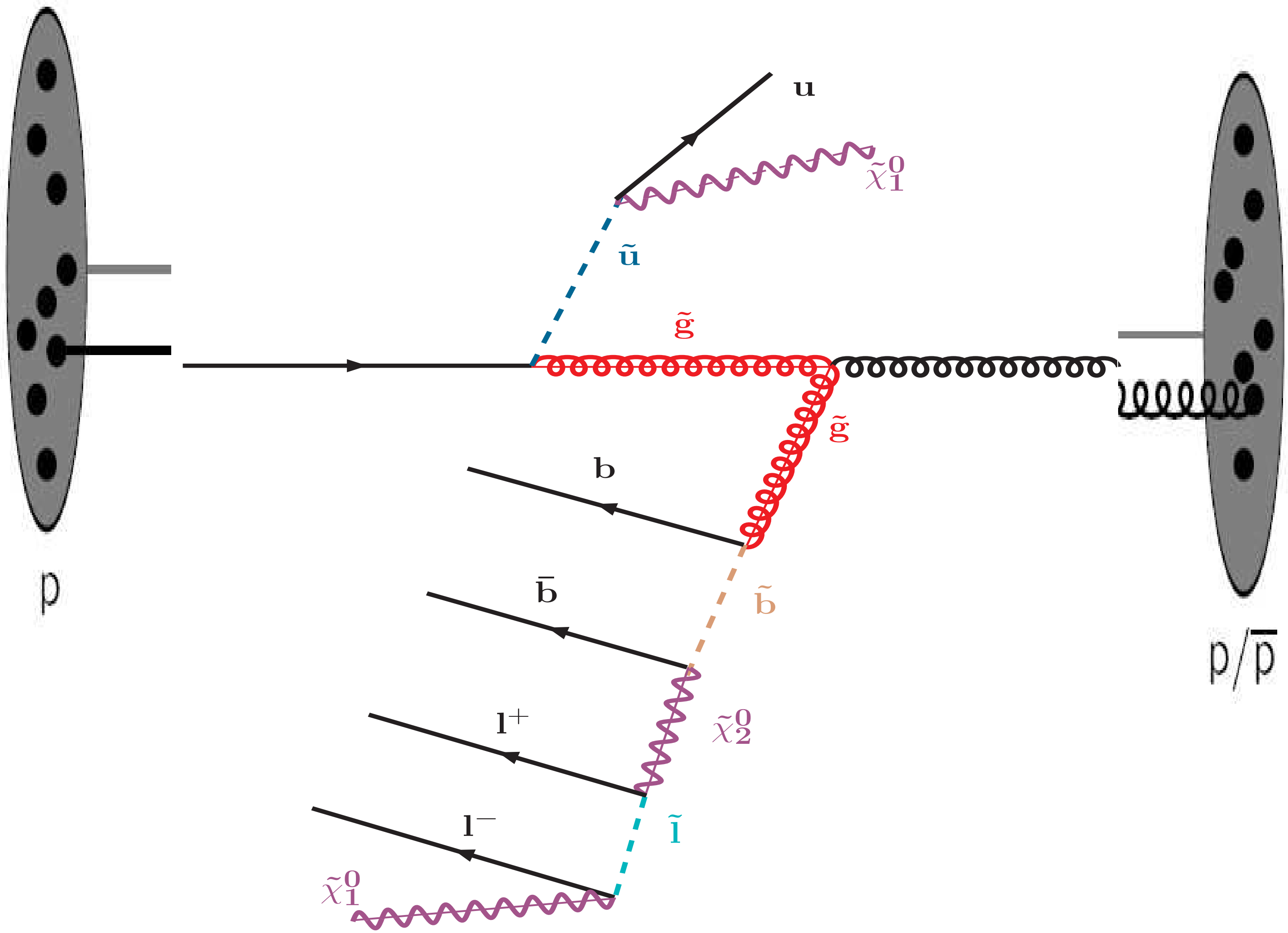
The Hard Process and MEG

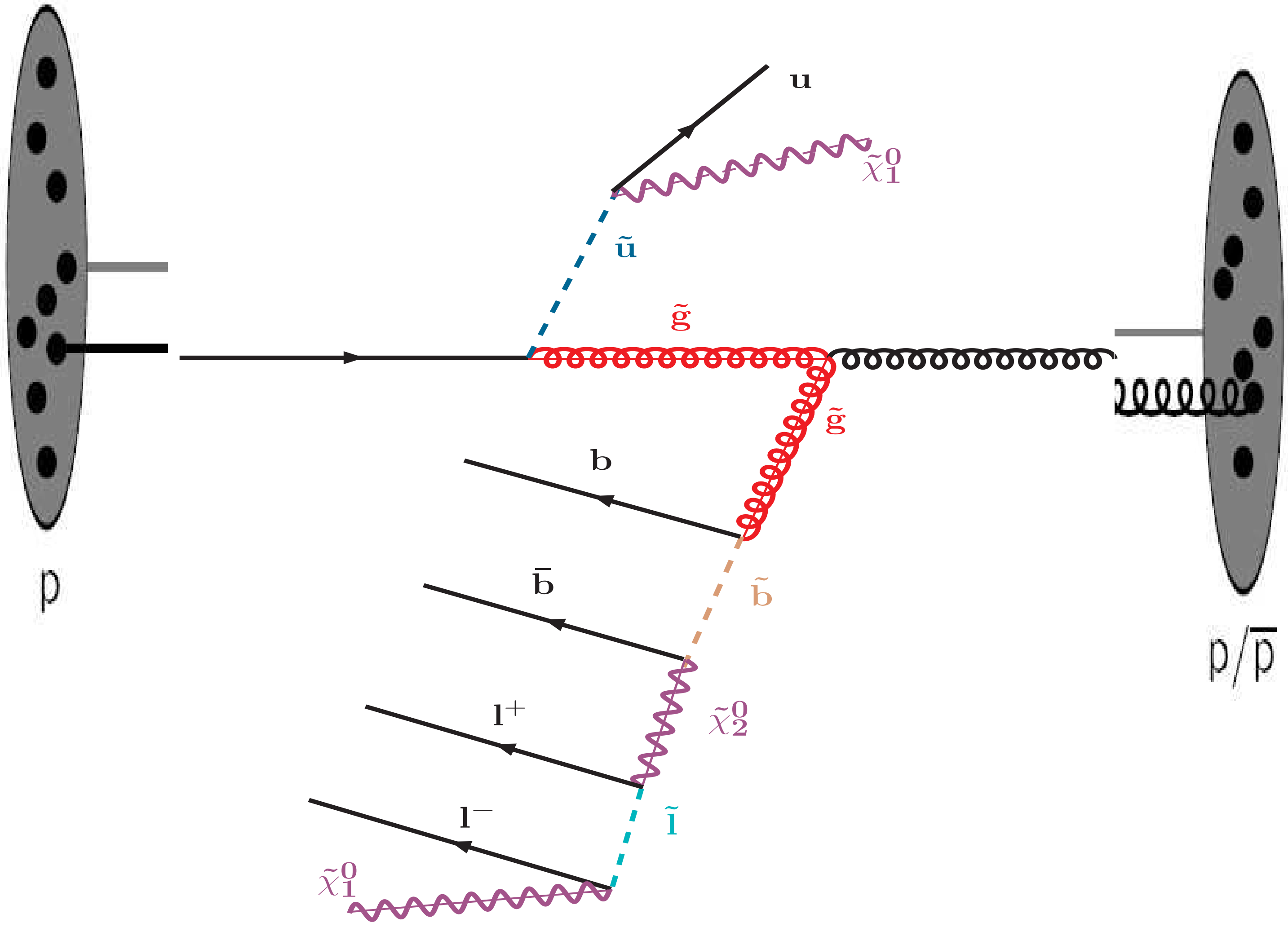
1. Get the partonic matrix element or matrix element squared
2. convolute over the PDF
3. integrate over phase space
4. 2 and 3 means integration which requires numerical evaluation $\rightsquigarrow$ MC techniques



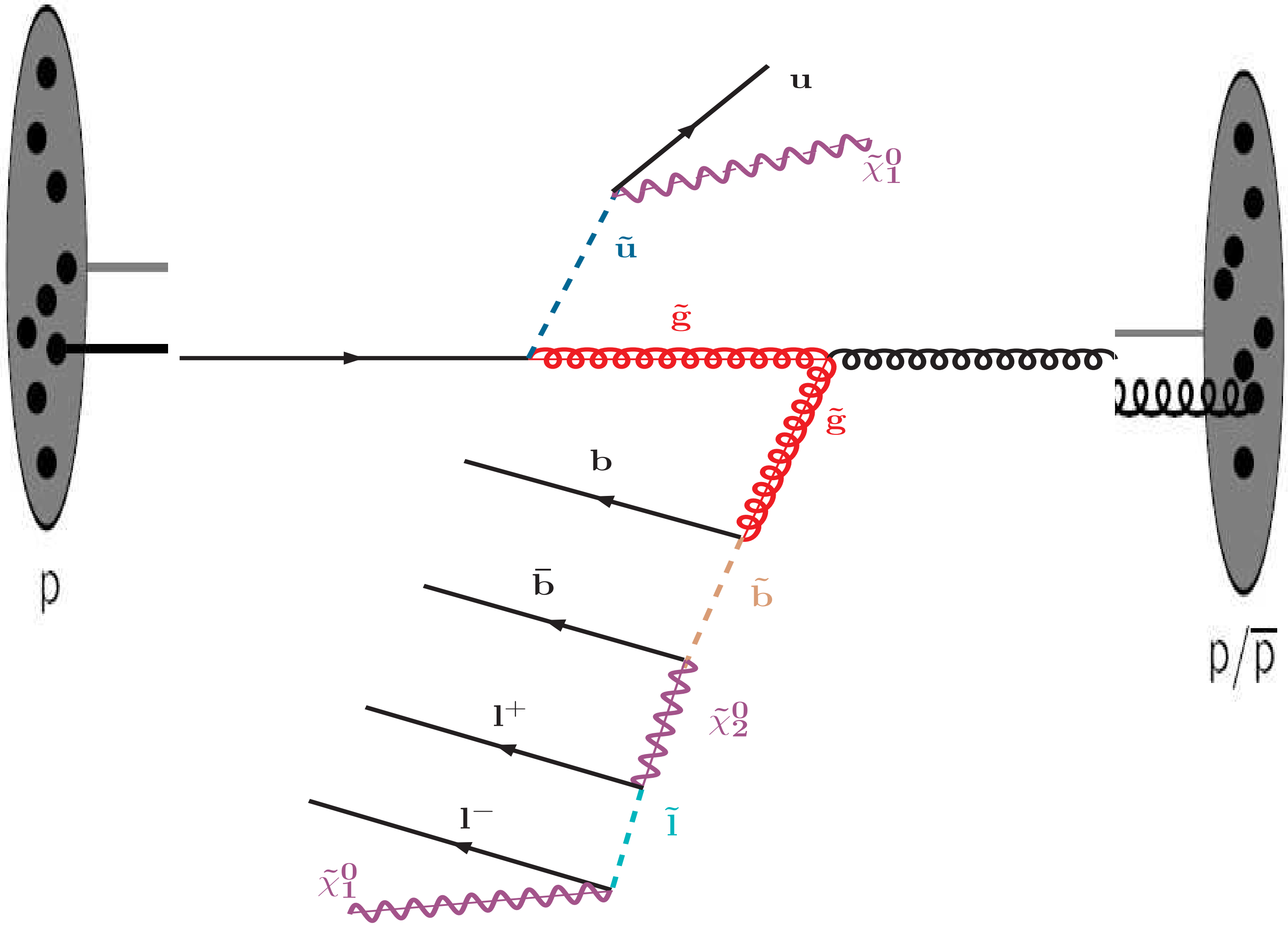
The hard (sub)process: at the heart of the physics

The Hard (sub)Process at the heart of the physics

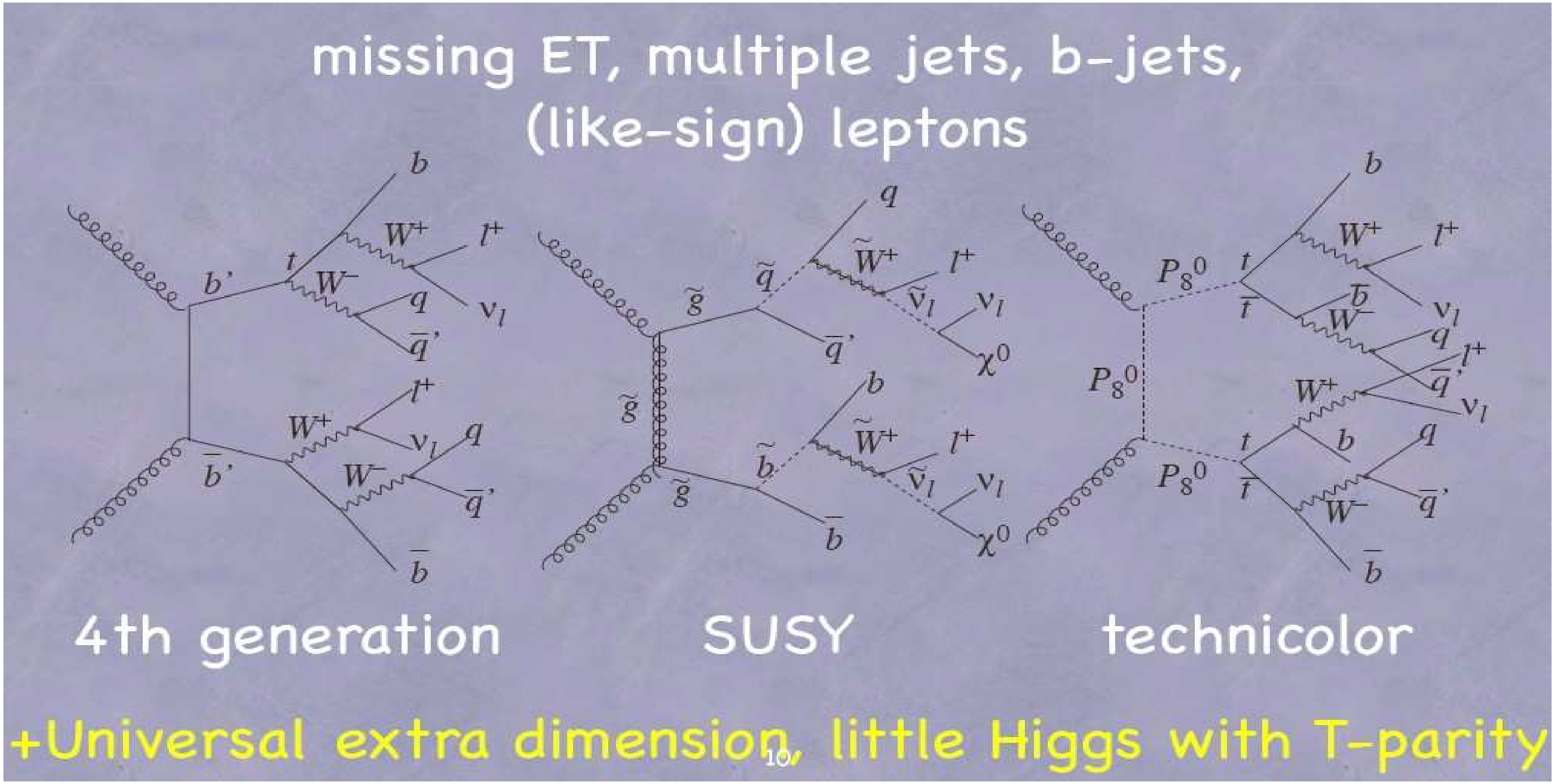




$ug \longrightarrow \tilde{u}\tilde{g}$ or?



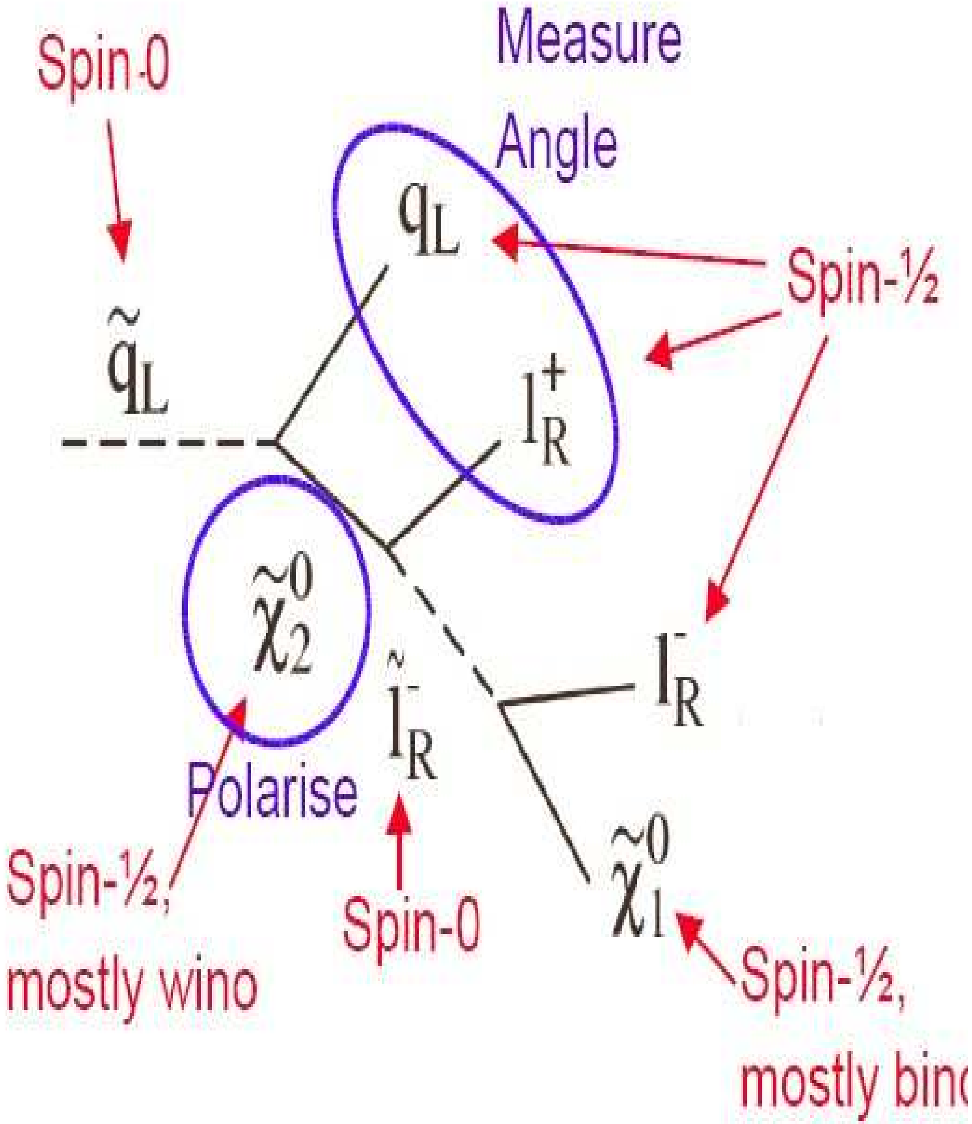
$2 \rightarrow 7$ process, successive decays, huge number of diagrams,...



Is it necessarily DM candidate?

