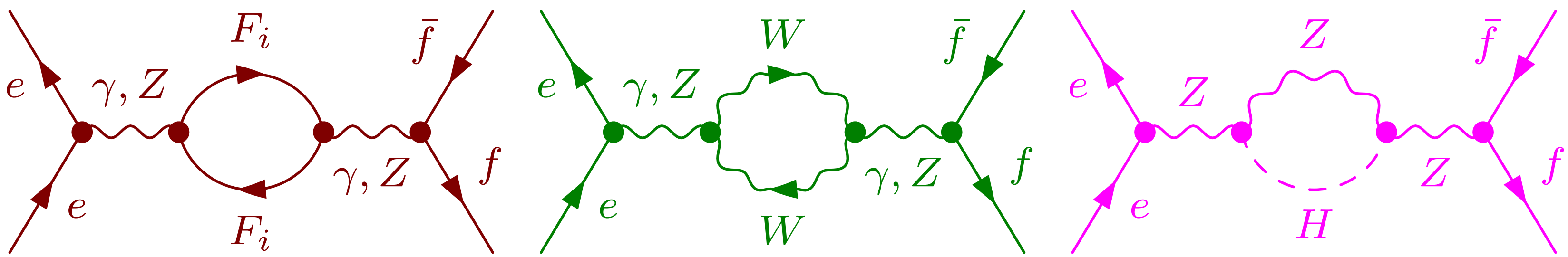
Elements of the Renormalisation of the Electroweak Theory

and

Precision Tests of the Standard Model



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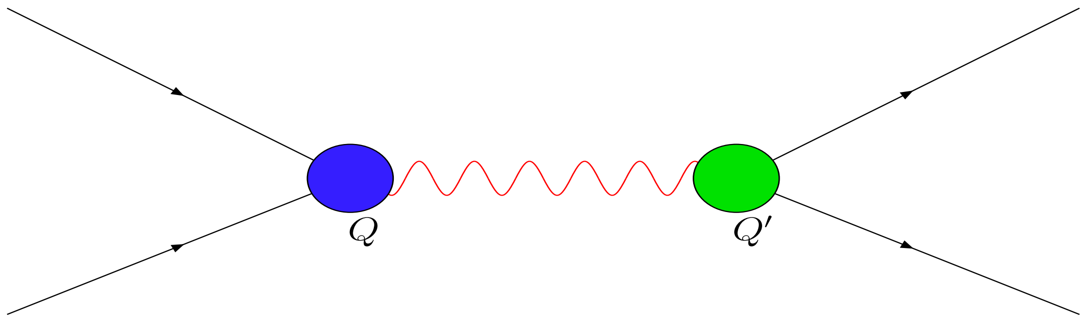
Recap: Getting finite results in Observables (1)

$e^+e^- \;\to f\bar{f}$ in QED

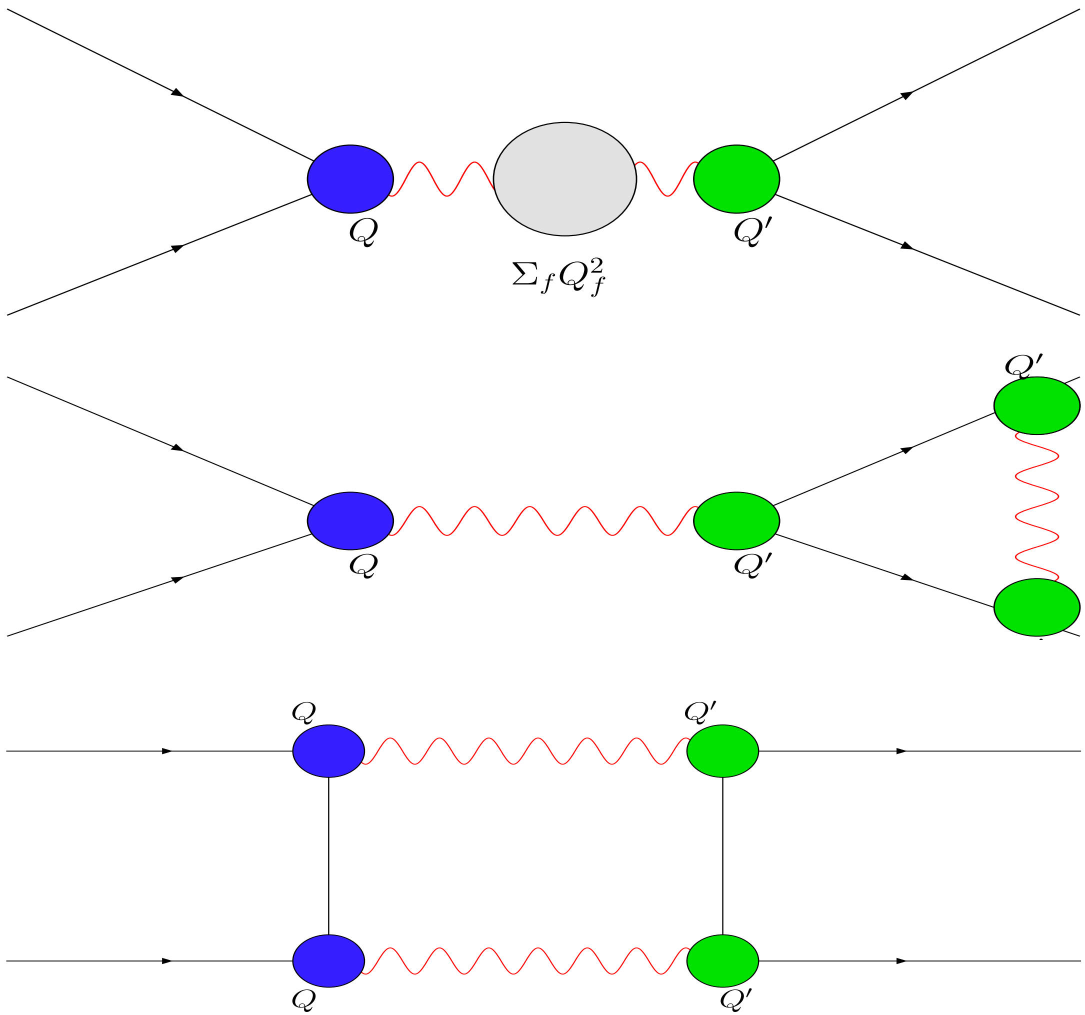
At tree-level



$$\mathcal{M}_0 = e_0 J_\mu^Q D_0^{\mu\nu} e_0 J_\mu^{Q'} \to e_0^2 Q \frac{1}{k^2} Q'$$

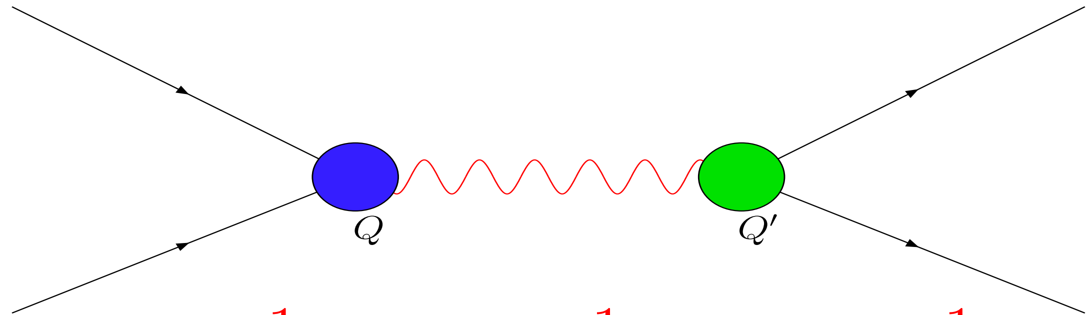
Recap: Getting finite results in Observables (2)

At one-loop: 3 types of contributions



Recap: Getting finite results in Observables (3)

The Universal contribution: Improved Born



$$\mathcal{M} = e_0 J_\mu^Q D^{\mu\nu} e_0 J_\mu^{Q'} = \frac{1}{k^2}\, e_0^2 Q \frac{1}{1 + e_0^2 \Pi_{QQ}} Q' \equiv \frac{1}{k^2}\, e_\star^2(k^2) Q Q'$$

$$\begin{aligned} e_\star^{-2}(k^2) &= e_0^{-2} + \Pi_{QQ}(k^2) \\ e_\star^{-2}(k^2) &= \left(e_0^{-2} + \Pi_{QQ}(k_R^2)\right) + \left(\Pi_{QQ}(k^2) - \Pi_{QQ}(k_R^2)\right) \\ &\equiv \underbrace{e^{-2}(k_R^2)}_{\text{input}} + \underbrace{\left(\Pi_{QQ}(k^2) - \Pi_{QQ}(k_R^2)\right)}_{finite} \end{aligned}$$

Recap: Getting finite results in Observables (3)

Choice of the scale k_R or $e^{-2}(k_R^2)$ is a renormalisation scheme. In QED and the SM: choose On-shell scheme: directly related to an observable. In the limit $k^2 \to 0$

$$\lim_{k^2 \to 0} \left(k^2 \mathcal{A}_{all\ contributions} \right) = \lim_{k^2 \to 0} k^2 \mathcal{M} = e^2 = e_\star^2(0)$$

$$e^{-2} = e_\star^{-2}(0) = e_0^{-2} + \Pi_{QQ}(0)$$

$$e_\star^2(k^2) = \frac{e^2}{1 + e^2(\Pi_{QQ}(k^2) - \Pi_{QQ}(0))}$$

Trading bare (e_0) with physical (input) parameter

leads to finite corrected observables

renormalisation and definition crucial

Recap: Getting finite results in Observables: Summary

- Lagrangian in terms of bare parameter $e_0 = Z_3 e e = e + \delta e$,
- Impose renormalisation condition
  $\frac{\delta e^2}{e^2} = \frac{\delta\alpha}{\alpha} = e^2 \Pi_{QQ}(0) = \Pi_{\gamma\gamma}(0)$
- Calculate matrix element at one-loop (with e_0)

