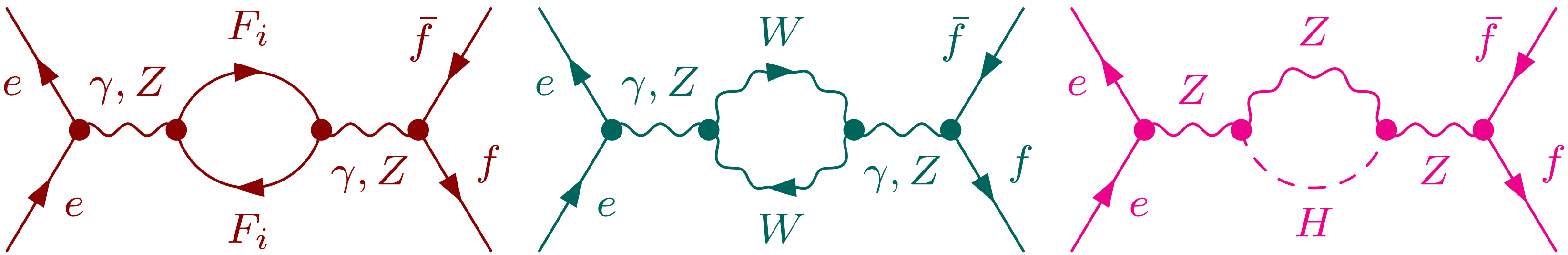
Elements of the Renormalisation of the Electroweak Theory

and

Precision Tests of the Standard Model

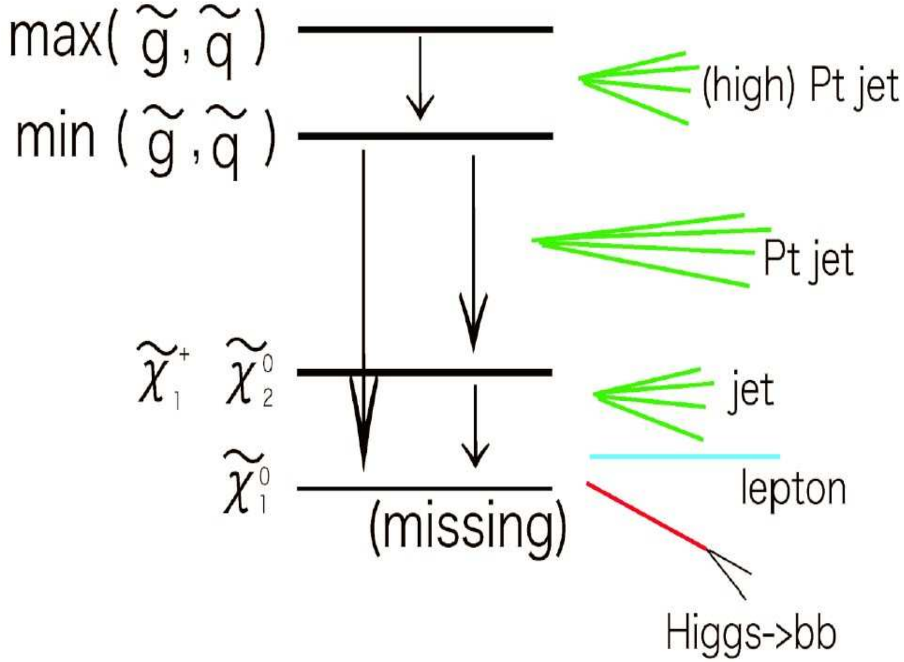


Fawzi BOUDJEMA

LAPTh-Annecy, France

Turn on the machine!

in 1998 we were told to expect an early SUSY discovery



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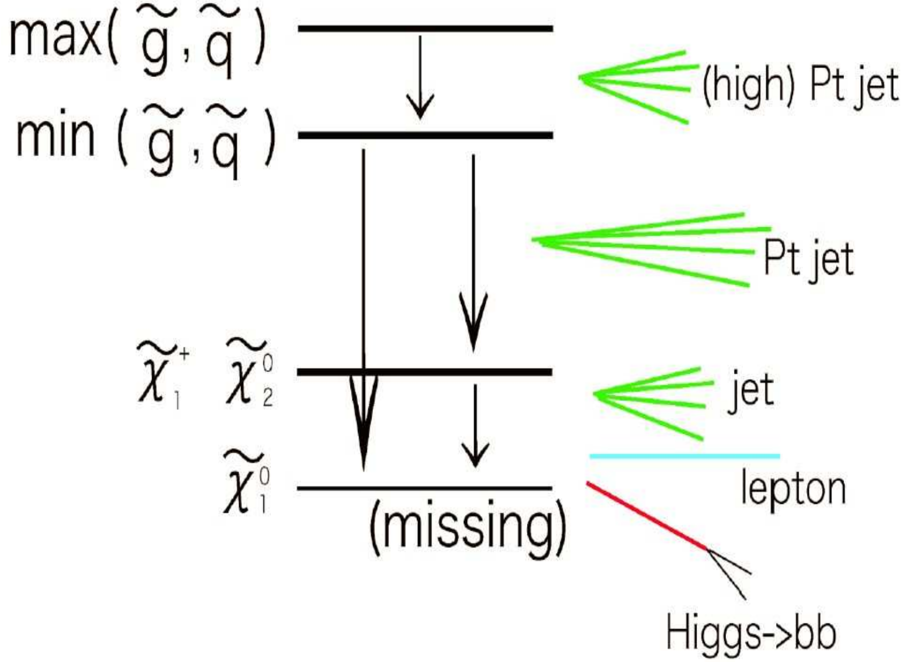
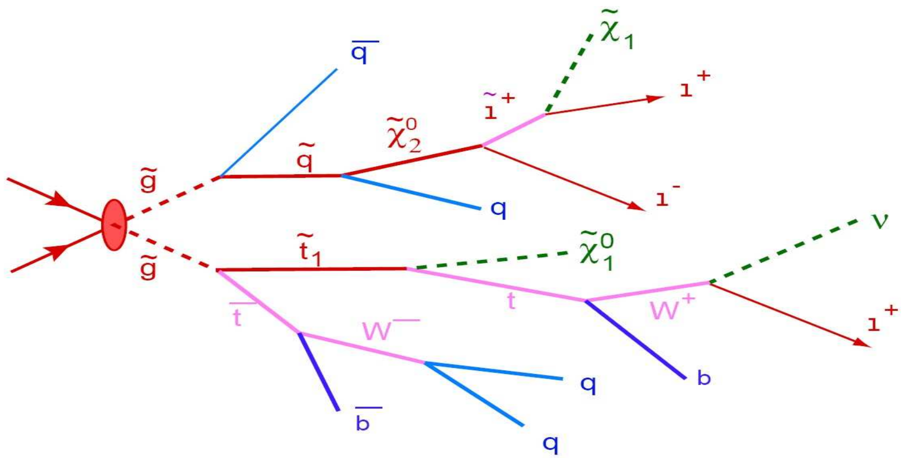
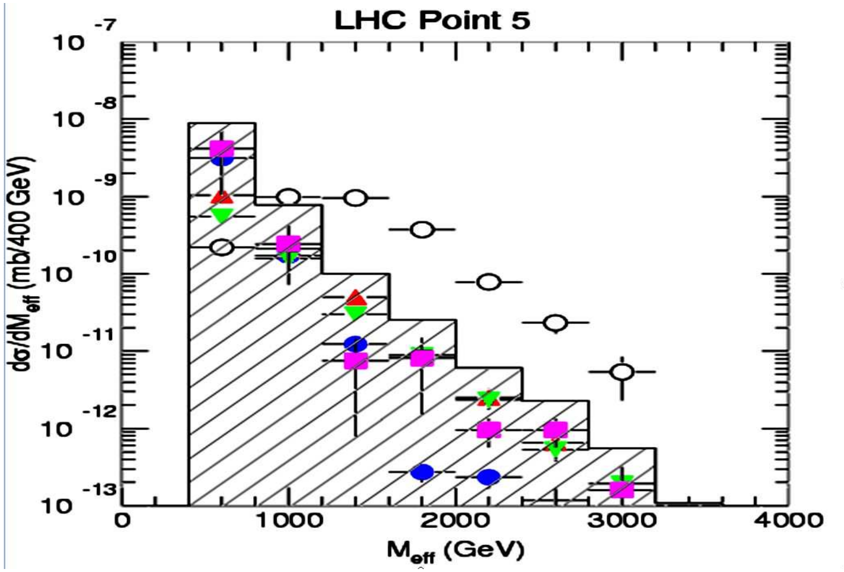




ATLAS TDR (same with CMS)



ATLAS TDR 98
(mSUGRA point, PreWMAP)

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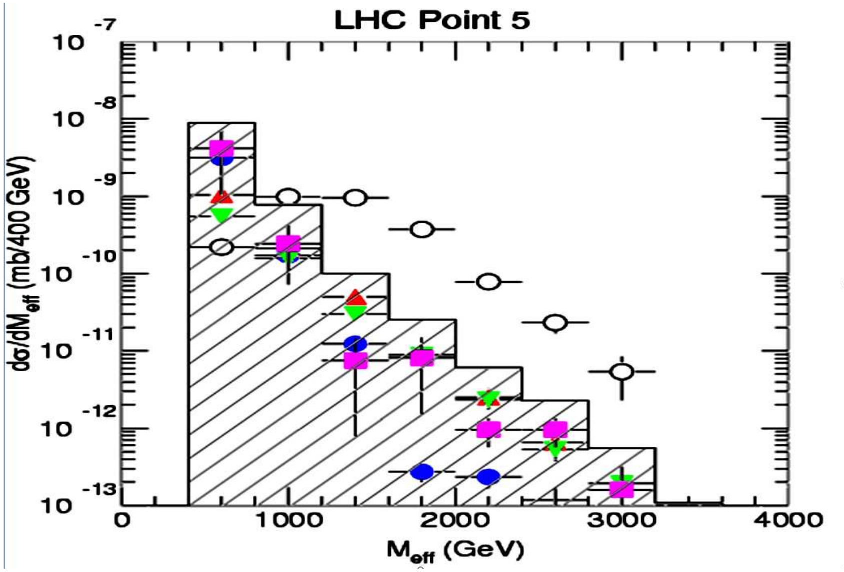

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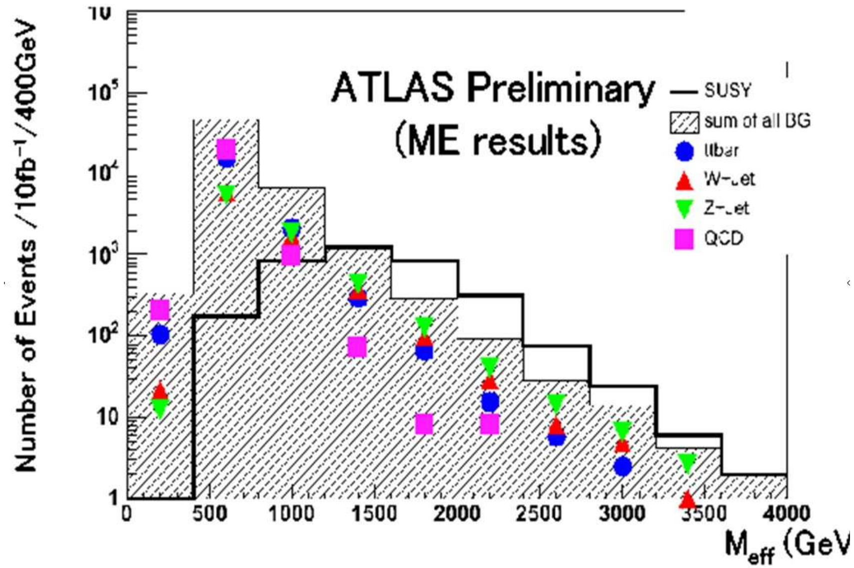

ATLAS 2006

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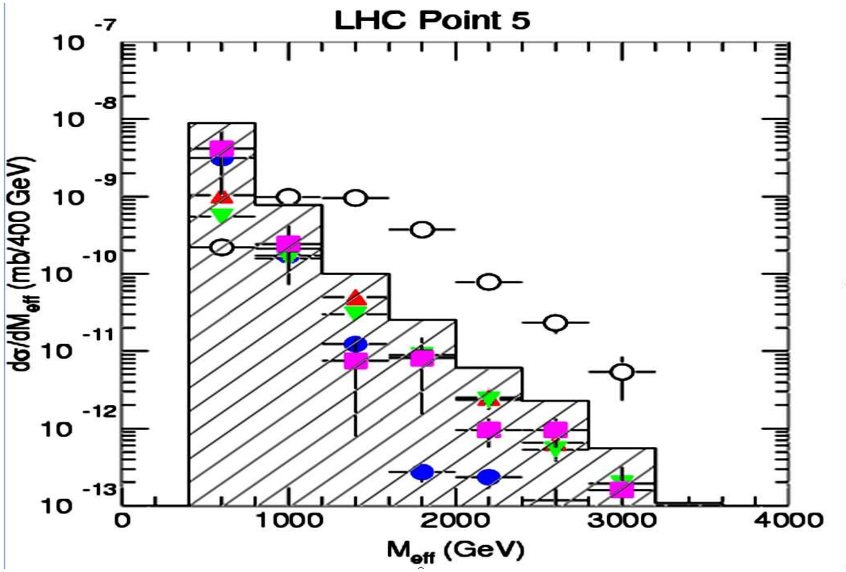

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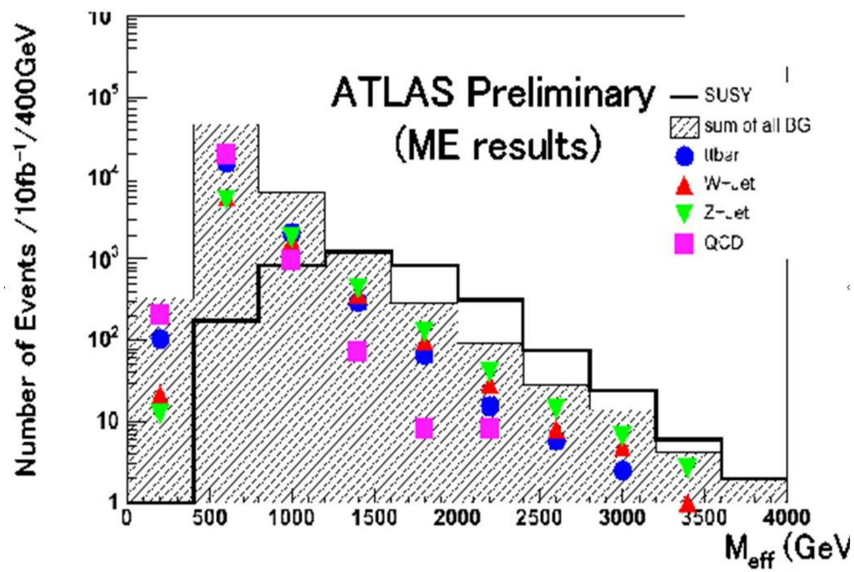
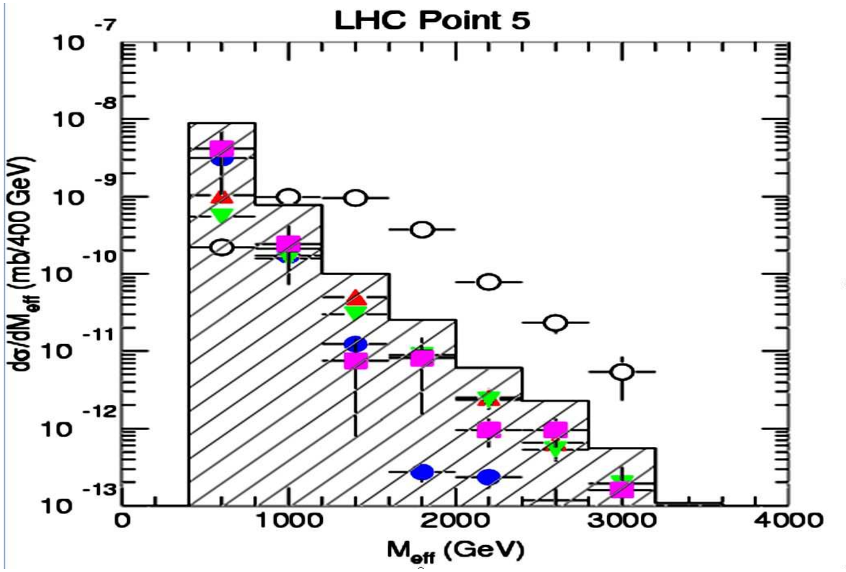


ATLAS 2006

What happened?

ATLAS TDR (same with CMS)



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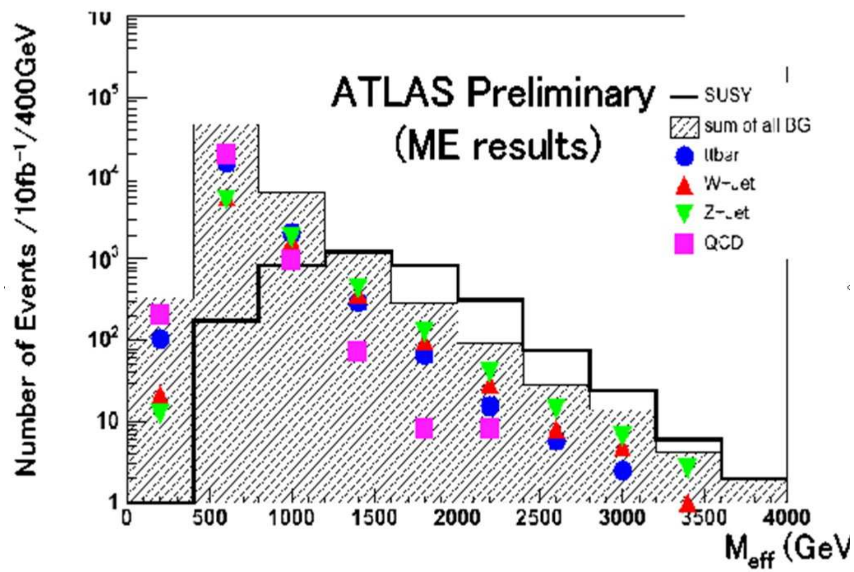
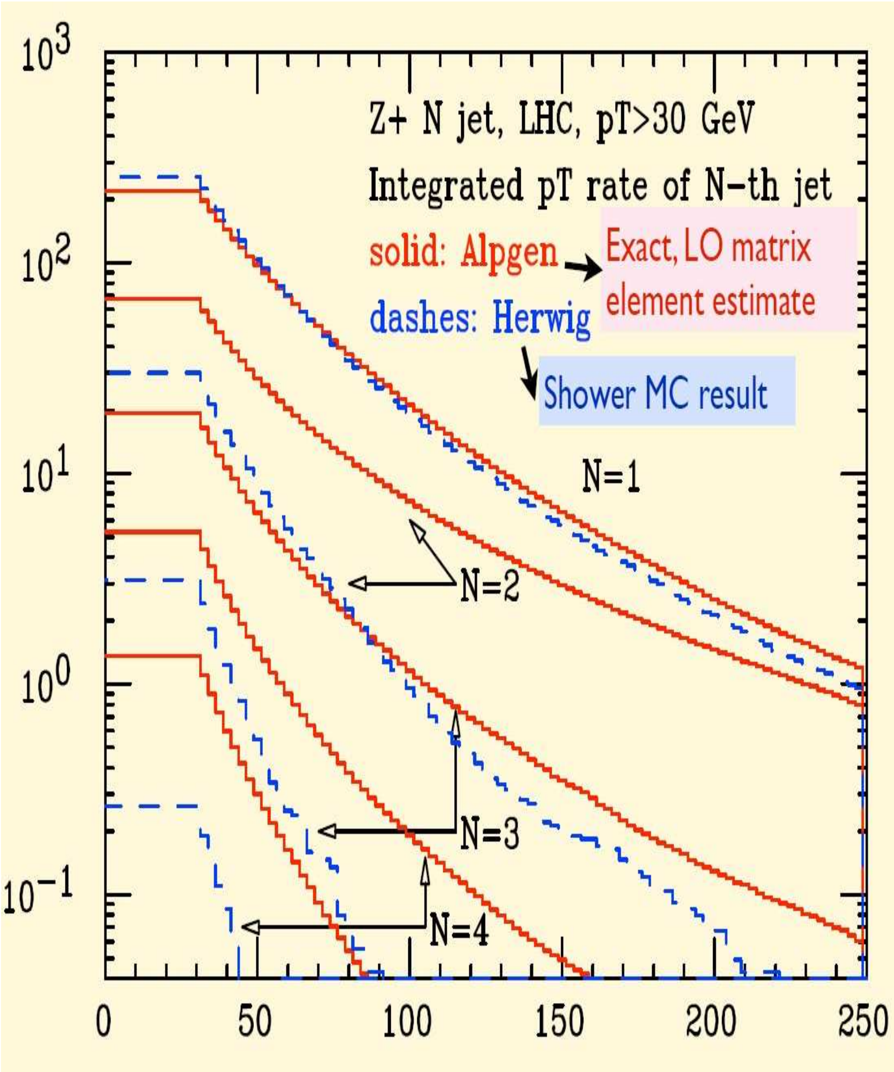