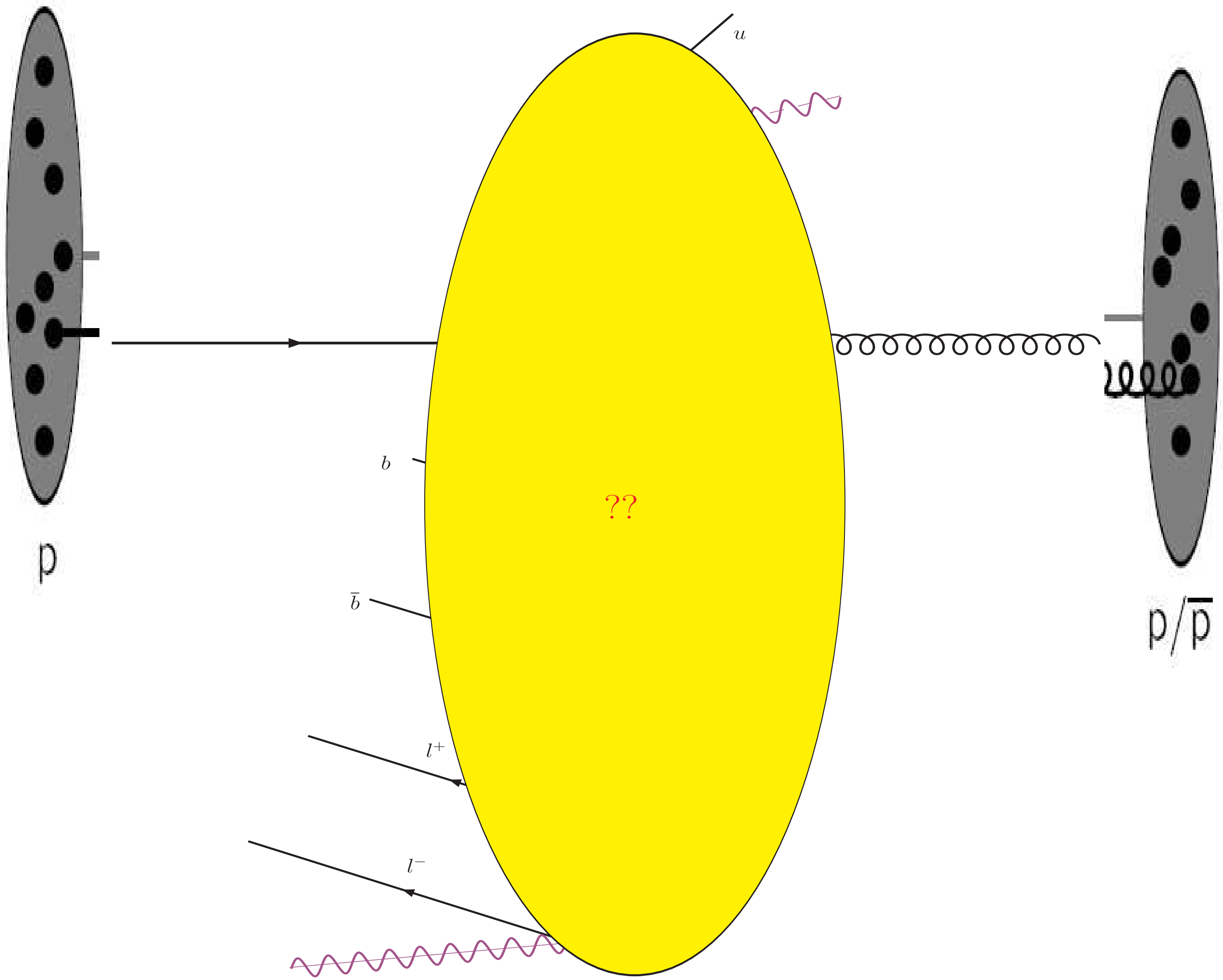
stable at the scale of LHC detectors, 1ms, not age of the Universe....

Next step: Properties of DM as an example, details of couplings and spin assignments.



$$\tilde{q}_L \to \tilde{\chi}_2^0 \quad q$$
$$\hookrightarrow \tilde{\ell}_R^\pm \quad \ell^\mp$$
$$\hookrightarrow \tilde{\chi}_1^0 \quad \ell^\pm$$

Integration: PDF and Cross sections



$$\sigma_{pp\to X} = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 f_a(x_1,\mu^2) f(x_2,\mu^2) \hat{\sigma}_{ab\to X}(\hat{s},\mu^2)$$

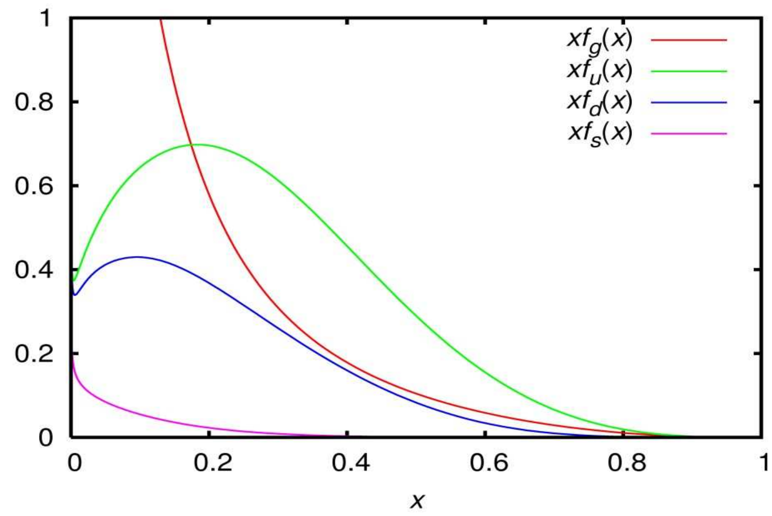
$f_i(x,\mu^2)$ is the **P**arton **D**istributions **F**unction

μ^2 is the factorisation scale !

Many libraries exist (CTEQ, MRSx)

reliable in the range

$10^{-3} < x < 0.8 \;\; (2\mathrm{GeV})^2 < \mu^2 < (1TeV)^2$



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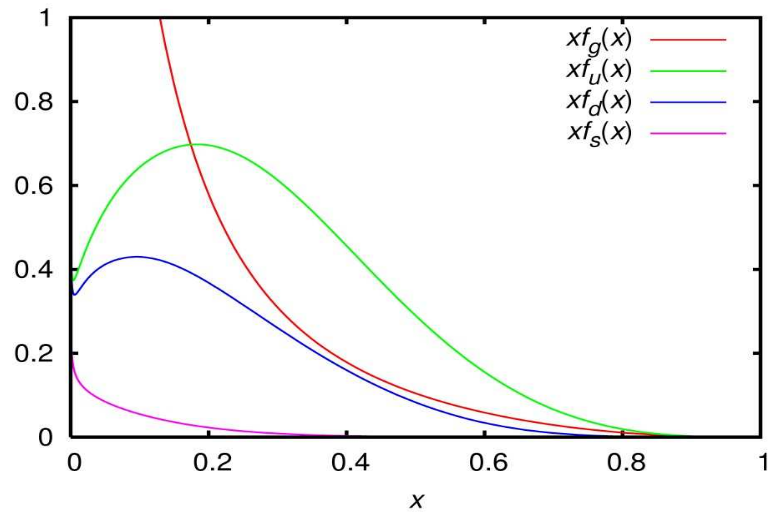

Phase Space

$$\hat{\sigma}_{ab\to X} = \frac{1}{2\hat{s}} \sum_{spin,..} \int_{\Phi_N} |\mathcal{M}|^2 d\Phi_N$$

$$\sigma_{pp\to X} = \sum_{a,b=q,\bar{q},g} \int \mathrm{d}x_1 \mathrm{d}x_2 f_a(x_1,\mu^2) f(x_2,\mu^2) \hat{\sigma}_{ab\to X}(\hat{s},\mu^2)$$

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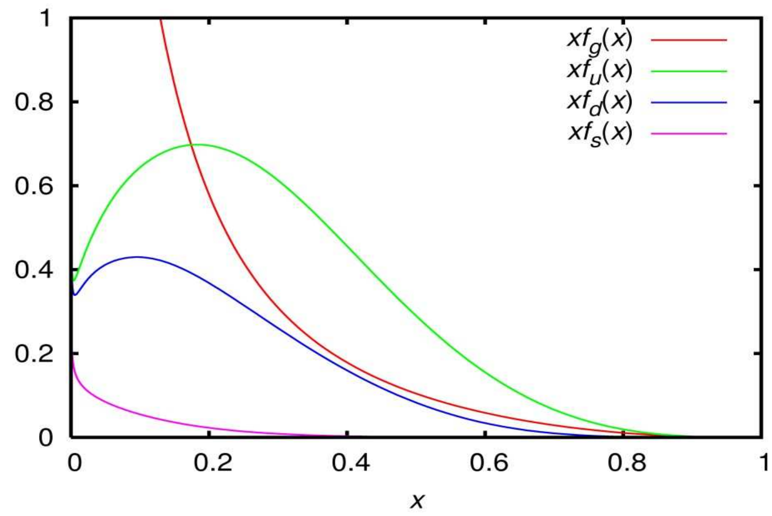

Phase Space

$$\hat{\sigma}_{ab\to X} = \frac{1}{2\hat{s}} \sum_{spin,..} \int_{\Phi_N} |\mathcal{M}|^2 \mathrm{d}\Phi_N$$

Integrals $\longrightarrow \int$

Matrix Elements Generators

Monte Carlo as Integrator

At the heart of the ME is the hard process, that is where the physics lies and that is what gives the probability of a particular event

For the hard process

- amplitude $\mathcal{M} \longrightarrow |\mathcal{M}|^2$ first part of what an MEG should get (the dynamics)
- $N_{\rm evt,cuts} \propto \int d\sigma = \int |\mathcal{M}|^2 {\rm d}\Phi(n)$
- Integration over a phase space with of large number n of dimensions, each particle $\rightarrow 3$ variables (momenta)
- $Dim[d\Phi(n)] \sim 3n$

$$\mathrm{d}\Phi(n) = \left(\prod_i n \frac{\mathrm{d}^2 p_i}{(2\pi)^3 (2E_i)}\right) (2\pi)^4 \delta\left(P_{in} - \sum_i^n p_i\right)$$

different MEG implement different techniques for the integration.

- MC is a numerical method for calculating/estimating an integral based on a random evaluation of the integrand
- Particularly useful because one deals with a large number of (integration) variables (momenta of particles)
- Limits of integration (cuts) are often complicated
- Integrand is a convolution of different functions

- MC integration through random number generation
- MC converges as $1/sqrtN$ for any d-dim integral
- Trapezium,Simpson better but only for d-dim small, not the case of HEP
- Importance sampling (change of variables)
- Importance + Stratified sampling (VEGAS/BASES)
- Multi-channel
- event weight, event generation

No new NEW PHYSICS in here

JUMP ▶

One dimension, example

$$I = \int_{x_1}^{x_2} f(x)dx = (x_2 - x_1) < f(x) > \quad \text{(usually } x_1 = 0, x_2 = 1)$$

The average can be calculated by selecting N values *randomly* $x_i, i = 1, \cdots N$ from *uniform distribution*, calculate $f(x_i)$

$$I = I_N = \frac{1}{N}(x_2 - x_1) \sum_{i=1}^{i=N} f(x_i) = \frac{1}{N} \sum_{i=1}^{i=N} W(x_i) \quad W(x_i) = \text{weight}$$

- Sum is invariant under reordering (randomize)
- Obviously approximation better if number of points N is larger
- Error given by the Central Limit Theorem

One dimension, example. The variance

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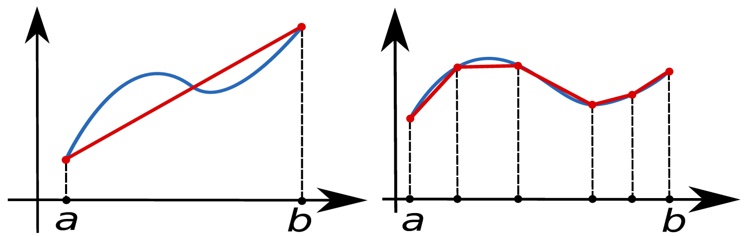

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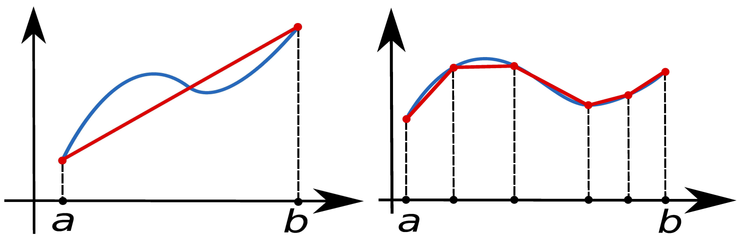
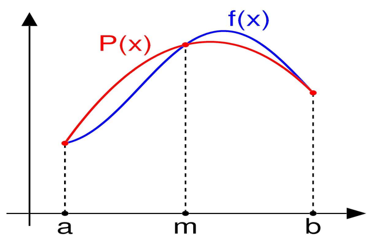


- Simpson (quadratic interpolation) $\propto 1/N^4$ (if derivative exists)



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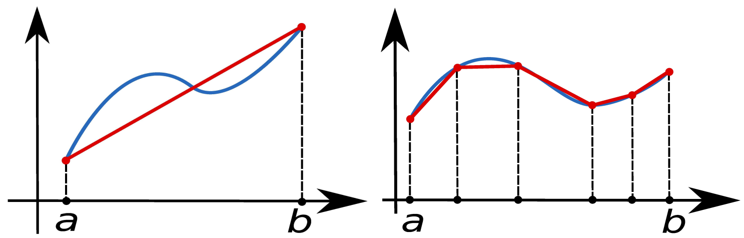

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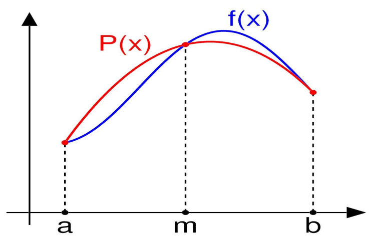


- but this is only in one dimension!

- Convergence may seem slow $\sqrt{1/N}$, but it can be estimated easily
- MC error does not depend on # of dimensions, d, $\propto 1/\sqrt{N}$

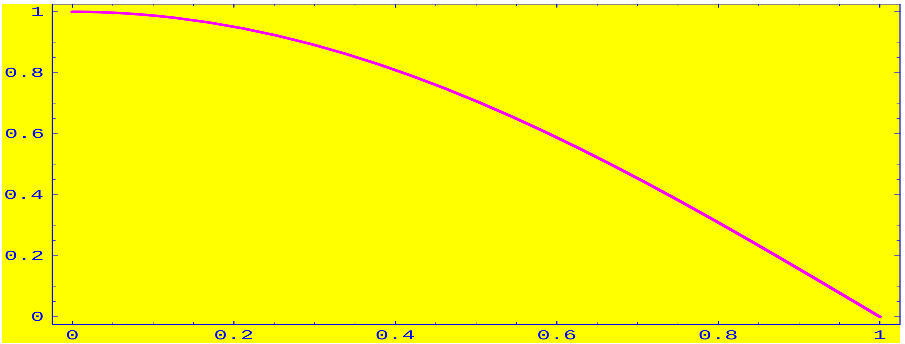
  Trapeze $\propto \rightarrow 1/N^{2/d}$

  Simpson $\propto \rightarrow 1/N^{4/d}$
- in MC one can improve convergence by minimising V_N while keeping the same number of points N
- **Importance Sampling**: non uniform sampling more efficient
- Convergence improved by putting more samples in regions where function is largest (where variance is largest)
- Hint: observe that if $f(x) = cste$ then $V_N = 0 \longrightarrow$ **make f as a close to a constant as possible!**

$$I \simeq I_N \pm \sqrt{\frac{V_N}{N}} = I_N \pm \sqrt{\frac{< f^2 > - < f >^2}{N}}$$

Example: Importance Sampling

Take $f(x) = cos\pi x/2$ then
$I = 2/\pi = 0.637$
MC, $I_N = 0.637 \pm 0.308/\sqrt{N}$
$(0.308 = \sqrt{V_N} = \sqrt{1/2 - (2/\pi)^2})$



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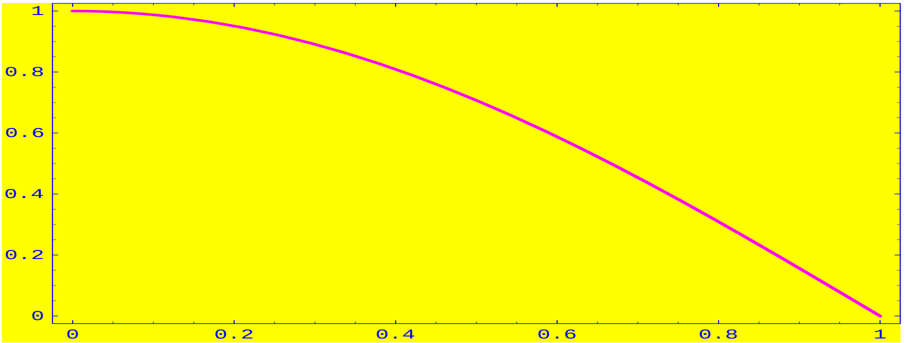
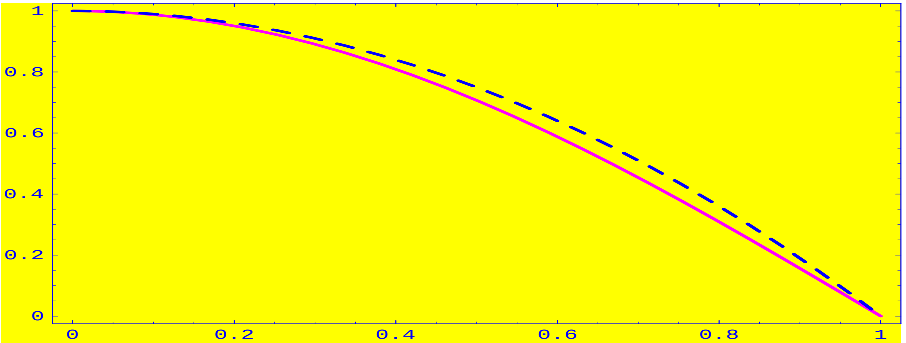


$$I = \int_0^1 dx(1-x^2)\frac{cos\pi x/2}{1-x^2} = \int_{y_1}^{y_2} dy \frac{cos\pi x[y]/2}{1-x[y]^2}$$



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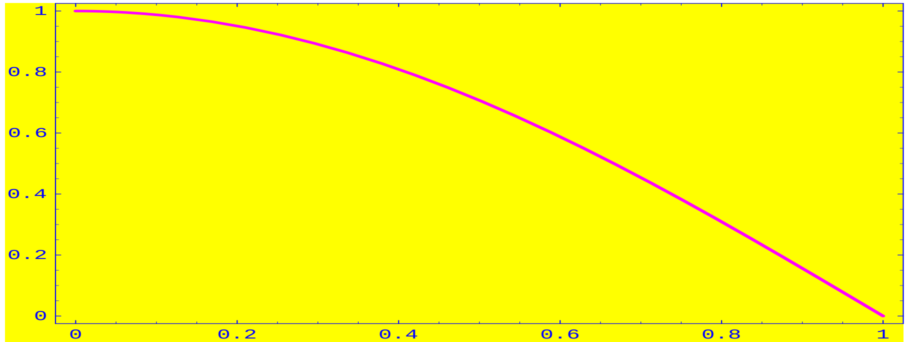

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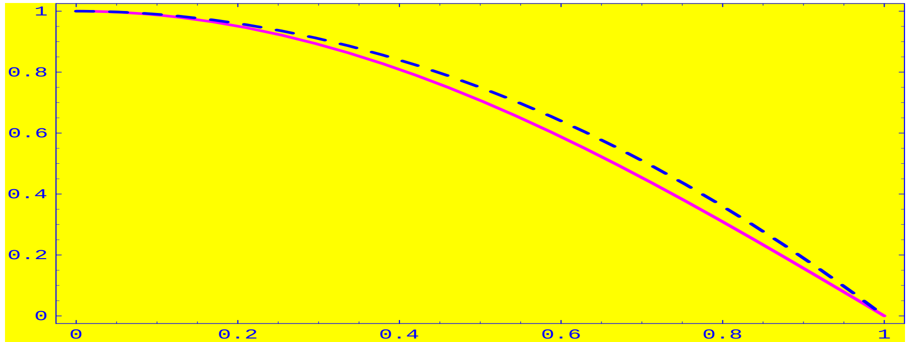

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- For the same accuracy $N \to N/100$ events
- We have in fact made a change of variables
- Note however that change of variables may be not so trivial and requires that one knows the function, here is relatively ok
  $y = x - x^3/3$!

in HEP many sharp peaks from resonances, apart from peaks due to forward scattering,..

$$
\begin{aligned}
I &= \int_{m^2_{min}}^{m^2_{max}} \mathrm{d}m^2 \frac{1}{(m^2-M^2)^2+M^2\Gamma^2}, \quad \Gamma/M \ll 1 \\
&= \frac{1}{M^2}\int_{x_{min}}^{x_{max}} \mathrm{d}x \frac{1}{(x-1)^2+\epsilon^2}, \quad x = m^2/M^2, \quad \epsilon = \Gamma/M
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Over a Breit-Wigner distribution

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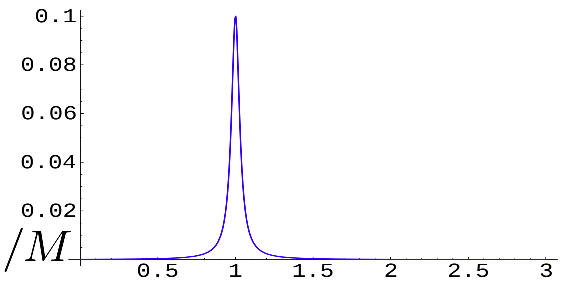

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The peak has been smoothed out completely.