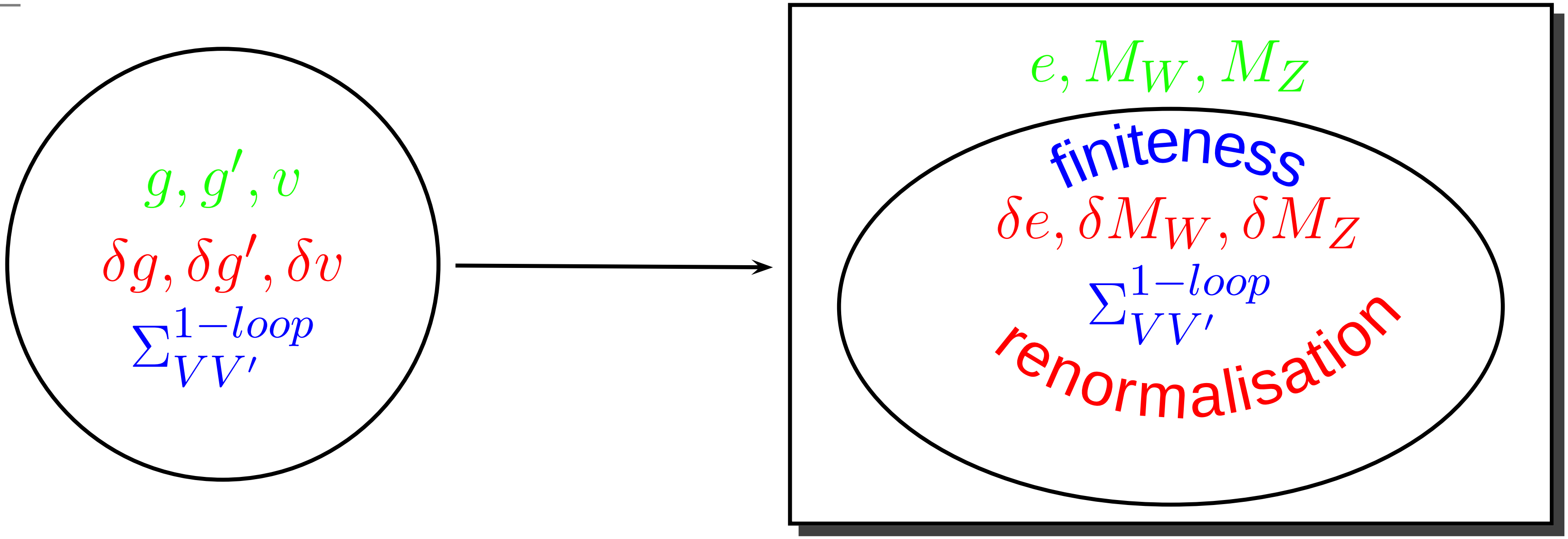
$$\mathcal{M}(k^2) = \frac{1}{k^2}\, e_0^2 Q \frac{1}{1 + e_0^2 \Pi_{QQ}} Q' \simeq \frac{QQ'}{k^2}\, \left(1 - e_0^2 \Pi_{QQ}\right) e_0^2$$

- trade-off bare with renormalised parameter

$$\mathcal{M}(k^2) = \frac{QQ'}{k^2}\, \left(1 - e_0^2 \Pi_{QQ} + \frac{\delta\alpha}{\alpha}\right) e^2$$

$$\sim \frac{QQ'}{k^2}\, \left(1 - e^2 \Pi_{QQ} + \frac{\delta\alpha}{\alpha}\right) e^2 \equiv \frac{Q e_\star^2(k^2) Q'}{k^2}$$

Fundamental parameters and physical parameters (1.)



$$\frac{\delta g'^{\,2}}{g'^{\,2}} = \frac{\delta e^2}{e^2} + \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}$$

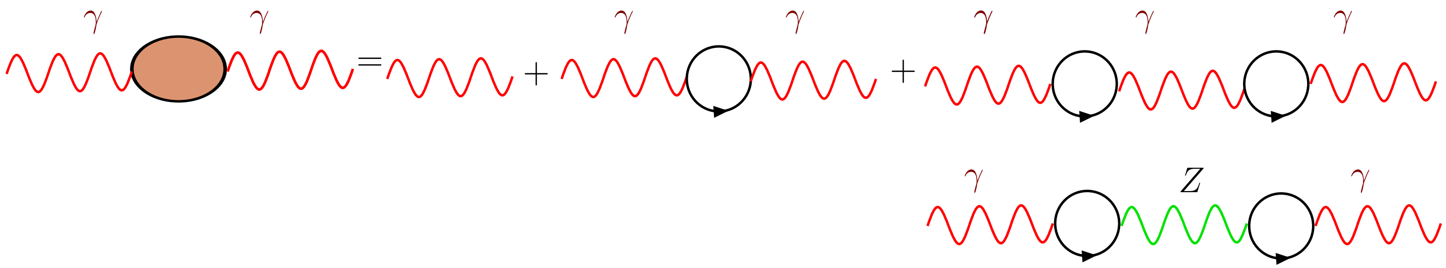
$$\frac{\delta g^2}{g^2} = \frac{\delta e^2}{e^2} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right)$$

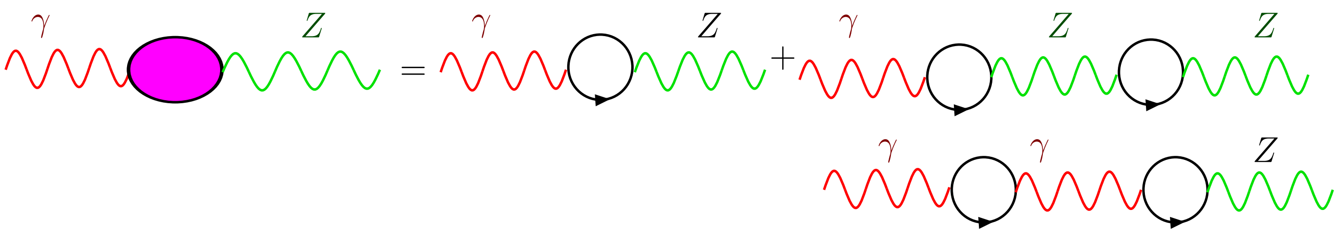
$$\frac{\delta v^2}{v^2} = -\frac{\delta e^2}{e^2} + \frac{c_W^2}{s_W^2}\frac{\delta M_Z^2}{M_Z^2} + \frac{s_W^2 - c_W^2}{s_W^2}\frac{\delta M_W^2}{M_W^2}$$

$$\left(s_M^2 = s_W^2 = 1 - \frac{M_W^2}{M_Z^2}\right) \blacktriangleright$$

$$\frac{\delta s_M^2}{s_M^2} = \frac{\delta s^2}{s^2} = -\frac{c_W^2}{s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) \quad , \quad \delta c^2 = -\delta s^2$$

Propagators and $Z - \gamma$ mixing





$$D_{\mu\nu}^{\gamma\gamma} = -\, i g_{\mu\nu} \frac{1}{k^2} \, \frac{1}{1 + \Pi_{\gamma\gamma}(k^2)} \qquad D_{\mu\nu}^{Z\gamma} = +\, i g_{\mu\nu} \, \frac{\Pi_{Z\gamma}(k^2)}{k^2 - M_{Z,0}^2}$$

$$D_{\mu\nu}^{ZZ} = -\, i g_{\mu\nu} \, \frac{1}{k^2 - M_{Z,0}^2 + G_{ZZ}(k^2)}$$

$$D_{\mu\nu}^{WW} = -\, i g_{\mu\nu} \, \frac{1}{k^2 - M_{W,0}^2 + G_{WW}(k^2)}$$

Definition of vector bosons masses: Pole structure

$$D^{WW}_{\mu\nu} = -\, i g_{\mu\nu} \frac{1}{k^2 - M^2_{W,0} + G_{WW}(k^2)}$$

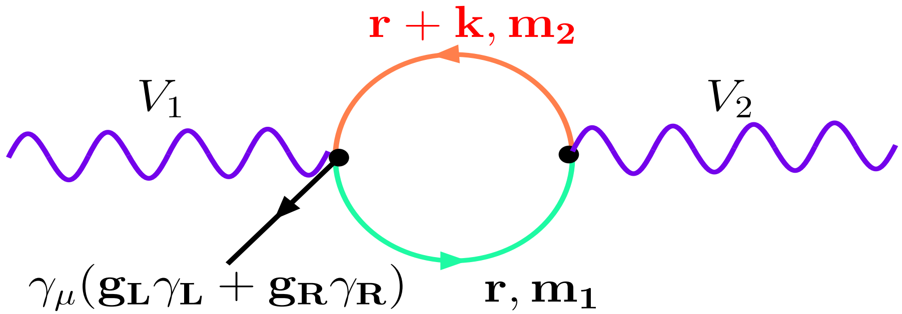
$\rightarrow 0$ for $k^2 = M_W^2$

Physical mass is defined from the zero of the real part of the inverse propagator, for $k^2 = M_W^2$

$$M_W^2 - \underbrace{M_W^2 \left(1 + \frac{\delta M_W^2}{M_W^2}\right)}_{M^2_{W,0}} + Re\left[G_{WW}(M_W^2)\right] = 0$$

$$\boxed{\delta M_W^2 = Re\left[G_{WW}(M_W^2)\right] = G_{WW}(0) + M_W^2 Re\left[\Pi_{WW}(M_W^2)\right]}$$

Chiral two-point functions (1)



$$
\begin{aligned}
-i\Sigma_{\mu\nu} &= \frac{(g_L^2 + g_R^2)}{2}\mu^{4-n}\int \frac{d^n r}{(2\pi)^n}\mathrm{Tr}\left(\frac{\gamma_\mu \not{r}\gamma_\nu(\not{r}+\not{k})}{(r^2-m_1^2)((r+k)^2-m_2^2)}\right) \\
&+ (g_L g_R) m_1 m_2 \mu^{4-n}\int \frac{d^n r}{(2\pi)^n}\mathrm{Tr}\left(\frac{\gamma_\mu\gamma_\nu}{(r^2-m_1^2)((r+k)^2-m}\right. \\
&\equiv g_L^2 i\Sigma_{\mu\nu}^{LL} + g_R^2 i\Sigma_{\mu\nu}^{RR} + g_L g_R(i\Sigma_{\mu\nu}^{LR} + i\Sigma_{\mu\nu}^{RL}) \\
i\Sigma_{\mu\nu}^{ij} &= -i(g_{\mu\nu}G_{ij}(k^2) - k_\mu k_\nu L(k^2)) \\
G_{ij}(k^2) &= G_{ij}(0) + k^2\Pi_{ij}
\end{aligned}
$$

Chiral two-point functions (2)

$$
\begin{aligned}
G_{LL,RR}(k^2) \quad &= \quad -\frac{4}{(4\pi)^2}\left\{\frac{C_{UV}}{4}(m_1^2+m_2^2)\ +\ k^2\left(B_2(m_1^2,m_2^2,k^2)-\frac{C_{UV}}{6}\right)\right.\\
&\qquad \left.-\frac{1}{2}\left(m_2^2 B_1(m_1^2,m_2^2,k^2)+m_1^2 B_1(m_2^2,m_1^2,k^2)\right)\right\}\\
G_{LR,RL}(k^2) \quad &= \quad \frac{2}{(4\pi)^2}m_1 m_2\left\{C_{UV}-B_0(m_1^2,m_2^2,k^2)\right\}\\
&\rightarrow \quad \Pi_{LR,RL}=\text{finite}\ \ ,\ \ \Pi_{LL}(m_1,m_2)-\Pi_{LL}(m_3,m_4)=\text{finite}\\
G_{LL}(0)+G_{LR}(0) \quad &= \quad \text{finite}\ \ \text{for}\ \ \ m_1=m_2\\
G_{LV} \quad &= \quad G_{LL}+G_{LR}=k^2(\Pi_{RR}+\Pi_{RL})\ \ \ ,\ \ G_{VV}=k^2(\Pi_{RR}+\Pi_{RL})
\end{aligned}
$$

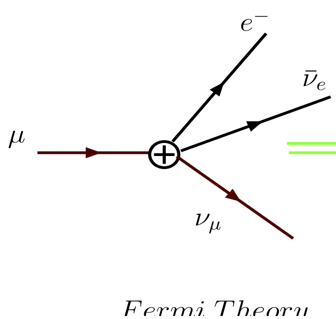
Chiral two-point functions (3)