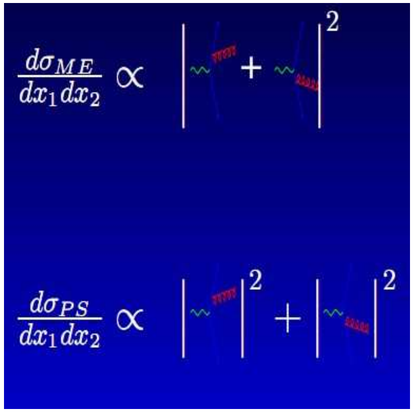
ATLAS 2006

QCD and SM processes can also produce hard jets! and these are/were lacking in PS/MC

ME vs PS: Limitations of PS

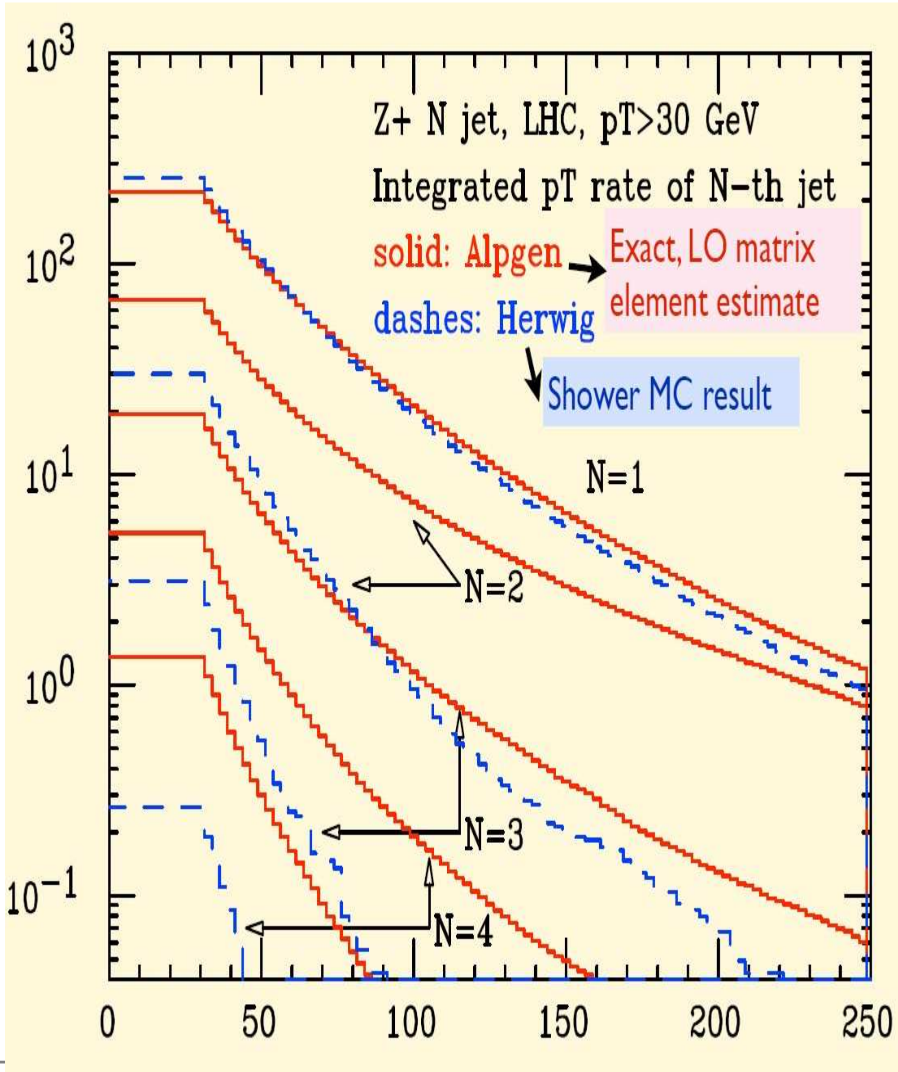


- PS do not describe hard jets
- ME do but in practice can not produce as many jets as PS
- ME evaluates the complete set of all diagrams/configurations: costly
- some real progress has been made in interfacing ME with PS



Still, all of this at leading order although very recent improvement MC/@NLO and POWHEG (higher multiplicity + NLO-Hamilton+Nason)

ME vs PS: Limitations of PS



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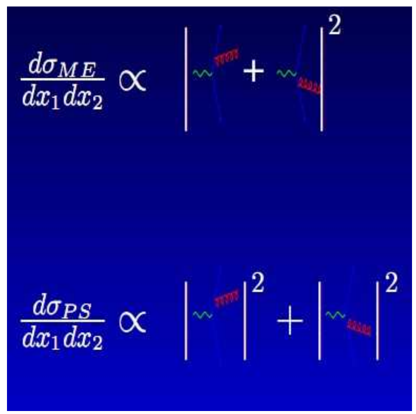
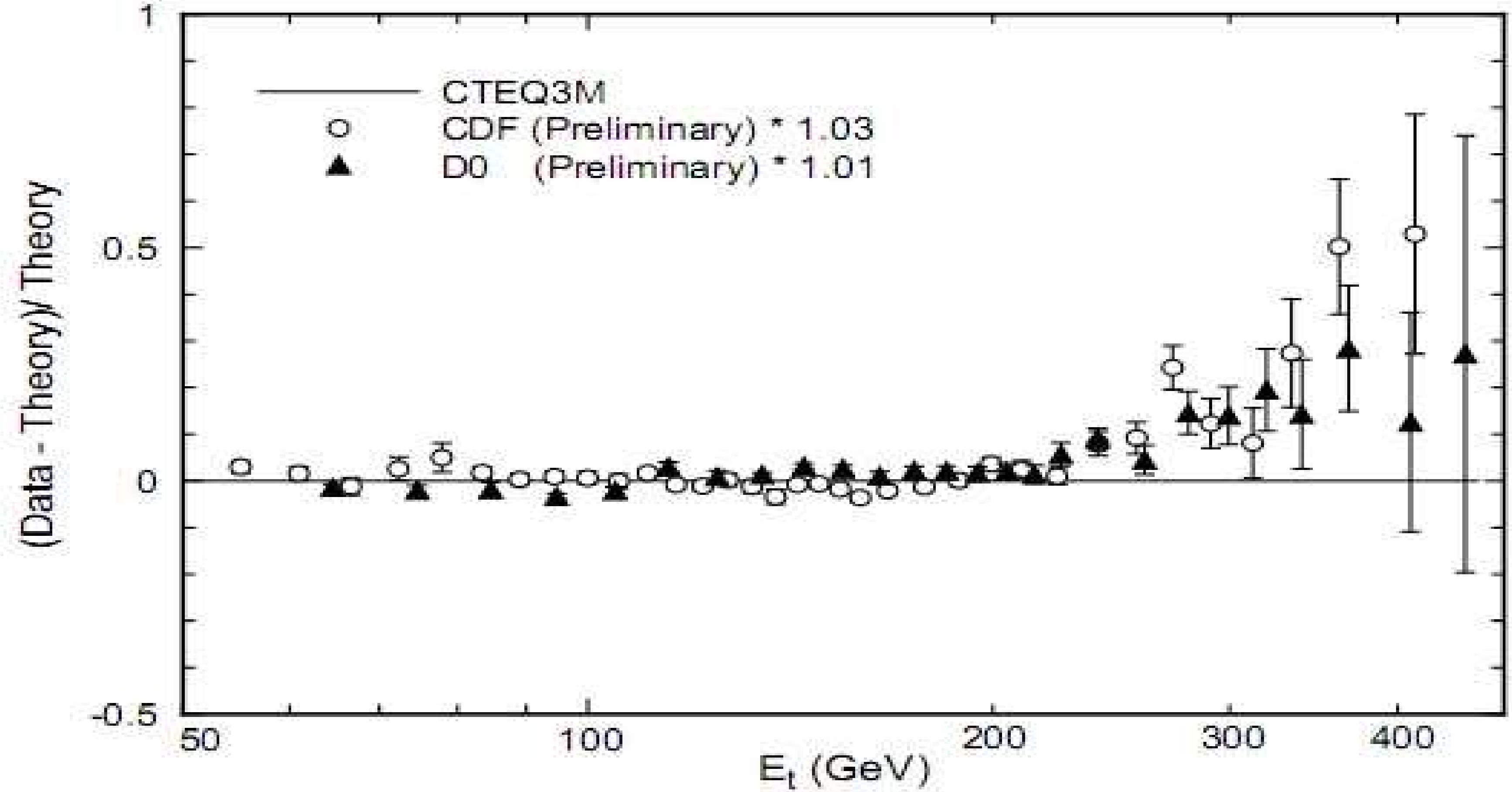
- ME evaluates the complete set of all diagrams/configurations: costly
- some real progress has been made in interfacing ME with PS
- Importance of NLO multileg, normalisation, etc...



Still, all of this at leading order although very

- Tevatron jet cross section at large E_T



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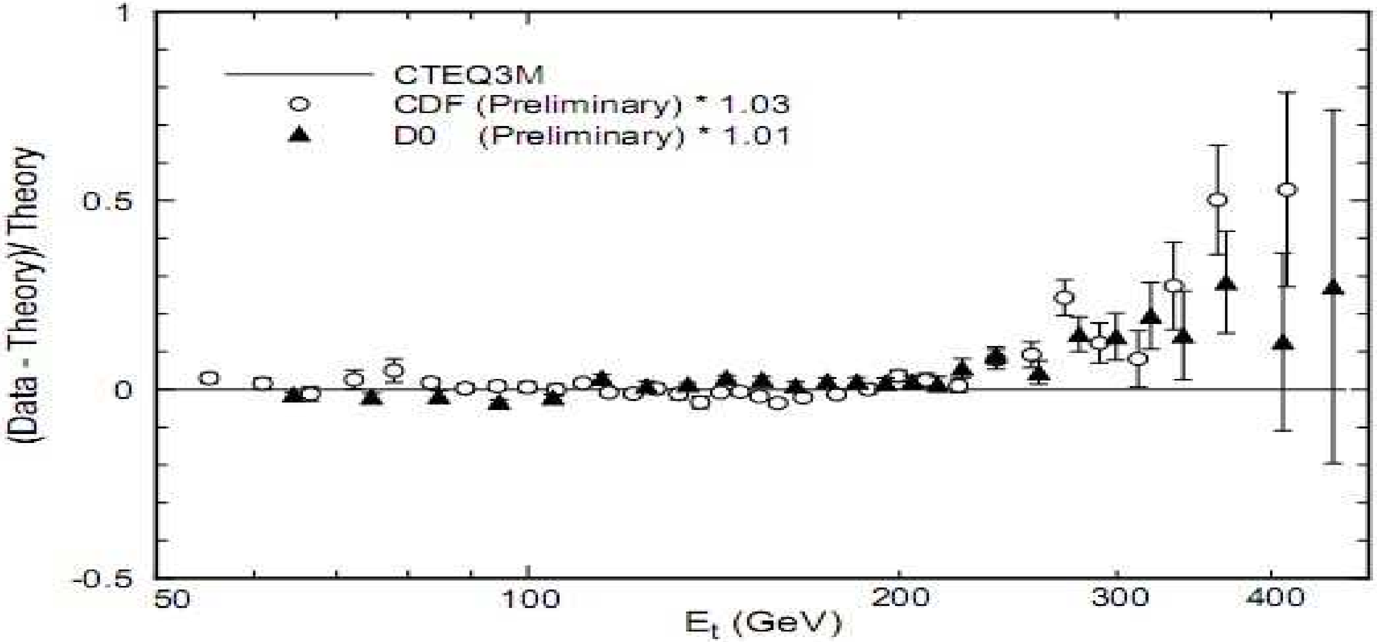
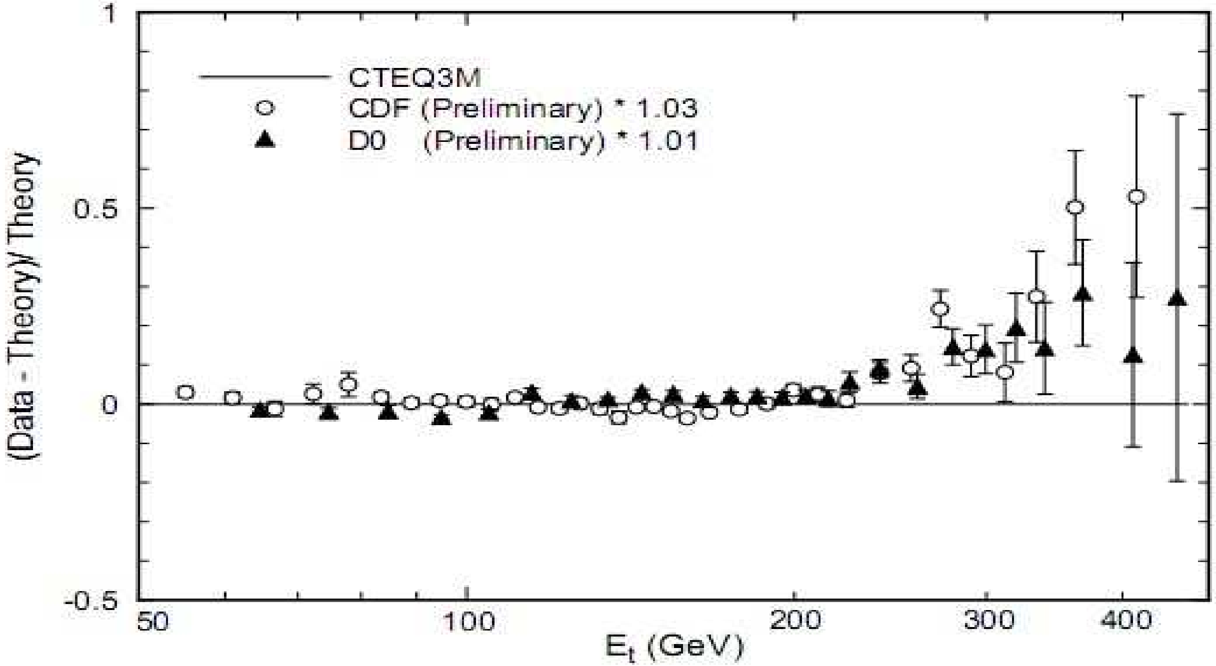


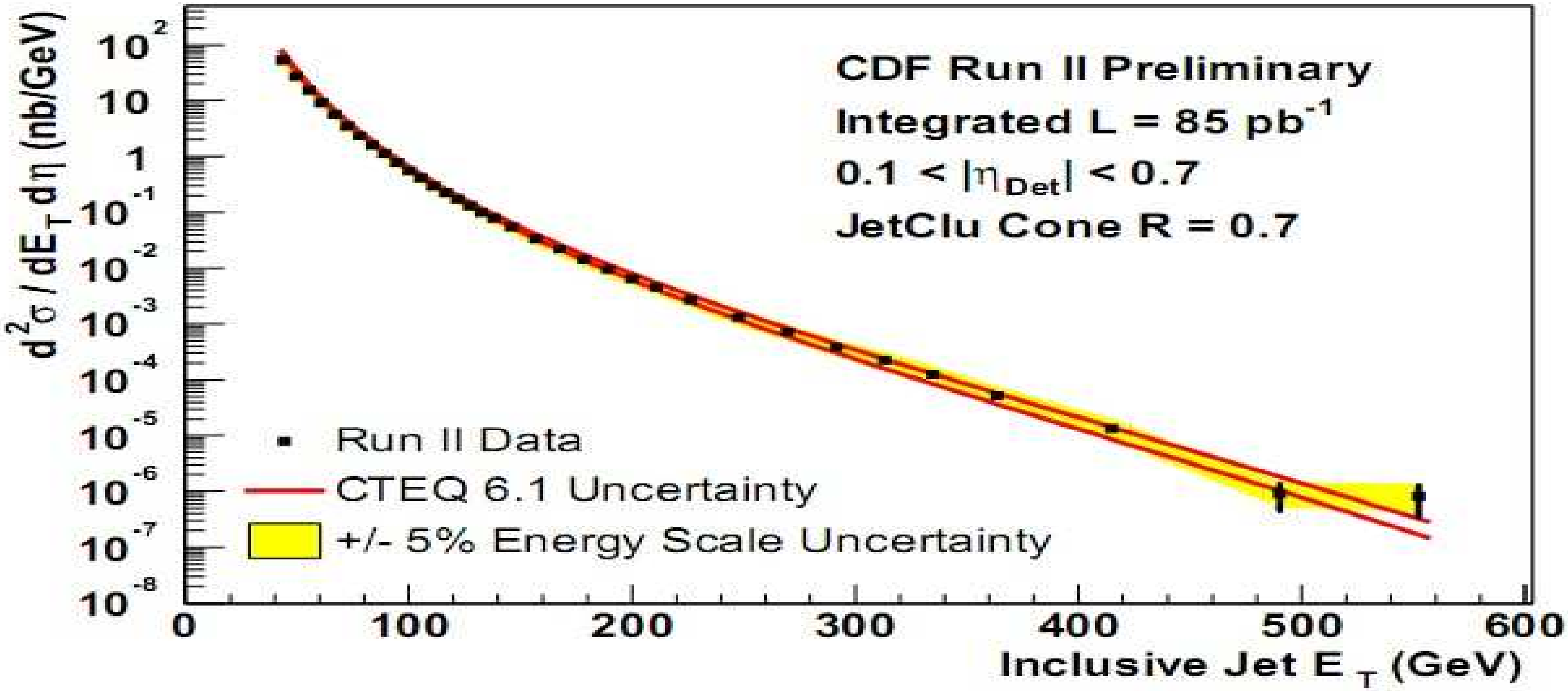
$\Rightarrow$ new physics?

- Tevatron jet cross section at large E_T



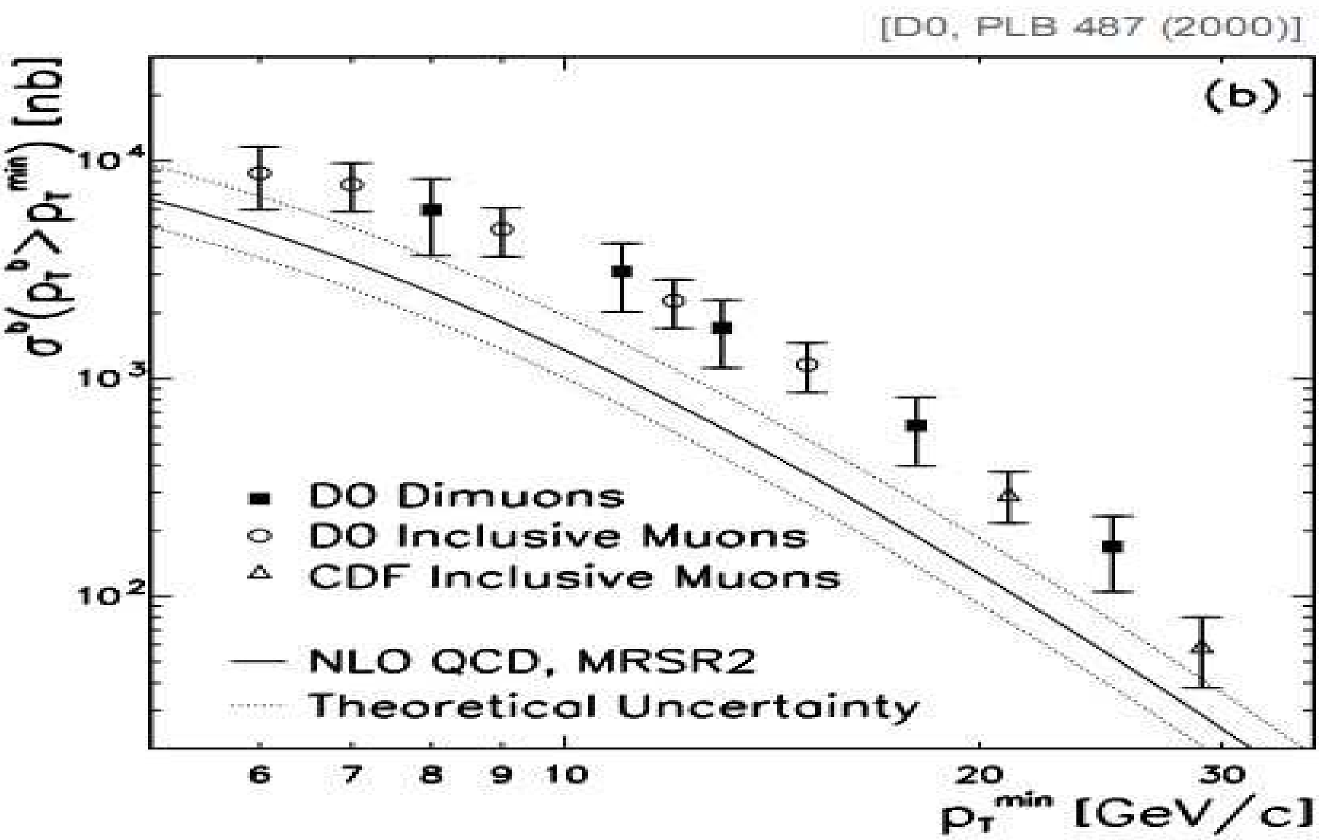
$$\mathcal{L}_{\rm BSM} \supseteq \frac{\tilde{g}^2}{M^2}\,\bar{\psi}\gamma^\mu\psi\,\bar{\psi}\gamma_\mu\psi \quad \Rightarrow \quad \frac{\text{data} - \text{theory}}{\text{theory}} \propto \tilde{g}^2\frac{E_T^2}{M^2}$$