Flat distribution, the error has been reduced to 0

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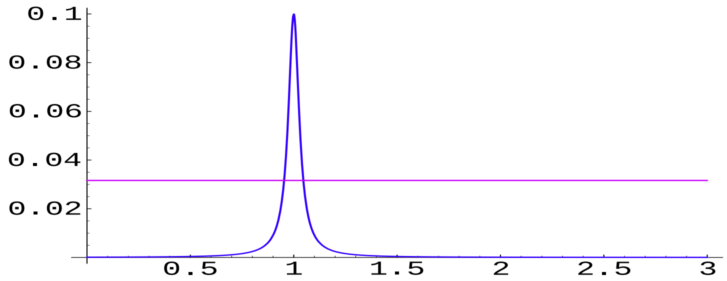

Unfortunately we can not always do the Jacobian trick efficiently, we do not always know $f(x)$

However, as we have seen, finding a simple function, $p(x)$, that approximate $f(x)$ reduces the error drastically

(up to normalisation) take

$$p(x), \int_{x_1}^{x_2} p(x) = 1, \qquad \rightarrow I = \int_{x_1}^{x_2} dx p(x) \frac{f(x)}{p(x)}$$

$$I = \langle \frac{f}{p} \rangle \pm \frac{1}{\sqrt{N}} \sqrt{\langle \frac{f^2}{p^2} \rangle - \langle \frac{f}{p} \rangle^2}$$

Sample according to $p(x)$ and make f/p as small as possible.

Unfortunately we usually do not know much about $f(x)$

But as we sample we can know more, reconstruct $p(x)$ piecemeal, with step function

$$p(x) = \frac{1}{N_b}\Delta x_i \quad \text{for} \quad x_i - \Delta x_i \leq x \leq x_i$$

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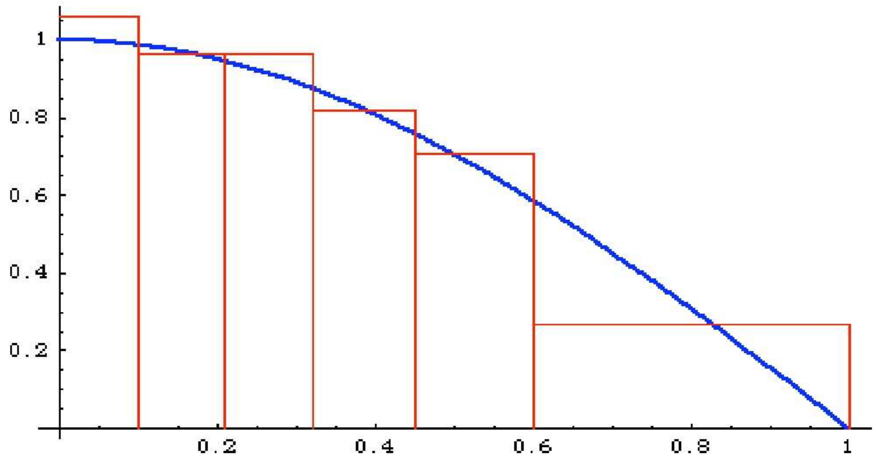

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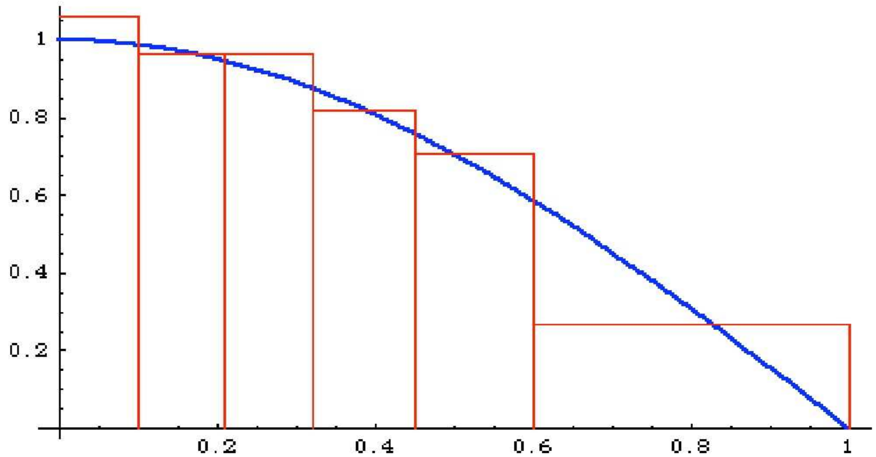

- Improve the fit by generating more points where $f(x)$ is large, *i.e* where the variance is large
- Adjust the bin size so that each bin has the same area

Iterative algorithm: VEGAS

Many variables, VEGAS bis

- The approach can be directly generalised to d dimensions if one can write the factorised from $p(\vec{x}) = p(x) \times p(y) \times \cdots$

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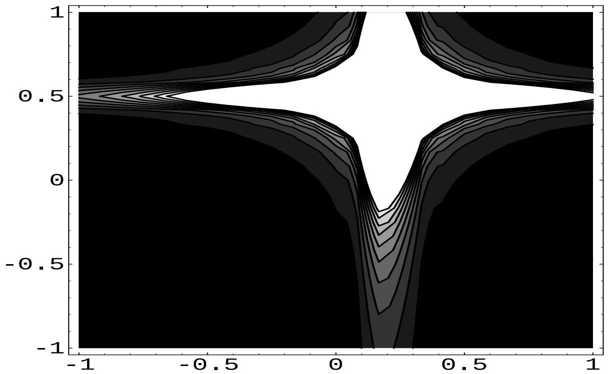
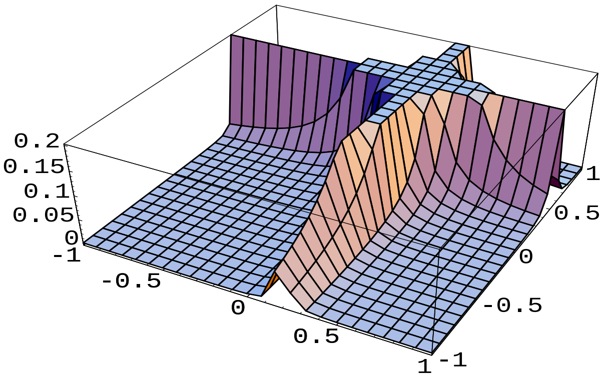
- this means 2^n possible kinematical invariants
- A scattering amplitude may have many peaks each aligned on a different invariant

VEGAS and alignment





$$\left(((x-0.2)^2+\epsilon)((y-0.5)^2+\epsilon)\right)^{-1}$$

$$\epsilon = 0.002$$

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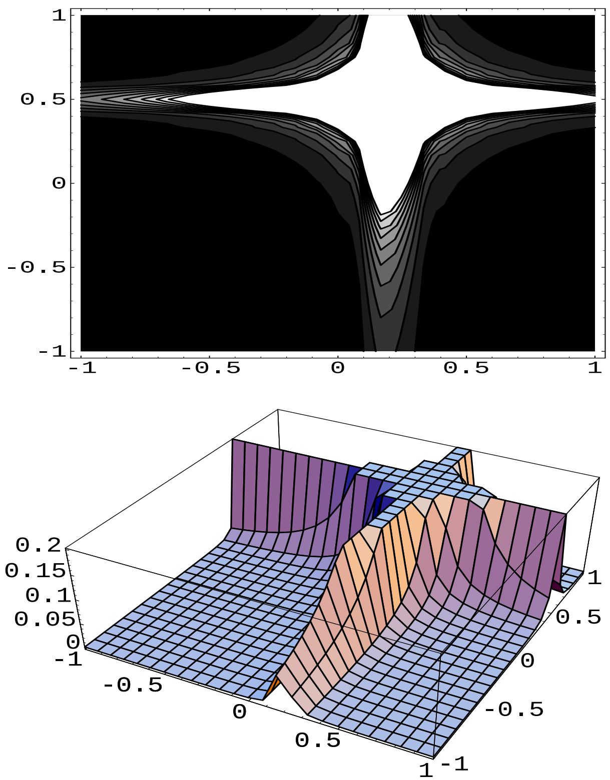

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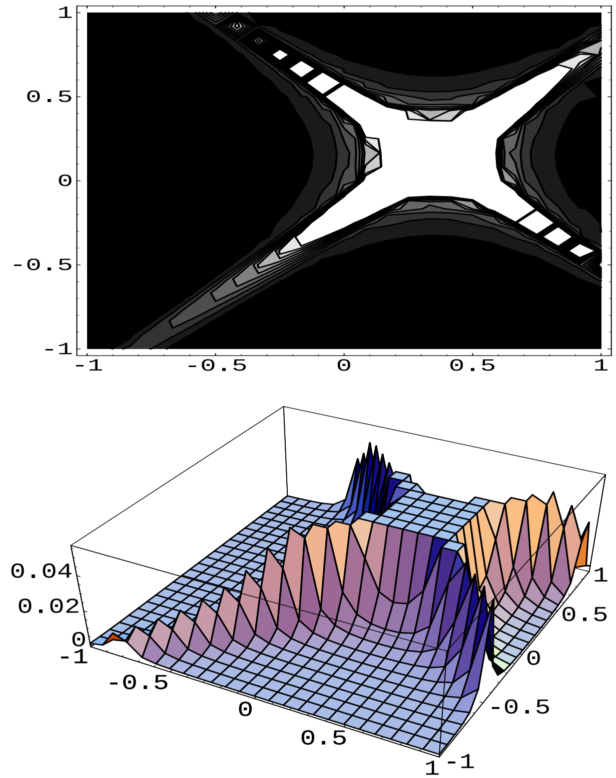

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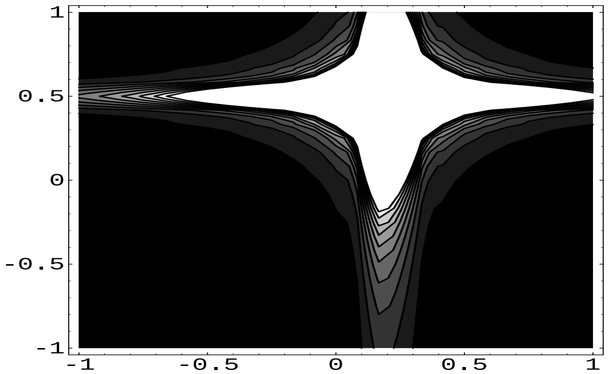

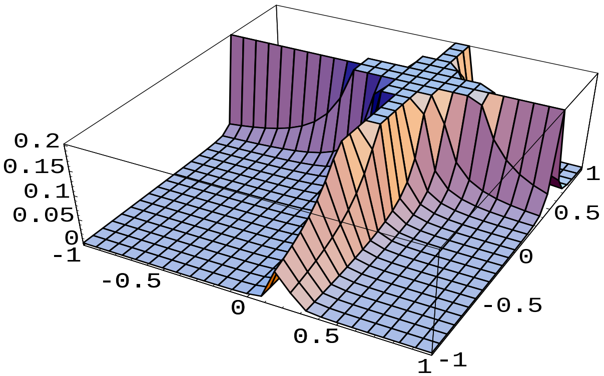

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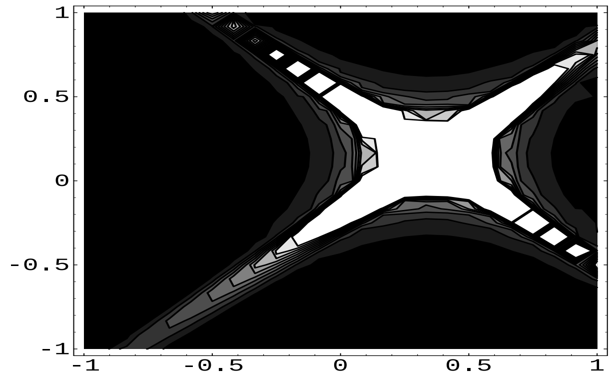

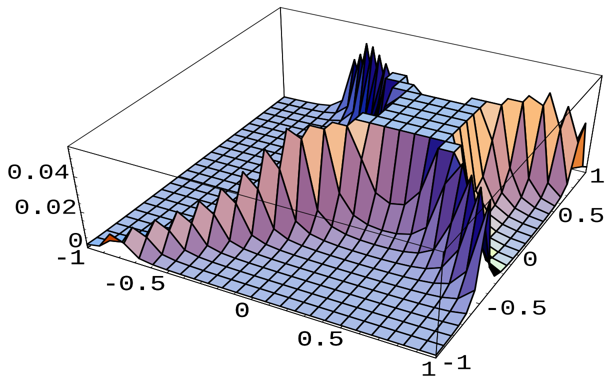

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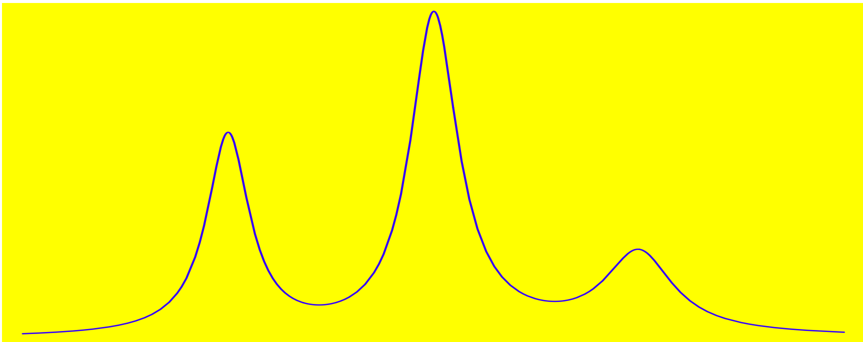

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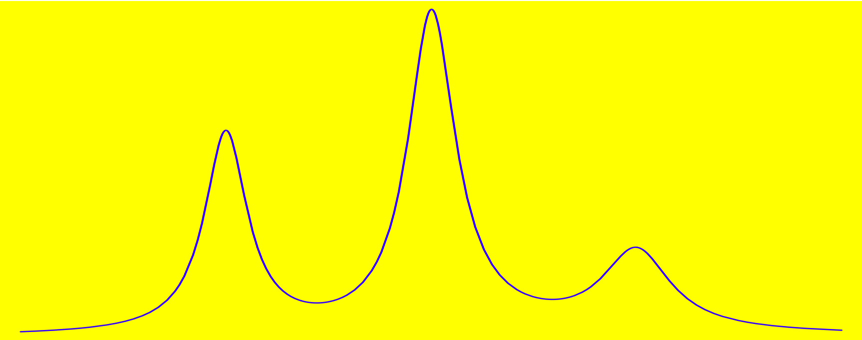

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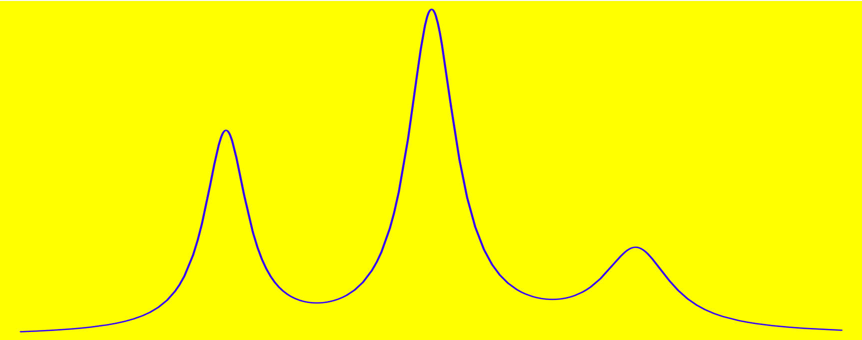

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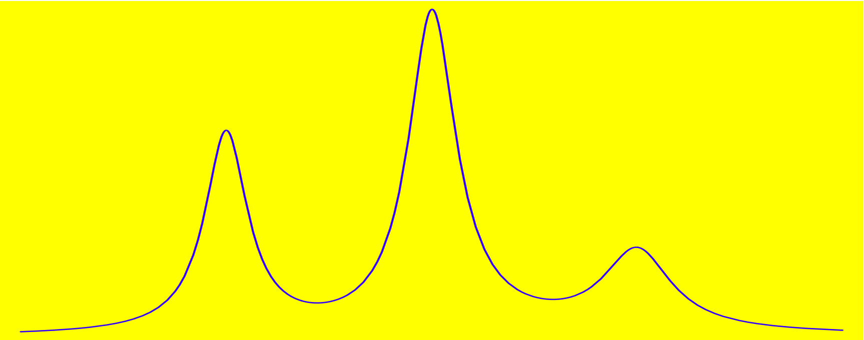

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Pick one of the integrals (channels) with prob α_i then calc. weight

α_i can be automatised. $\int$ does not depend on α_i but V_N does

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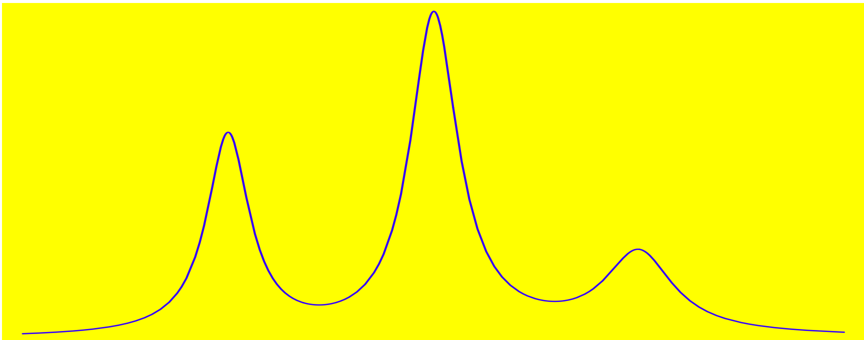

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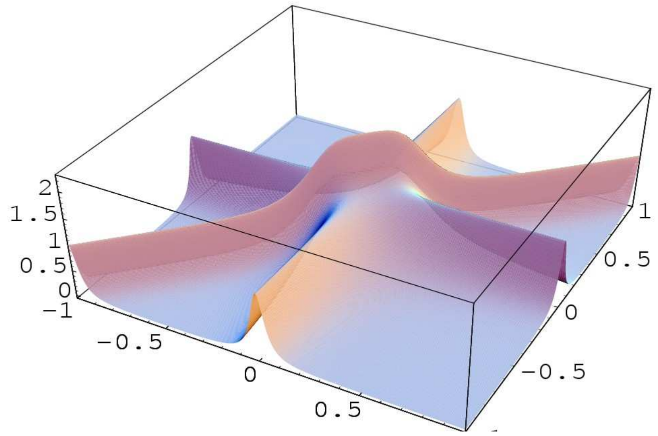
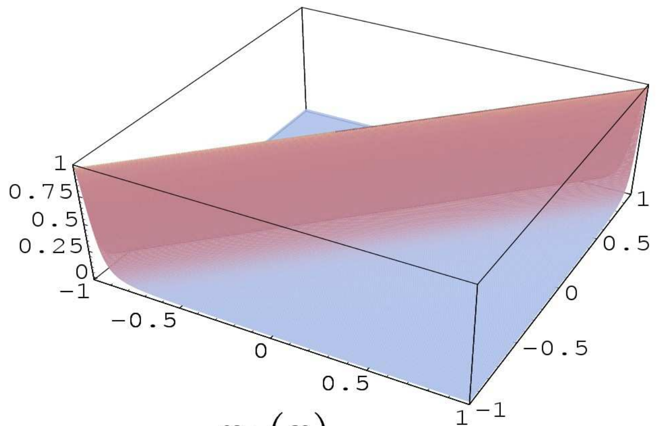
In general not each channel is invertible $\rightarrow g_i$ (peaking may be more complicated)
N coupled equations for α_i, so best when number of channels small.