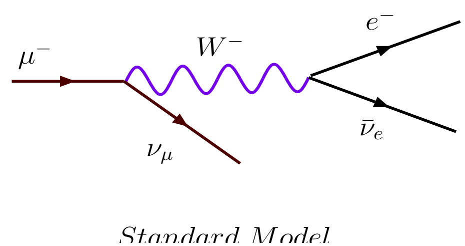
$$
\begin{aligned}
D_\mu &= \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ W_\mu^- T^-) - i\frac{g}{c_W} Z_\mu (T_3 - s_W^2 Q) - ieA_\mu Q \\
33, 11 &\rightarrow LL \quad Q \rightarrow V = L + R \\
\Pi_{\gamma\gamma} &= e^2 \Pi_{QQ} \\
\Pi_{\gamma Z} &= \left(\frac{e^2}{s_W c_W}\right) (\Pi_{3Q} - s_W^2 \Pi_{QQ}) \\
\Pi_{ZZ} &= \left(\frac{e^2}{s_W^2 c_W^2}\right) (\Pi_{33} - 2 s_W^2 \Pi_{3Q} + s_W^4 \Pi_{QQ}) \\
\Pi_{WW} &= \left(\frac{e^2}{s_W^2}\right) \Pi_{11} = g^2 \Pi_{11} \\
G_{WW} &= g^2 G_{11} \\
G_{ZZ} &= \frac{g^2}{c_W^2} G_{33}
\end{aligned}
$$

Finiteness if only combinations are

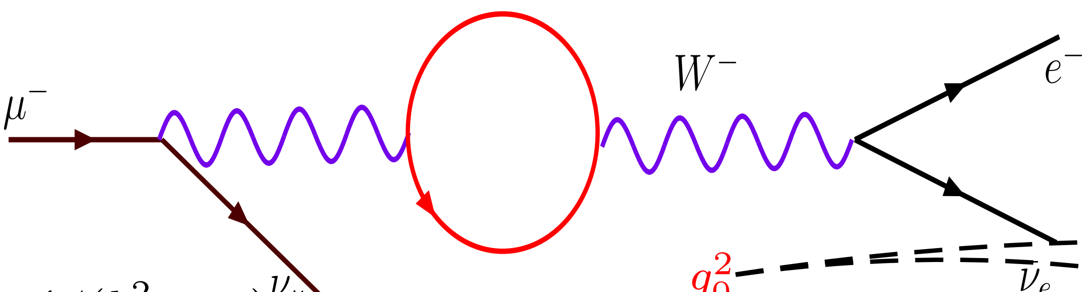
$$\left(\Pi_{ij}(k^2) - \Pi_{i',j'}(k'^2)\right) \quad \text{or} \quad \left(G_{33}(0) - G_{11}(0)\right)$$

Muon decay and Δr





$$\mathcal{M}_\mathbf{0} = \frac{\mathbf{g_0^2}}{\mathbf{k^2 - M^2_{W,0}}} \quad \text{with } \mathbf{k^2 \to 0}$$



at one-loop

$$\mathcal{M}(k^2 \to 0) = \frac{g_0^2}{k^2_{\hookrightarrow 0} - M^2_{W,0} + G_{WW}(k^2_{\hookrightarrow 0})} = -\frac{g^2}{M_W^2} \frac{1 + \frac{\delta g^2}{g^2}}{1 + \frac{\delta M_W^2}{M_W^2} - \frac{G_{WW}(0)}{M_W^2}}$$

$$= -\frac{g^2}{M_W^2}\left(1 + \frac{\delta g^2}{g^2}\right)\left(1 + \Re e[\Pi_{WW}(M_W^2)]\right)^{-1}$$

$$\simeq -\frac{g^2}{M_W^2}\left(1 + \underbrace{\frac{\delta g^2}{g^2} - \Re e[\Pi_{WW}(M_W^2)]}_{\Delta\hat{r} = finite} + \delta r_{V+B}\right) \equiv -\frac{g^2}{M_W^2}(1 + \Delta r)$$

Finiteness of $\Delta\hat{r}$

$$\Delta\hat{r} = \frac{\delta g^2}{g^2} - \Re e[\Pi_{WW}(M_W^2)] = \frac{\delta e^2}{e^2} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) - \Re e[\Pi_{WW}(M_W^2$$

$$= \Pi_{\gamma\gamma}(0) + \frac{c_W^2}{s_W^2}\left\{\underbrace{\left(\frac{G_{WW}(0)}{M_W^2} - \frac{G_{ZZ}(0)}{M_Z^2}\right)}_{\text{finite}} + \left(\Re e\Pi_{WW}(M_W^2) - \Re e\Pi_{ZZ}(M_Z^2)\right)\right\} - \Re e[\Pi_{WW}($$

$$\Delta\hat{r} = \left(\Pi_{\gamma\gamma}(0) - \Re e\Pi_{\gamma\gamma}(M_Z^2)\right) \; + \; g^2\frac{c_W^2}{s_W^2}\left(\frac{G_{11} - G_{33}}{M_W^2}\right)$$

$$+\frac{g^2}{s_W^2}\left\{-\Re e\Pi_{33}(M_Z^2) + 2s_W^2\Re e\Pi_{3Q}(M_Z^2) + (c_W^2 - s_W^2)\Re e\Pi_{11}(M_W^2)\right\}$$

$$= \left(\Pi_{\gamma\gamma}(0) - \Re e\Pi_{\gamma\gamma}(M_Z^2)\right) \; + \; g^2\frac{c_W^2}{s_W^2}\left(\frac{G_{11} - G_{33}}{M_W^2}\right)$$

$$+g^2\left\{2\Re e[\Pi_{3Q}(M_Z^2) - \Pi_{33}(M_Z^2)] \; + \; \frac{c_W^2 - s_W^2}{s_W^2}\Re e[\Pi_{11}(M_W^2) - \Pi_{33}(M_Z^2)]\right\}$$

$$\Delta\alpha(M_Z^2) = \left(\Pi_{\gamma\gamma}(0) - \Re e\Pi_{\gamma\gamma}(M_Z^2)\right) \equiv e^2\left(\Pi_{QQ}(0) - \Re e\Pi_{QQ}(M_Z^2)\right)$$

Δr: correcting a tree-level formula (1)

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1+\Delta r) \quad \Delta r = \Delta\hat{r} + \delta r_{V+B} \qquad r_{V+B} \sim 6.43\ 10^{-3}$$

$$\Delta\hat{r} = \Delta\alpha(M_Z^2) - \frac{c_W^2}{s_W^2}\varepsilon_1 + \frac{c_W^2 - s_W^2}{s_W^2}\varepsilon_2 + 2\varepsilon_3$$

$\boxed{\varepsilon_{1,2,3} \text{ or } S,T,U\text{: non-decoupling effects}}$

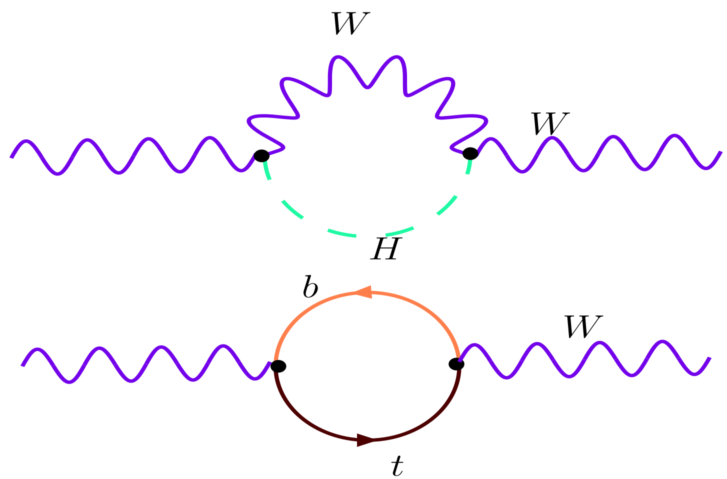
$$\varepsilon_1 = \Delta\rho = \alpha T = g^2\left(\frac{G_{33}-G_{11}}{M_W^2}\right) \propto m_t^2$$

SU(2) breaking

$$\varepsilon_2 = -\frac{\alpha}{4s_W^2}U = g^2\left(\Re e[\Pi_{11}(M_W^2)] - \Re e[\Pi_{33}(M_Z^2)]\right)$$

$$\varepsilon_3 = \frac{\alpha}{4s_W^2}S = g^2\left(\Re e[\Pi_{3Q}(M_Z^2)] - \Re e[\Pi_{33}(M_Z^2)]\right) = -\frac{g^2}{2}\Re e[\Pi_{3Y}(M_Z^2)] \rightarrow \text{chiral symm. b}$$

Δr Probe of M_H and m_t

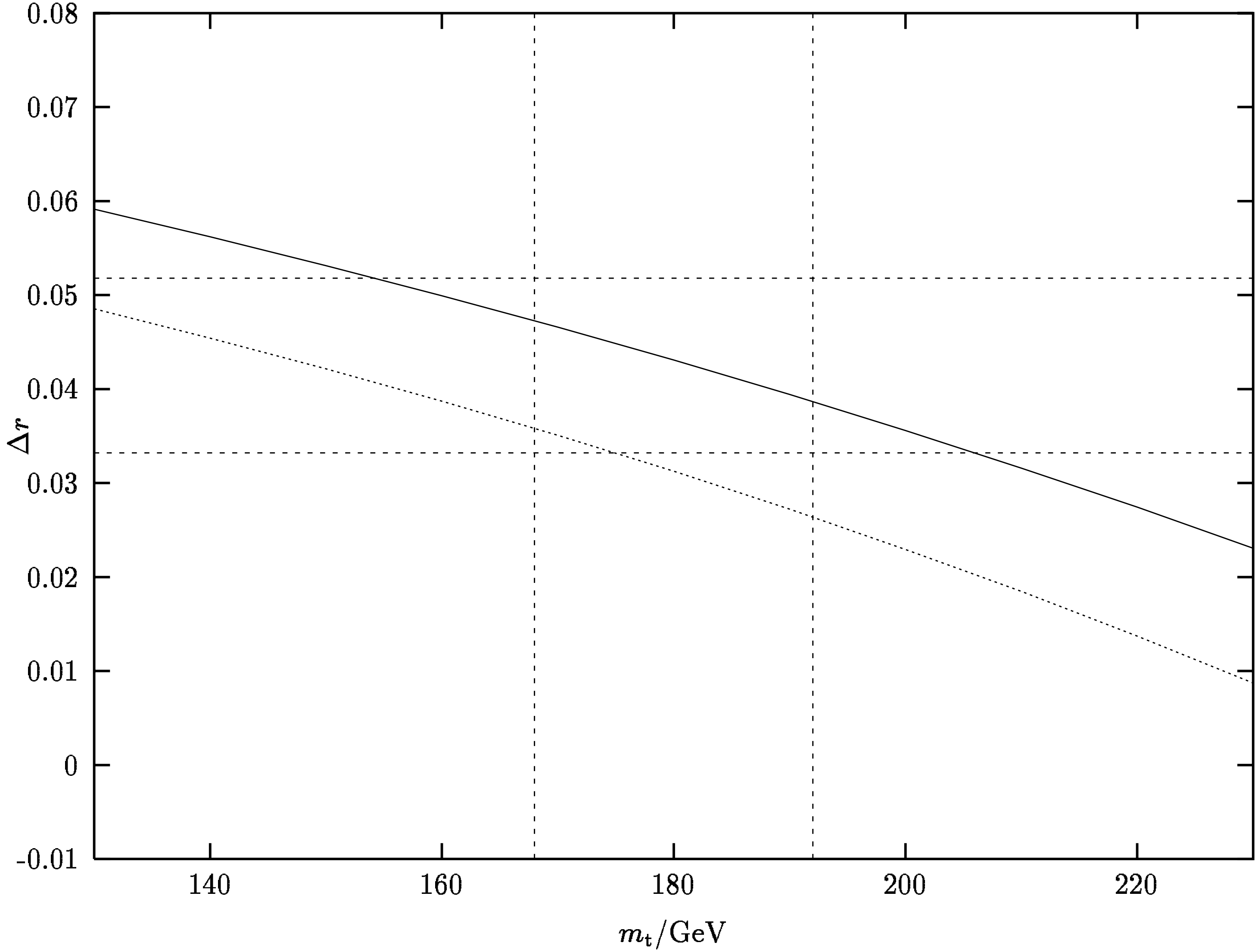


$$\varepsilon_{\mathbf{1}} = \frac{\mathbf{3G}_\mu \mathbf{M_Z^2}}{\mathbf{8}\pi^{\mathbf{2}}\sqrt{\mathbf{2}}} \left\{ \left(\frac{\mathbf{m_t^2}}{\mathbf{M_Z^2}} - \mathbf{1} \right) - \mathbf{2s_M^2} \ln(\mathbf{M_H}/\mathbf{M_Z}) \right\} + \varepsilon_{\mathbf{1}}^{\mathrm{NP}}$$