- Tevatron jet cross section at large E_T



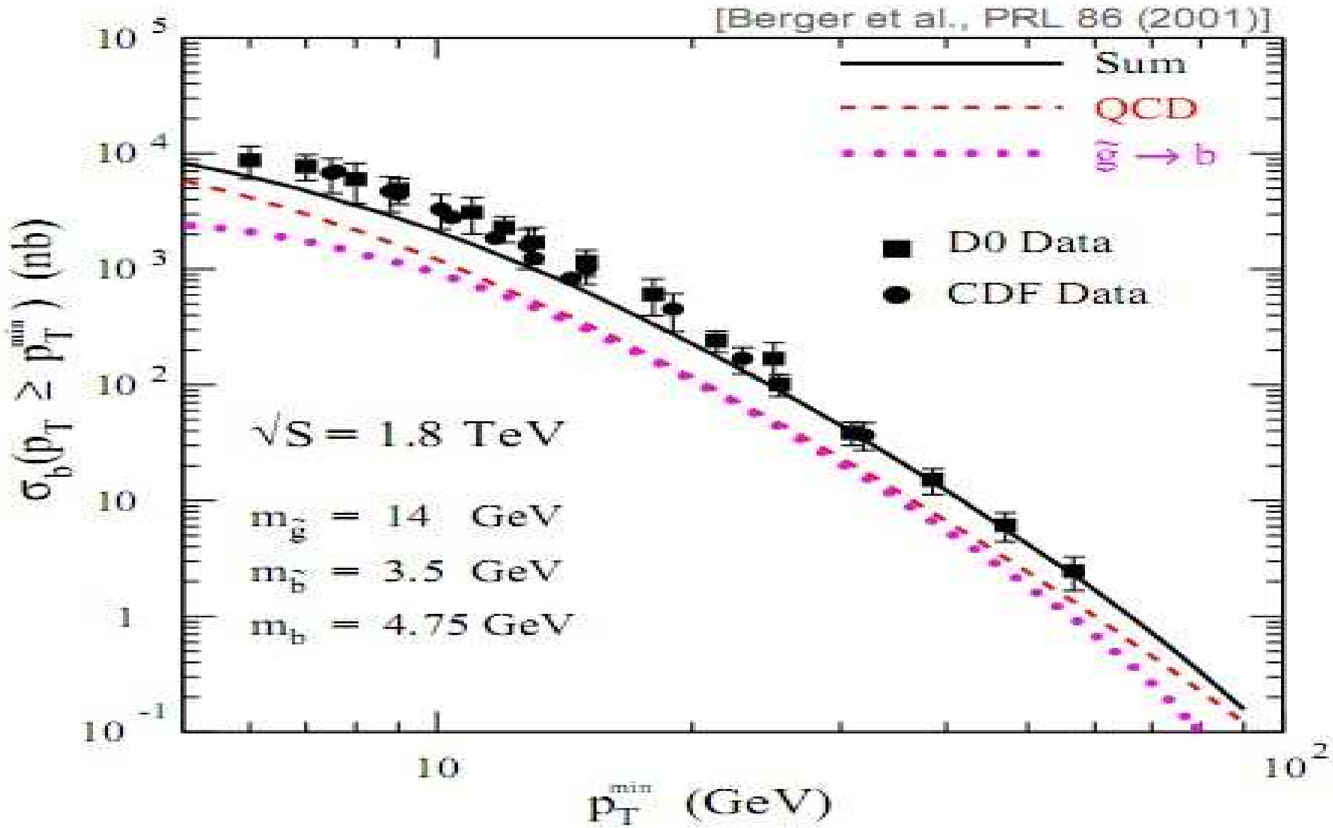
$\Rightarrow$ just QCD. . . (uncertainty in gluon pdf at large x)

- Tevatron bottom quark cross section



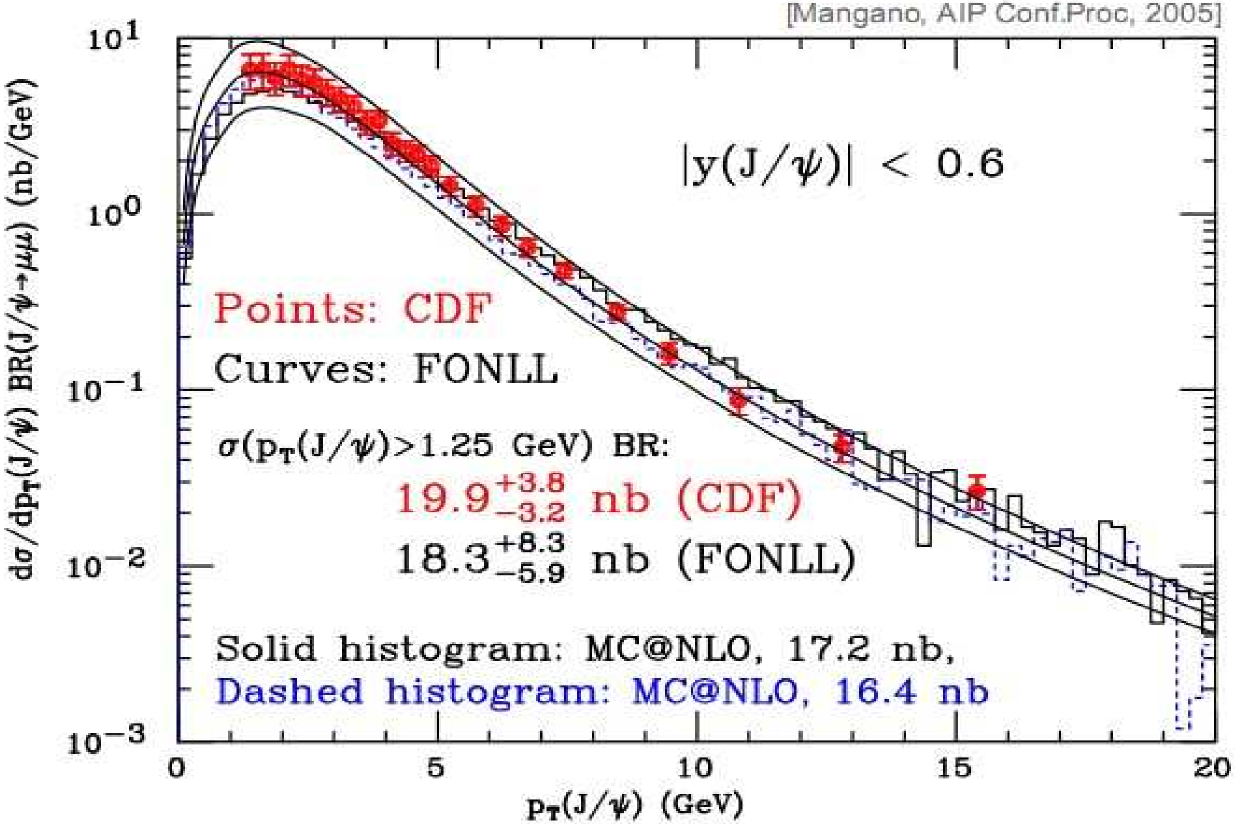
$\Longrightarrow$ new physics?

Tevatron bottom quark cross section



$\Rightarrow$ light gluino/sbottom production?

- Tevatron bottom quark cross section



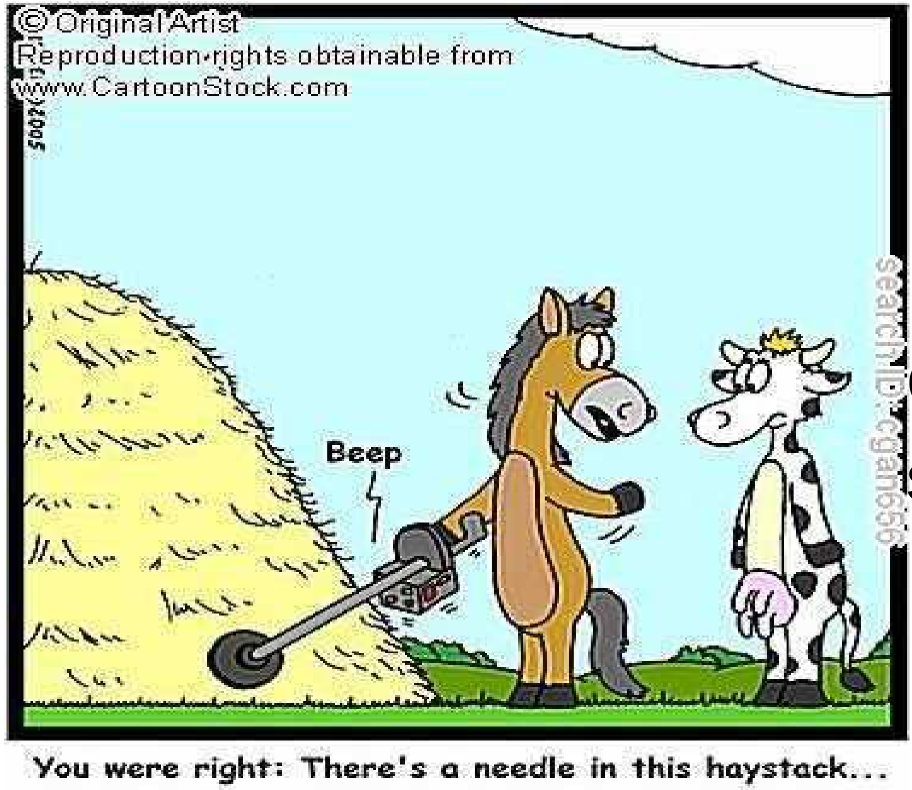
$\Rightarrow$ just QCD… (α_s, low-x gluon pdf, $b \longrightarrow B$ fragmentation & exp. analyses)

Approche du phénoménologue

What we hope for!

Magic Needle!

Approche du phénoménologue



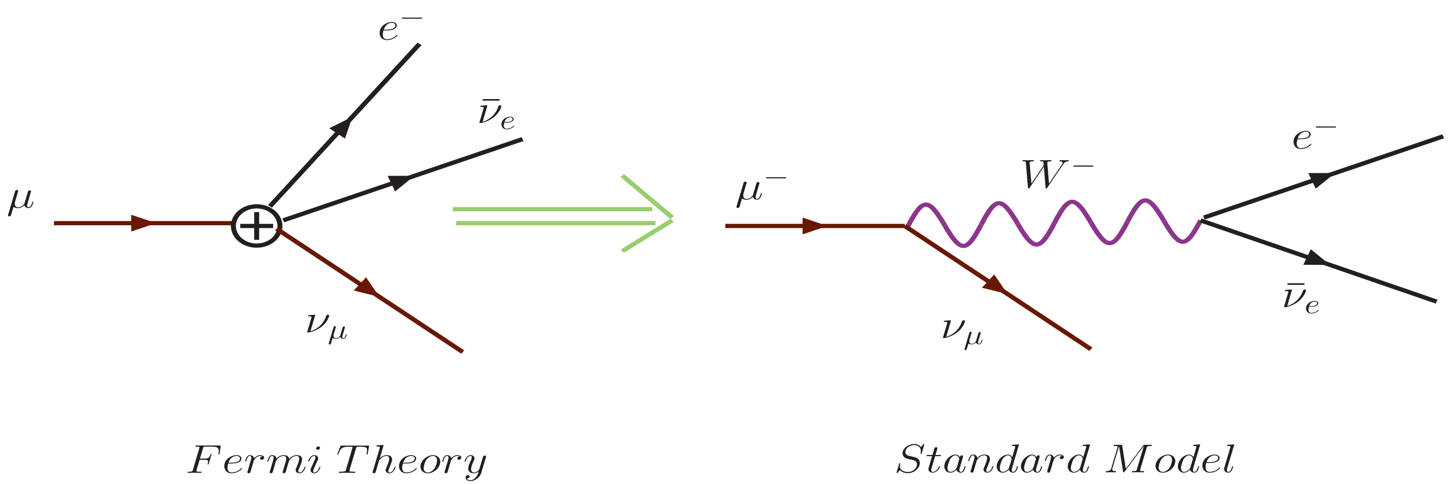
Clever Enough to find special signatures

Approche du phénoménologue

most probably count each straw to weigh the haystack precisely

Electroweak Observables

- $M_W = \frac{gv}{2}$ and $M_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2} = \frac{M_W}{\cos\theta_W}$
-
  - $$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$



-

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2}}\right) \qquad s_W^2 = s_M^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2}}\right)$$

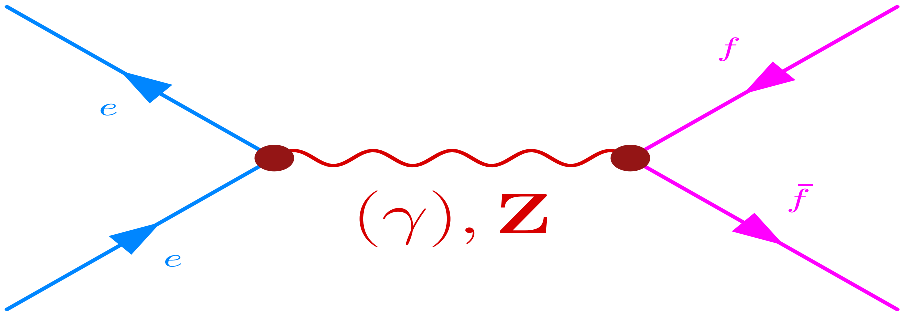
Neutral currents:

$$\begin{aligned}
\mathcal{L}_0 &= \sum_i Q^i \bar{\psi}_i \gamma^\mu \psi_i A_\mu \quad (QED) \\
&+ \frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma^5) \psi_i Z_\mu \quad (NC) \\
g_V^i &= T_3(i) - 2Q_i s_W^2; , \qquad g_A^i = T_3(i)
\end{aligned}$$

◀

$$\begin{aligned}
\mathcal{L}_{NC} &= \frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i \gamma^\mu \left(g_L^i (1-\gamma^5) + g_R^i (1+\gamma^5) \right) \psi_i Z_\mu \\
g_V &= g_L + g_R \qquad g_A = g_L - g_R
\end{aligned}$$

Phenomenology at the Z pole (1)



$$\frac{d\sigma_Z^f}{d\Omega} = \frac{9}{4}\frac{\left(s/M_Z^2\right)\Gamma_{ee}\Gamma_{f\bar{f}}}{(s-M_Z^2)^2+s^2\Gamma_Z^2/M_Z^2}\Big[(1+\cos^2\theta)(1-P_e A_e) + 2\cos\theta A_f(-P_e+A_e)\Big]$$

$$\Gamma(Z\to f\bar{f}) = \frac{\alpha M_Z}{3}\frac{1}{s_W^2 c_W^2}({g_V^f}^2+{g_A^f}^2)\ N_c^f$$

$$A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2+(g_A^f)^2} = \frac{(g_L^f)^2-(g_R^f)^2}{(g_L^f)^2+(g_R^f)^2}\ ,\quad A_\ell = \frac{2(1-4s_W^2)}{1+(1-4s_W^2)^2}\sim 0.15$$

Phenomenology at the Z pole (2)

$$
\begin{aligned}
A^f_{FB} &\equiv \frac{\sigma^f_F - \sigma^f_B}{\sigma^f_F + \sigma^f_B} = \frac{3}{4} A_e A_f, \\
A^f_{LR} &\equiv \frac{1}{P_e} \frac{\sigma^f(-|P_e|) - \sigma^f(+|P_e|)}{\sigma^f(-|P_e|) + \sigma^f(+|P_e|)} = A_e \\
\bar{A}^f_{FB} &\equiv \frac{\left(\sigma^f_F(-|P_e|) - \sigma^f_B(-|P_e|)\right) - \left(\sigma^f_F(+|P_e|) - \sigma^f_B(+|P_e|)\right)}{\sigma^f_F(-|P_e|) + \sigma^f_B(-|P_e|) + \sigma^f_F(+|P_e|) + \sigma^f_B(+|P_e|)} \\
&= \frac{3}{4} P_e A_f.
\end{aligned}
$$

Z Lineshape: scan at the pole

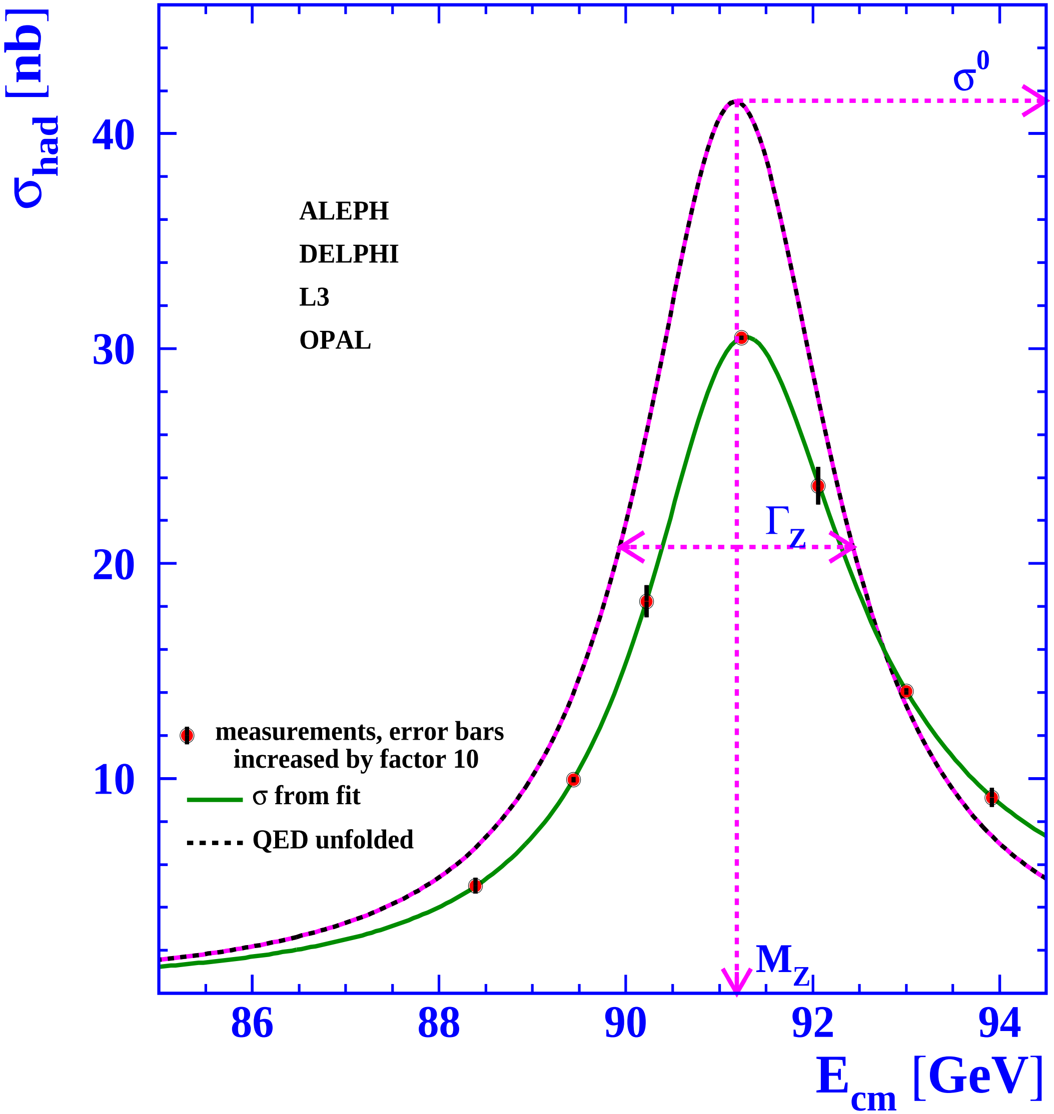
$$\sigma_{had} = 12\pi \frac{\Gamma_{ee}\Gamma_{had}}{M_Z^2 \Gamma_Z^2}$$

$$R_\ell \equiv \frac{\Gamma_{had}}{\Gamma_\ell} \sim 21$$

$$R_{b,c} \equiv \frac{\Gamma_{bb,cc}}{\Gamma_{had}}$$

$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had,e,\mu,\tau}$$

$$N_\nu = \frac{\Gamma_{inv}/\Gamma_\ell}{(\Gamma_\nu/\Gamma_\ell)_{SM}}$$



$$\sin^2\theta_{\mathbf{W}}$$

- From measured W/Z masses:

$s_W^2 = s_M^2 = 1 - M_W^2/M_Z^2 \rightarrow$

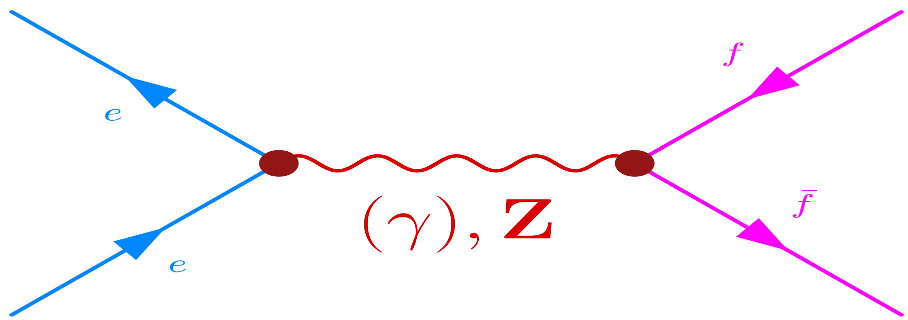
$$s_M^2 = .22164 \pm .00079 \;\; (i.e \;\; .4\% \text{ precision})$$

10σ!!!!