This is the method (multi-channel) used in the most sophisticated codes.

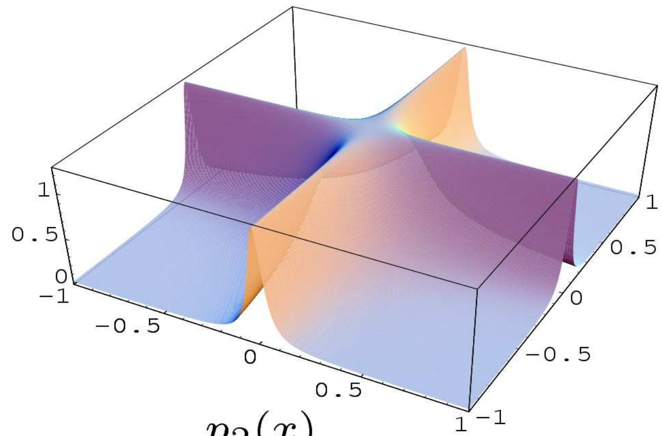
Multichannel, many dimensions



- what to do here?
- decompose into different channels
- $\alpha_1 p_1(x) + \alpha_2 p_2(x)$



$p_1(x)$



$p_2(x)$

For physical processes we usually know where the peaks are

cross section integrator vs event generator

$$\mathrm{d}\sigma(u\bar{u} \to Z^0 \to d\bar{d}) = \frac{1}{\hat{s}}|\mathcal{M}|^2 \frac{\mathrm{d}\cos\theta \mathrm{d}\phi}{8(2\pi)^2}$$

- sample the phase space (2-dim) $-1 < \cos\theta < 1,\, 0 < \phi < 2\pi$
- choosing $\cos\theta, \phi$ variables using uniformly distributed random number generator defines a candidate event
- $\mathrm{d}\sigma$ is the event weight (probability of the event)
- $< \mathrm{d}\sigma > \sim \int \mathrm{d}\sigma$ converges to the cross section
- at this point candidate events $\theta\phi$ are distributed flat and carry no physics

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If function to be integrated is a probability density (positive definite, $f(x) > 0$) one can convert it to arrive at a simulation of physical processes or Event Generator

- In addition to calculating the integral we often also want to select values of x (momenta,..) *at random according to $f(x)$*. This is easy provided that we know the maximum value of the function in the region we are integrating over.
- Then we randomly generate values of x in the integration region and keep them with probability

$$\mathcal{P} = \frac{f(x)}{f_{\max}} \leq R$$

- which is easy to implement by generating a random number between 0 and 1 and keeping the value of x if the random number R is less than the probability.
- This is called unweighting.

Selection of x according to $f(x)$, in a random probabilistic way, event as they occur in Nature

$$\int_{x_{min}}^{x} f(z)dz = R \int_{x_{min}}^{x_{max}} f(z)dz = RI$$

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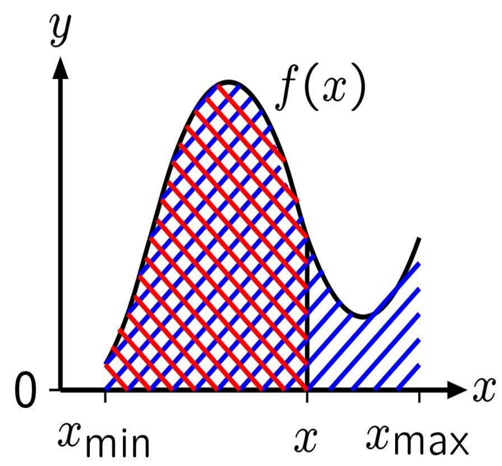
- **Analytical (assumes primitive and its inverse known)**

$x = F^{-1}(F(x_{min}) + R\, I)$



Selection of x according to $f(x)$, in a random probabilistic way, event as they occur in Nature

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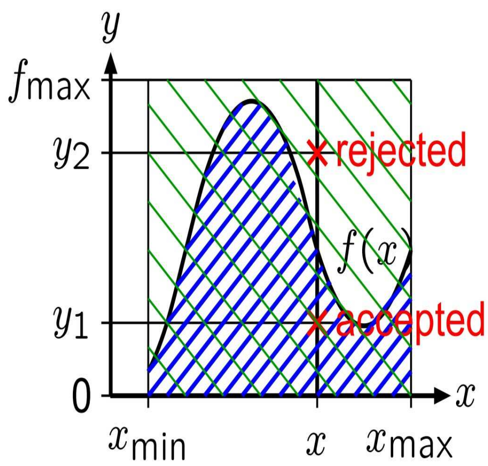
●● **Hit and miss: assumes f_{max} known**



$$I = \int_{x_{min}}^{x_{max}} f(x)dx = f_{max}(x_{max} - x_{min})\frac{N_{acc}}{N_{tries}}$$

$$\frac{N_{acc}}{N_{tries}} = \text{efficiency}$$

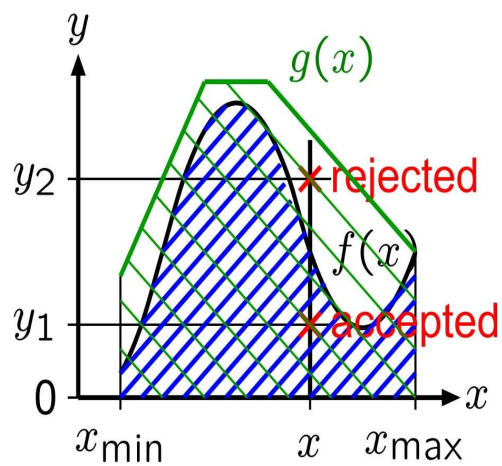
MC $\longrightarrow$ Event Generator, involves acceptance/rejection

Selection of x according to $f(x)$, in a random probabilistic way, event as they occur in Nature

$$\int_{x_{min}}^{x} f(z)dz = R \int_{x_{min}}^{x_{max}} f(z)dz = RI$$

• • •**Importance Sampling: take $f(x) < g(x)$ where $G(x)$ and G^{-1} simple**

1. select x according to $g(x)$ 2. select $y = Rg(x)$ (new R) if $y > f(x)$ go back to 1

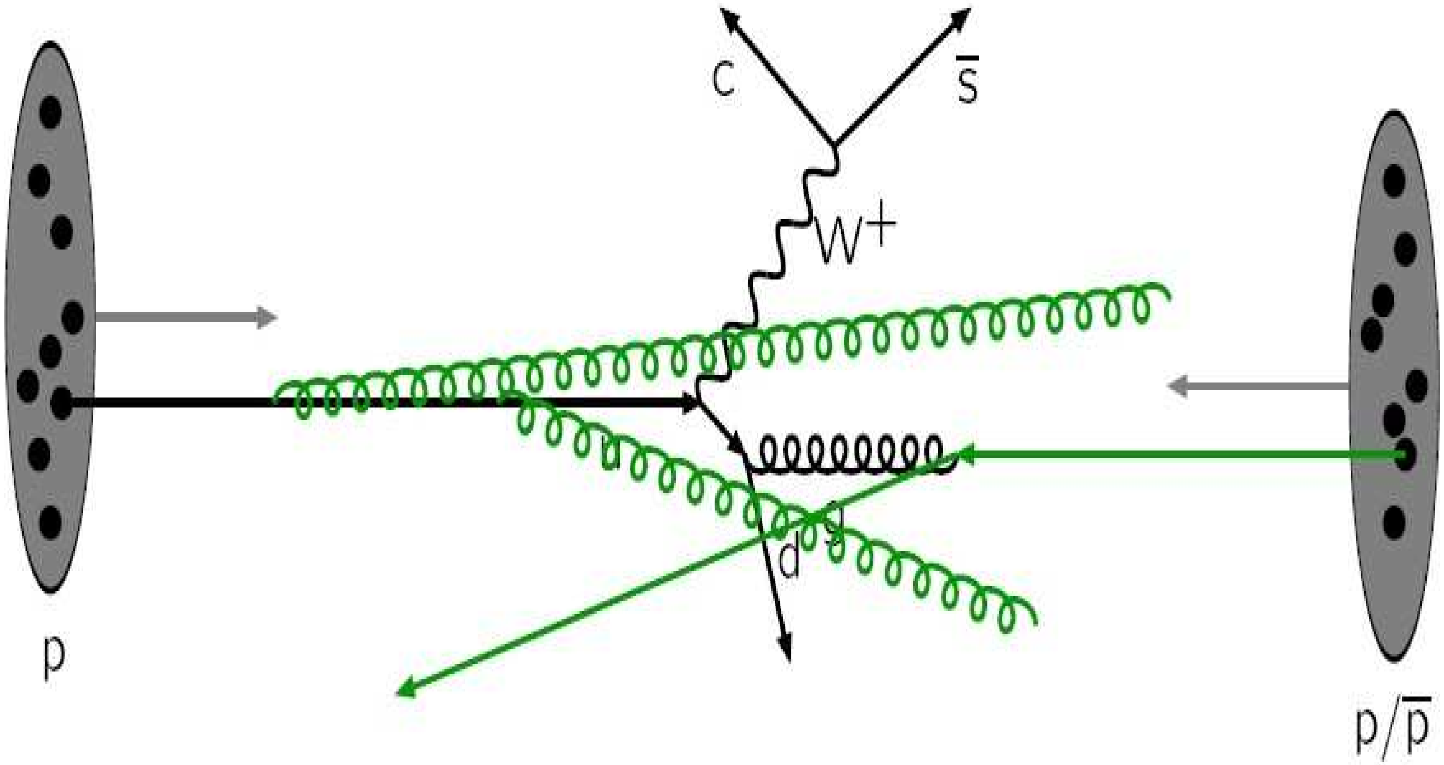


- Advantages of Monte Carlo Fast convergence in many dimensions
  - Arbitrarily complex integration regions
  - Few points needed to get first estimate
  - Each additional point improves the accuracy
  - Easy error estimate
  - More than one quantity can be evaluated at once.
- Disadvantages of Monte Carlo Slow convergence in few dimensions, but that is hardly the case in particle physics
- MC is well suited for particle physics where phase space integration involves a lot of variables with a complicated often not smooth function representing the cross section

- With an integrand that is positive definite, which is the case for MC at LO, one deals with a probability. This lends itself to an event generator
- Allows a fully exclusive treatment exactly like *real data*
- At the most basic level a Monte Carlo event generator is a program which simulates particle physics events with the same probability as they occur in nature.
- In essence it performs a large number of integrals and then unweights to give the momenta of the particles which interact with the detector

Event Generators

Parton Showers

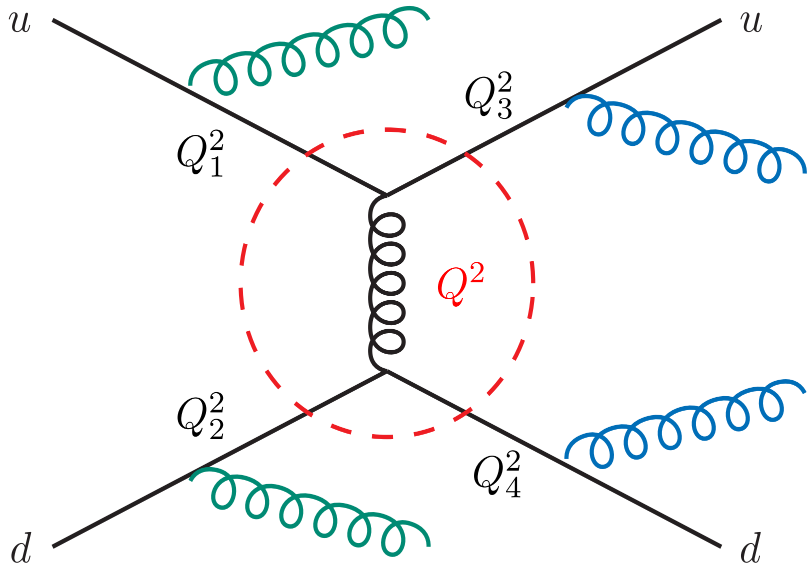


ISR: Initial State Radiation

Parton Shower Approach

$\mathcal{P}_{\rm ISR/FSR}$ Accelerated charged particles radiate

$2 \rightarrow n = (2 \rightarrow 2)_{\rm On\ Shell}$ + ISR + FSR



- FSR is time-like shower $Q_i^2 > 0$ decreasing, relatively simple
- ISR is space-like shower $Q_i^2 < 0$ increasing, physics complicated

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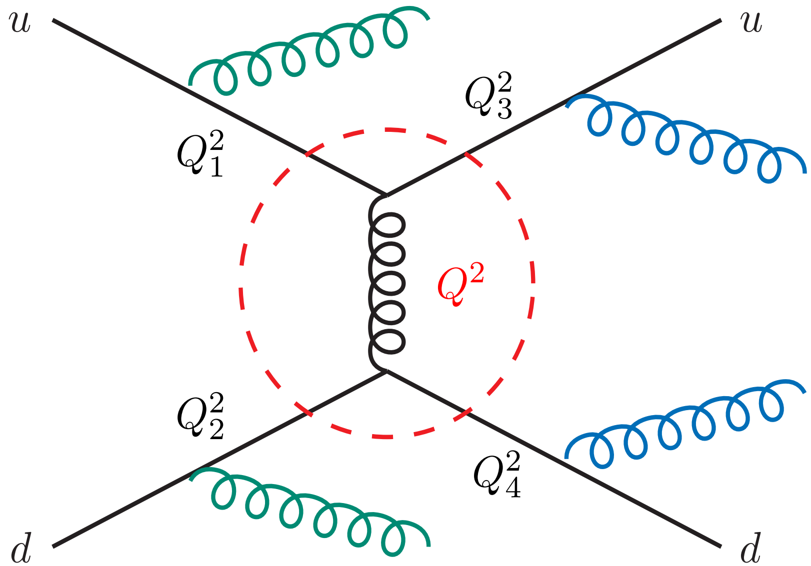

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- Shower is viewed as a probabilistic process which occurs with unit probability
- The (total) cross section is not affected but indirectly it is since the event shape is changed
- Obviously it is an approximation to the full process

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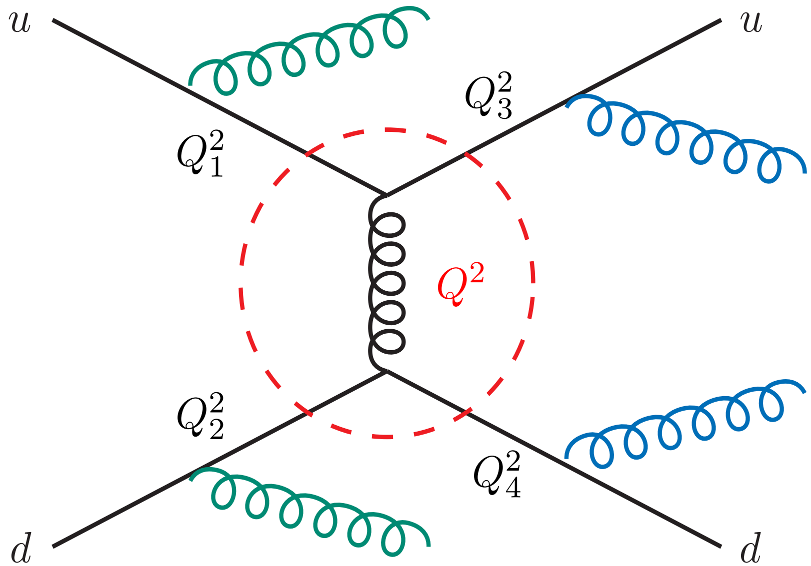