$$\varepsilon_{\mathbf{2}} = -\frac{\mathbf{G}_\mu \mathbf{M_W^2}}{\mathbf{2}\pi^{\mathbf{2}}\sqrt{\mathbf{2}}} \ln(\mathbf{m_t}/\mathbf{M_Z}) + \varepsilon_{\mathbf{2}}^{\mathrm{NP}}$$

$$\varepsilon_{\mathbf{3}} = \frac{\mathbf{G}_\mu \mathbf{M_W^2}}{\mathbf{12}\pi^{\mathbf{2}}\sqrt{\mathbf{2}}} \{\ln(\mathbf{M_H}/\mathbf{M_Z}) - \ln(\mathbf{m_t^2}/\mathbf{M_Z^2})\} + \varepsilon_{\mathbf{3}}^{\mathrm{NP}}$$

Δr: correcting a tree-level formula (2)



s_Z^2

With Δr we predict M_W

$$\frac{\mathbf{M_W^2}}{\mathbf{M_Z^2}} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} + \sqrt{\mathbf{1} - \frac{\mathbf{4}\pi\alpha(\mathbf{1}+\boldsymbol{\Delta}\mathbf{r})}{\sqrt{\mathbf{2}}\mathbf{G}_\mu \mathbf{M_Z^2}}}\right) \quad ; \quad \mathbf{s_M^2} = \mathbf{s_W^2} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} - \sqrt{\mathbf{1} - \frac{\mathbf{4}\pi\alpha(\mathbf{1}+\boldsymbol{\Delta}\mathbf{r})}{\sqrt{\mathbf{2}}\mathbf{G}_\mu \mathbf{M_Z^2}}}\right)$$

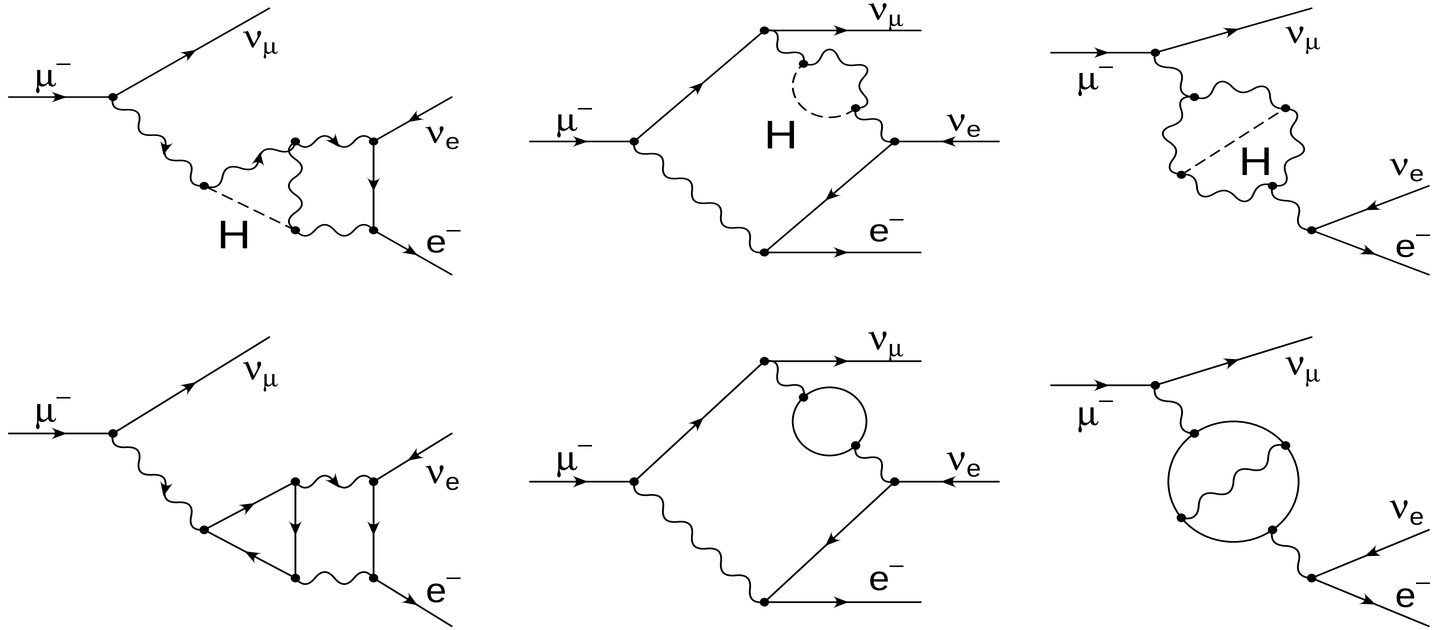
Improved s_W^2

$$\mathbf{s_Z^2} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} - \sqrt{\mathbf{1} - \frac{\mathbf{4}\pi\alpha_\mathbf{Z}}{\sqrt{\mathbf{2}}\mathbf{G}_\mu \mathbf{M_Z^2}}}\right) \quad \alpha_\mathbf{Z} = \frac{\alpha}{\mathbf{1} - \boldsymbol{\Delta}\alpha_\mathbf{Z}}$$

$$\begin{cases} s_Z^2 = .212154 & \text{for} \quad \alpha_Z = \alpha(0) \\ s_Z^2 = .231086 & \text{for} \quad \alpha_Z = 128.907 \\ s_Z^2 = .231019 & \text{for} \quad \alpha_Z = 128.944 \end{cases}$$

$s^2_{\rm eff}|_{1991} = .2329 \pm^{.0034}_{.0029}$ compatible with any of the above values with a running α, but not with $\alpha(0) = \alpha$.

Δr beyond one-loop



$$
\begin{aligned}
M_W &= M_W^0 - c_1\,\mathrm{dH} - c_2\,\mathrm{dH}^2 + c_3\,\mathrm{dH}^4 + c_4(\mathrm{dh}-1) - c_5\,\mathrm{d}\alpha + c_6\,\mathrm{dt} \\
&\quad - c_7\,\mathrm{dt}^2 - c_8\,\mathrm{dH}\,\mathrm{dt} + c_9\,\mathrm{dh}\,\mathrm{dt} - c_{10}\,\mathrm{d}\alpha_s + c_{11}\,\mathrm{dZ},
\end{aligned}
$$

$$
\mathrm{dH} = \ln\left(\frac{M_H}{100\ \mathrm{GeV}}\right), \qquad \mathrm{dh} = \left(\frac{M_H}{100\ \mathrm{GeV}}\right)^2, \quad \mathrm{dt} = \left(\frac{m_t}{174.3\ \mathrm{GeV}}\right)^2 - 1,
$$

$$
\mathrm{dZ} = \frac{M_Z}{91.1875\ \mathrm{GeV}} - 1, \qquad \mathrm{d}\alpha = \frac{\Delta\alpha(M_Z^2)}{0.05907} - 1, \quad \mathrm{d}\alpha_s = \frac{\alpha_s(M_Z^2)}{0.119} - 1.
$$

Corrections to the neutral current observables

$$\mathcal{M}_0 = \underbrace{-e^2 Q \frac{1}{k^2} Q'}_{\gamma\ \text{exch.}} \underbrace{- \frac{e^2}{s_W^2 c_W^2}(T_3 - s_W^2 Q)\frac{1}{k^2 - M_Z^2}(T_3' - s_W^2 Q')}_{Z\ \text{exch.}}$$

$$\mathcal{M}^{1-\text{loop}} = -e^2 \underbrace{\left(1 + \frac{\delta e^2}{e^2}\right)}_{counterterm} Q \underbrace{\frac{1}{k^2\,(1 + \Pi_{\gamma\gamma}(k^2))}}_{\gamma\gamma\ 2-pt} Q'$$

$$- \frac{e^2}{s_W^2 c_W^2} \underbrace{\frac{\left(1 + \frac{\delta e^2}{e^2}\right)}{\left(1 + \frac{\delta s^2}{s^2} - \frac{s_W^2}{c_W^2}\frac{\delta s^2}{s^2}\right)}}_{counterterm} \underbrace{\frac{\Big(T_3 - s_W^2(1 + \frac{\delta s^2}{s^2})Q\Big) \times \Big(T_3' - s_W^2(1 + \frac{\delta s^2}{s^2})Q'}{k^2 - M_Z^2\left(1 + \frac{\delta M_Z^2}{M_Z^2}\right) + G_{ZZ}(k^2)}}_{ZZ\ 2-pt}$$

$$+ \frac{e^2}{s_W c_W}(T_3 - s_W^2 Q)\underbrace{\frac{\Pi_{\gamma Z}(k^2)}{k^2 - M_Z^2}}_{Z\gamma\ 2-pt}(Q') \qquad + \frac{e^2}{s_W c_W}(Q)\underbrace{\frac{\Pi_{\gamma Z}(k^2)}{k^2 - M_Z^2}}_{Z\gamma\ 2-pt}(T_3' - s_W^2 Q')$$

Corrections to the neutral current observables (2)combine ZZ with $Z\gamma$

$$+ \quad \frac{e^2}{s_W c_W}(T_3 - s_W^2 Q)\frac{\Pi_{\gamma Z}(k^2)}{k^2 - M_Z^2}(Q')$$

$$= \quad -\frac{e^2}{s_W^2 c_W^2}\,(T_3 - s_W^2 Q)\left[\frac{c_W}{s_W}\Pi_{\gamma Z}(k^2)\right](-s_W^2 Q') \quad \rightarrow$$

$$- \quad \frac{e^2}{s_W^2 c_W^2}\frac{\left(1+\frac{\delta e^2}{e^2}\right)}{\left(1+\frac{\delta s^2}{s^2}-\frac{s_W^2}{c_W^2}\frac{\delta s^2}{s^2}\right)}\frac{\left(T_3 - s_W^2(1+\frac{\delta s^2}{s^2})Q\right)\times\left(\frac{c_W}{s_W}\Pi_{\gamma Z}(k^2)\right)\times\left(-s_W^2(1+\frac{\delta s^2}{s^2}\right.}{k^2 - M_Z^2\left(1+\frac{\delta M_Z^2}{M_Z^2}\right)+G_{ZZ}(k^2)}$$

$$-\mathcal{M}^{ZZ+Z\gamma} = -\mathcal{M}^Z$$

$$\frac{\mathbf{e^2}}{\mathbf{s_W^2 c_W^2}}\frac{\left(\mathbf{1}+\frac{\delta \mathbf{e^2}}{\mathbf{e^2}}\right)}{\left(\mathbf{1}+\frac{\delta \mathbf{s^2}}{\mathbf{s^2}}-\frac{\mathbf{s_W^2}}{\mathbf{c_W^2}}\frac{\delta \mathbf{s^2}}{\mathbf{s^2}}\right)}\frac{\left(\mathbf{T_3} - \boxed{s_W^2(1+\frac{\delta s^2}{s^2}+\frac{c_W}{s_W}\Pi_{\gamma Z}(k^2))}\,\mathbf{Q}\right)\left(\mathbf{T_3'} - \mathbf{s_W^2}(\mathbf{1}+\frac{\delta \mathbf{s^2}}{\mathbf{s^2}}\frac{\mathbf{c_W}}{\mathbf{s_w}}\mathbf{\Pi}\right.}{\mathbf{k^2} - \mathbf{M_Z^2}\left(\mathbf{1}+\frac{\delta \mathbf{M_Z^2}}{\mathbf{M_Z^2}}\right)+\mathbf{G_{ZZ}(0)}+\mathbf{k^2\Pi_{ZZ}(k^2)}}$$