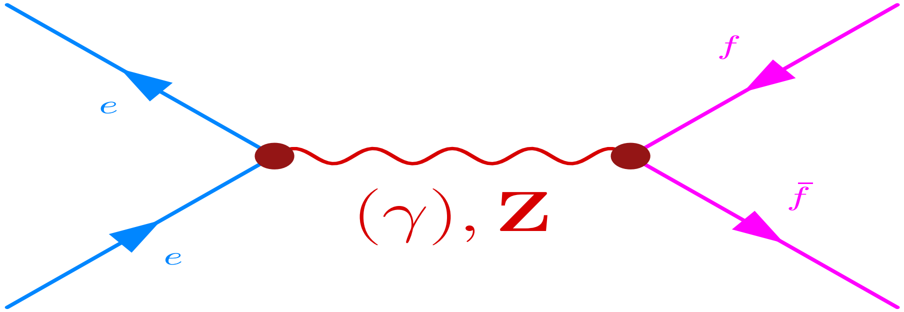
- From Leptonic Z observables, *e.g*:

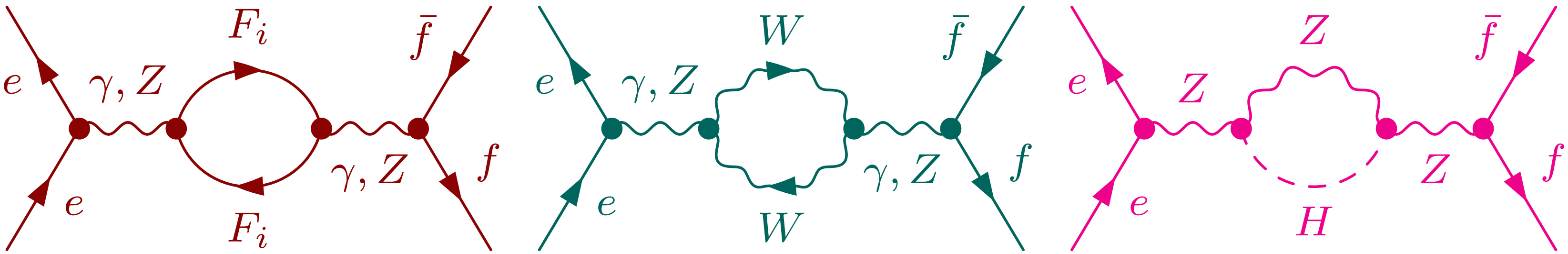
$A_{LR}^e = \frac{2(1-4s_{\text{eff}}^2)}{1+(1-4s_{\text{eff}}^2)^2} \rightarrow$

$$s_{\text{eff},l}^2 = .23159 \pm .00018 \;\; (i.e \;\; .08\% \text{ precision})$$


e
e
f
$\bar{f}$
$(\gamma), \mathbf{Z}$

Loop calculations: Precision and Indirect effects





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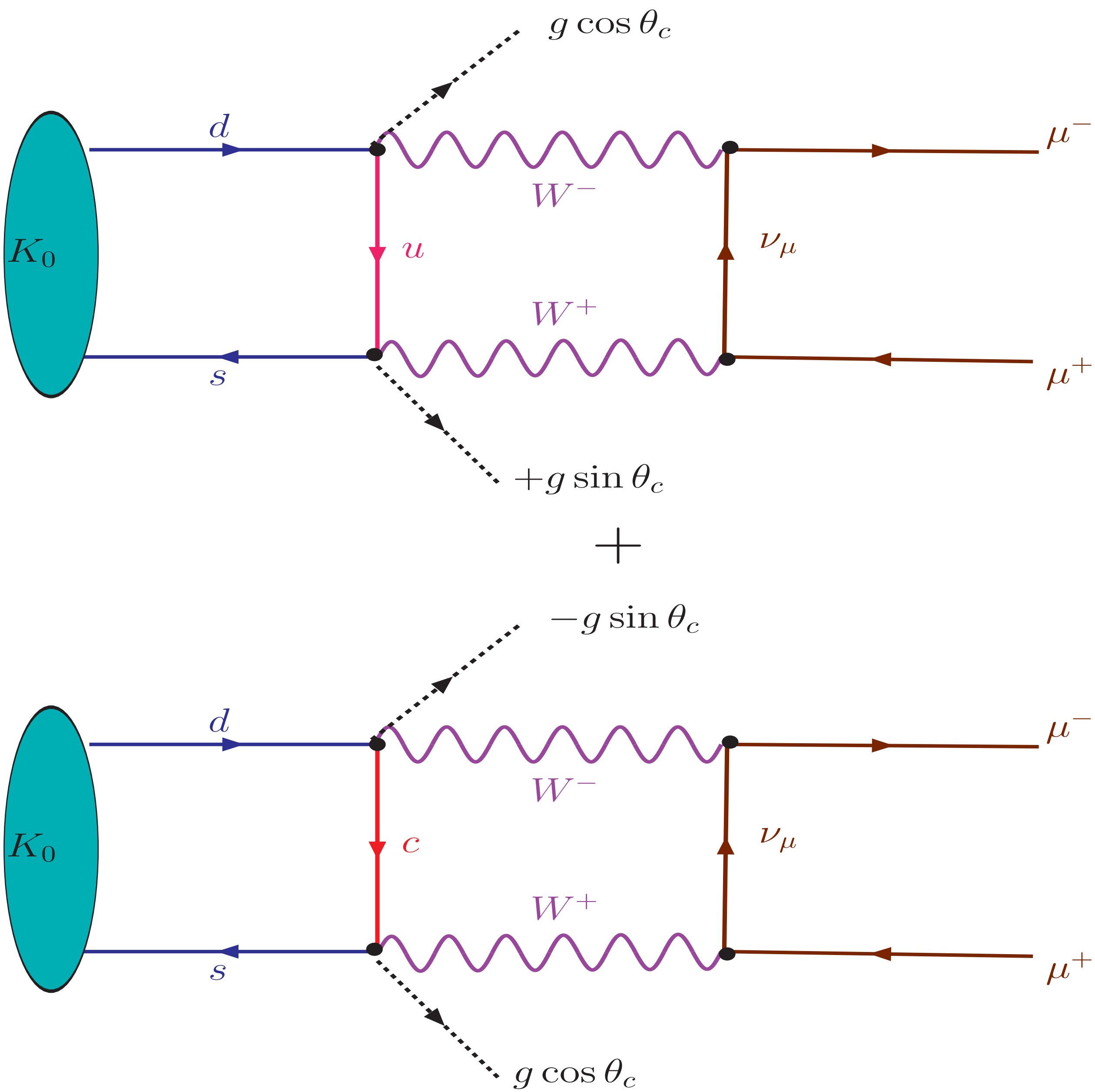

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- relation from input to fundamental parameters as direct and transparent as possible

Lagrangian parameters and physical parameters (2.)

In the SM $\Longrightarrow$ $g, g', v \quad (y_f \lambda)$

Trade-of

$$\mathbf{M_W = \frac{gv}{2}} \qquad M_W = 80.450 \pm .039\mathrm{GeV} \qquad \frac{\Delta M_W}{M_W} \sim 5.10^{-4}$$

$$\mathbf{M_Z = \frac{\sqrt{g^2+g'^2}v}{2}} \qquad M_Z = 91.1875 \pm .0021\mathrm{GeV} \qquad \frac{\Delta M_Z}{M_Z} \sim 2.10^{-5}$$

$$\mathbf{e^{-2} = g^{-2} + g'^{-2}} \qquad \alpha^{-1} = \underbrace{137.03599235(73)}_{(g-2)_e} \qquad \frac{\Delta\alpha^{-1}}{\alpha^{-1}} \sim 5.10^{-9}$$

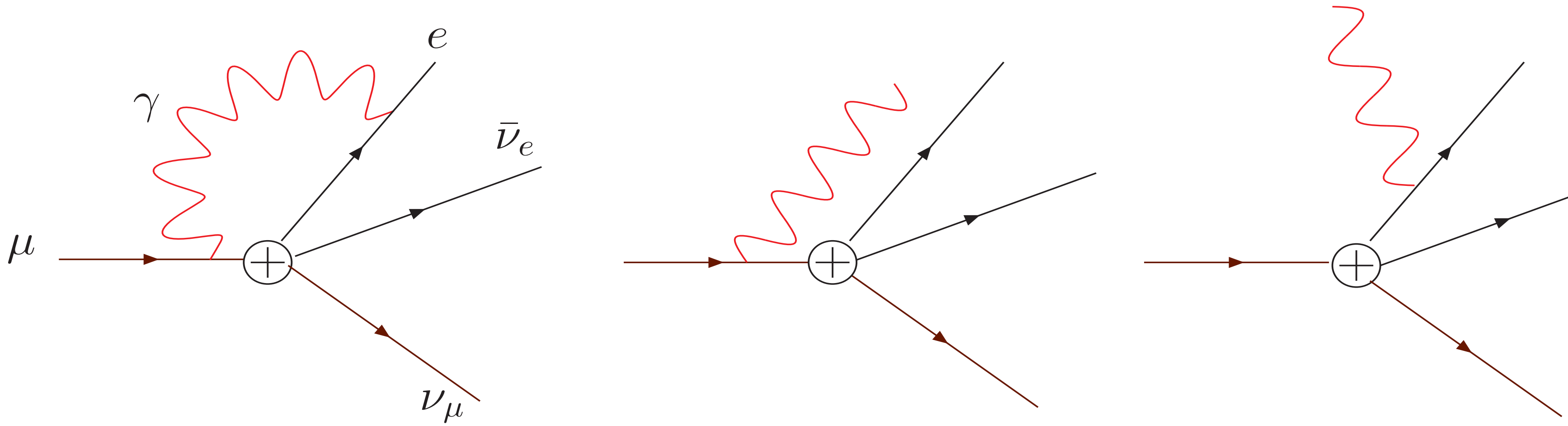
start of LEP1 $\Rightarrow$

$$M_Z = 91.174 \pm .02\mathrm{GeV} \qquad \frac{\Delta M_Z}{M_Z} \sim 2.10^{-4}$$

$$M_W = 79.91 \pm .39\mathrm{GeV} \qquad \frac{\Delta M_W}{M_W} \sim 5.10^{-3}$$

$$G_\mu = (1.16637 \pm .00001)10^{-5} GeV^{-2} \qquad \frac{\Delta G_\mu}{G_\mu} \sim 8.6\ 10^{-6}$$

Trade-of G_μ vs M_W



$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F(\frac{m_e^2}{m_\mu^2}) \times \left[1 - 1.8098 \frac{\alpha(m_\mu)}{\pi} + 6.7427 \left(\frac{\alpha(m_\mu)}{\pi}\right)^2\right]$$

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \qquad \alpha^{-1}(\mu) \simeq 136$$

Need for Radiative Corrections

$\sin^2\theta_W$

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Need for Radiative Corrections: $\alpha,\, G_\mu,\, s_W^2$

$$\Gamma_1(Z \to f\bar{f}) = \frac{\sqrt{2}G_\mu M_Z^3}{3\pi}\left({g_V^f}^2 + {g_A^f}^2\right) N_c^f \stackrel{?}{=}$$

$$\Gamma_2(Z \to f\bar{f}) = \frac{\alpha M_Z}{3}\frac{1}{s_\theta^2 c_\theta^2}\left({g_V^f}^2 + {g_A^f}^2\right) N_c^f$$

$$\frac{\Delta\Gamma}{\Gamma} = \frac{\Gamma_2}{\Gamma_1} - 1 \qquad \frac{\Gamma_2}{\Gamma_1} = \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}\frac{1}{s_\theta^2 c_\theta^2}$$

$$\frac{\Delta\Gamma}{\Gamma} \xrightarrow{s_\theta^2 = s_{\rm eff}^2} 6\%$$

$$\frac{\Delta\Gamma}{\Gamma} \xrightarrow{s_\theta^2 = s_M^2} 3\%$$

Infinities and regularisation

To gain more precision we need to go beyond tree-level

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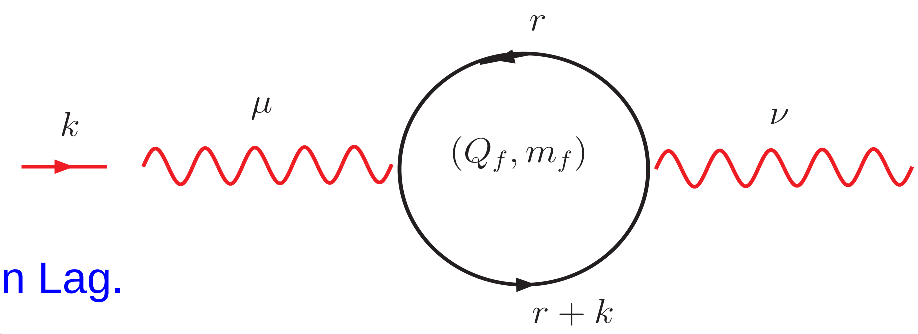
But, almost invariably

Dirac: corrections are small but infinite!

Infinities and regularisation

▶ Example: Vacuum Polarisation in QED



bare coupling in Lag.

$$\mathbf{i\Sigma_{\mu\nu}} = \underbrace{(-1)}_{\mathbf{fermion}} \; (-\mathbf{ie_0})^{\mathbf{2}}(\mathbf{Q_f})^{\mathbf{2}} \int \frac{\mathbf{d^4r}}{(\mathbf{2}\pi)^{\mathbf{4}}} \mathrm{Tr}\left(\gamma_\mu \frac{\mathbf{i}}{\not{\mathbf{r}} - \mathbf{m}} \gamma_\nu \frac{\mathbf{i}}{\not{\mathbf{r}} + \not{\mathbf{k}} - \mathbf{m}}\right)$$

$$infinite: \int d^4r/r^2 \to \int^\Lambda d^4r/r^2 \to \int^\Lambda r^2 \to \Lambda^2 \;\; \text{with} \;\; \Lambda \to \infty.$$

Dimensional regularisation: keeping track of infinities

This procedure keeps track of infinities in a GI way

Continuation in the number of space-time dimension $4 \to n$

$\mathbf{r}_\mu = (\mathbf{r_0}, \mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}) \to (\mathbf{r_0}, \mathbf{r_1}, \cdots, \mathbf{r_{n-1}})$

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Step-0

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Step-0

render all propagator rational: $(\not{\mathbf{r}} - \mathbf{m})^{-1} \to \frac{\not{\mathbf{r}}+\mathbf{m}}{\mathbf{r^2}-\mathbf{m^2}}$

Step-1

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Step-0

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Step-1

- ${g_\mu}^\mu = g_{\mu\nu} g^{\nu\mu} = \mathrm{Tr}(1) = n$
- Dirac algebra in n-dimension implies *e.g* $\gamma_\alpha \gamma_\mu \gamma_\nu \gamma^\alpha = 4 g_{\mu\nu}\, \mathbf{1} \; - \; (4-n)\, \gamma_\mu \gamma_\nu$
- scale of couplings $\int \frac{\mathbf{d^4 r}}{(\mathbf{2}\pi)^\mathbf{4}} \to \mu^{\mathbf{4-n}} \int \frac{\mathbf{d^n r}}{(\mathbf{2}\pi)^\mathbf{n}}$

Dimensional regularisation: keeping track of infinities (2)

Step-2 Wick Rotation

Move to Euclidian space:

$$\mathbf{r_0 = i r_n}, \quad \mathbf{d^n r = i d^n r_E}, \quad \mathbf{r^2 = -r_E^2}$$

Step-3: Combine all propagators into one

$$\frac{\mathbf{1}}{\mathbf{D_1 D_2 \cdots D_N}} = \mathbf{\Gamma(N)} \int_0^1 \mathbf{dx_1} \cdots \int_0^1 \mathbf{dx_{N-1}} \frac{\mathbf{1}}{(\mathcal{D}_\mathbf{N})^\mathbf{N}}$$

$$\mathcal{D}_N = D_1 x_1 + \cdots D_N (1 - \sum^{N-1} x_i) = r^2 - 2r.P(x_i) + M^2(x_i)$$

shift momenta $\quad r - P(x_i) \to r$

$$(r^2 - 2r.P(x_i) + M^2(x_i)) \to r^2 + (M^2 - P^2) \equiv \left(r^2_{(E)} + \Delta_{(E)}\right)$$

Dimensional regularisation: keeping track of infinities (3)

Step-4 Integration over the loop momenta

First $\int d^n r_E \int dx_i \to \int dx_i \int d^n r_E$

$$\begin{aligned} d^n r_E &= r_E dr_E d\Omega_n \\ d\Omega_n &= d\phi(\sin\theta_1 d\theta_1)(\sin^2\theta_2 d\theta_2)\cdots(\sin^{n-2}\theta_{n-2}d\theta_{n-2}) \end{aligned}$$

$$\int d\Omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}$$

$$\int d^n r_E \frac{(r_E^2)^\alpha}{(r_E^2+\Delta_E)^N} = \frac{\pi^{n/2}}{\Gamma(N)}\Gamma(N-n/2)\Delta_E^{-(N-n/2)} = \tilde{I}^{(N)}\Gamma(N-n/2)$$

Collect $1/\varepsilon$ and do parametric integrals

Step-5: Get back to Minkowski and collect divergences

- $\varepsilon = \frac{4-n}{2}$
- $\Gamma(-m+\varepsilon) = \frac{(-1)^m}{m!}\left(\frac{1}{\varepsilon} \; + \; \sum_1^m \frac{1}{k} - \gamma_E + \mathcal{O}(\varepsilon)\right)$
  $\gamma_E = -\Gamma'(1) X^\varepsilon = 1 + \varepsilon \ln X + \mathcal{O}(\varepsilon^2)$
- $\Gamma\left(\frac{\epsilon}{2}\right) = \frac{2}{\epsilon} - \gamma_E + \mathcal{O}(\epsilon) \qquad \Gamma\left(\frac{\epsilon-1}{2}\right) = -\frac{2}{\epsilon} - (1-\gamma_E) + \mathcal{O}(\epsilon)$
- Useful: $C_{UV} = \frac{1}{\varepsilon} \; + \; \ln(4\pi) - \gamma_E$

Step-6: Do parametric integrals: $\int dx_i$

Following the 6-step recipe for the two-point function

$$i\,\Sigma_{\mu\nu} = i\left(k_\mu k_\nu - k^2 g_{\mu\nu}\right)\Pi_{\gamma\gamma}(k^2) \quad , \quad k^\mu\,\Sigma_{\mu\nu} = 0$$

- $$\begin{aligned}
\Pi_{\gamma\gamma} &= e_0^2 \Pi_{QQ} = e_0^2 \sum_f N_c^f Q_f^2 \Pi^f_{\gamma\gamma}(k^2) \\
\Pi^f_{\gamma\gamma}(k^2) &= \frac{1}{12\pi^2}\left(C_{UV} - 6\int_0^1 dx x(1-x)\ln\left(\frac{m_f^2 - k^2 x(1-x)}{\mu^2}\right)\right) \\
&= \frac{1}{12\pi^2}\left(C_{UV} + \frac{5}{3} - \ln(-k^2/\mu^2) + \cdots\right) \quad |k^2| \gg m_f^2 \\
&= \frac{1}{12\pi^2}\left(C_{UV} - \ln(m_f^2/\mu^2) + \frac{k^2}{5m^2} + \cdots\right) \quad |k^2| \ll m_f^2
\end{aligned}$$

in all cases $\boxed{\Pi^f_{\gamma\gamma}(k^2) - \Pi^f_{\gamma\gamma}(Q^2) = finite}$

The resummed propagator

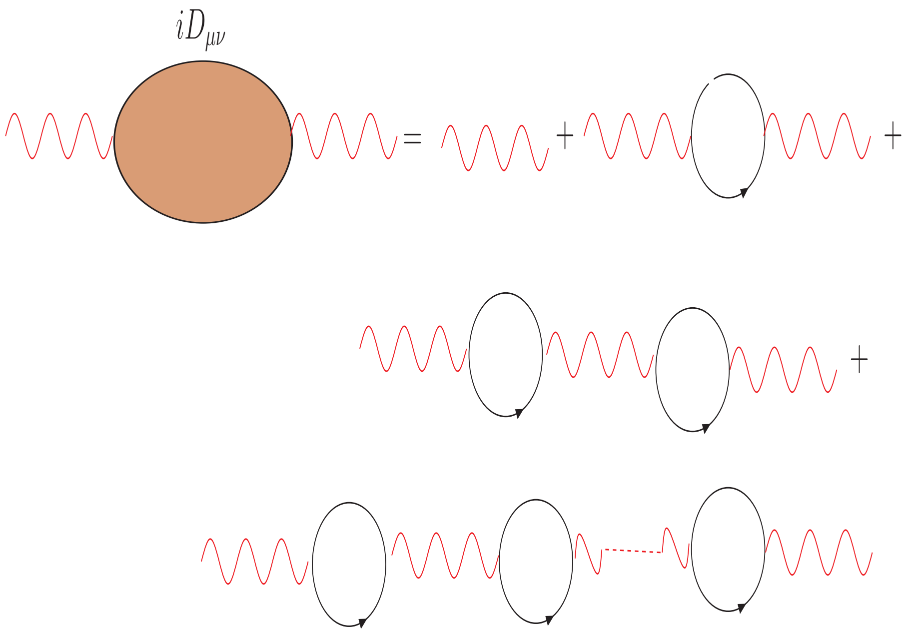
At tree-level a general propagator for particle of mass M_0:

$$iD^0_{\mu\nu} = -i\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2-\xi M_0^2}\right)\frac{1}{k^2-M_0^2}$$