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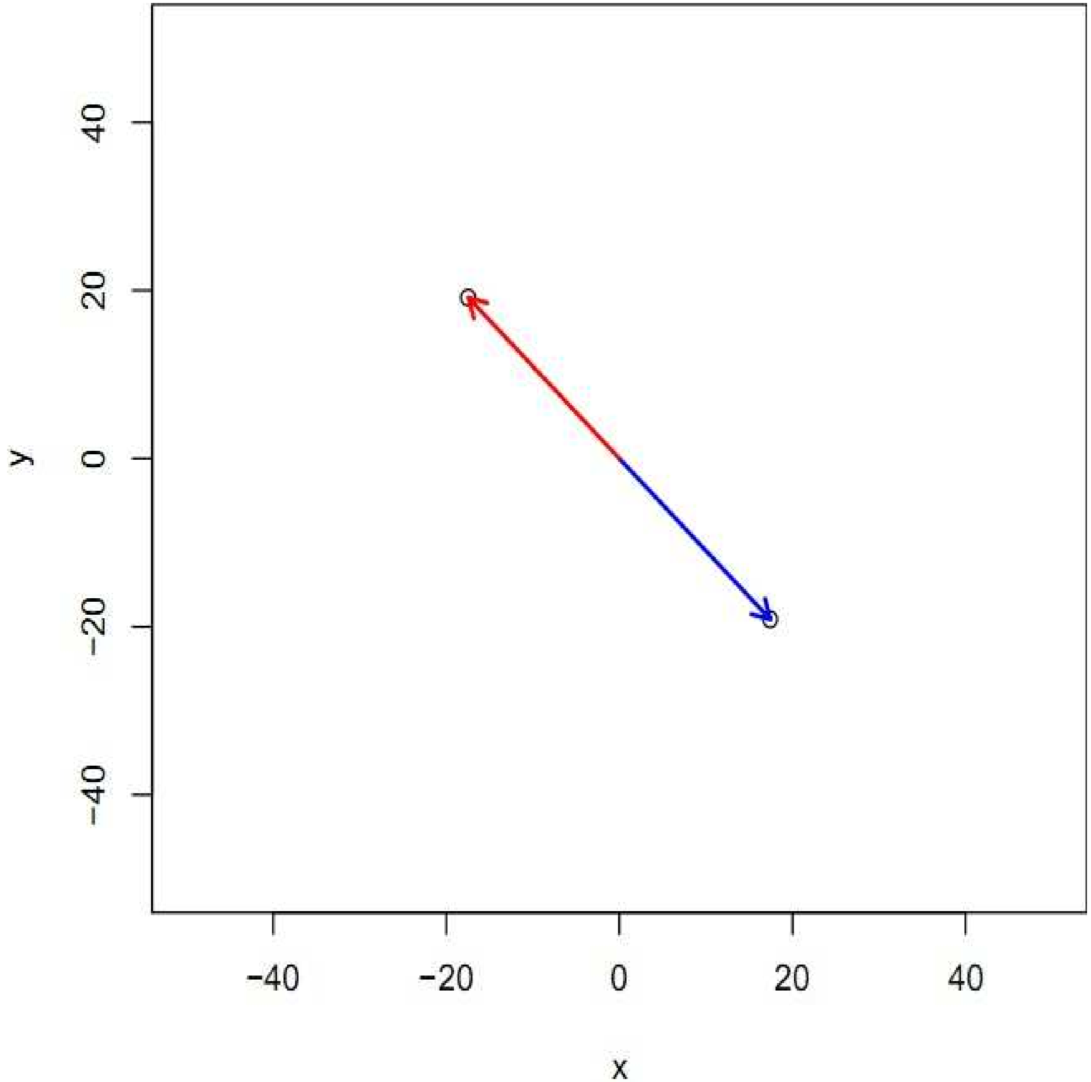
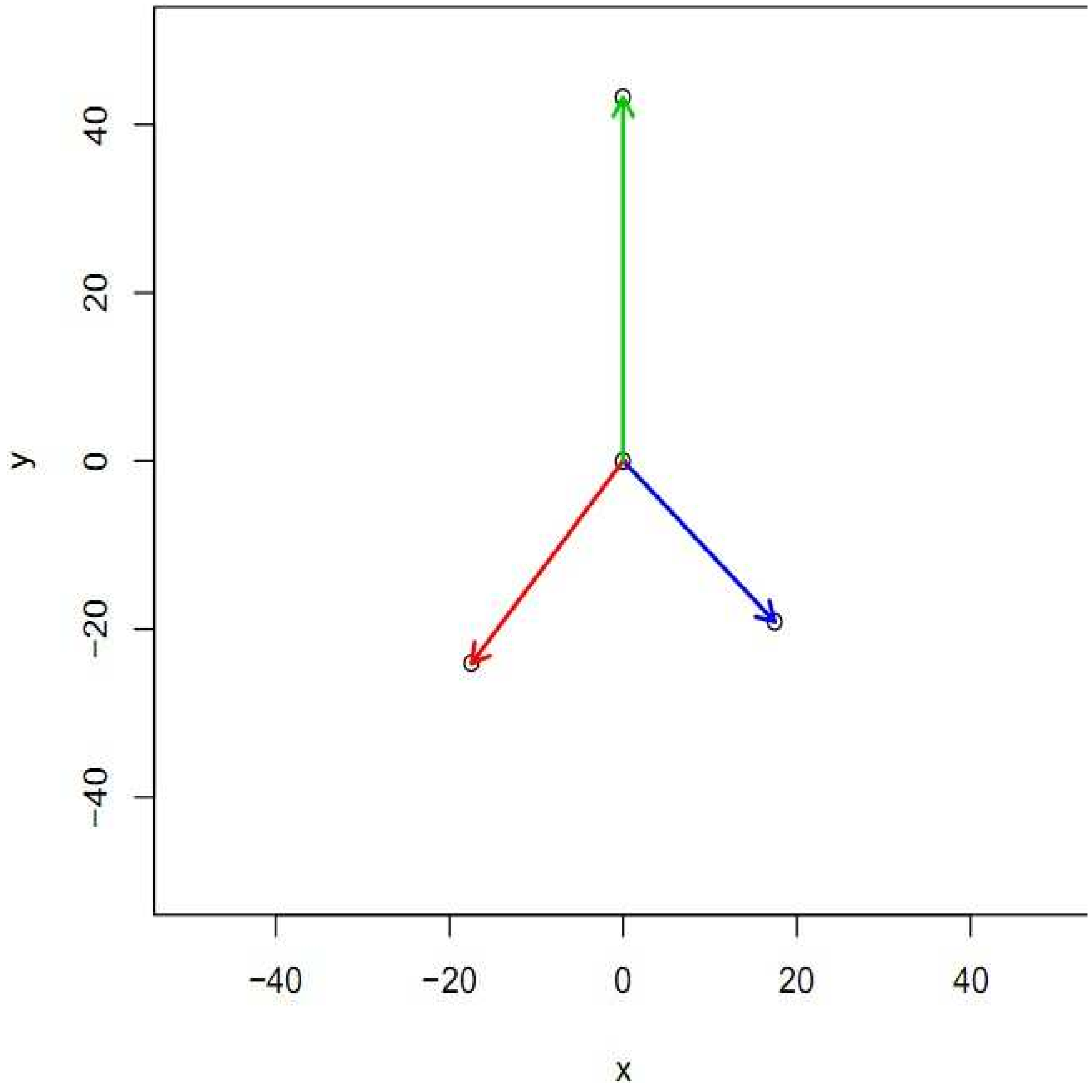
watch

Parton Shower movie



Parton Shower movie



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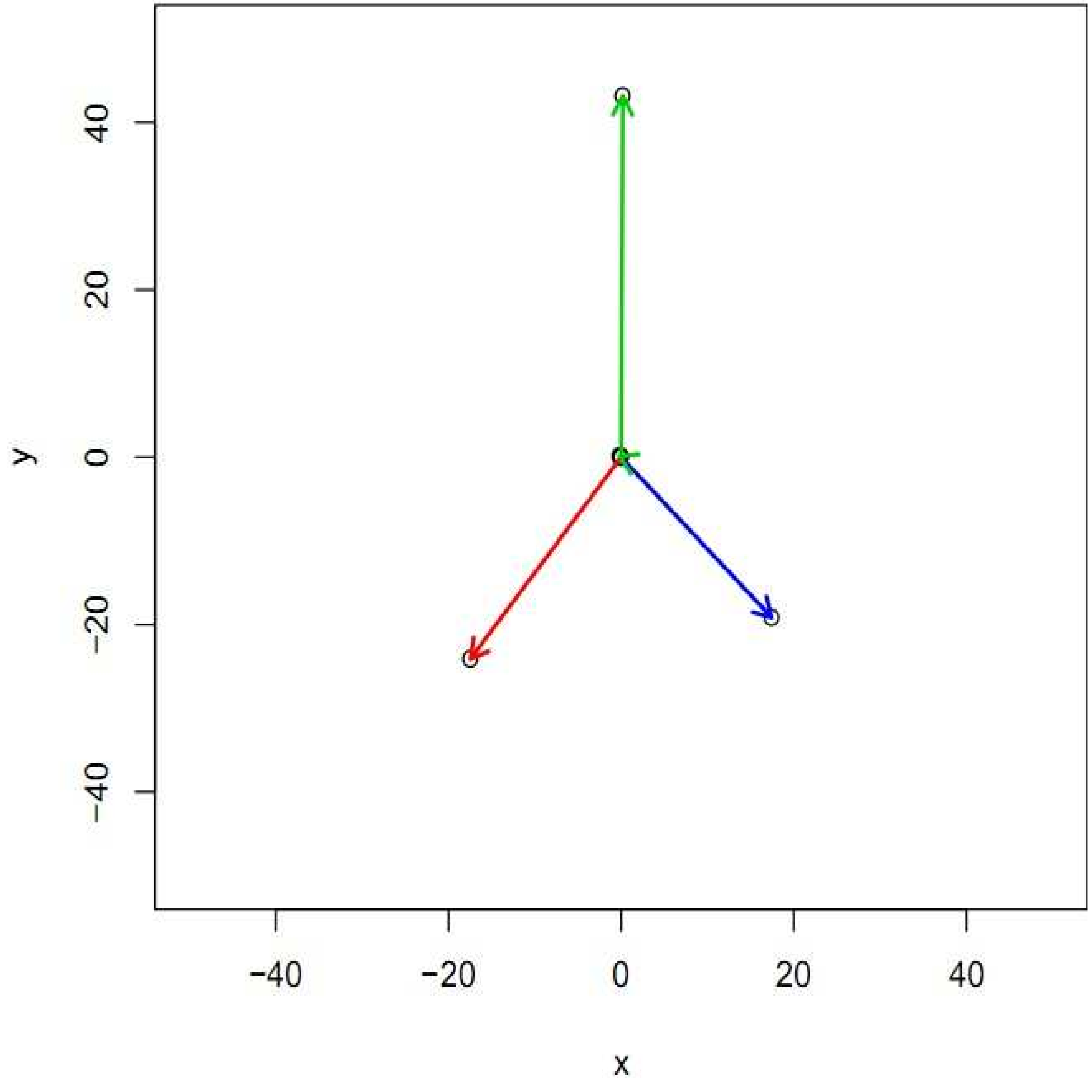

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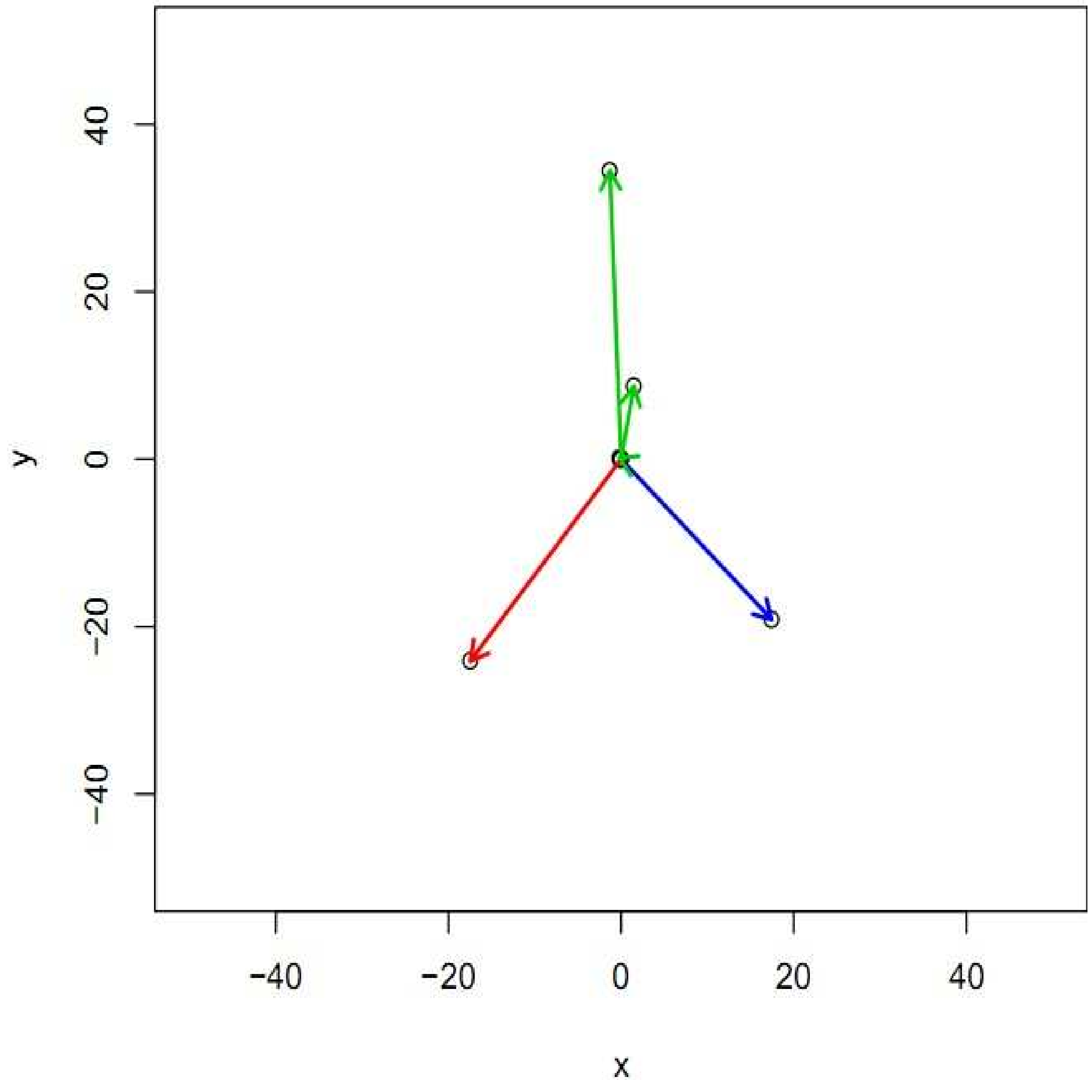

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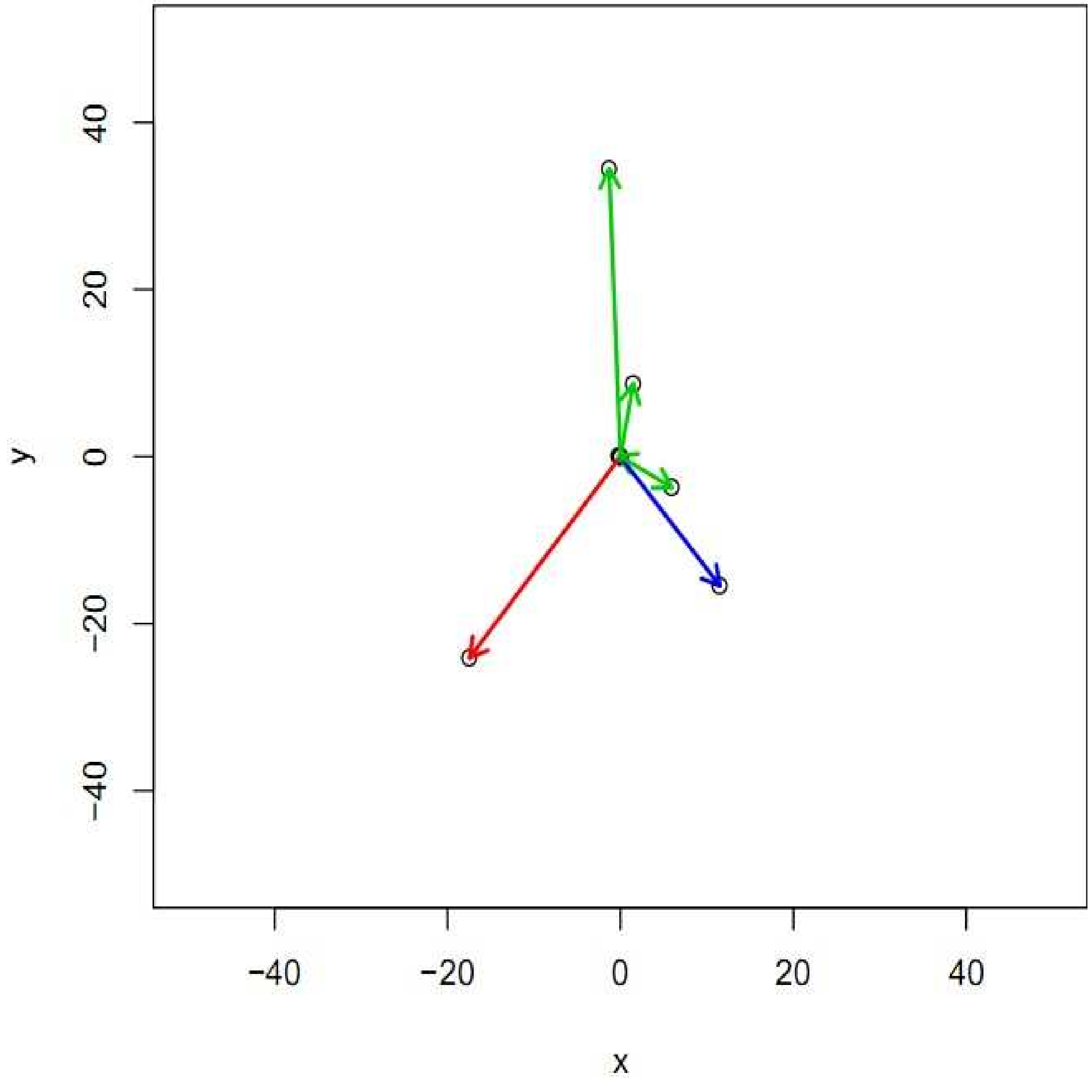

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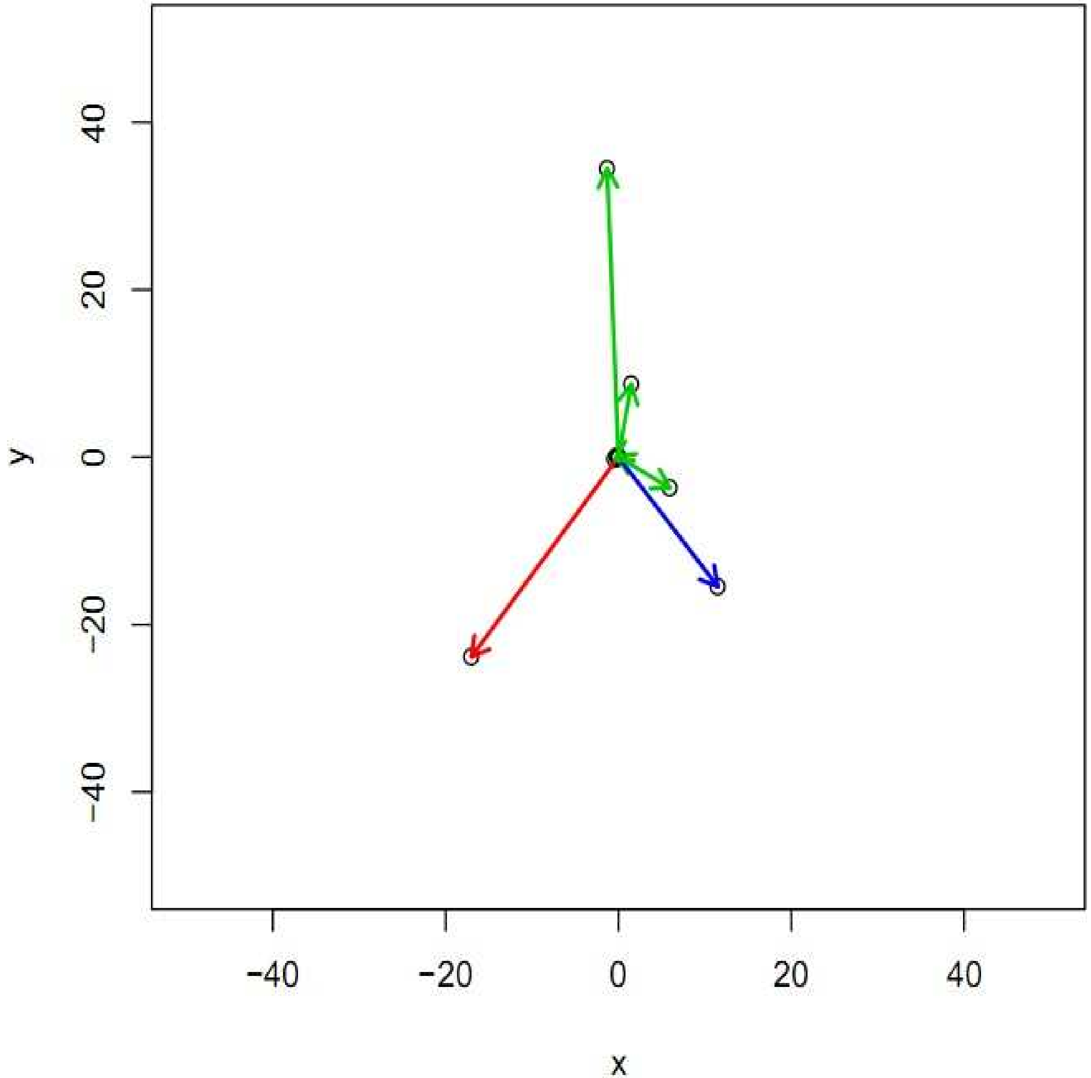