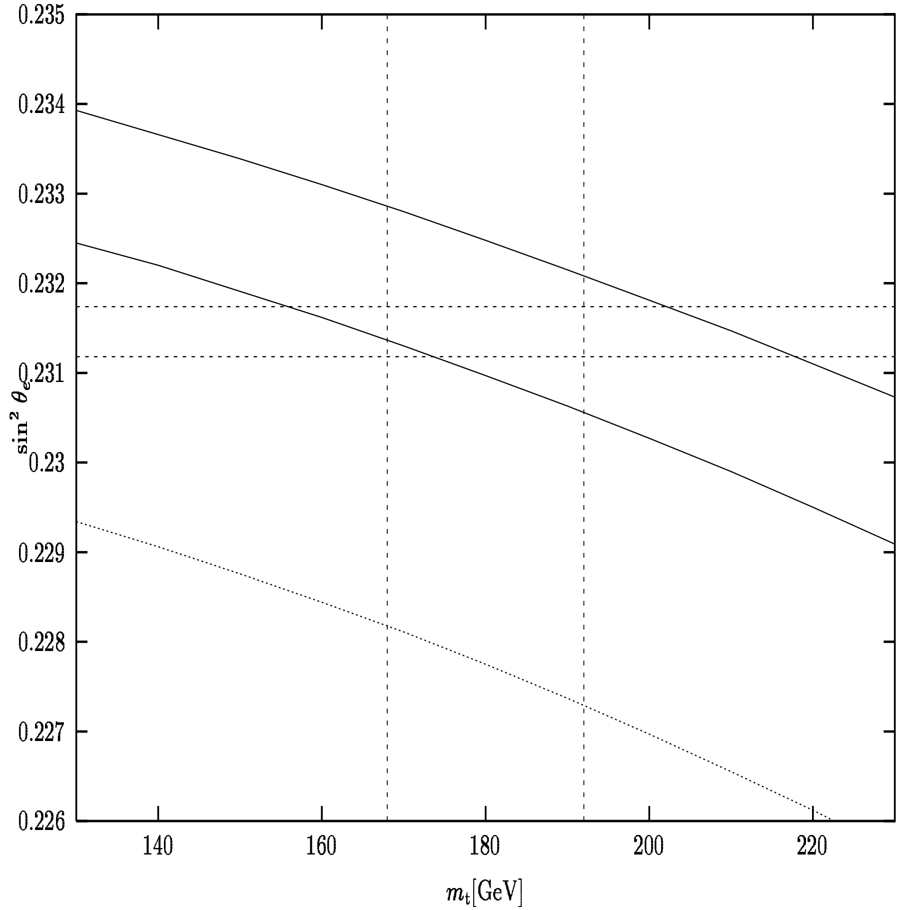
$$\mathcal{M}^{\mathbf{Z}} = -\frac{\mathbf{e_\star^2}}{\mathbf{s_\star^2 c_\star^2}}(\mathbf{T_3} - \boxed{s_\star^2}\,\mathbf{Q})\frac{\mathbf{Z_\star(k^2)}}{\mathbf{k^2} - \mathbf{M_{Z\star}^2(k^2)}}(\mathbf{T_3'} - \mathbf{s_\star^2 Q'}) = \mathbf{Z(k^2)}(\mathbf{T_3} - \mathbf{s_\star^2 Q})(\mathbf{T_3'} - \mathbf{s_\star^2 Q'})$$

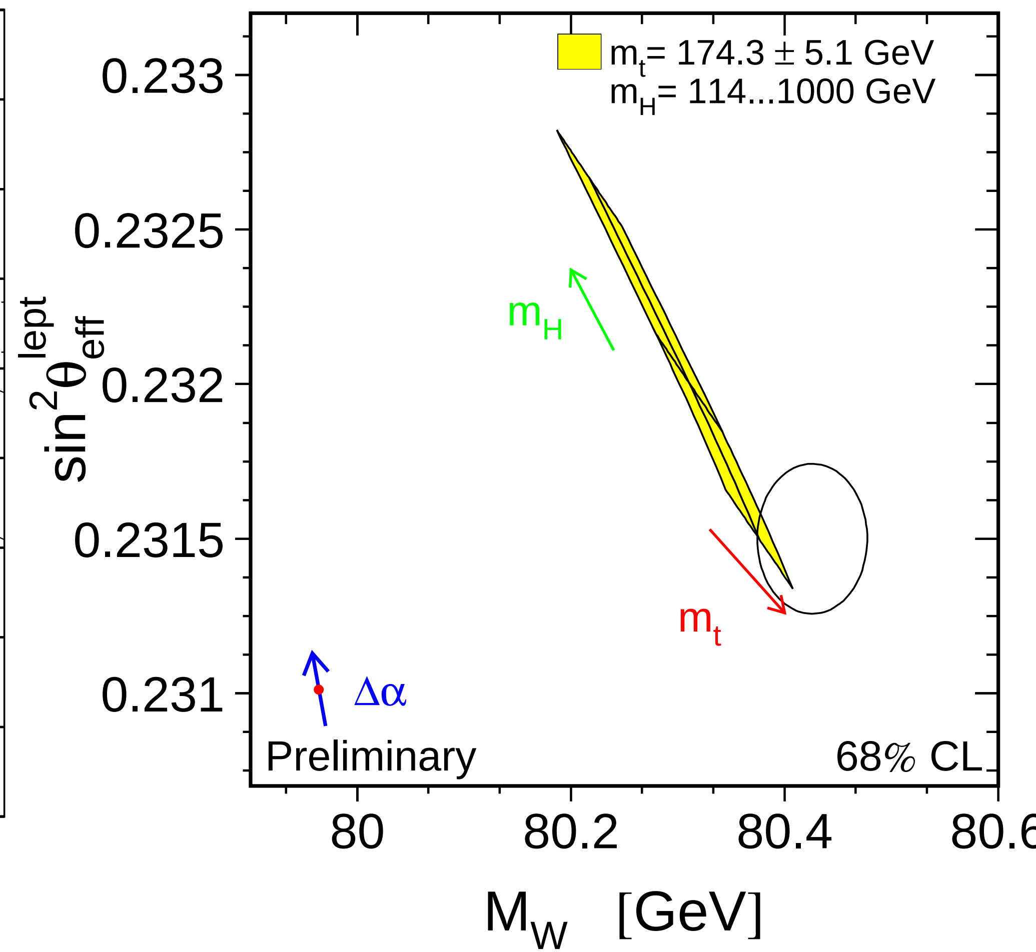
$\sin^2\theta$ in Z couplings

$$
\begin{aligned}
s^2_\star(k^2) &= s^2_Z\Bigg\{1+(\Delta\alpha(k^2)-\Delta\alpha(M_Z^2)) + \frac{c_W^2}{c_W^2-s_W^2}(\Delta\alpha_{NP}(M_Z^2)-\Delta\rho+\delta r_{V+B})\\
&+ \frac{e^2}{s_W^2}\frac{1}{c_W^2-s_W^2}\left(\Re e\Pi_{3Q}(M_Z^2)-\Re e\Pi_{33}(M_Z^2)\right)+\frac{e^2}{s_W^2}\left(\Pi_{3Q}(k^2)-\Re e\Pi_{3Q}(M_Z^2)\right)\Bigg\}\\
s^2_\star(M_Z^2) &= s^2_Z\left\{1-\frac{c_W^2}{c_W^2-s_W^2}\left(\Delta\rho-\frac{\varepsilon_3}{c_W^2}-\Delta\alpha_{NP}(M_Z^2)-\delta r_{V+B}\right)\right\}\\
s^2_\star(M_Z^2) &= s^2_Z(1+\Delta\kappa')
\end{aligned}
$$

$$
\begin{aligned}
s^2_\star(M_Z^2) &= .23122 \quad m_t=174.3\mathrm{GeV} \quad M_H=300\mathrm{GeV}\\
s^2_\star(M_Z^2) &= .23018 \quad m_t=200\mathrm{GeV} \quad M_H=300\mathrm{GeV}\\
s^2_\star(M_Z^2) &= .23457 \quad m_t=50\mathrm{GeV} \quad M_H=300\mathrm{GeV}
\end{aligned}
$$

$sin^2\theta_{eff}$





Z couplings at the peak

$$\mathbf{Z(k^2)} = \frac{\mathbf{e_\star^2(k^2)}}{\mathbf{s_\star^2(k^2)c_\star^2(k^2)}}\left(\mathbf{1} + \frac{\mathbf{c_W^2 - s_W^2}}{\mathbf{s_W c_W}}\mathbf{\Pi}_{\gamma\mathbf{Z}}\mathbf{(k^2)} + \mathbf{\Pi}_{\gamma\gamma}\mathbf{(k^2)}\right) \times \mathbf{D_Z}$$

$$\mathbf{D_Z} = \frac{\mathbf{1}}{\mathbf{k^2 - M_Z^2 + k^2\Pi_{ZZ}(k^2) - M_Z^2\Pi_{ZZ}(M_Z^2)}}$$

$$\mathbf{M_Z^\star}^{\mathbf{2}}\mathbf{(k^2)} = \left\{\mathbf{M_Z^2 - (k^2 - M_Z^2)(\Pi_{ZZ}(k^2) - \Pi_{ZZ}(M_Z^2))}\right\}$$

$$\mathbf{M_Z^\star}^{\mathbf{2}}\mathbf{(M_Z^2) = M_Z^2} \quad \text{and} \quad \frac{\mathbf{dM_Z^\star}^{\mathbf{2}}\mathbf{(k^2)}}{\mathbf{dk^2}}_{|\mathbf{k^2=M_Z^2}} = \mathbf{0}$$