In general a two-point function for vector bosons is

$$i\Sigma_{\mu\nu} = -i\left(g_{\mu\nu}G(k^2) - k_\mu k_\nu L(k^2)\right)$$

$$G(k^2) = G(0) + k^2\Pi(k^2) \equiv G + k^2\Pi(k^2)$$

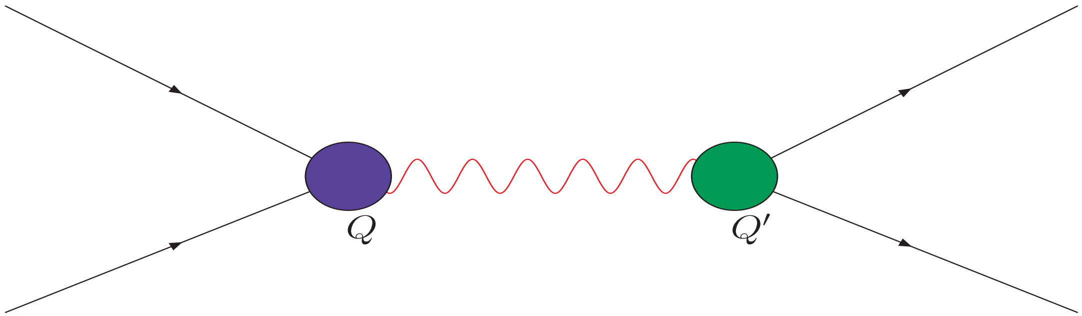


$$iD_{\mu\nu} = \frac{-ig_{\mu\nu}}{k^2 - M_0^2 + (G(0) + k^2\Pi(k^2))}$$

Getting finite results in Observables (1)

$e^+e^- \rightarrow f\bar{f}$ in QED

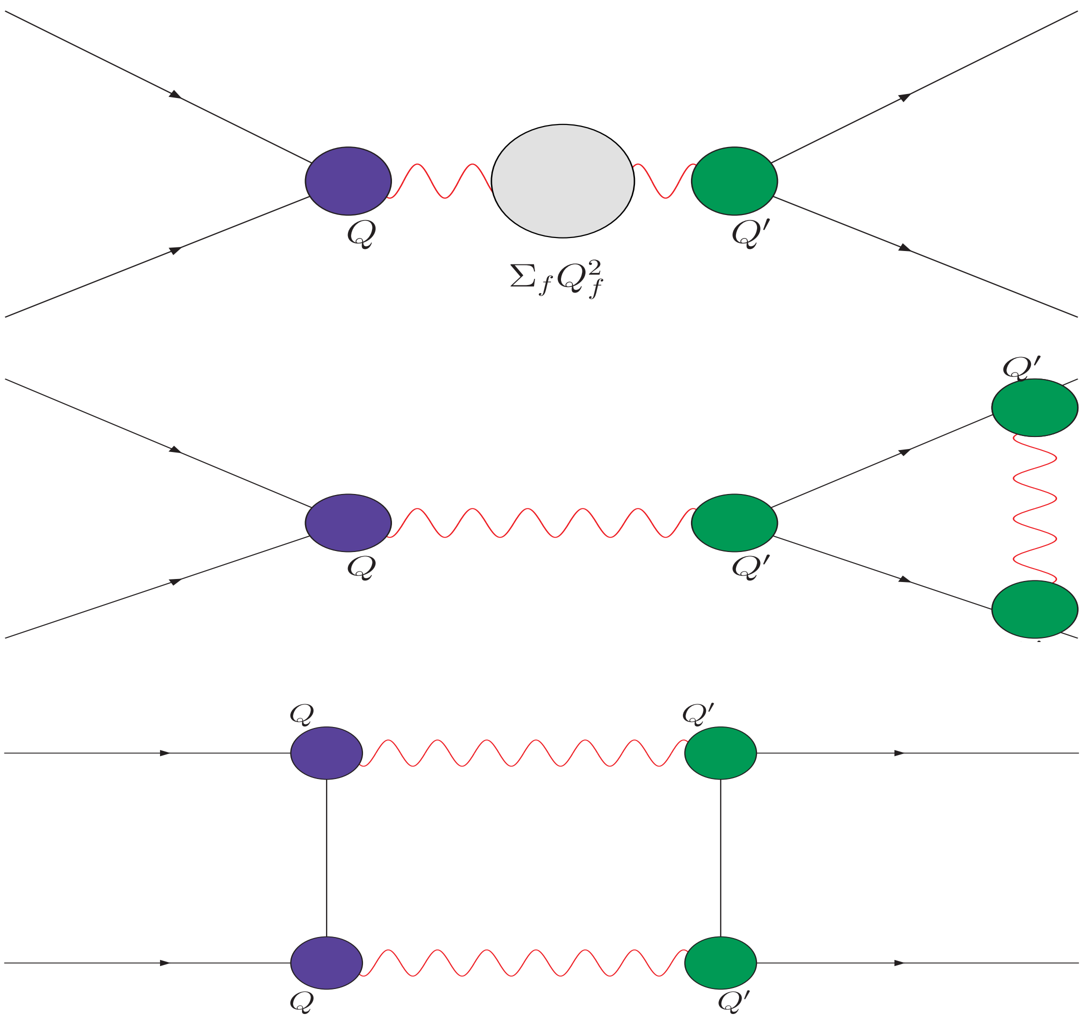
At tree-level



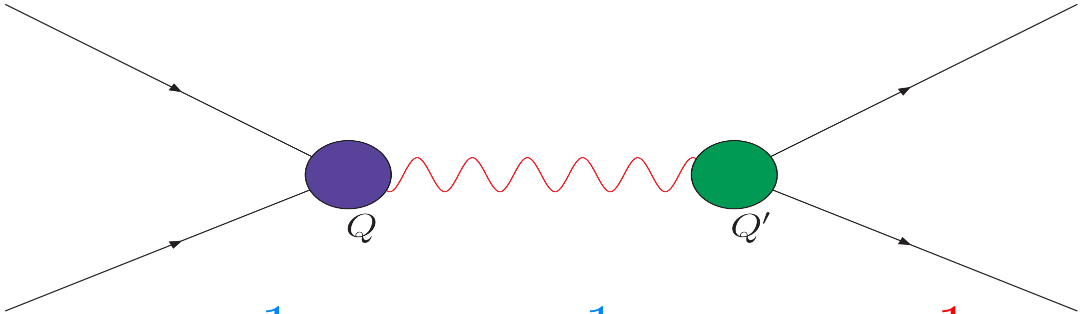
$$\mathcal{M}_0 = e_0 J_\mu^Q D_0^{\mu\nu} e_0 J_\mu^{Q'} \rightarrow e_0^2 Q \frac{1}{k^2} Q'$$

Getting finite results in Observables (2)

At one-loop: 3 types of contributions



The Universal contribution: Improved Born



$$\mathcal{M} = e_0 J^Q_\mu D^{\mu\nu} e_0 J^{Q'}_\mu = \frac{1}{k^2}\, e_0^2 Q \frac{1}{1 + e_0^2 \Pi_{QQ}} Q' \equiv \frac{1}{k^2}\, e_\star^2(k^2) Q Q'$$

$$\begin{aligned}
e_\star^{-2}(k^2) &= e_0^{-2} + \Pi_{QQ}(k^2) \\
e_\star^{-2}(k^2) &= \left(e_0^{-2} + \Pi_{QQ}(k_R^2)\right) + \left(\Pi_{QQ}(k^2) - \Pi_{QQ}(k_R^2)\right) \\
&\equiv \underbrace{e^{-2}(k_R^2)}_{\text{input}} + \underbrace{\left(\Pi_{QQ}(k^2) - \Pi_{QQ}(k_R^2)\right)}_{finite}
\end{aligned}$$

Getting finite results in Observables (3)

Choice of the scale k_R or $e^{-2}(k_R^2)$ is a renormalisation scheme.
In QED and the SM: choose On-shell scheme: directly related to an observable. In the limit $k^2 \to 0$

$$\lim_{k^2 \to 0} \left(k^2 \mathcal{A}_{all\ contributions}\right) = \lim_{k^2 \to 0} k^2 \mathcal{M} = e^2 = e_\star^2(0)$$

$$e^{-2} = e_\star^{-2}(0) = e_0^{-2} + \Pi_{QQ}(0)$$

$$e_\star^2(k^2) = \frac{e^2}{1 + e^2(\Pi_{QQ}(k^2) - \Pi_{QQ}(0))}$$

Trading bare (e_0) with physical (input) parameter

leads to finite corrected observables

renormalisation and definition crucial

Getting finite results in Observables: Summary

- Lagrangian in terms of bare parameter $e_0 = Z_3 e = e + \delta e$,
- Impose renormalisation condition
  $\frac{\delta e^2}{e^2} = \frac{\delta\alpha}{\alpha} = e^2 \Pi_{QQ}(0) = \Pi_{\gamma\gamma}(0)$
- Calculate matrix element at one-loop (with e_0)

$$\mathcal{M}(k^2) = \frac{1}{k^2}\, e_0^2 Q \frac{1}{1 + e_0^2 \Pi_{QQ}} Q' \simeq \frac{QQ'}{k^2}\, \left(1 - e_0^2 \Pi_{QQ}\right) e_0^2$$

- trade-off bare with renormalised parameter

$$\begin{aligned} \mathcal{M}(k^2) &= \frac{QQ'}{k^2}\, \left(1 - e_0^2 \Pi_{QQ} + \frac{\delta\alpha}{\alpha}\right) e^2 \\ &\sim \frac{QQ'}{k^2} \left(1 - e^2 \Pi_{QQ} + \frac{\delta\alpha}{\alpha}\right) e^2 \equiv \frac{Q e_\star^2(k^2) Q'}{k^2} \end{aligned}$$

$\Delta\alpha$ (1)

$$\alpha_\star(k^2) = \frac{\alpha}{1-\Delta\alpha(k^2)} \quad , \qquad \Delta\alpha(k^2) = \Pi_{\gamma\gamma}(0) - \Re e[\Pi_{\gamma\gamma}(k^2)]$$

Perturbative Calculations give

$$\frac{1}{\alpha_\star(k^2)} = \frac{1}{\alpha} - \frac{1}{3\pi}\sum_{k^2 \gg m_f^2} N_c Q_f^2 \left(\ln(|k^2|/m_f^2) - \frac{5}{3} \right)$$

$$+ \frac{1}{3\pi}\sum_{m_f^2 \gg k^2} N_c Q_f^2 \frac{k^2}{m_f^2}$$

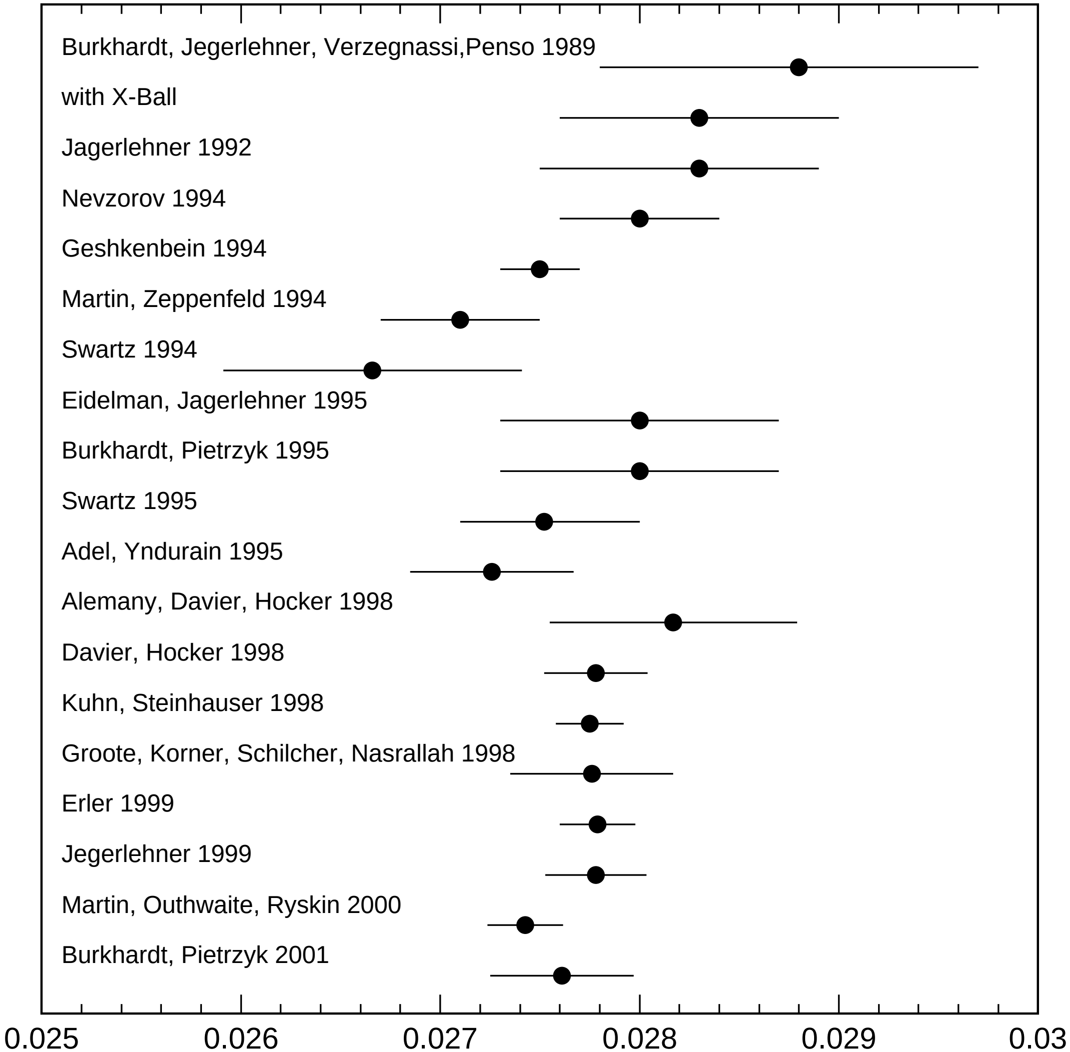
$\Delta\alpha(M_Z^2)$

$$\begin{aligned}
\Delta\alpha(M_Z^2) &= \Delta\alpha_{e,\mu,\tau}(M_Z^2) + \Delta\alpha^{(5)}_{\rm had}(M_Z^2) + \Delta\alpha_{\rm NP}(M_Z^2) \qquad \text{and} \\
\alpha_Z &= \frac{\alpha}{1 - \Delta\alpha_{e,\mu,\tau}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(M_Z^2)}
\end{aligned}$$

$$\begin{aligned}
\Delta\alpha_{e,\mu,\tau}(M_Z^2) &= 314.98\ 10^{-4} \\
10^4 \times \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 276.3 \pm 1.6 \\
\alpha_Z^{-1} &= \alpha^{-1}(M_Z^2) = 128.944 \pm 0.022
\end{aligned}$$

$$\frac{\alpha(M_Z^2)}{\alpha} - 1 \simeq +6\%$$

$\Delta\alpha(M_Z^2)$



$\Delta\alpha$

The electromagnetic coupling grows with energy.

$$\alpha_\star(Q^2) = \frac{\alpha}{1 - \underbrace{\frac{1}{3\pi} \sum_{Q^2 \gg m_f^2} N_c Q_f^2 \left(\ln(Q^2/m_f^2) \right)}_{>0}}$$

at some scale Q^2, the denominator will vanish! This corresponds to a Landau pole. Taking the electron alone, this pole $Q_{\rm LP} \simeq e^{173} TeV$. Something must happen before then...this is much higher than the Planck scale 10^{19}TeV.

• In our on-shell scheme, the renormalisation point, *i.e., the point where the physical value was taken was at* $k^2 = 0$

It is possible to chose another point/scale, $k^2 = \mu_R^2$*, then*

$$\alpha_\star(k^2) = \underbrace{\left(e_0^{-2} + \Pi_{QQ}(\mu_R^2)\right)}_{e^{-2}(\mu_R^2)} + \left(\Pi_{QQ}(k^2) - \Pi_{QQ}(\mu_R^2)\right)$$

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• *In massless theories or even in QED/EW where* $m_f \ll Q^2$*, it is always better to take a renormalisation point such that* $\mu_R^2 \gg m_f^2$*, then at some other point* $\mu_R'^2$

$$e^2(\mu_R'^2) = e^2(\mu_R^2) + \frac{e^4}{12\pi^2} \sum_f N_c Q_f^2 \left(\ln(\mu_R'^2/\mu_R^2)\right)$$

- In massless theories or even in QED/EW where $m_f \ll Q^2$, it is always better to take a renormalisation point such that $\mu_R^2 \gg m_f^2$, then at some other point $\mu_R'^2$

$$e^2(\mu_R'^2) = e^2(\mu_R^2) + \frac{e^4}{12\pi^2}\sum_f N_c Q_f^2 \left(\ln(\mu_R'^2/\mu_R^2)\right)$$

- the behaviour of the charge as a function of scale is contained in the β function

$$\begin{aligned} \beta &= \mu'_R \frac{\partial e(\mu'_R)}{\partial \mu'_R} = \frac{\partial e(k^2)}{\partial t} \quad t = \frac{1}{2}\ln k^2/\mu^2 \\ \beta_{\rm QED} &= +\frac{e^3}{12\pi^2}\sum_f N_c Q_f^2 > 0 \end{aligned}$$

• In massless theories or even in QED/EW where $m_f \ll Q^2$, it is always better to take a renormalisation point such that $\mu_R^2 \gg m_f^2$, then at some other point $\mu_R'^2$

$$e^2(\mu_R'^2) = e^2(\mu_R^2) + \frac{e^4}{12\pi^2} \sum_f N_c Q_f^2 \left(\ln(\mu_R'^2/\mu_R^2)\right)$$

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• β corresponds to the coefficient of the divergence in Π_{QQ} controlled by $1/\epsilon$. It is therefore (relatively) easy to compute.

Imagine that we renormalise at a scale $\mu^2 \gg m_f^2$. This particularly useful when

- The theory contains massless particles
- Is not perturbative $\left(\alpha(k^2) \gg 1\right)$ at small k^2
- for *noble reasons we know α at higher values of k^2 and want to predict α at lower values.*

$\overline{MS}$ scheme

Imagine that we renormalise at a scale $\mu^2 \gg m_f^2$. Minimal subtraction is defined as

$$e^2_{\overline{MS}} = e_0^{-2} + \frac{1}{12\pi^2} \sum_f N_c Q_f^2 \overbrace{\left(\underbrace{\frac{1}{\epsilon}}_{MS} + \ln 4\pi - \gamma_E \right)}^{\overline{MS}}$$

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$$\Longrightarrow \alpha_\star(k^2)^{-2} = \alpha^{-2}_{\overline{MS}}(\mu) + \frac{1}{3\pi}\sum_f N_c Q_f^2 \left(\frac{5}{3} + \ln \mu^2/k^2\right)$$

$$\alpha_\star^{-2}(k^2) = \alpha^{-2}_{\overline{MS}}(k^2) + \frac{5}{9\pi}\sum_f N_c Q_f^2$$

The large logs have been absorbed!