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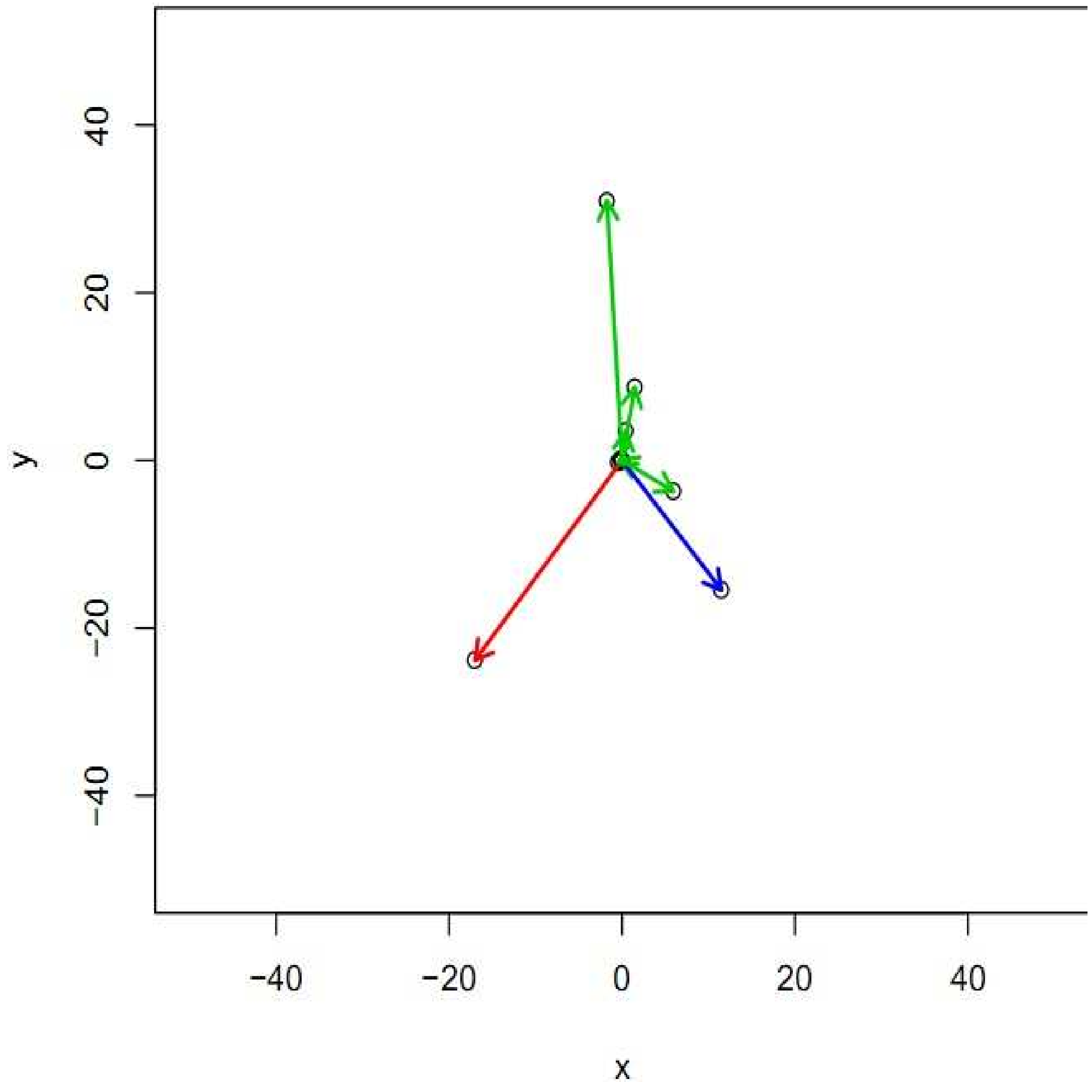

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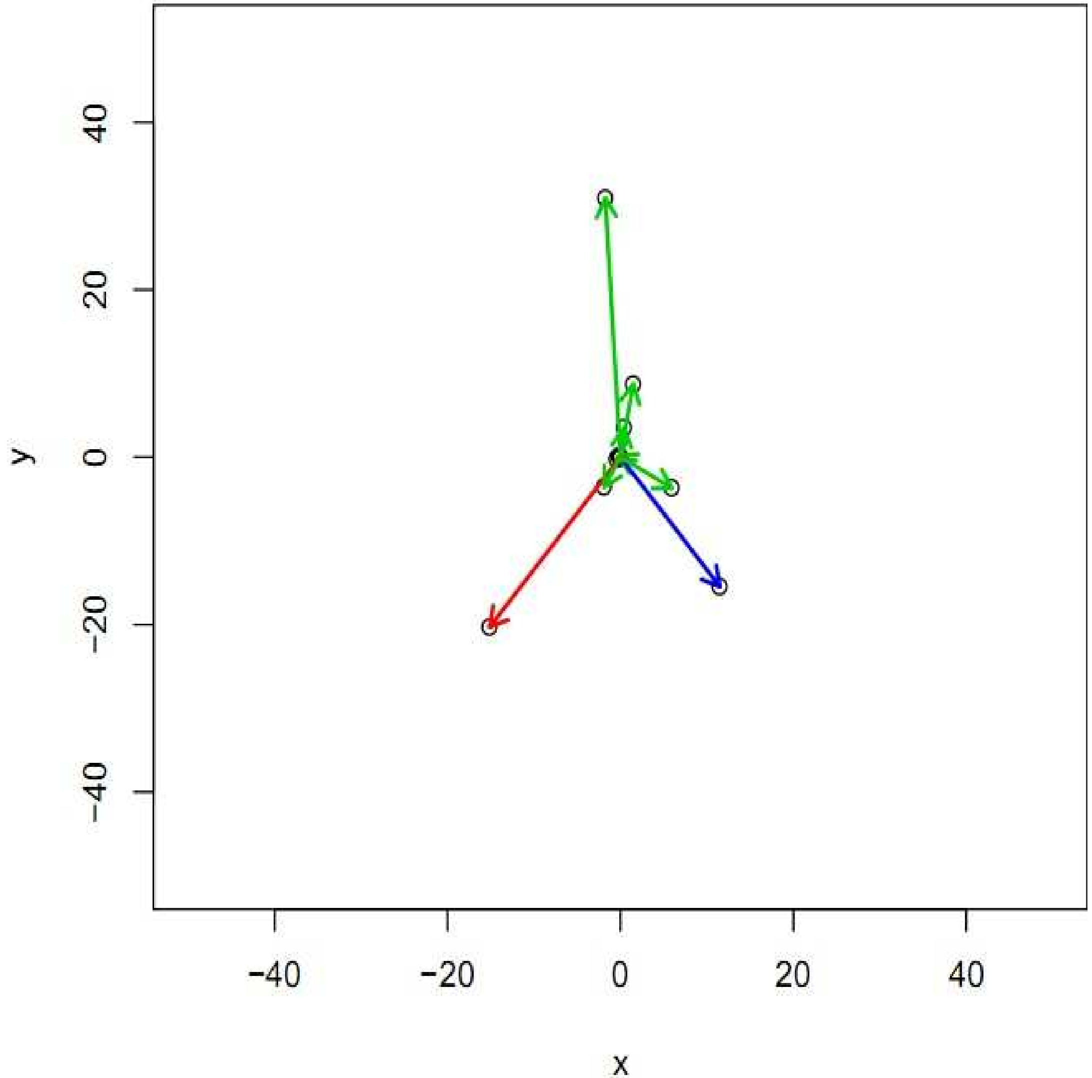

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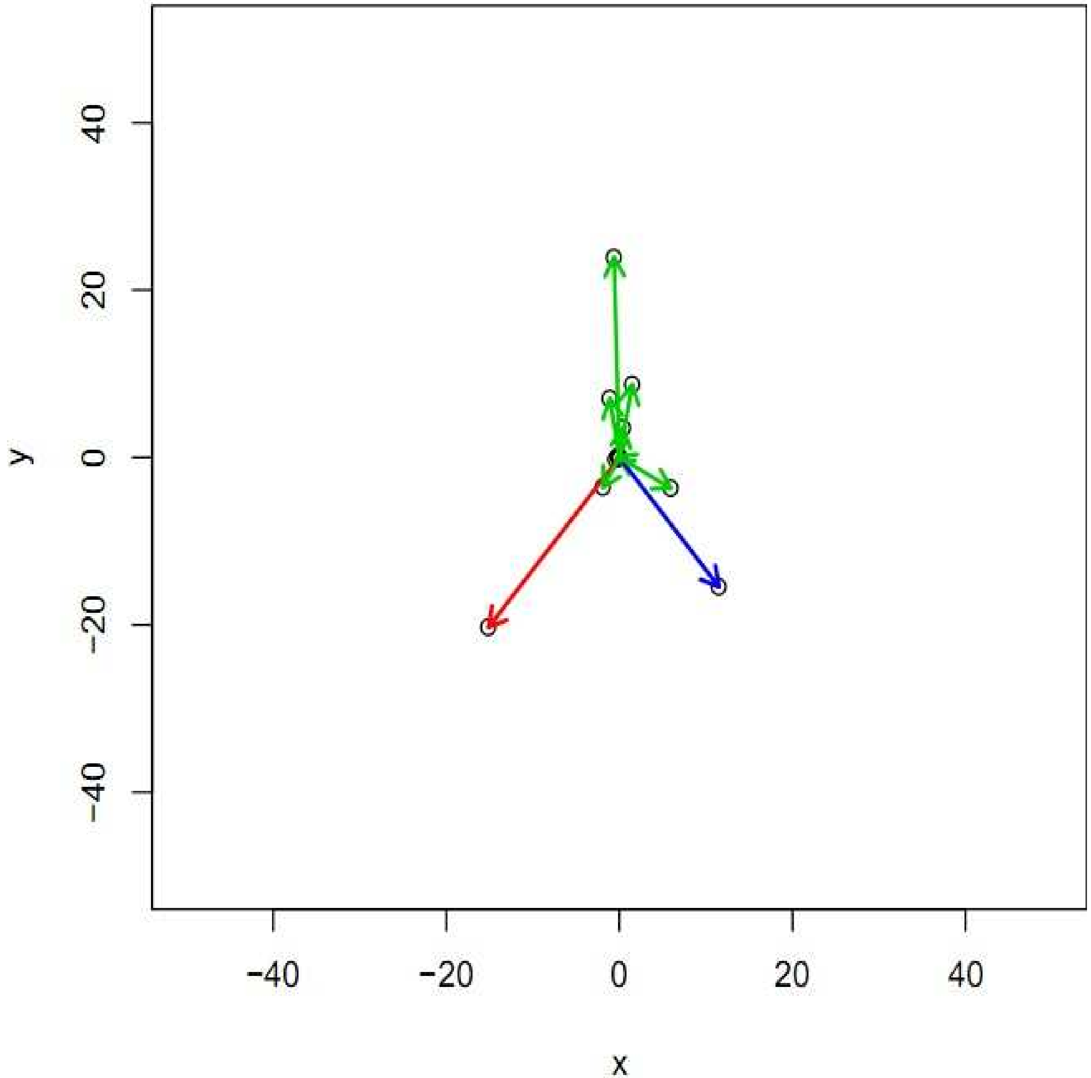

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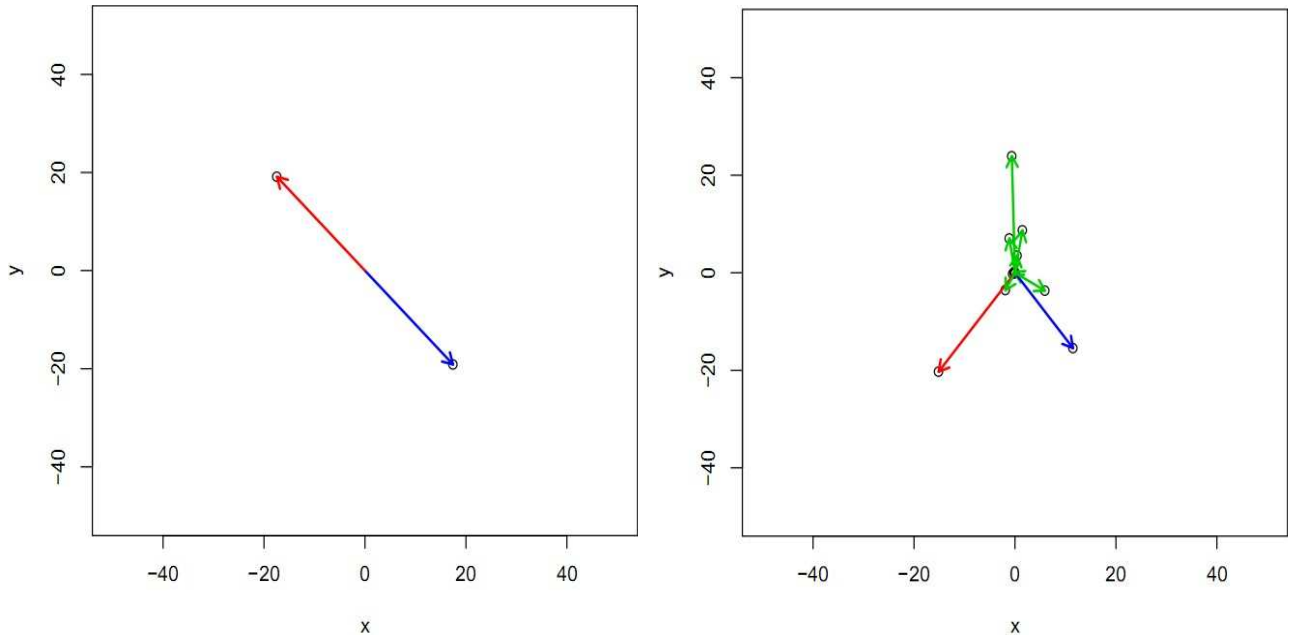
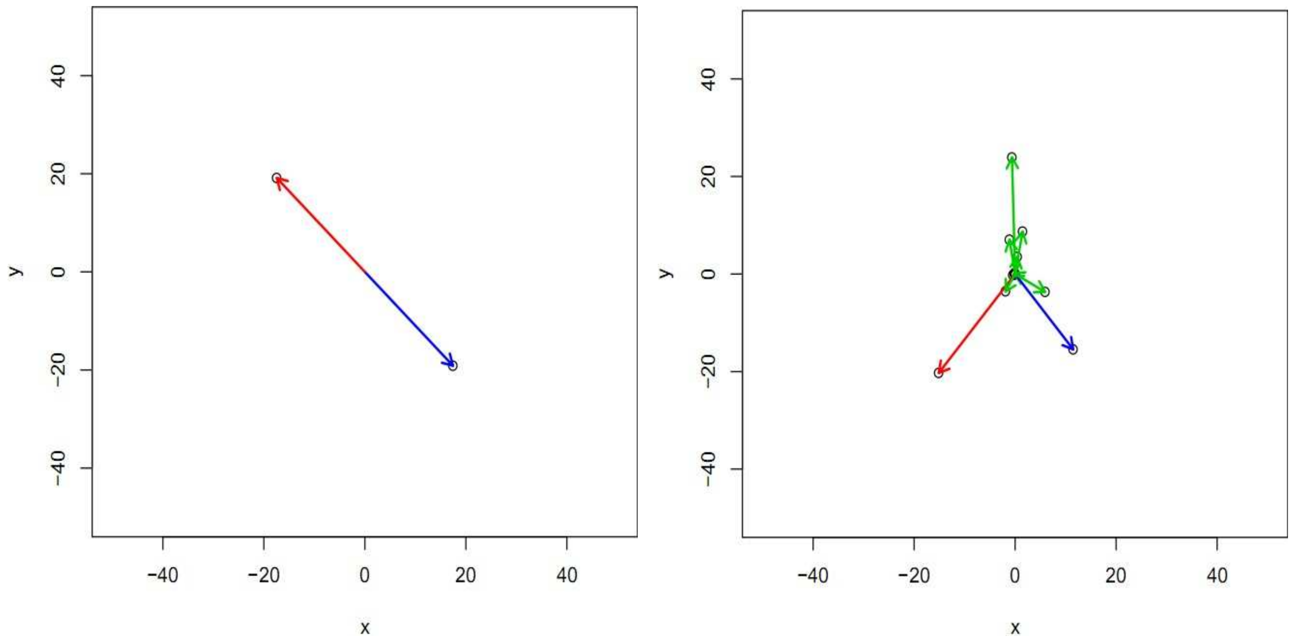


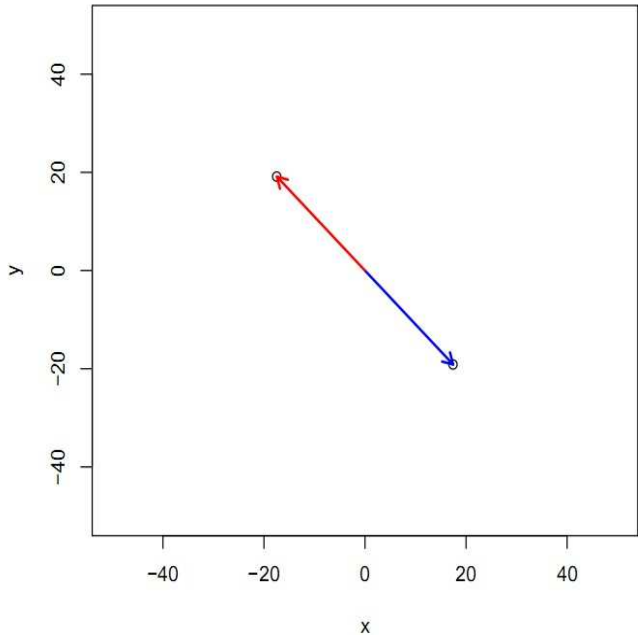
- The topology generated by the PS can be quite complicated