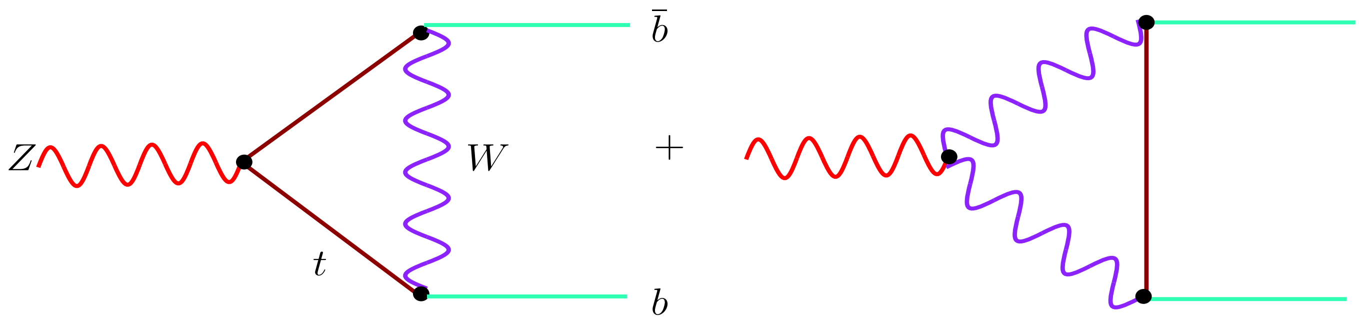
$$Z_\star(k^2) = 1 + \frac{c_W^2 - s_W^2}{s_W c_W}\Pi_{\gamma Z}(k^2) + \Pi_{\gamma\gamma}(k^2) - \Pi_{ZZ}(k^2) + (k^2 - 2M_Z^2)\tilde{\Pi}'_{ZZ}(k^2)$$

$$\Pi_{ZZ}(k^2) = \Pi_{ZZ}(M_Z^2) + (k^2 - M_Z^2)\tilde{\Pi}'_{ZZ}(k^2)$$

$$\frac{\mathbf{4}\pi\alpha_\star\mathbf{(M_Z^2)}}{\mathbf{s_\star^2(M_Z^2)c_\star^2(M_Z^2)}}\mathbf{Z_\star(M_Z^2) = 4\sqrt{2}G}_\mu\mathbf{M_Z^2}\left(\mathbf{1 + \Delta}\rho - \mathbf{M_Z^2\tilde{\Pi}'_{ZZ}(M_Z^2)}\right) \simeq \mathbf{4\sqrt{2}G}_\mu\mathbf{M_Z^2\,(1 + \Delta}\rho)$$

$$\mathbf{g_A}^{\text{lepton}} = -\frac{\mathbf{1}}{\mathbf{2}}\sqrt{\mathbf{1 + \Delta}\rho} \quad , \quad \mathbf{g_V}^{\text{lepton}} = \mathbf{g_A}^{\text{lepton}}(\mathbf{1 - 4s^2_{\text{eff}\ ,\text{lepton}}})$$

$Z_{b\bar{b}}$: An important vertex correction



$$\mathcal{M}_{\mathbf{Zb_L b}} = -\mathbf{i}\frac{\mathbf{e_\star^2}}{\mathbf{c_\star^2 s_\star^2}}\mathbf{Z}_\mu \bar{\mathbf{b}}\gamma_\mu \mathbf{b_L}\left((\frac{\mathbf{1}}{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{s_\star^2}) - \delta\mathbf{v_b}\right)$$

m_t^2 dependence in width almost washed away

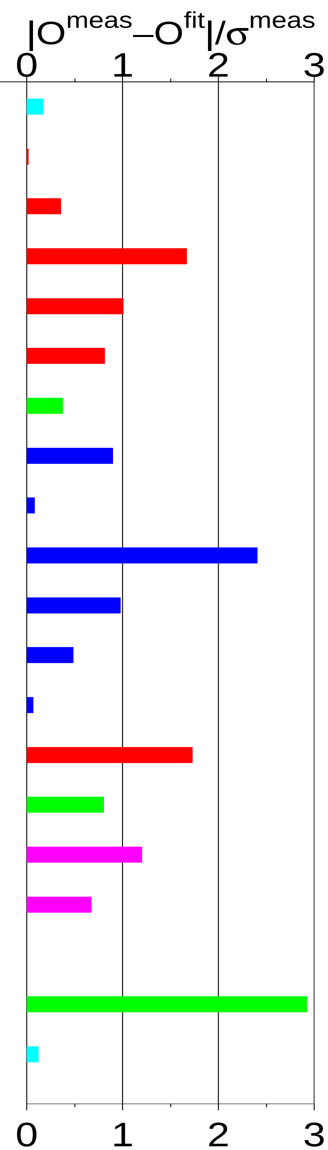
$$\Gamma_{\mathbf{Zb\bar{b}}} = \Gamma^{\mathbf{0}}_{\mathbf{Zb\bar{b}}}(\mathbf{G}_\mu, \mathbf{M_Z}, \alpha(\mathbf{M_Z^2}))\left(\mathbf{1} + \frac{\mathbf{19}}{\mathbf{13}}(\boldsymbol{\Delta}\rho + \delta\mathbf{v_b})\right)$$

$$\boldsymbol{\Delta}\rho \sim +\frac{\alpha}{\pi}\frac{\mathbf{m_t^2}}{\mathbf{M_Z^2}} \text{ while } \delta\mathbf{v_b} = -\frac{\mathbf{20}\alpha}{\mathbf{19}\pi}\left(\frac{\mathbf{m_t^2}}{\mathbf{M_Z^2}} + \frac{\mathbf{13}}{\mathbf{6}}\ln(\mathbf{m_t^2}/\mathbf{M_Z^2})\right)$$

LEP Pulls

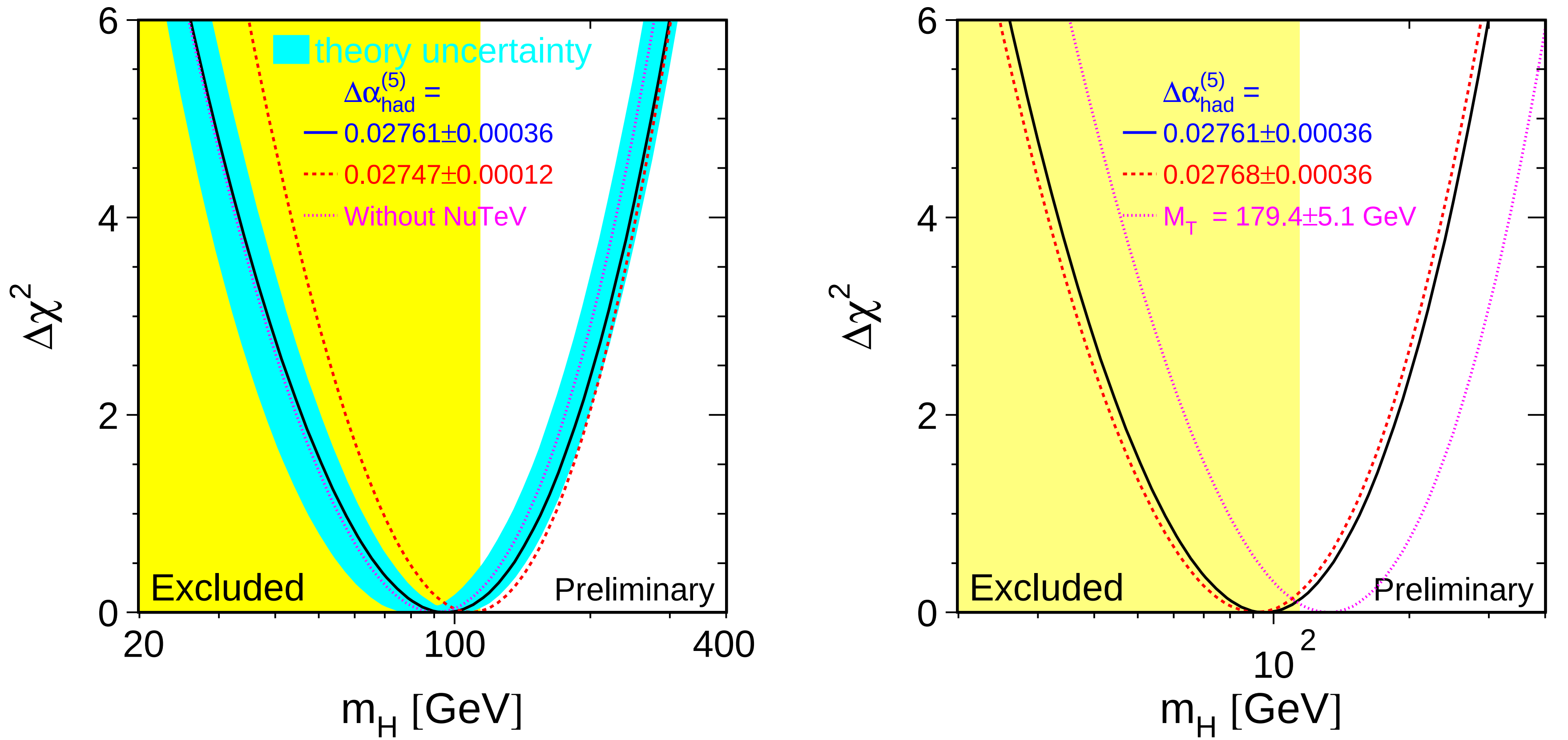
Summer 2003

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4960
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01636
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1477
R_b	0.21638 ± 0.00066	0.21579
R_c	0.1720 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0997 ± 0.0016	0.1036
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0740
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1477
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.426 ± 0.034	80.385
Γ_W [GeV]	2.139 ± 0.069	2.093
m_t [GeV]	174.3 ± 5.1	174.3
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	0.2229
$Q_W(Cs)$	-72.84 ± 0.46	-72.90



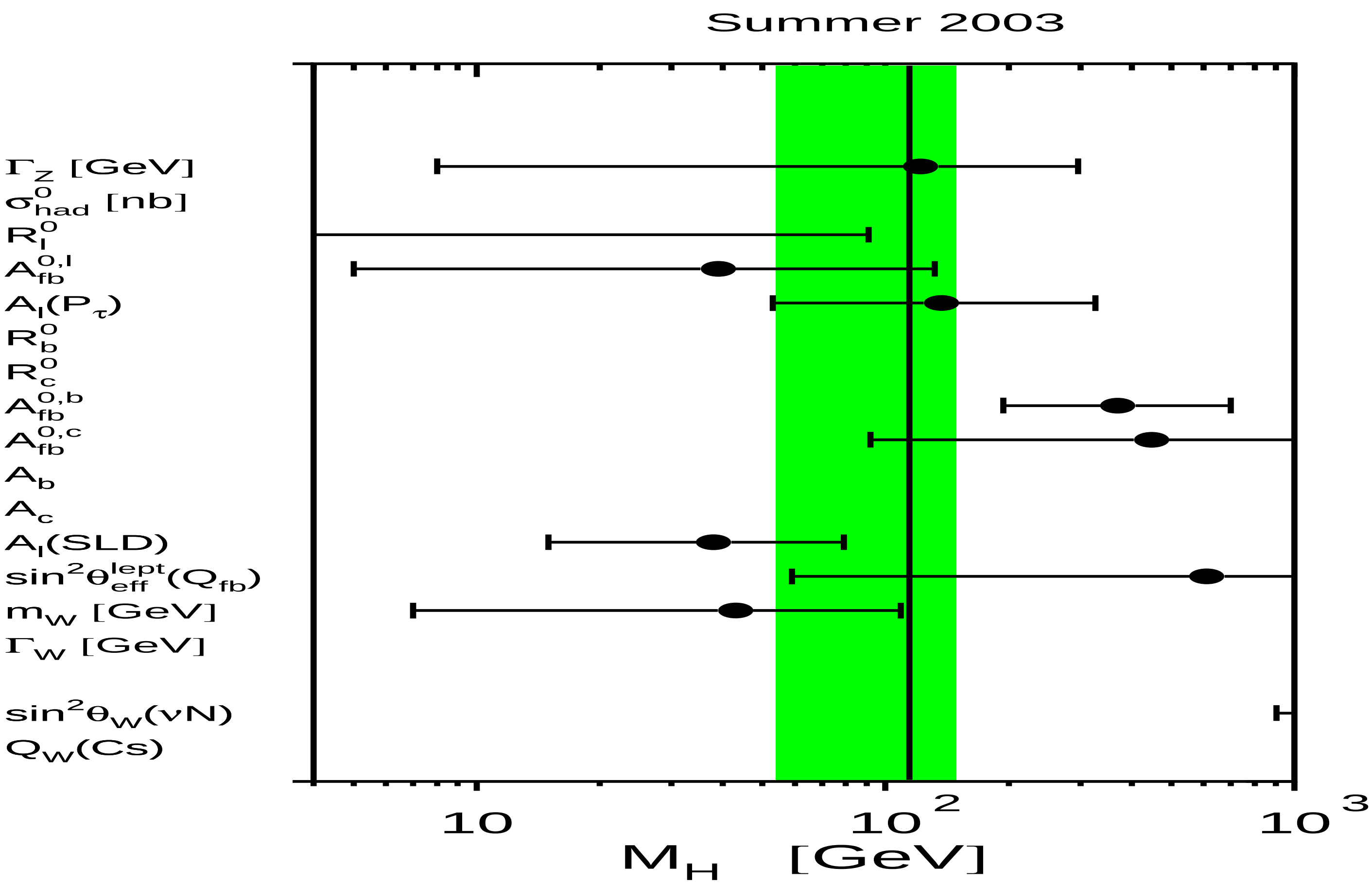
$\sigma^{meas.}$ is the error on $\mathcal{O}^{meas.}$