I must collect some thousands logs and pile them

Prospero in the Tempest, Act III, Shakespare

$\overline{MS}$ scheme

Imagine that we renormalise at a scale $\mu^2 \gg m_f^2$. Minimal subtraction is defined as

$$e^2_{\overline{MS}} = e_0^{-2} + \frac{1}{12\pi^2} \sum_f N_c Q_f^2 \overbrace{\left(\underbrace{\frac{1}{\epsilon}}_{MS} + \ln 4\pi - \gamma_E \right)}^{\overline{MS}}$$

$$\begin{aligned}
\Longrightarrow \alpha_\star(k^2)^{-2} &= \alpha^{-2}_{\overline{MS}}(\mu) + \frac{1}{3\pi} \sum_f N_c Q_f^2 \left(\frac{5}{3} + \ln \mu^2/k^2 \right) \\
\alpha_\star^{-2}(k^2) &= \alpha^{-2}_{\overline{MS}}(k^2) + \frac{5}{9\pi} \sum_f N_c Q_f^2 \\
\alpha_\star(k^2) &= \alpha_{\overline{MS}}(k^2) \left(1 - \frac{\alpha_{\overline{MS}}}{3\pi} \sum_f \frac{5}{3} N_c Q_f^2 \right)
\end{aligned}$$

$\overline{MS}$ scheme, higher orders

$$\alpha_\star(k^2) \quad = \quad \alpha_{\overline{MS}}(k^2)\left(1 - \frac{\alpha_{\overline{MS}}}{3\pi}\sum_f \frac{5}{3} N_c Q_f^2\right)$$

scale ambiguous, need to go to higher loops

$\beta_{\rm QCD}$, $\Lambda_{\rm QCD}$, asymptotic freedom

$$\beta_{\rm QCD} \quad = \quad \mu'_R \frac{\partial g_s(\mu'_R)}{\partial \mu'_R} = \left(-11 + \frac{2}{3} n_f \right) \frac{g_s^3}{16\pi^2}$$

$$\beta_{\rm QCD} = \mu'_R \frac{\partial g_s(\mu'_R)}{\partial \mu'_R} = \left(-11 + \frac{2}{3} n_f \right) \frac{g_s^3}{16\pi^2}$$

- from self-interaction of gluons, non-Abelian effect

+ Fermion contribution, with top included: $n_f = 6$

with $n_f = 6$ flavours

$$\beta_{\rm QCD} < 0$$

$$\alpha_s(k^2) = \frac{\alpha_s(\mu_R^2)}{1 + b_s^{n_f} \frac{\alpha_s(\mu_R^2)}{4\pi} \ln(k^2/\mu_R^2)} \quad \alpha_s(\mu) \text{ in } \overline{MS}$$

$$b_s^{n_f} = 11 - \frac{2n_f}{3} > 0$$

$\Lambda_{\rm QCD}$ is a physical parameter, defined for a value of k^2 where $\alpha_s(k^2)$ is non perturbative. It is the Landau pole of the coupling. It is a dynamical scale.
Obviously it depends on the number of flavours.

$$\Lambda_{\rm QCD}^{n_f} = \mu exp\left(- \frac{2\pi}{b_s^{n_f} \alpha_s(\mu)} \right), \qquad \frac{{\rm d}\Lambda}{{\rm d}\mu} = 0$$

then

$$\alpha_s(k^2) = \frac{4\pi}{b_s^{n_f} \left(\ln k^2/\Lambda_f \right)}$$

Experiments give

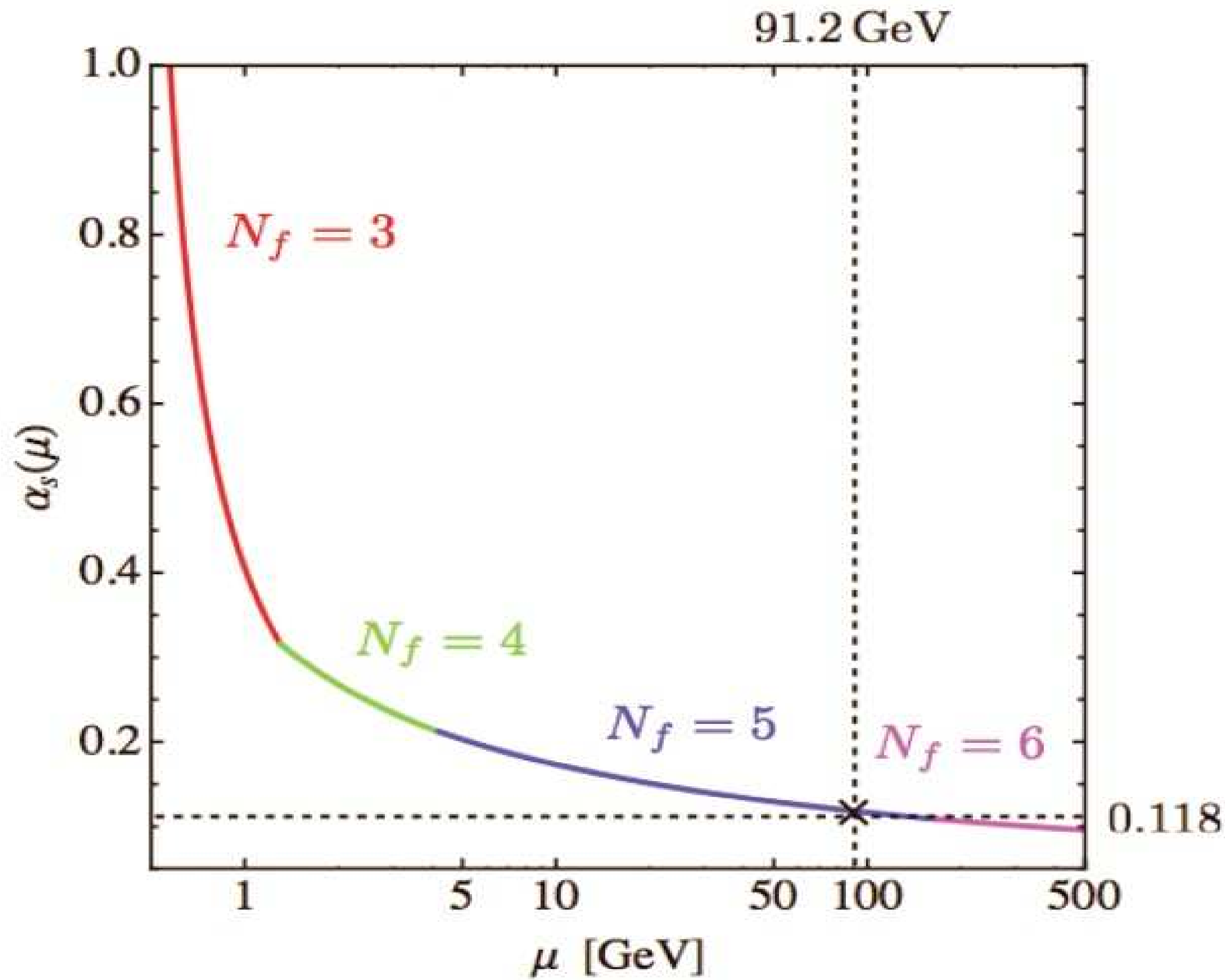
$$\Lambda^{n_f=4,5}_{\overline{MS}}$$

Indicating that the value is extracted from data fitted with a perturbative calculation done up to a certain order, in a particular renormalisation scheme, assuming n_f flavours.

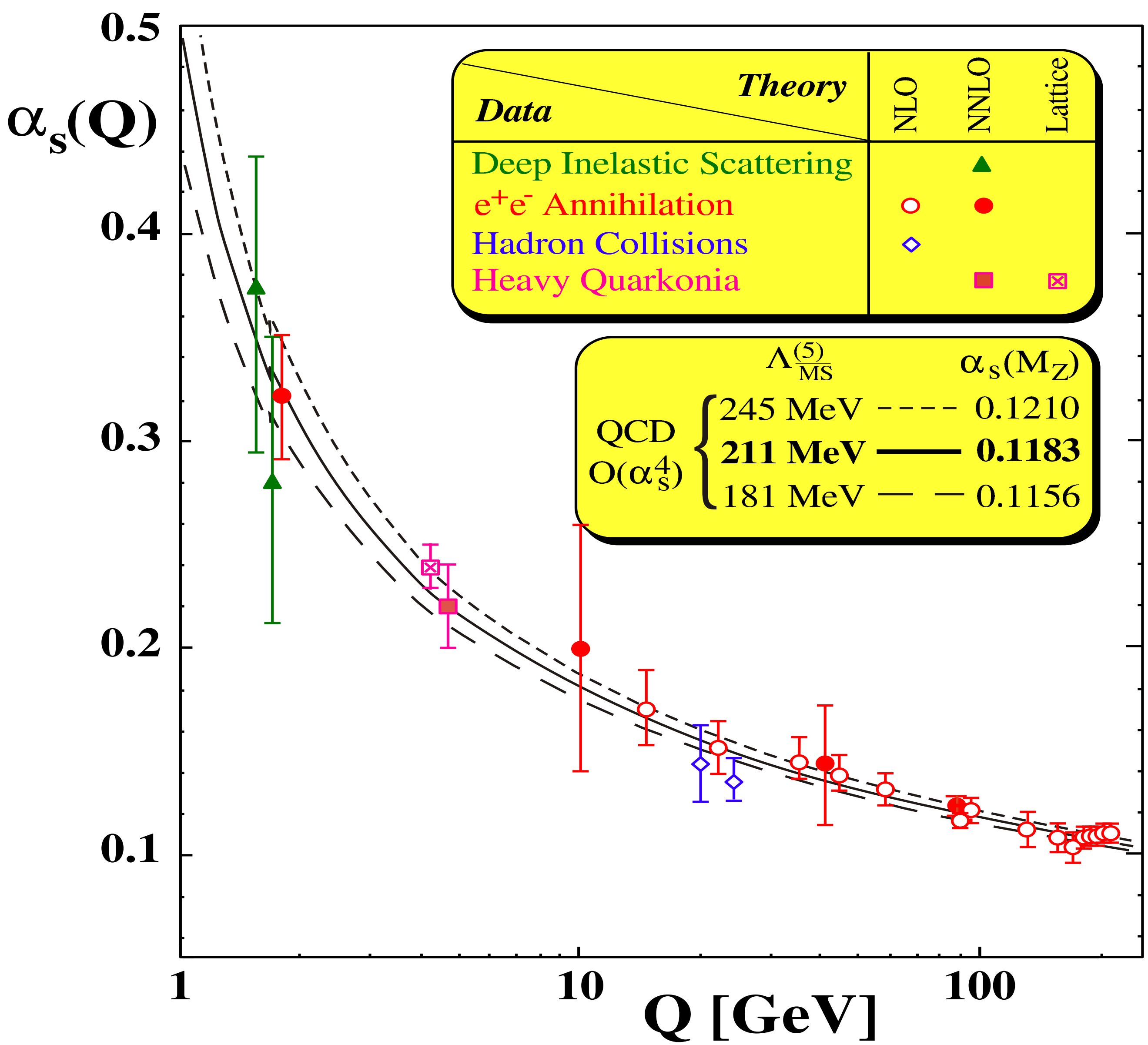
- $k^2 < m_b^2 \rightleftharpoons n_f = 4$, bottom considered as heavy
- $k^2 < m_t^2 \rightleftharpoons n_f = 5$

when crossing a threshold one imposes

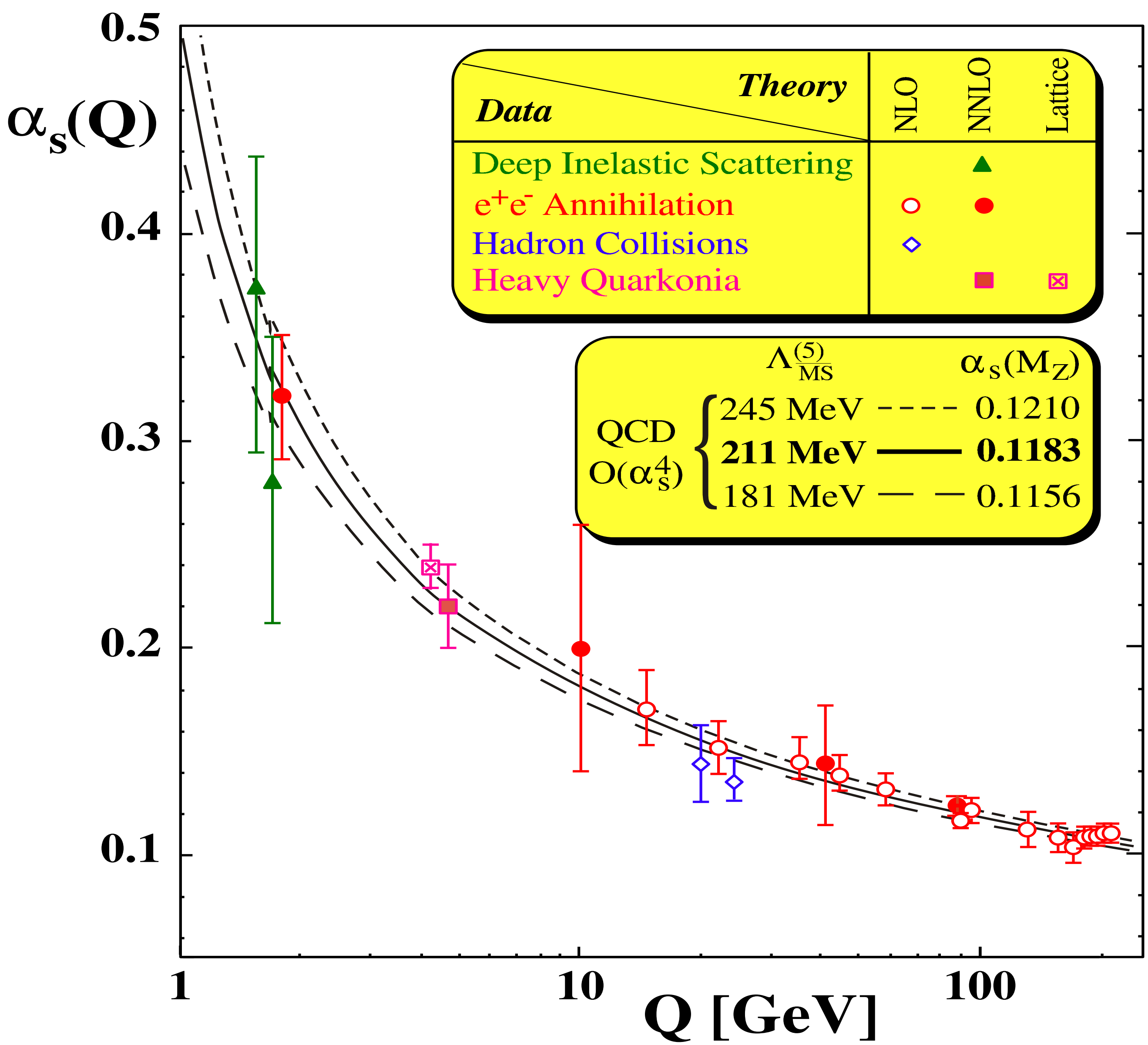
$$\alpha_s^{n_f=4}(m_b) = \alpha_s^{n_f=5}(m_b) \Rightarrow \Lambda_4 = \Lambda_5 \left(\frac{m_b}{\Lambda_5}\right)^{2/25}$$


91.2 GeV
1.0
0.8
0.6
0.4
0.2
$\alpha_s(\mu)$
$N_f = 3$
$N_f = 4$
$N_f = 5$
$N_f = 6$
0.118
1
5
10
50
100
500
μ [GeV]

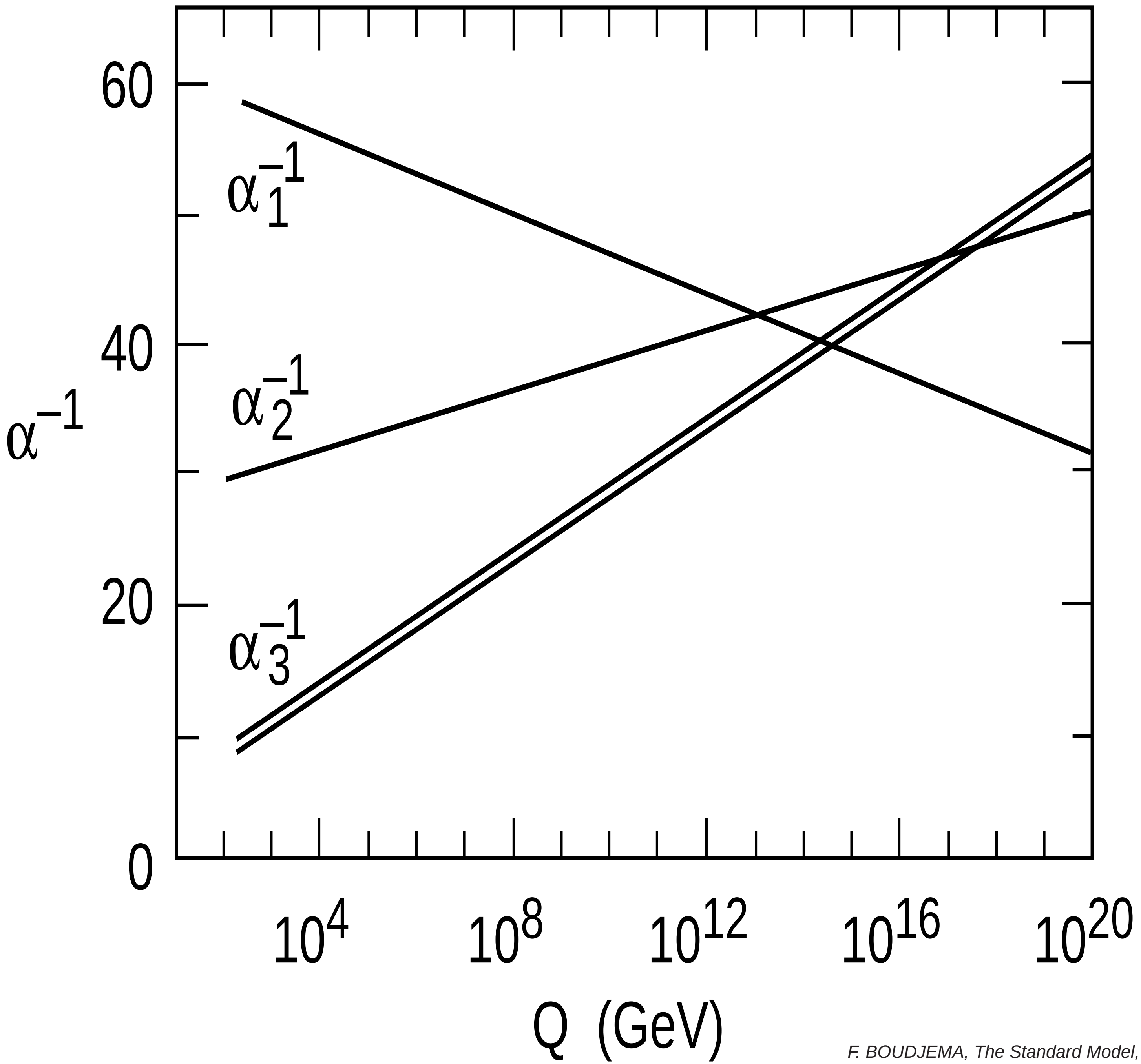
$\beta_{\rm QCD}$, $\Lambda_{\rm QCD}$, asymptotic freedom



α_s running: experimental confirmation

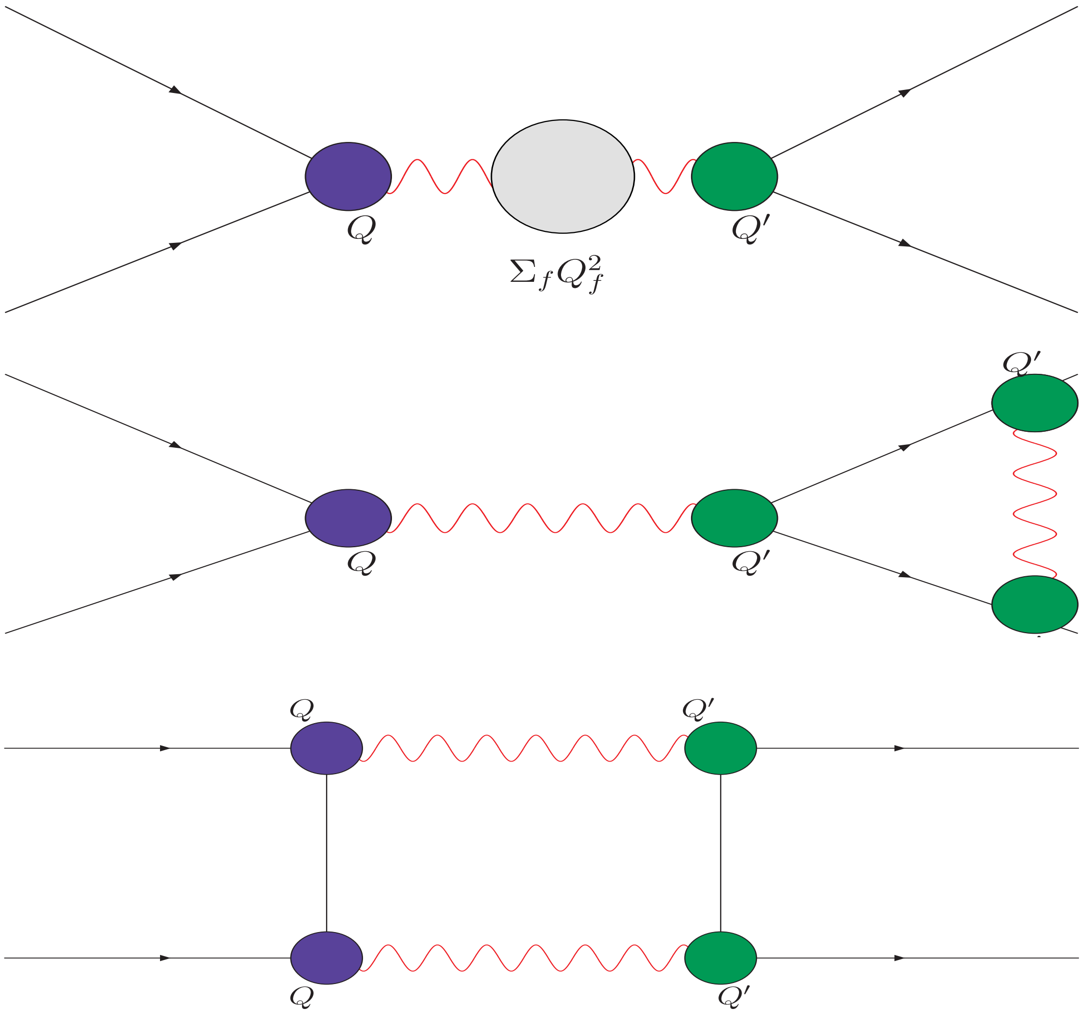


Coupling Constant Unification?