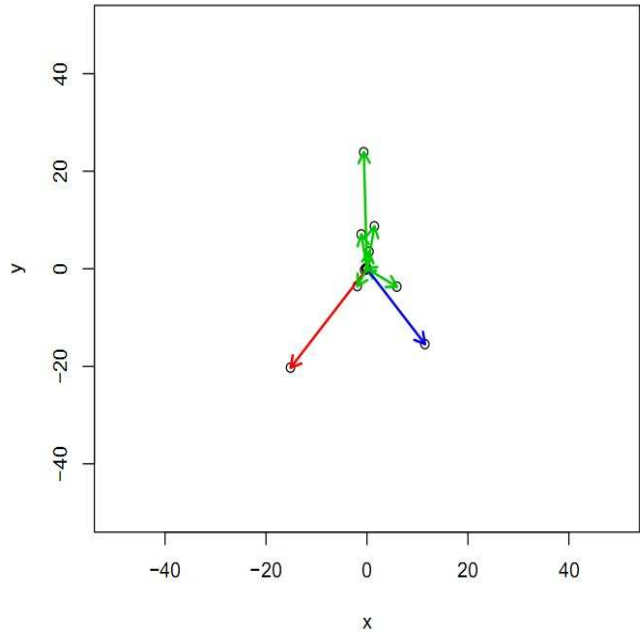
Parton Shower movie



- The topology generated by the PS can be quite complicated
- These are events shape that can not be described by fixed order pert. calculations

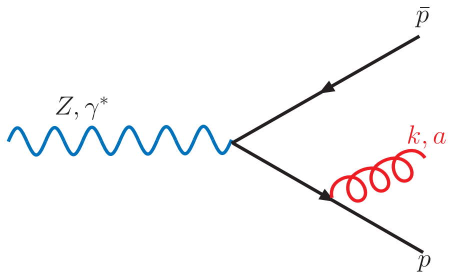
Parton Shower movie

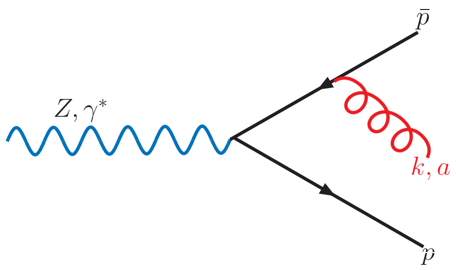




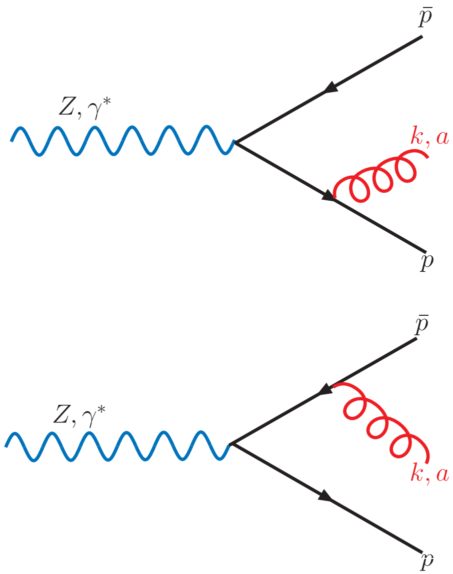
- The topology generated by the PS can be quite complicated
- These are events shape that can not be described by fixed order pert. calculations
- Total cross section still given by hard scattering (usually LO), experiments usually normalise to data

Origin and justification of PS: soft and collinear divergencies





Origin and justification of PS: soft and collinear divergencies

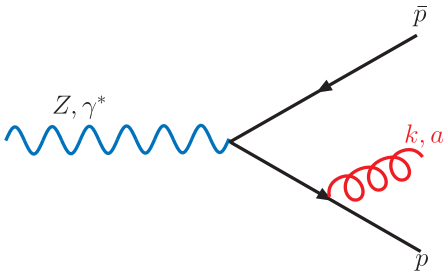


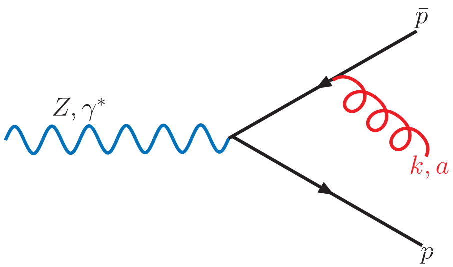
$$\mathcal{A}_\mu = \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \qquad m_q = 0$$

$$+ \; \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon}\, v(\bar{p})$$

$$= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{\bar{p}}+\not{k})\not{\epsilon}\, v(\bar{p})}{2\bar{p}.k}\right)t_a$$

$$2p.k = 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \;\rightarrow\; 0 \text{ for } E_g \rightarrow 0, \quad \theta_{pk} \rightarrow 0$$





$$\mathcal{A}_\mu = \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \quad m_q = 0$$

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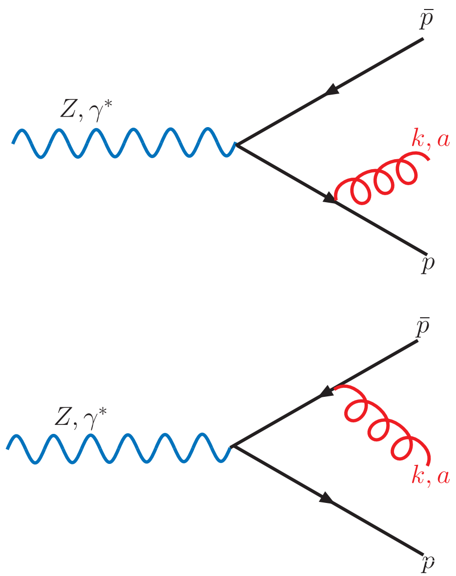
$$2p.k = 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \longrightarrow 0 \text{ for } E_g \to 0, \quad \theta_{pk} \to 0$$

$$\mathcal{A}_{\rm soft}(k \to 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_0 \quad \text{diverges } k \to 0$$



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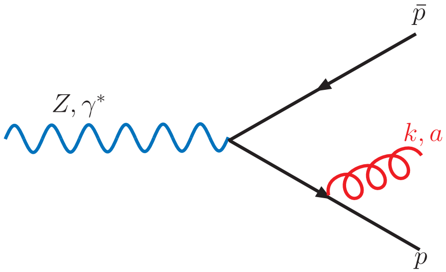
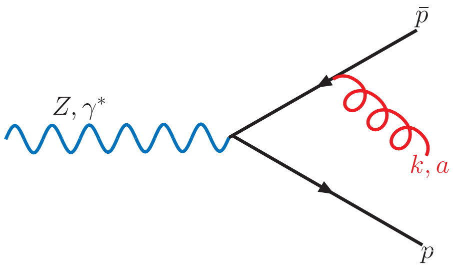
$$2p.k = 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \rightarrow 0 \text{ for } E_g \rightarrow 0, \quad \theta_{pk} \rightarrow 0$$

$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_{0g}$$





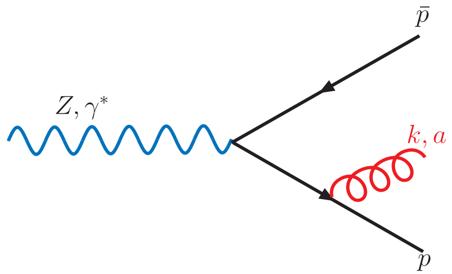
$$\begin{aligned}
\mathcal{A}_\mu &= \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \qquad m_q = 0 \\
&+ \bar{u}(p)\Gamma_\mu \frac{i}{\bar{\not{p}}+\not{k}}(-ig_s t_a)\not{\epsilon}v(\bar{p}) \\
&= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon}v(\bar{p})}{2\bar{p}.k}\right)t_a \\
2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \;\rightarrow\; 0 \text{ for } E_g \rightarrow 0, \quad \theta_{pk} \rightarrow 0
\end{aligned}$$

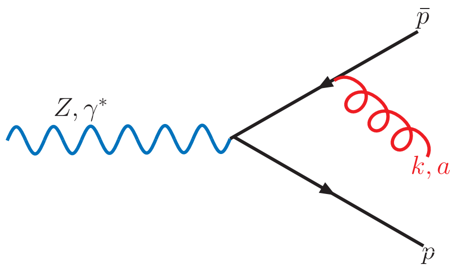
$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_{0g}$$

Universal Radiator Factor

We have **factorisation** of the soft emission (long distance) from the short distance *i.e.* the hard process

Squaring soft/collinear





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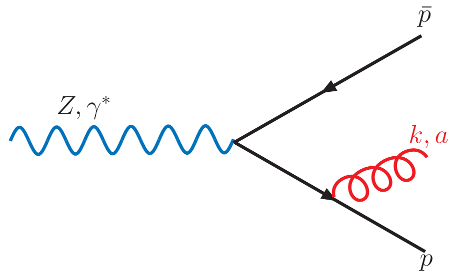
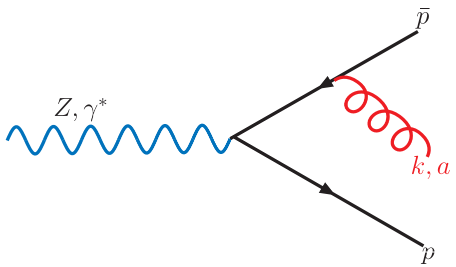
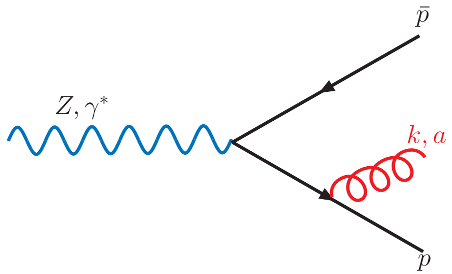
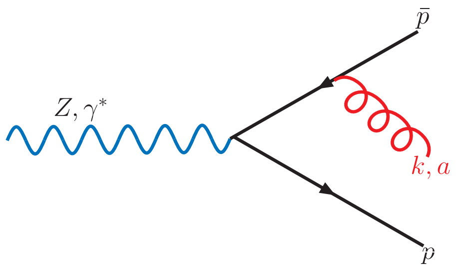




$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

Squaring soft/collinear

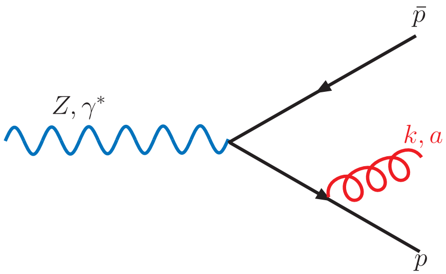




$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

$$|\mathcal{M}_{1g}|^2 = \sum_{a,pol.(\epsilon)} |\mathcal{A}_{1g}(k \to 0)|^2 = C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k} |\mathcal{M}_{0g}|^2$$

Squaring soft/collinear



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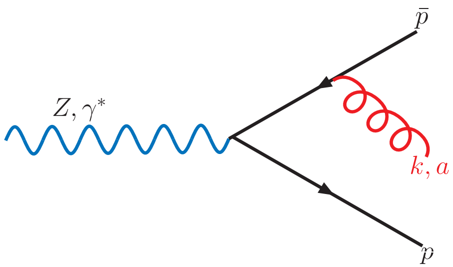
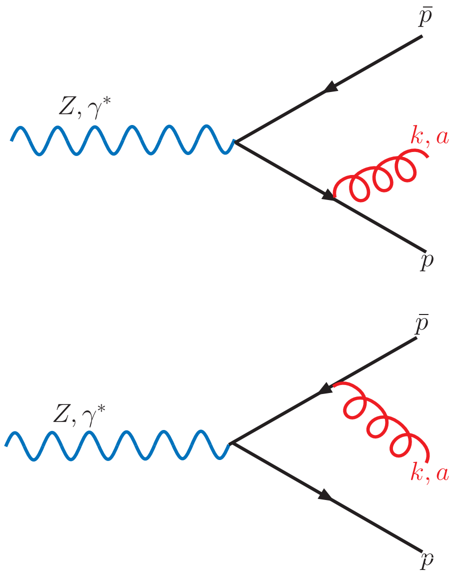


Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}} \right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

Squaring soft/collinear



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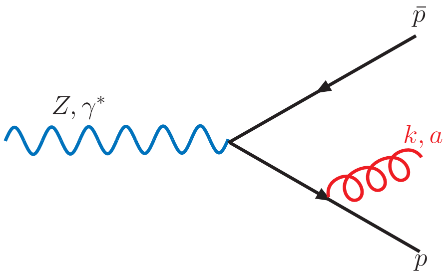
Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

$$\theta = \theta_{\angle pk} \ , \ \phi = \text{azimuth}$$

$$d\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$



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Phase Space

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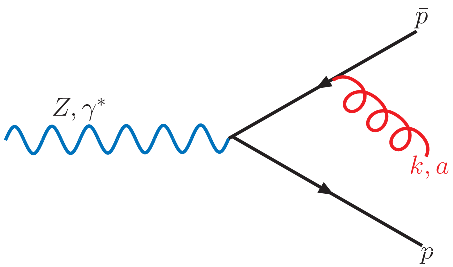
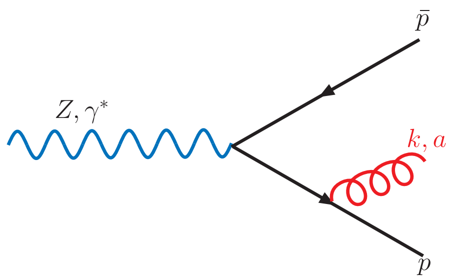


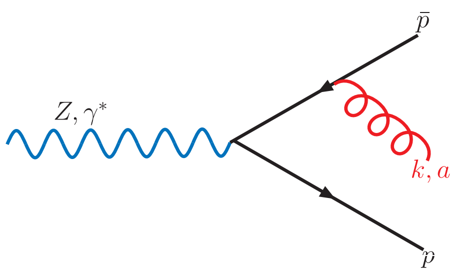
$$\theta = \theta_{\angle pk}\ ,\ \phi = \text{azimuth}$$

$$\mathrm{d}\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$

- $\mathrm{d}\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$, collinear divergence

$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$





Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k \; \bar{p}.k}$$

$$\theta = \theta_{\angle pk} \ , \ \phi = \text{azimuth}$$

$$d\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$$x_i = 2E_i/E_{\rm tot} \quad p \to 1, \quad k \to 3$$

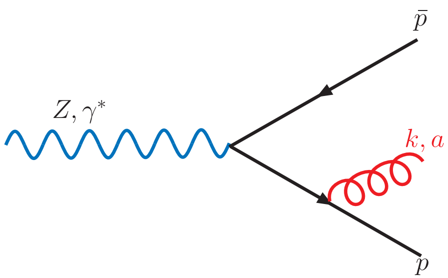
$$d\mathcal{S}_\phi = \frac{\alpha_s C_F}{2\pi} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

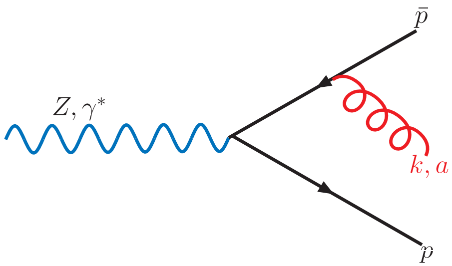
$$= \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right) d\cos\theta \, dx_3$$

- $d\mathcal{S}$ diverges for $\omega \to 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $d\mathcal{S}$ diverges for $\theta \to 0$ and $\theta \to \pi$, collinear divergence

Squaring soft/collinear

$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right) \mathcal{A}_{0g}$$





Phase Space

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$$\mathrm{d}\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$

$$x_i = 2E_i/E_{\text{tot}} \quad p \to 1, \quad k \to 3$$

$$\mathrm{d}\mathcal{S}_\phi \quad = \quad \frac{\alpha_s C_F}{2\pi} \mathrm{d}x_1 \mathrm{d}x_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$= \quad \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3\right) \mathrm{d}\cos\theta \mathrm{d}x_3$$

- $\mathrm{d}\mathcal{S}$ diverges for $\omega \to 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \to 0$ and $\theta \to \pi$, collinear divergence
- collinear divergence for $x_1 \to 1$ or $x_2 \to 1$ and Infrared divergence for $x_3 \to 0$

Splitting

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta}=\frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}}\sim\frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2} \quad \text{for}\ \ \theta,\bar{\theta}\sim 0 \ll 1$$

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

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q and $\bar q$ as independent emitters, notion of splitting as a probability

$$\mathrm{d}\sigma_{1g} \sim \mathrm{d}\sigma_{0g} \sum_{q\to qg}^{\bar q\to\bar q g} C_F \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\frac{1+(1-z)^2}{z}\mathrm{d}z \qquad (z\equiv x_3)$$

Splitting

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

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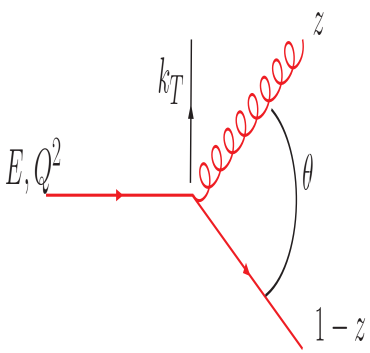

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\,\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

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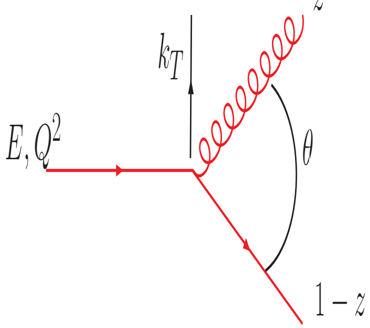


different choices of the evolution variables, equivalent in the collinear limit (diff. in practice/different codes)

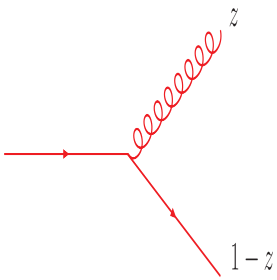
$$Q^2 = E^2 z(1-z)\theta^2 \qquad k_T^2 = E^2 z^2 (1-z)^2\theta^2$$

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}Q^2}{Q^2} = \frac{\mathrm{d}k_T^2}{k_T^2}$$

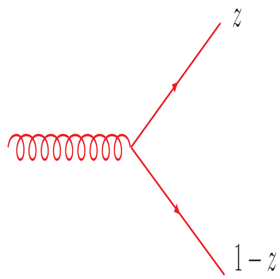
DGLAP

This generalises to different parton branching (gluon, quarks)

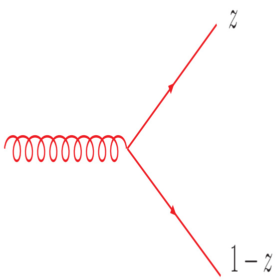
$$d\sigma_{bc} \sim d\sigma_a \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} P_{a\to bc}(z)dz$$



$$P_{gq}(z) = C_F\left(\frac{1+z^2}{1-z}\right)$$

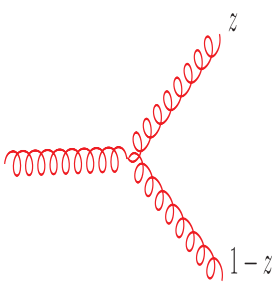


$$P_{qg}(z) = C_F\left(\frac{1+(1-z)^2}{z}\right)$$



$$P_{qg}(z) = T_R\left(z^2+(1-z)^2\right) \qquad T_R = \frac{n_f}{2}$$

(divergences at $z = 0, 1$ dealt with soft/virtual corr.)