Higgs and the blue-band



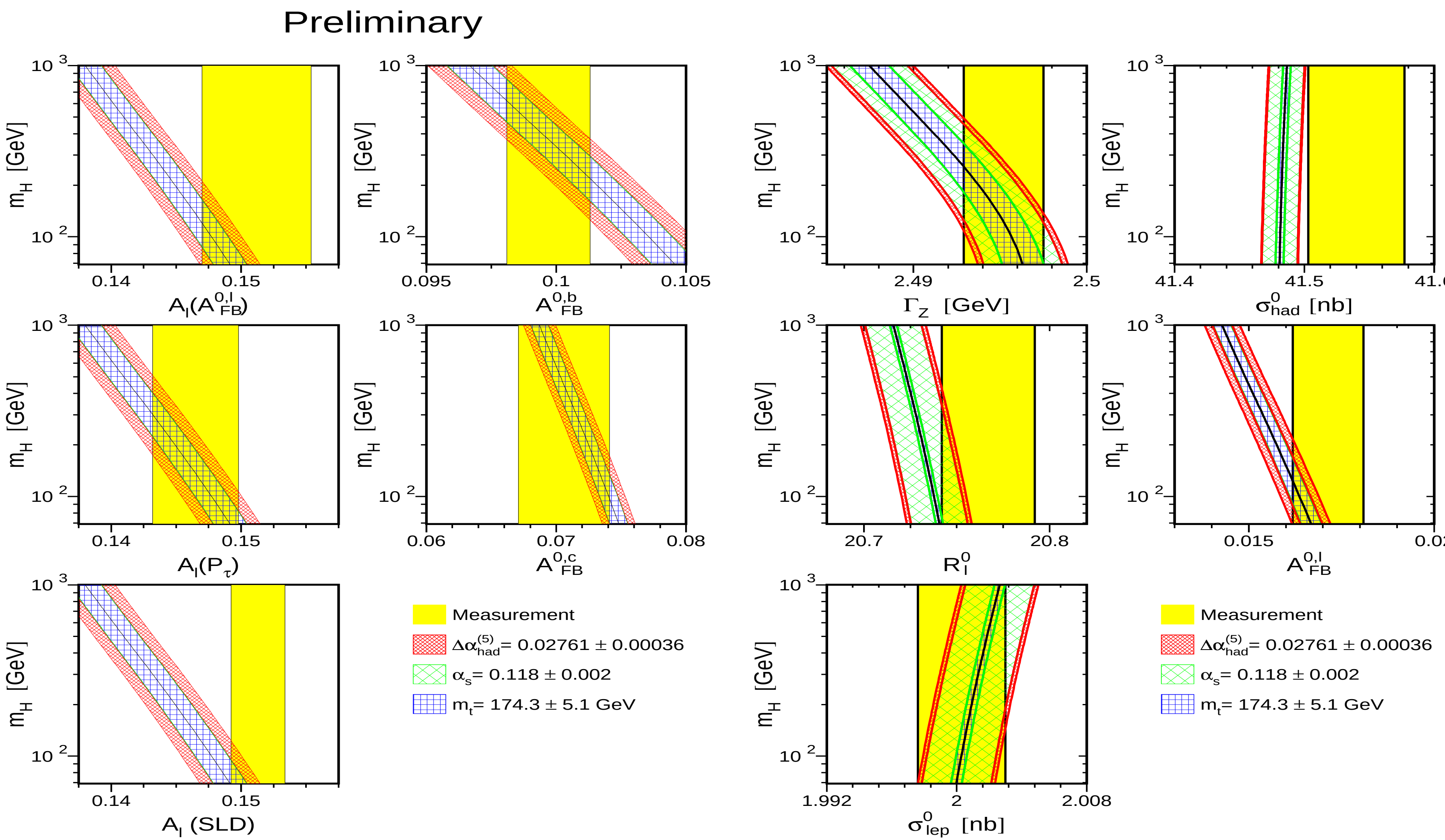
The famous blue band. This gives the limit on the Higgs mass from all precision data and also after excluding the anomalous NuTeV result. The area shaded in yellow is the mass excluded from a direct search. The blue band refers to the estimated theoretical uncertainty from missing higher order terms. The χ^2 is given for 2 values of $\Delta\alpha(M_Z^2)$. From LEP electroweak group. The second plot shows what happens if one increases the average value of the top mass within 1σ.

M_H sensitivity (1)

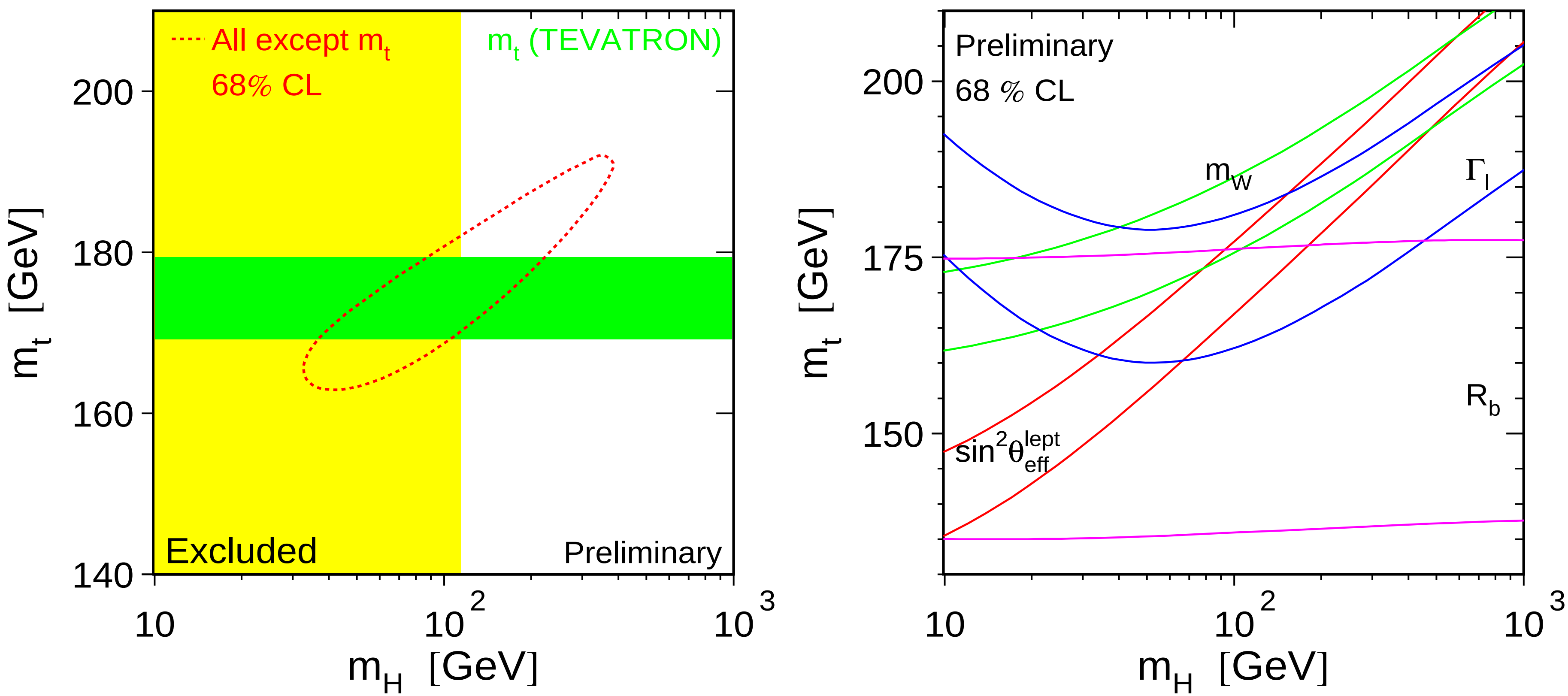


M_H extraction from different observables. Not all point to a low value of M_H. Some point to values much smaller than the direct limit.

M_H sensitivity (2)