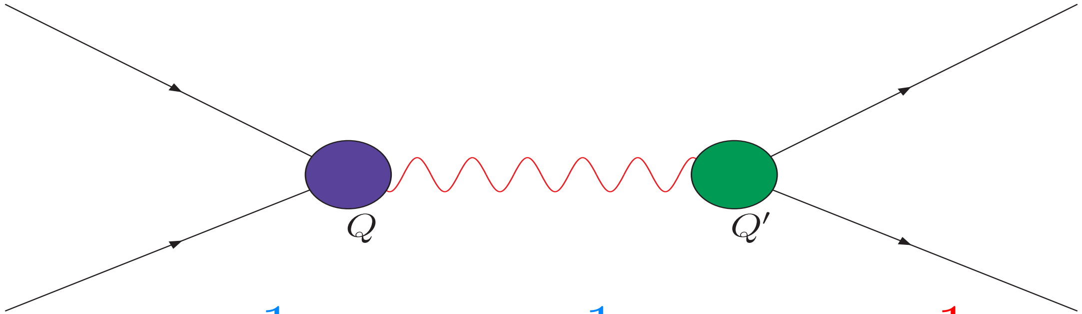
Recap: Getting finite results in Observables (2)

At one-loop: 3 types of contributions



Recap: Getting finite results in Observables (3)

The Universal contribution: Improved Born



$$\mathcal{M} = e_0 J^Q_\mu D^{\mu\nu} e_0 J^{Q'}_\mu = \frac{1}{k^2}\, e_0^2 Q \frac{1}{1+e_0^2 \Pi_{QQ}} Q' \equiv \frac{1}{k^2}\, e_\star^2(k^2) Q Q'$$

$$\begin{aligned} e_\star^{-2}(k^2) &= e_0^{-2} + \Pi_{QQ}(k^2) \\ e_\star^{-2}(k^2) &= \left(e_0^{-2} + \Pi_{QQ}(k_R^2)\right) + \left(\Pi_{QQ}(k^2) - \Pi_{QQ}(k_R^2)\right) \\ &\equiv \underbrace{e^{-2}(k_R^2)}_{\text{input}} + \underbrace{\left(\Pi_{QQ}(k^2) - \Pi_{QQ}(k_R^2)\right)}_{finite} \end{aligned}$$

Recap: Getting finite results in Observables (3)

Choice of the scale k_R or $e^{-2}(k_R^2)$ is a renormalisation scheme.
In QED and the SM: choose On-shell scheme: directly related to an observable. In the limit $k^2 \to 0$

$$\lim_{k^2\to 0}\left(k^2 \mathcal{A}_{all\ contributions}\right) = \lim_{k^2\to 0} k^2 \mathcal{M} = e^2 = e_\star^2(0)$$

$$e^{-2} = e_\star^{-2}(0) = e_0^{-2} + \Pi_{QQ}(0)$$

$$e_\star^2(k^2) = \frac{e^2}{1 + e^2(\Pi_{QQ}(k^2) - \Pi_{QQ}(0))}$$

Trading bare (e_0) with physical (input) parameter

leads to finite corrected observables

renormalisation and definition crucial

Recap: Getting finite results in Observables: Summary

- Lagrangian in terms of bare parameter $e_0 = Z_3 e e = e + \delta e$,
- Impose renormalisation condition
  $\frac{\delta e^2}{e^2} = \frac{\delta\alpha}{\alpha} = e^2 \Pi_{QQ}(0) = \Pi_{\gamma\gamma}(0)$
- Calculate matrix element at one-loop (with e_0)

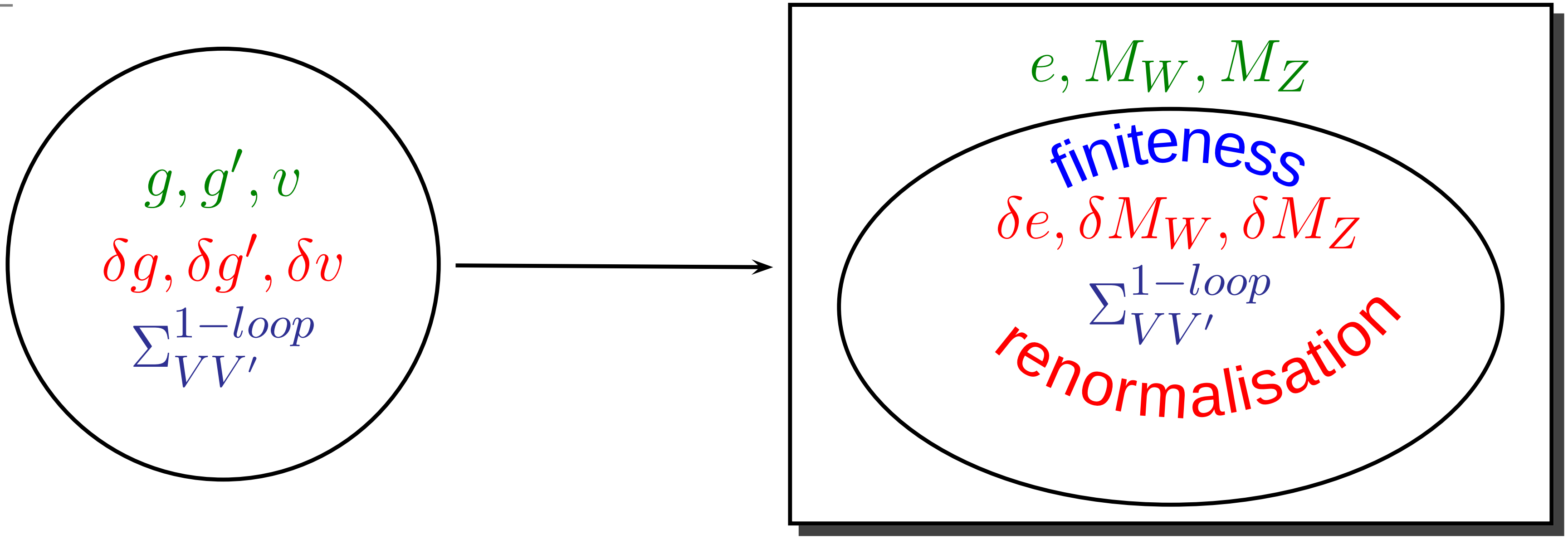
$$\mathcal{M}(k^2) = \frac{1}{k^2}\, e_0^2 Q \frac{1}{1 + e_0^2 \Pi_{QQ}} Q' \simeq \frac{QQ'}{k^2}\, \left(1 - e_0^2 \Pi_{QQ}\right) e_0^2$$

- trade-off bare with renormalised parameter

$$\mathcal{M}(k^2) = \frac{QQ'}{k^2}\, \left(1 - e_0^2 \Pi_{QQ} + \frac{\delta\alpha}{\alpha}\right) e^2$$

$$\sim \frac{QQ'}{k^2} \left(1 - e^2 \Pi_{QQ} + \frac{\delta\alpha}{\alpha}\right) e^2 \equiv \frac{Q e_\star^2(k^2) Q'}{k^2}$$

Lagrangian parameters and physical parameters (1.)



Lagrangian parameters and physical parameters (1.)

g, g', v

$\delta g, \delta g', \delta v$

$\Sigma_{VV'}^{1-loop}$

$\longrightarrow$

e, M_W, M_Z

finiteness

$\delta e, \delta M_W, \delta M_Z$

$\Sigma_{VV'}^{1-loop}$

renormalisation

$$\frac{\delta g'^{\,2}}{g'^{\,2}} = \frac{\delta e^2}{e^2} + \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}$$

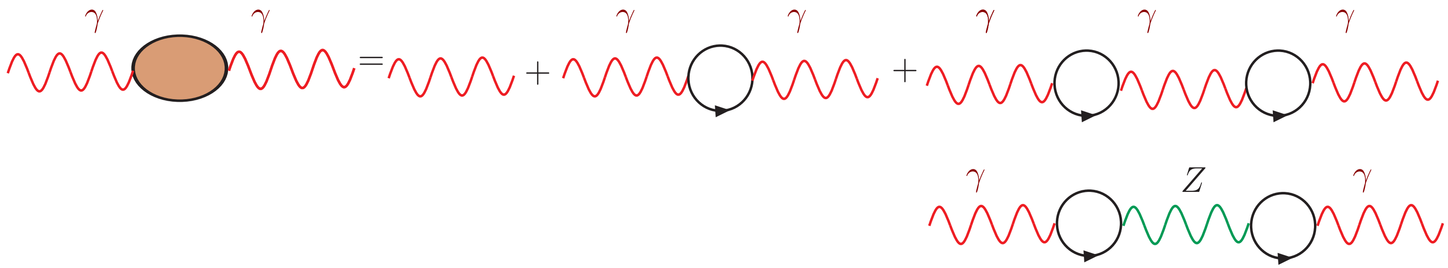
$$\frac{\delta g^2}{g^2} = \frac{\delta e^2}{e^2} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right)$$

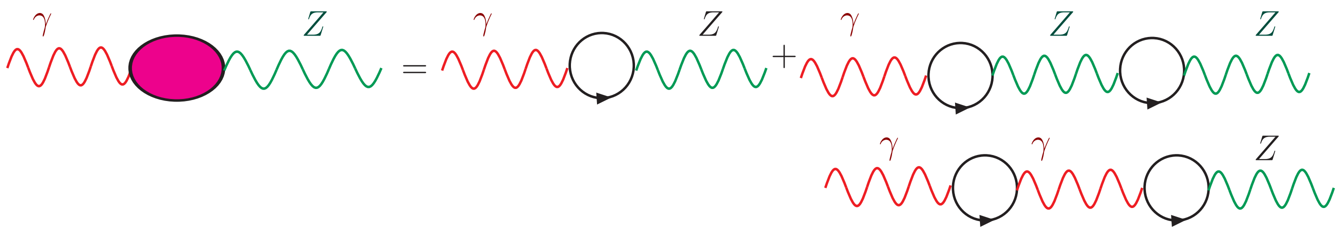
$$\frac{\delta v^2}{v^2} = -\frac{\delta e^2}{e^2} + \frac{c_W^2}{s_W^2}\frac{\delta M_Z^2}{M_Z^2} + \frac{s_W^2 - c_W^2}{s_W^2}\frac{\delta M_W^2}{M_W^2}$$

$$\left(s_M^2 = s_W^2 = 1 - \frac{M_W^2}{M_Z^2}\right) \blacktriangleright$$

$$\frac{\delta s_M^2}{s_M^2} = \frac{\delta s^2}{s^2} = -\frac{c_W^2}{s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) \quad , \quad \delta c^2 = -\delta s^2$$

Propagators and $Z-\gamma$ mixing





$$D^{\gamma\gamma}_{\mu\nu} \;=\; -\,ig_{\mu\nu}\frac{1}{k^2}\,\frac{1}{1+\Pi_{\gamma\gamma}(k^2)} \qquad D^{Z\gamma}_{\mu\nu} = +\,ig_{\mu\nu}\,\frac{\Pi_{Z\gamma}(k^2)}{k^2-M^2_{Z,0}}$$

$$D^{ZZ}_{\mu\nu} \;=\; -\,ig_{\mu\nu}\,\frac{1}{k^2-M^2_{Z,0}+G_{ZZ}(k^2)}$$

$$D^{WW}_{\mu\nu} \;=\; -\,ig_{\mu\nu}\,\frac{1}{k^2-M^2_{W,0}+G_{WW}(k^2)}$$

$$D^{WW}_{\mu\nu} = -\, i g_{\mu\nu} \frac{1}{k^2 - M^2_{W,0} + G_{WW}(k^2)}$$

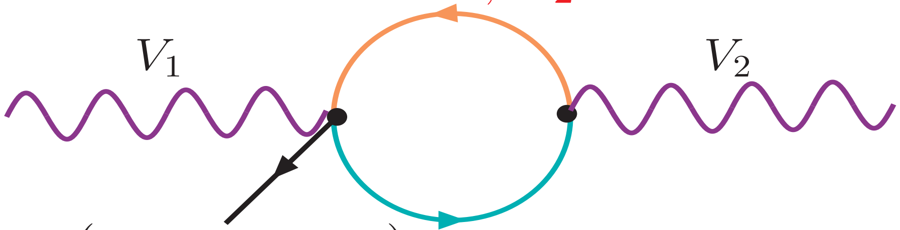
$\rightarrow 0$ **for** $k^2 = M^2_W$

Physical mass is defined from the zero of the real part of the inverse propagator, for $k^2 = M^2_W$

$$M^2_W - \underbrace{M^2_W \left(1 + \frac{\delta M^2_W}{M^2_W}\right)}_{M^2_{W,0}} + Re\left[G_{WW}(M^2_W)\right] = 0$$

$$\boxed{\delta M^2_W = Re\left[G_{WW}(M^2_W)\right] = G_{WW}(0) + M^2_W Re\left[\Pi_{WW}(M^2_W)\right]}$$

Chiral two-point functions (1)



$$
\begin{aligned}
-i\Sigma_{\mu\nu} &= \frac{(g_L^2+g_R^2)}{2}\mu^{4-n}\int\frac{d^nr}{(2\pi)^n}\mathrm{Tr}\left(\frac{\gamma_\mu \not{r}\gamma_\nu(\not{r}+\not{k})}{(r^2-m_1^2)((r+k)^2-m_2^2)}\right)\\
&+ (g_Lg_R)m_1m_2\mu^{4-n}\int\frac{d^nr}{(2\pi)^n}\mathrm{Tr}\left(\frac{\gamma_\mu\gamma_\nu}{(r^2-m_1^2)((r+k)^2-m_2^2)}\right)\\
&\equiv g_L^2 i\Sigma_{\mu\nu}^{LL}+g_R^2 i\Sigma_{\mu\nu}^{RR}+g_Lg_R(i\Sigma_{\mu\nu}^{LR}+i\Sigma_{\mu\nu}^{RL})\\
i\Sigma_{\mu\nu}^{ij} &= -i(g_{\mu\nu}G_{ij}(k^2)-k_\mu k_\nu L(k^2))\\
G_{ij}(k^2) &= G_{ij}(0)+k^2\Pi_{ij}
\end{aligned}
$$

Chiral two-point functions (2)

$$
\begin{aligned}
G_{LL,RR}(k^2) &= -\frac{4}{(4\pi)^2}\left\{\frac{C_{UV}}{4}(m_1^2+m_2^2) \;+\; k^2\left(B_2(m_1^2,m_2^2,k^2)-\frac{C_{UV}}{6}\right)\right. \\
&\quad \left. -\frac{1}{2}\left(m_2^2B_1(m_1^2,m_2^2,k^2)+m_1^2B_1(m_2^2,m_1^2,k^2)\right)\right\} \\
G_{LR,RL}(k^2) &= \frac{2}{(4\pi)^2}m_1m_2\left\{C_{UV}-B_0(m_1^2,m_2^2,k^2)\right\} \\
&\rightarrow \Pi_{LR,RL}=\text{finite} \;\;,\;\; \Pi_{LL}(m_1,m_2)-\Pi_{LL}(m_3,m_4)=\text{finite} \\
G_{LL}(0)+G_{LR}(0) &= \text{finite for } \; m_1=m_2 \\
G_{LV} &= G_{LL}+G_{LR}=k^2(\Pi_{RR}+\Pi_{RL}) \;\;,\;\; G_{VV}=k^2(\Pi_{RR}+\Pi_{RL})
\end{aligned}
$$

Chiral two-point functions (3)

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu T^+ W^-_\mu T^-) - i\frac{g}{c_W} Z_\mu (T_3 - s_W^2 Q) - ieA_\mu Q$$

$$33, 11 \rightarrow LL \quad Q \rightarrow V = L + R$$

$$\Pi_{\gamma\gamma} = e^2 \Pi_{QQ}$$

$$\Pi_{\gamma Z} = \left(\frac{e^2}{s_W c_W}\right)(\Pi_{3Q} - s_W^2 \Pi_{QQ})$$

$$\Pi_{ZZ} = \left(\frac{e^2}{s_W^2 c_W^2}\right)(\Pi_{33} - 2s_W^2 \Pi_{3Q} + s_W^4 \Pi_{QQ})$$

$$\Pi_{WW} = \left(\frac{e^2}{s_W^2}\right)\Pi_{11} = g^2 \Pi_{11}$$

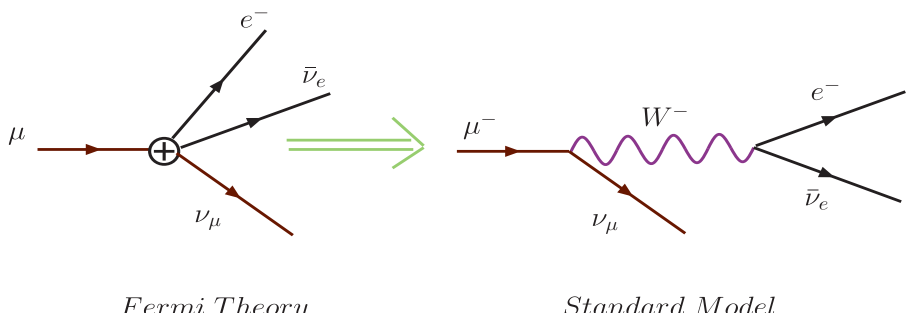
$$G_{WW} = g^2 G_{11}$$

$$G_{ZZ} = \frac{g^2}{c_W^2} G_{33}$$

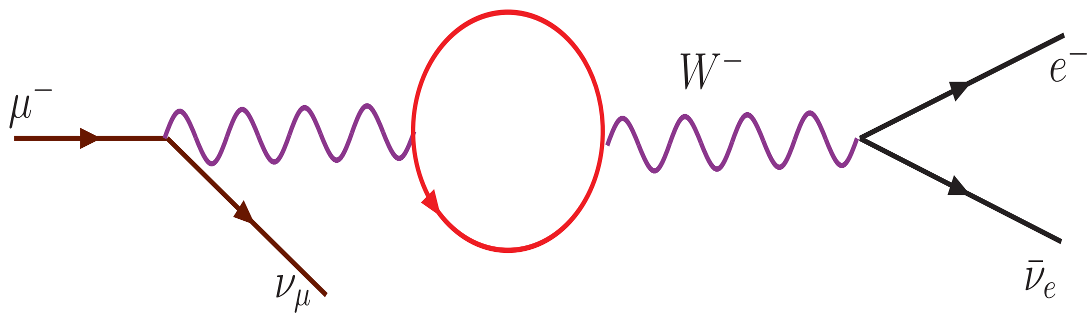
Finiteness if only combinations are

$$\left(\mathbf{\Pi_{ij}(k^2) - \Pi_{i',j'}(k'^2)}\right) \quad \text{or} \quad \left(\mathbf{G_{33}(0) - G_{11}(0)}\right)$$

Muon decay and Δr



$$\mathcal{M}_0 = \frac{g_0^2}{k^2 - M_{W,0}^2} \quad \text{with } \mathbf{k^2 \to 0}$$



at one-loop

$$\mathcal{M}(k^2 \to 0) = \frac{g_0^2}{k^2_{\hookrightarrow 0} - M^2_{W,0} + G_{WW}(k^2_{\hookrightarrow 0})} = -\frac{g^2}{M_W^2}\,\frac{1+\frac{\delta g^2}{g^2}}{1+\frac{\delta M_W^2}{M_W^2} - \frac{G_{WW}(0)}{M_W^2}}$$

$$= -\frac{g^2}{M_W^2}\left(1+\frac{\delta g^2}{g^2}\right)\left(1+\Re e[\Pi_{WW}(M_W^2)]\right)^{-1}$$

$$\simeq -\frac{g^2}{M_W^2}\left(1+\underbrace{\frac{\delta g^2}{g^2} - \Re e[\Pi_{WW}(M_W^2)]}_{\Delta\hat{r}=finite} + \delta r_{V+B}\right) \equiv -\frac{g^2}{M_W^2}(1+\Delta r)$$

Finiteness of $\Delta\hat{r}$

$$\Delta\hat{r} = \frac{\delta g^2}{g^2} - \Re e[\Pi_{WW}(M_W^2)] = \frac{\delta e^2}{e^2} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) - \Re e[\Pi_{WW}(M_W^2)]$$

$$= \Pi_{\gamma\gamma}(0) + \frac{c_W^2}{s_W^2}\left\{\underbrace{\left(\frac{G_{WW}(0)}{M_W^2} - \frac{G_{ZZ}(0)}{M_Z^2}\right)}_{\text{finite}} + \left(\Re e\Pi_{WW}(M_W^2) - \Re e\Pi_{ZZ}(M_Z^2)\right)\right\} - \Re e[\Pi_{WW}$$

$$\Delta\hat{r} = \left(\Pi_{\gamma\gamma}(0) - \Re e\Pi_{\gamma\gamma}(M_Z^2)\right) + g^2\frac{c_W^2}{s_W^2}\left(\frac{G_{11}-G_{33}}{M_W^2}\right)$$

$$+\frac{g^2}{s_W^2}\left\{-\Re e\Pi_{33}(M_Z^2) + 2s_W^2\Re e\Pi_{3Q}(M_Z^2) + (c_W^2 - s_W^2)\Re e\Pi_{11}(M_W^2)\right\}$$

$$= \left(\Pi_{\gamma\gamma}(0) - \Re e\Pi_{\gamma\gamma}(M_Z^2)\right) + g^2\frac{c_W^2}{s_W^2}\left(\frac{G_{11}-G_{33}}{M_W^2}\right)$$

$$+g^2\left\{2\Re e[\Pi_{3Q}(M_Z^2) - \Pi_{33}(M_Z^2)] + \frac{c_W^2 - s_W^2}{s_W^2}\Re e[\Pi_{11}(M_W^2) - \Pi_{33}(M_Z^2)]\right\}$$

$$\Delta\alpha(M_Z^2) = \left(\Pi_{\gamma\gamma}(0) - \Re e\Pi_{\gamma\gamma}(M_Z^2)\right) \equiv e^2\left(\Pi_{QQ}(0) - \Re e\Pi_{QQ}(M_Z^2)\right)$$