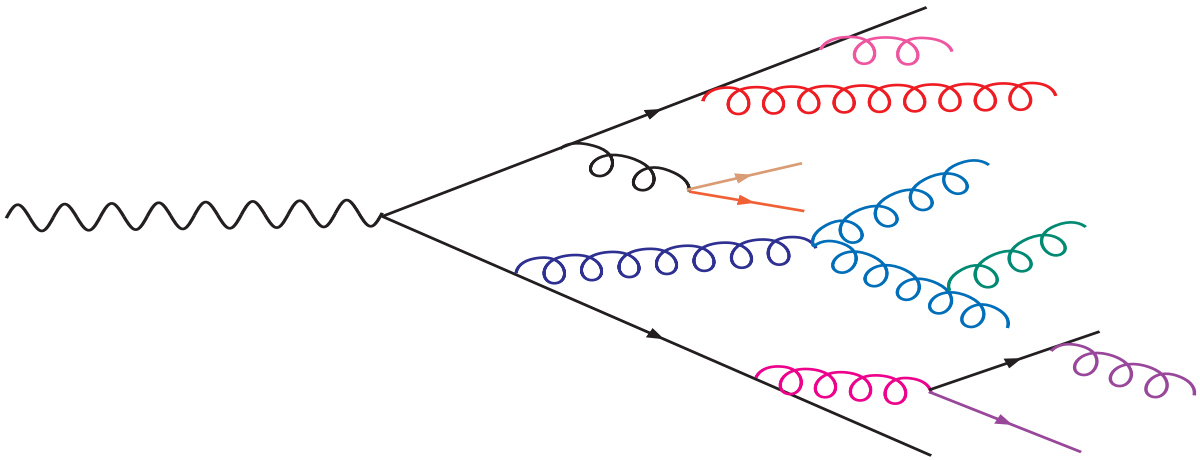


$$P_{gg}(z) = C_A\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right) \qquad C_A = 3 \qquad (C_F = 4/3)$$

Gluons radiate the most

$P(z,\phi)$ can be defined for polarisation effects

Ex. Final State PS



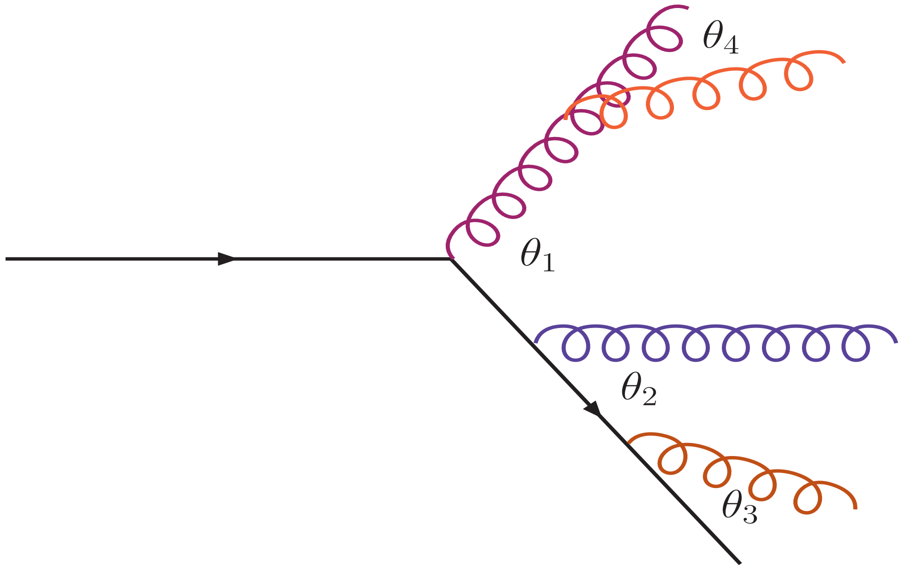
Need soft/collinear cut-offs to stay away from non perturbative physics. Details are model/code dependent

$Q > m_0 = min(m_{ij}) \sim 1$GeV

$z_{min}(E,Q) < z < z_{max}(E,Q)$

$k_T > k_{T,min} \sim 0.5$GeV

Radiation is angle ordered



$\theta_1 > \theta_2 > \theta_3$ and $\theta_1 > \theta_4$

On average, emissions have decreasing angles with respect to emitters

the jet is squeezed

The Probability of real emission exponentiates

- Conservation of total probability $\mathcal{P}_{\rm something} + \mathcal{P}_{\rm nothing} = 1$!
- Product of probabilities as time evolves $T \sim 1/Q$ evolves
  $\mathcal{P}_{\rm nothing}(0 < t < T) = \mathcal{P}_{\rm nothing}(0 < t \leq T_1)\mathcal{P}_{\rm nothing}(T_1 < t \leq T)$
- subdivide further $T_i = (i/n)T, 0 \leq i \leq n$

$$\begin{aligned}
\mathcal{P}_{\rm nothing}(0 < t < T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\rm nothing}(T_i < t < T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\rm something}(T_i < t < T_{i+1})) \\
&= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\rm something}(T_i < t < T_{i+1})\right) \\
&= \exp\left(-\int_0^T \frac{{\rm d}\mathcal{P}_{\rm something}(t)}{{\rm d}t}{\rm d}t\right)
\end{aligned}$$

The Probability of real emission exponentiates

- Conservation of total probability $\mathcal{P}_{\text{something}} + \mathcal{P}_{\text{nothing}} = 1$!
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  $\mathcal{P}_{\text{nothing}}(0 < t < T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1)\mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$
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\end{aligned}$$

$$\mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T) \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t} \mathrm{d}t\right)$$

The Probability of real emission exponentiates

$$\mathrm{d}\mathcal{P}_{\mathrm{first}}(T) = \mathrm{d}\mathcal{P}_{\mathrm{something}}(T)\exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\mathrm{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

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$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\mathrm{d}z\; \boxed{\exp\left(-\sum_{bc}\int_{Q^2}^{Q^2_{\rm max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int_{Q_0^2/Q^2}^{1-Q_0^2/Q^2}\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\mathrm{d}z'\right)}$$

Q_0 =low cut-off scale

The Probability of real emission exponentiates

$$\mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T)\exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

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Q_0 =low cut-off scale

$\Delta(Q^2, Q^2_{\max})$, Sudakov form factor

(probability of emitting no radiation between these 2 scales)

$\mathcal{P}_{\text{nothing}}$

(a given parton only branches once)

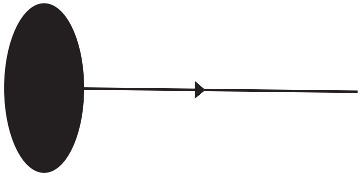
- Start with a parton at high Q^2_{max} (typical of hard process)
- Work out the scale of the next branching, Q^2 by generating a random number $R \in [0,1]$ and solving $R = \Delta(Q^2_{max}, Q^2)$
- if no solution $Q^2 > Q^2_0$ `stop`
- otherwise work out the type of the branching
- generate the momenta of the decay products using the splitting functions
- repeat the procedure for the newly produced partons

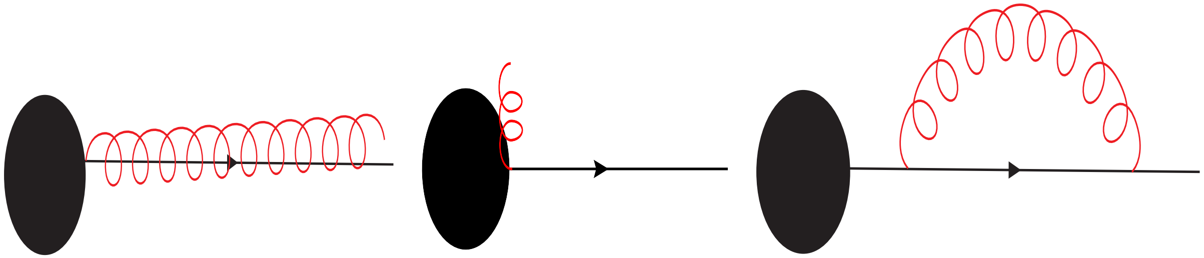
(some) differences between the MC for PS

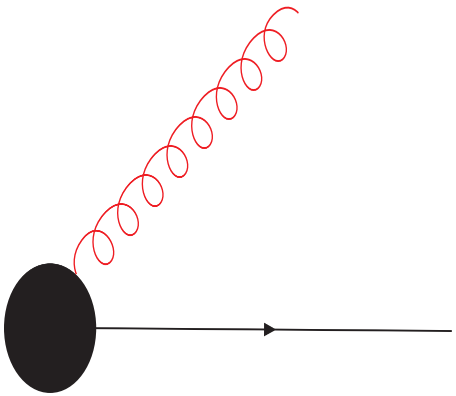
- key difference is the evolution/scale variable
  - Angle θ (ordering `HERWIG`
  - Virtuality Q^2
  - Transverse momentum k_T
- $\int dQ^2/Q^2 = \log Q^2$, LL the same but important sub-leading differences
- soft emission (coherence), recall this factorises at the amplitude level...

Resolved and

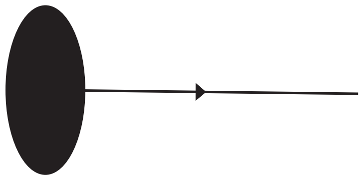
what difference?



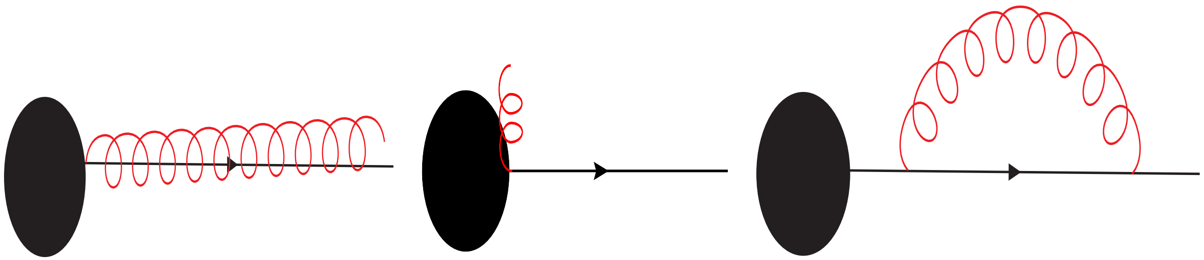


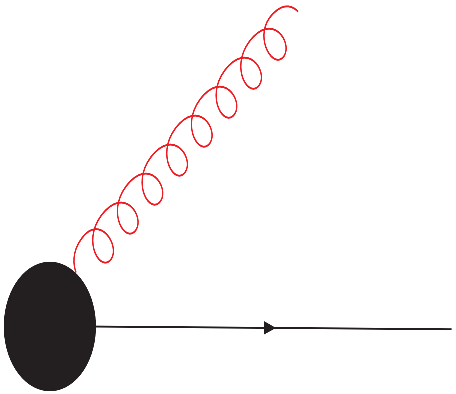


what difference?



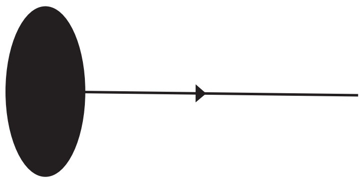
no radiation



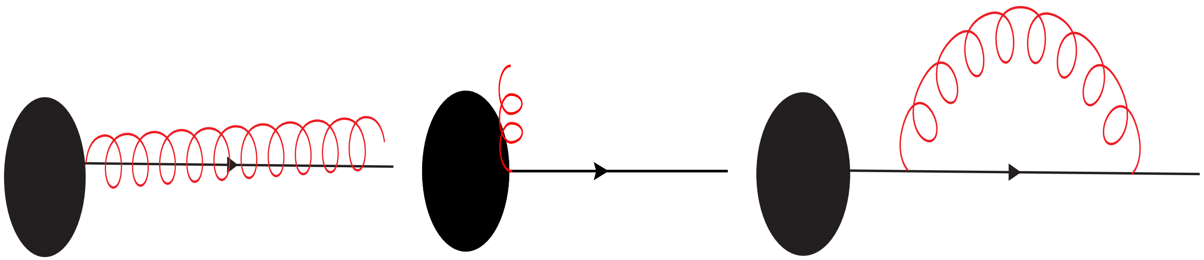


Resolved and

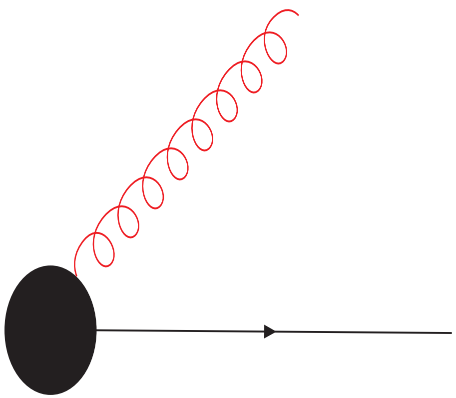
what difference?



no radiation



collinear soft virtual/loop

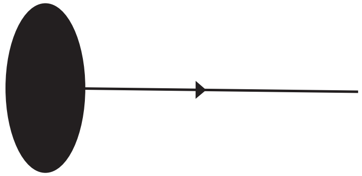


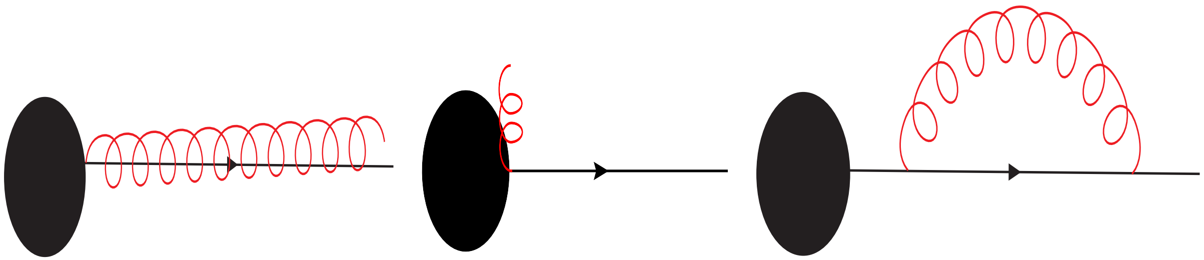
Resolved $k_T > Q_0$

Resolved and

what difference?

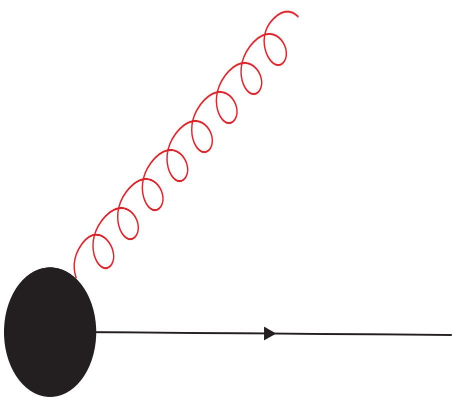
no radiation





collinear soft virtual/loop

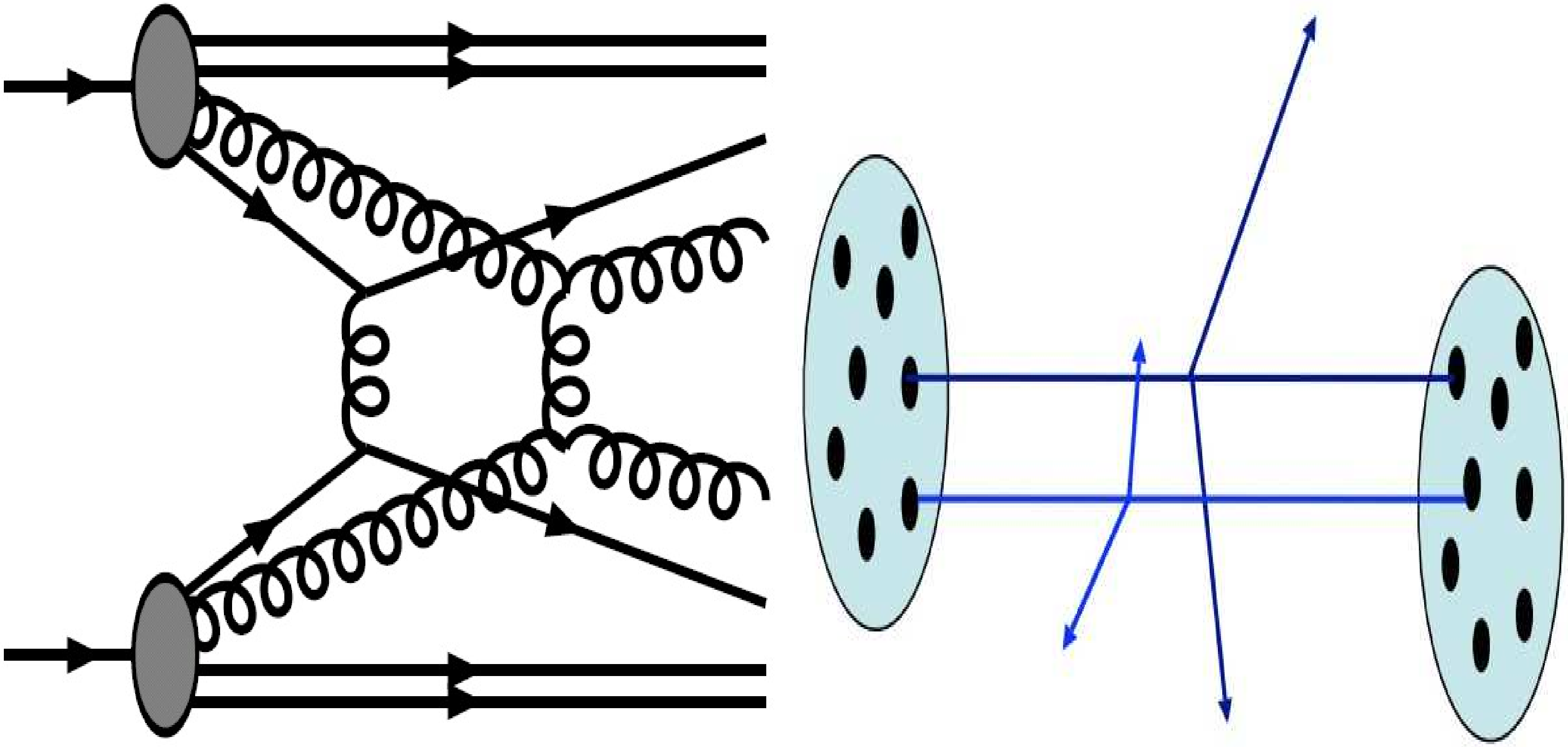
Unresolved from no radiation, $k_T < Q_0$. With addition of virtual, divergence tamed



Resolved $k_T > Q_0$

Event Generators

Underlying Events and such



- For small $p_{T,\min}$ and high energy inclusive parton-parton cross section is larger than proton-proton cross section
- More than one parton (per proton) scatter
- calls for a model of spatial distribution within the proton (perturbation theory gives n-scatter distribution
- UE is not understood. It is modelled through some data that do not give strong constraints and hence extrapolation can be doubted.(minimum bias)