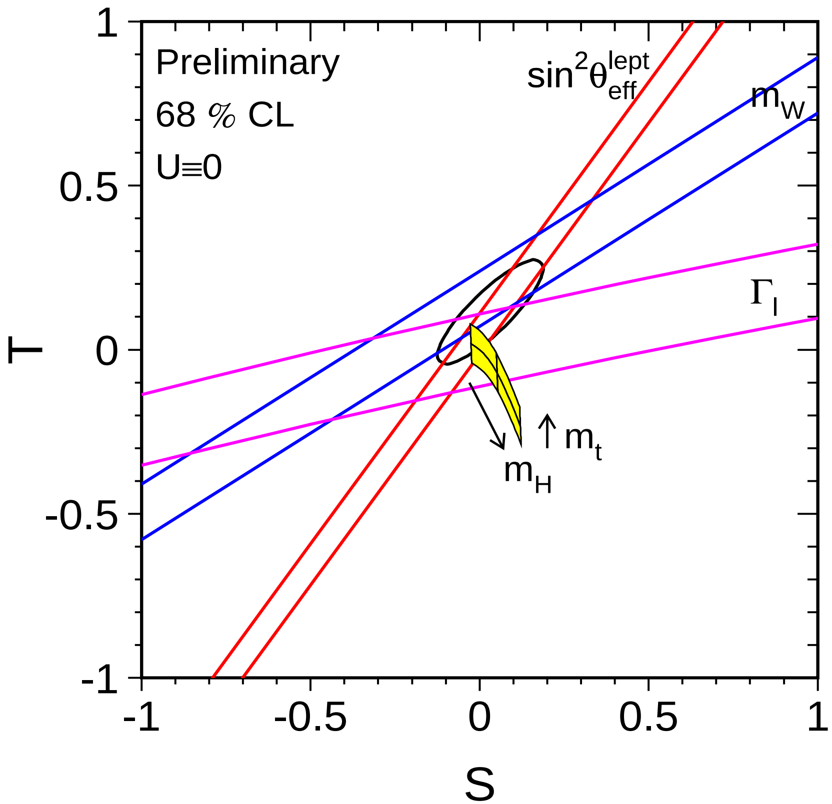


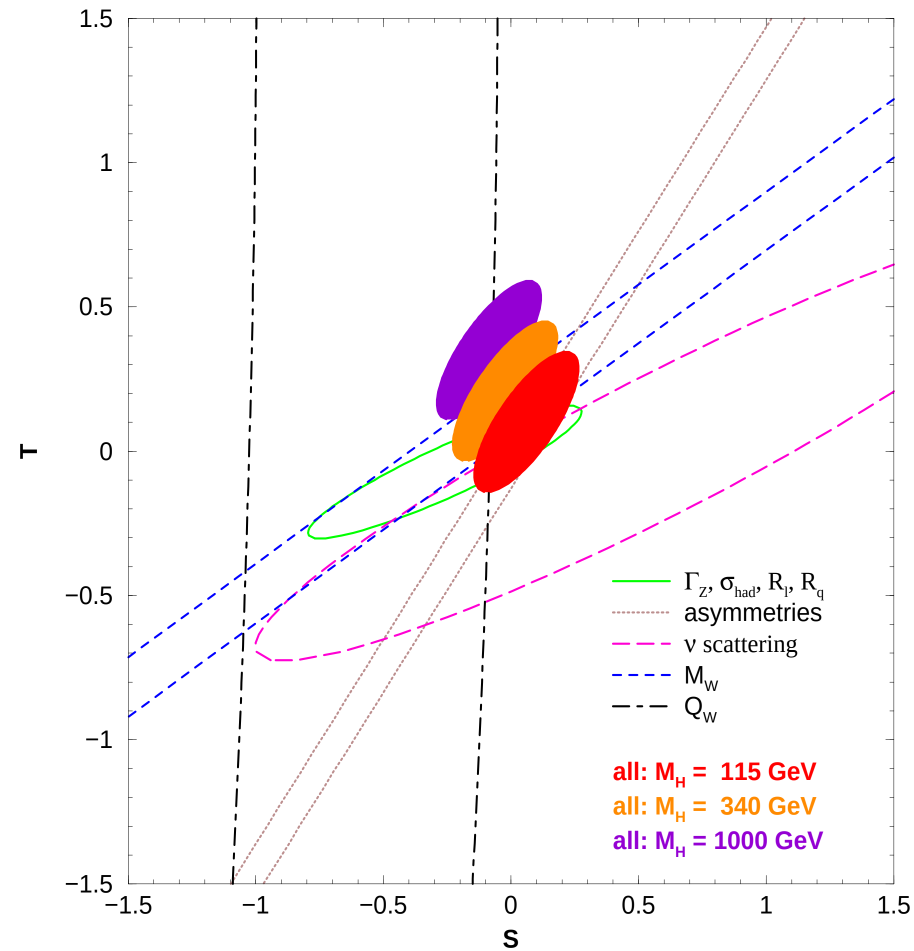
m_t vs M_H sensitivity



Prediction of the top mass from the precision measurement and how it compares with the direct limit on the top mass from the TEVATRON. The plot on the left shows how the different precision observables constraint in the M_H vs m_t plan.

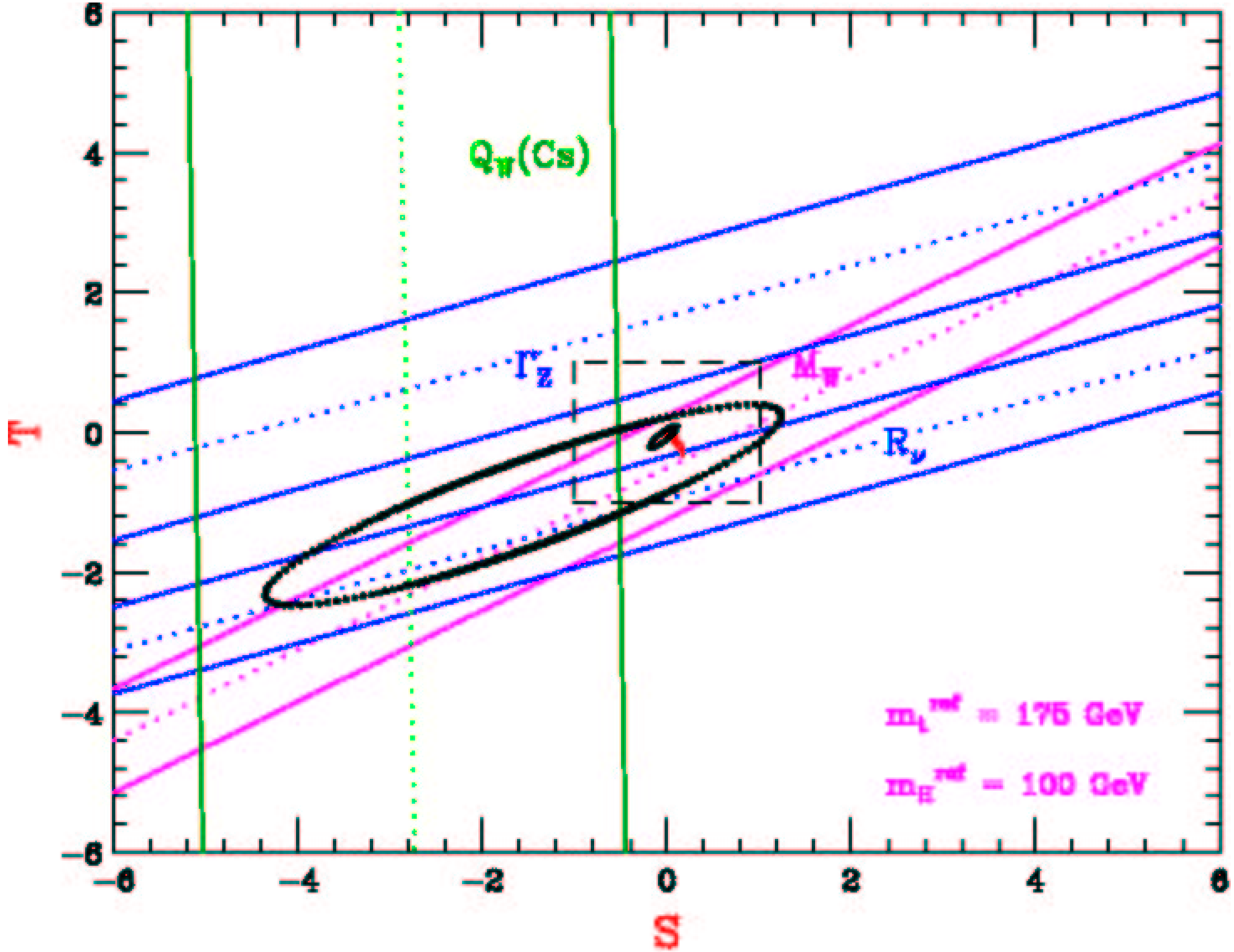
The S-T plane



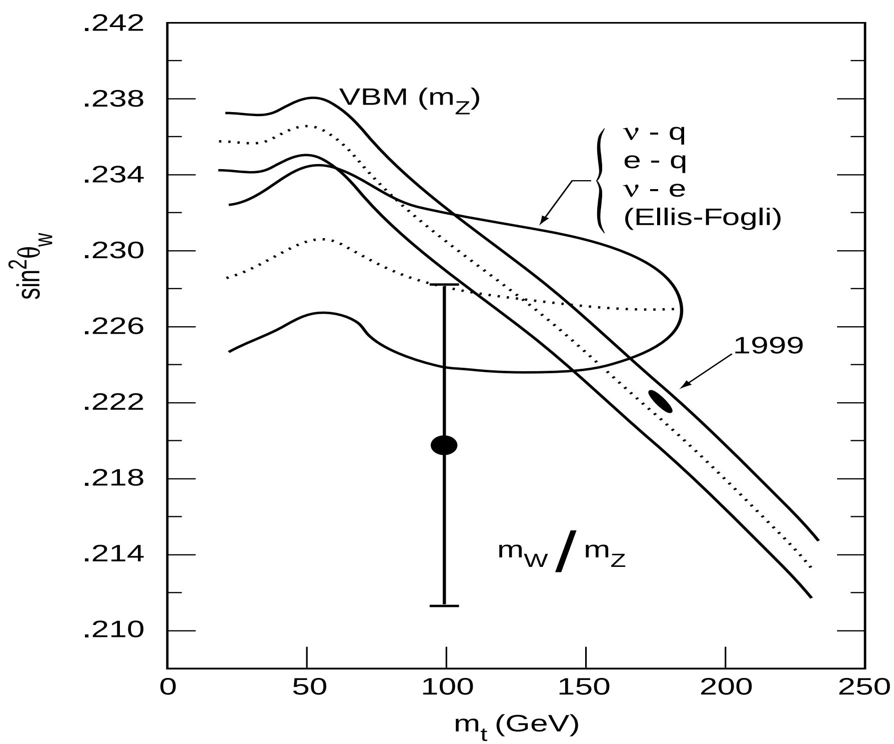


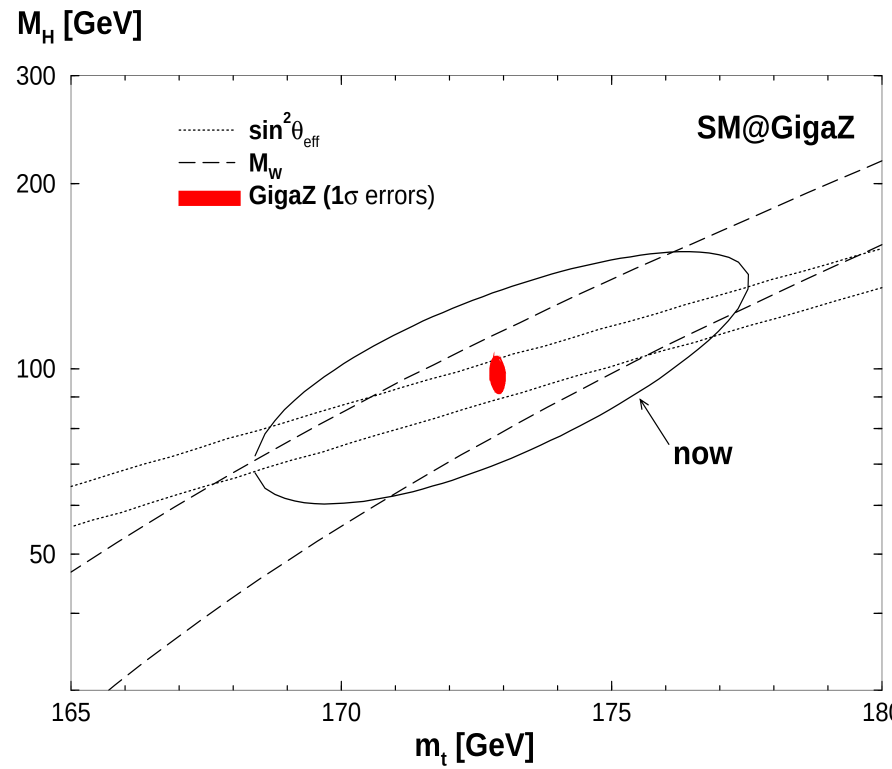
Constraint in on the S and T variables. The variation from m_t within its 1σ value and M_H is shown. The second graph, which fits for different values of M_H, shows that a high value of M_H is more comfortable with $S < 0$ but $T > 0$. T

The S-T plane: 1989 *VS* 1999



How far we've come and how far will go





A global fit on the mixing angle just before the start of LEP and how it compares with the situation in 1999!. If a linear collider is built and is run as a Z factory, the second plot shows how much improvement one expects.

End

That's all folks for α/π's do go through the notes

and send typos to

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