Δr: correcting a tree-level formula (1)

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1+\Delta r) \quad \Delta r = \Delta\hat{r} \;+\; \delta r_{V+B} \qquad r_{V+B} \sim 6.43\;10^{-3}$$

$$\Delta\hat{r} = \Delta\alpha(M_Z^2) \;-\; \frac{c_W^2}{s_W^2}\varepsilon_1 \;+\; \frac{c_W^2 - s_W^2}{s_W^2}\varepsilon_2 \;+\; 2\varepsilon_3$$

$\varepsilon_{1,2,3}$ **or** S, T, U**:** non-decoupling effects

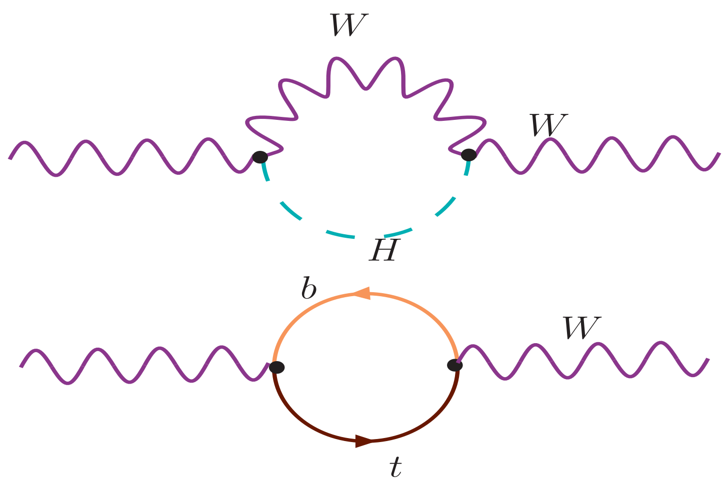
$$\varepsilon_1 \;=\; \Delta\rho = \alpha T = g^2\left(\frac{G_{33}-G_{11}}{M_W^2}\right) \propto m_t^2$$

SU(2) breaking

$$\varepsilon_2 \;=\; -\frac{\alpha}{4s_W^2}U = g^2\left(\Re e[\Pi_{11}(M_W^2)] - \Re e[\Pi_{33}(M_Z^2)]\right)$$

$$\varepsilon_3 \;=\; \frac{\alpha}{4s_W^2}S = g^2\left(\Re e[\Pi_{3Q}(M_Z^2)] - \Re e[\Pi_{33}(M_Z^2)]\right) = -\frac{g^2}{2}\Re e[\Pi_{3Y}(M_Z^2)] \rightarrow \text{chiral symm.}$$

Δr Probe of M_H and m_t

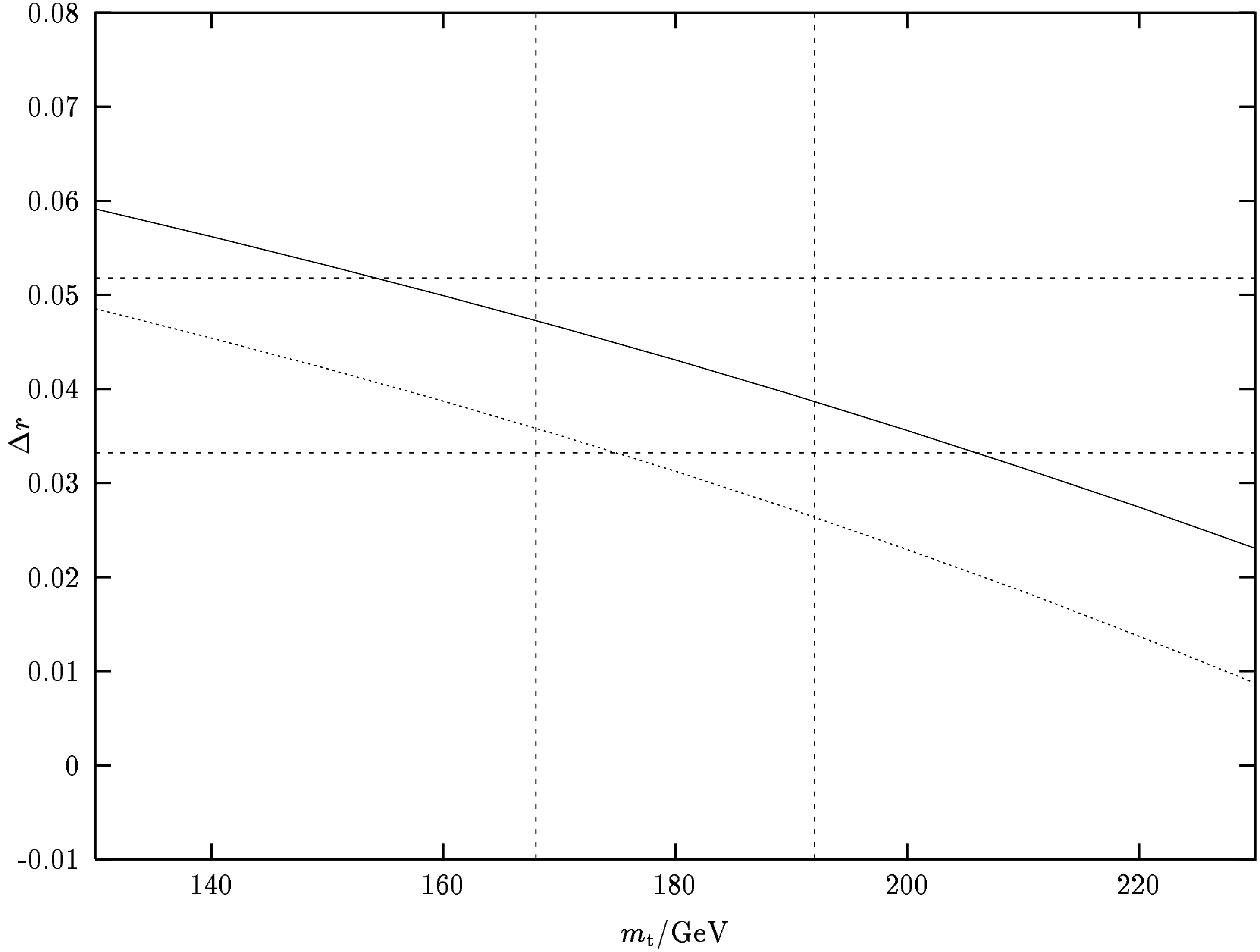


$$\varepsilon_1 = \frac{3G_\mu M_Z^2}{8\pi^2\sqrt{2}} \left\{ \left(\frac{m_t^2}{M_Z^2} - 1 \right) - 2s_M^2 \ln(M_H/M_Z) \right\} + \varepsilon_1^{NP}$$

$$\varepsilon_2 = -\frac{G_\mu M_W^2}{2\pi^2\sqrt{2}} \ln(m_t/M_Z) + \varepsilon_2^{NP}$$

$$\varepsilon_3 = \frac{G_\mu M_W^2}{12\pi^2\sqrt{2}} \{\ln(M_H/M_Z) - \ln(m_t^2/M_Z^2)\} + \varepsilon_3^{NP}$$

Δr: correcting a tree-level formula (2)



s_Z^2

With Δr we predict M_W

$$\frac{\mathbf{M_W^2}}{\mathbf{M_Z^2}} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} + \sqrt{\mathbf{1} - \frac{\mathbf{4}\pi\alpha(\mathbf{1}+\mathbf{\Delta r})}{\sqrt{\mathbf{2}}\mathbf{G}_\mu \mathbf{M_Z^2}}}\right) \quad ; \quad \mathbf{s_M^2} = \mathbf{s_W^2} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} - \sqrt{\mathbf{1} - \frac{\mathbf{4}\pi\alpha(\mathbf{1}+\mathbf{\Delta r})}{\sqrt{\mathbf{2}}\mathbf{G}_\mu \mathbf{M_Z^2}}}\right)$$

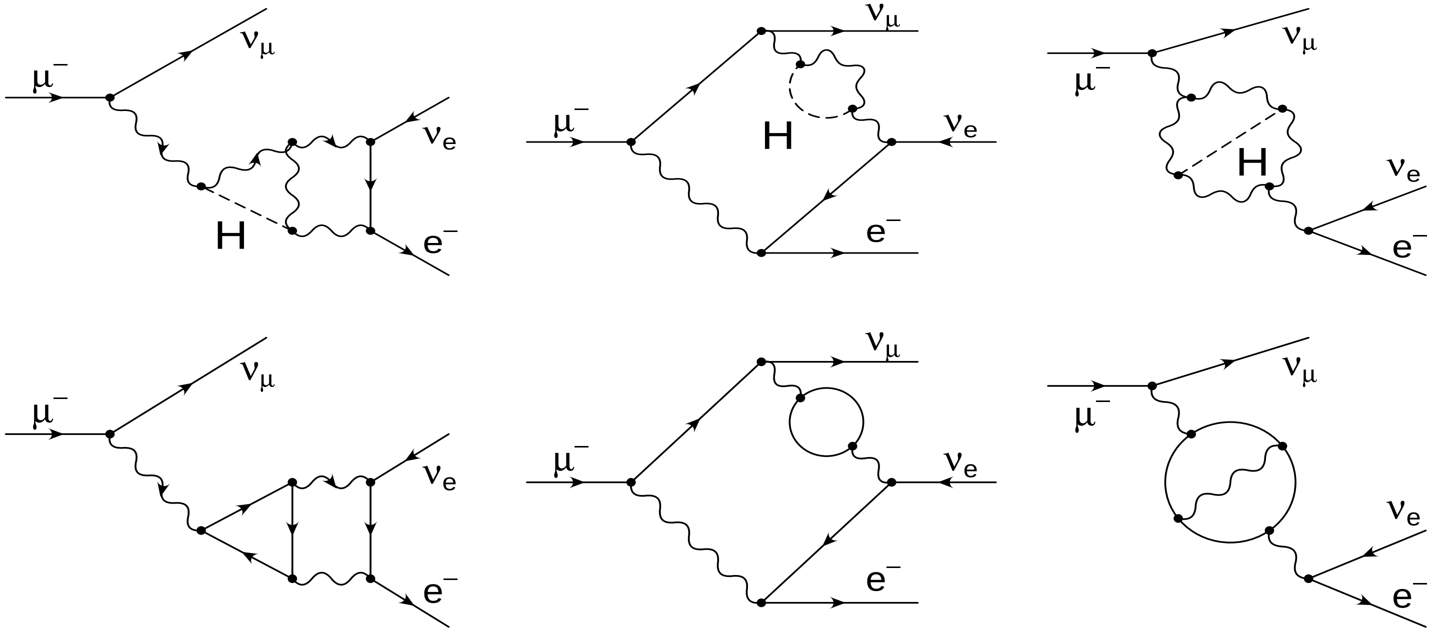
Improved s_W^2

$$\mathbf{s_Z^2} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} - \sqrt{\mathbf{1} - \frac{\mathbf{4}\pi\alpha_\mathbf{Z}}{\sqrt{\mathbf{2}}\mathbf{G}_\mu \mathbf{M_Z^2}}}\right) \quad \alpha_\mathbf{Z} = \frac{\alpha}{\mathbf{1} - \mathbf{\Delta}\alpha_\mathbf{Z}}$$

$$\begin{cases} s_Z^2 = .212154 & \text{for} \quad \alpha_Z = \alpha(0) \\ s_Z^2 = .231086 & \text{for} \quad \alpha_Z = 128.907 \\ s_Z^2 = .231019 & \text{for} \quad \alpha_Z = 128.944 \end{cases}$$

$s^2_{\rm eff}|_{1991} = .2329 \pm^{.0034}_{.0029}$ compatible with any of the above values with a running α, but not with $\alpha(0) = \alpha$.

Δr beyond one-loop



$$M_W = M_W^0 - c_1\,\mathrm{dH} - c_2\,\mathrm{dH}^2 + c_3\,\mathrm{dH}^4 + c_4(\mathrm{dh}-1) - c_5\,\mathrm{d}\alpha + c_6\,\mathrm{dt} - c_7\,\mathrm{dt}^2 - c_8\,\mathrm{dH}\,\mathrm{dt} + c_9\,\mathrm{dh}\,\mathrm{dt} - c_{10}\,\mathrm{d}\alpha_s + c_{11}\,\mathrm{dZ},$$

$$\mathrm{dH} = \ln\left(\frac{M_H}{100\ \mathrm{GeV}}\right), \qquad \mathrm{dh} = \left(\frac{M_H}{100\ \mathrm{GeV}}\right)^2, \quad \mathrm{dt} = \left(\frac{m_t}{174.3\ \mathrm{GeV}}\right)^2 - 1,$$

$$\mathrm{dZ} = \frac{M_Z}{91.1875\ \mathrm{GeV}} - 1, \qquad \mathrm{d}\alpha = \frac{\Delta\alpha(M_Z^2)}{0.05907} - 1, \quad \mathrm{d}\alpha_s = \frac{\alpha_s(M_Z^2)}{0.119} - 1.$$

Corrections to the neutral current observables

$$\mathcal{M}_0 = \underbrace{-e^2 Q \frac{1}{k^2} Q'}_{\gamma \text{ exch.}} \underbrace{- \frac{e^2}{s_W^2 c_W^2}(T_3 - s_W^2 Q)\frac{1}{k^2 - M_Z^2}(T_3' - s_W^2 Q')}_{Z \text{ exch.}}$$

$$\begin{aligned}
\mathcal{M}^{1-\text{loop}} \quad = \quad & -e^2 \underbrace{\left(1 + \frac{\delta e^2}{e^2}\right)}_{counterterm} Q \underbrace{\frac{1}{k^2\left(1 + \Pi_{\gamma\gamma}(k^2)\right)}}_{\gamma\gamma\ 2-pt} Q' \\
- \quad & \frac{e^2}{s_W^2 c_W^2} \underbrace{\frac{\left(1 + \frac{\delta e^2}{e^2}\right)}{\left(1 + \frac{\delta s^2}{s^2} - \frac{s_W^2}{c_W^2}\frac{\delta s^2}{s^2}\right)}}_{counterterm} \underbrace{\frac{\left(T_3 - s_W^2(1 + \frac{\delta s^2}{s^2})Q\right) \times \left(T_3' - s_W^2(1 + \frac{\delta s^2}{s^2})Q\right.}{k^2 - M_Z^2\left(1 + \frac{\delta M_Z^2}{M_Z^2}\right) + G_{ZZ}(k^2)}}_{ZZ\ 2-pt} \\
+ \quad & \frac{e^2}{s_W c_W}(T_3 - s_W^2 Q)\underbrace{\frac{\Pi_{\gamma Z}(k^2)}{k^2 - M_Z^2}}_{Z\gamma\ 2-pt}(Q') \qquad + \frac{e^2}{s_W c_W}(Q)\underbrace{\frac{\Pi_{\gamma Z}(k^2)}{k^2 - M_Z^2}}_{Z\gamma\ 2-pt}(T_3' - s_W^2 Q')
\end{aligned}$$

Corrections to the neutral current observables (2)combine ZZ with $Z\gamma$

$$+ \quad \frac{e^2}{s_W c_W}(T_3 - s_W^2 Q)\frac{\Pi_{\gamma Z}(k^2)}{k^2 - M_Z^2}(Q')$$

$$= \quad -\frac{e^2}{s_W^2 c_W^2}\left(T_3 - s_W^2 Q\right)\left[\frac{c_W}{s_W}\Pi_{\gamma Z}(k^2)\right]\left(-s_W^2 Q'\right) \quad \rightarrow$$

$$- \quad \frac{e^2}{s_W^2 c_W^2}\frac{\left(1+\frac{\delta e^2}{e^2}\right)}{\left(1+\frac{\delta s^2}{s^2}-\frac{s_W^2}{c_W^2}\frac{\delta s^2}{s^2}\right)}\frac{\left(T_3 - s_W^2(1+\frac{\delta s^2}{s^2})Q\right)\times\left(\frac{c_W}{s_W}\Pi_{\gamma Z}(k^2)\right)\times\left(-s_W^2(1+\frac{\delta}{s}\right.}{k^2 - M_Z^2\left(1+\frac{\delta M_Z^2}{M_Z^2}\right)+G_{ZZ}(k^2)}$$

$$-\mathcal{M}^{ZZ+Z\gamma} = -\mathcal{M}^Z$$

$$\frac{\mathbf{e^2}}{\mathbf{s_W^2 c_W^2}}\frac{\left(\mathbf{1}+\frac{\delta \mathbf{e^2}}{\mathbf{e^2}}\right)}{\left(\mathbf{1}+\frac{\delta \mathbf{s^2}}{\mathbf{s^2}}-\frac{\mathbf{s_W^2}}{\mathbf{c_W^2}}\frac{\delta \mathbf{s^2}}{\mathbf{s^2}}\right)}\frac{\left(\mathbf{T_3} - \boxed{s_W^2(1+\frac{\delta s^2}{s^2}+\frac{c_W}{s_W}\Pi_{\gamma Z}(k^2))}\,\mathbf{Q}\right)\left(\mathbf{T_3'} - \mathbf{s_W^2}(\mathbf{1}+\frac{\delta \mathbf{s^2}}{\mathbf{s^2}}\frac{\mathbf{c_W}}{\mathbf{s_w}}\mathbf{\Pi}\right.}{\mathbf{k^2 - M_Z^2}\left(\mathbf{1}+\frac{\delta \mathbf{M_Z^2}}{\mathbf{M_Z^2}}\right)+\mathbf{G_{ZZ}(0)+k^2\Pi_{ZZ}(k^2)}}$$

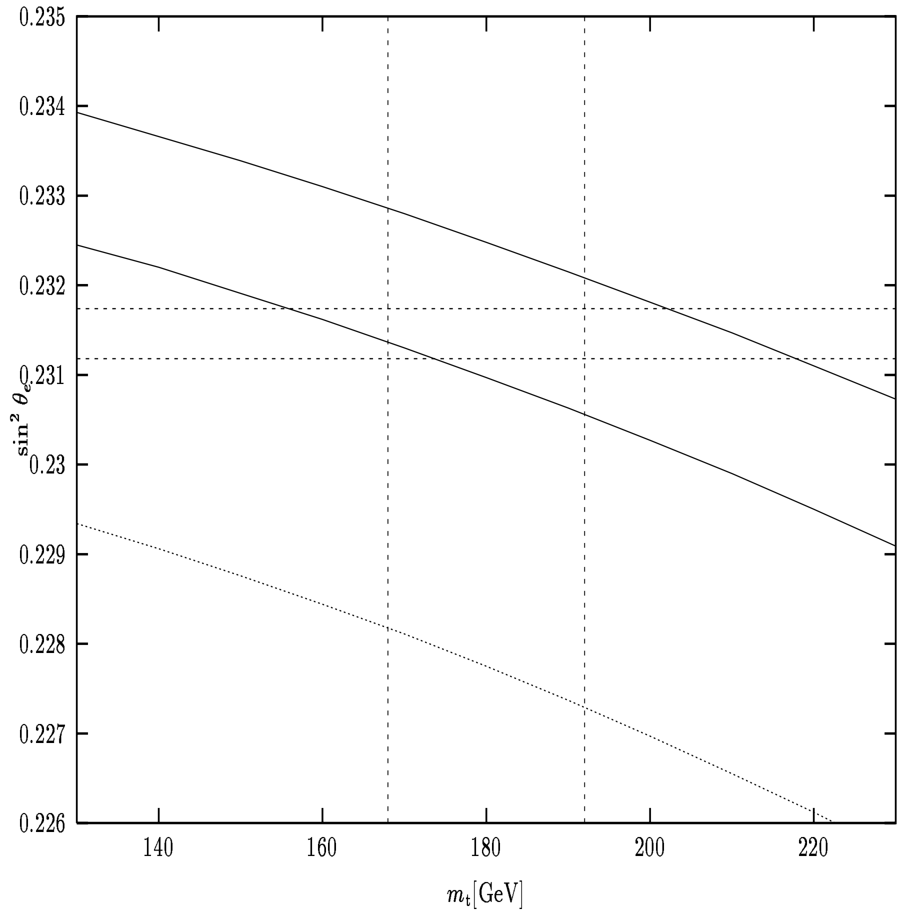
$$\mathcal{M}^{\mathbf{Z}} = -\frac{\mathbf{e_\star^2}}{\mathbf{s_\star^2 c_\star^2}}(\mathbf{T_3} - \boxed{s_\star^2}\,\mathbf{Q})\frac{\mathbf{Z_\star(k^2)}}{\mathbf{k^2 - M_{Z\star}^2(k^2)}}(\mathbf{T_3'} - \mathbf{s_\star^2 Q'}) = \mathbf{Z(k^2)}(\mathbf{T_3} - \mathbf{s_\star^2 Q})(\mathbf{T_3'} - \mathbf{s_\star^2 Q'})$$

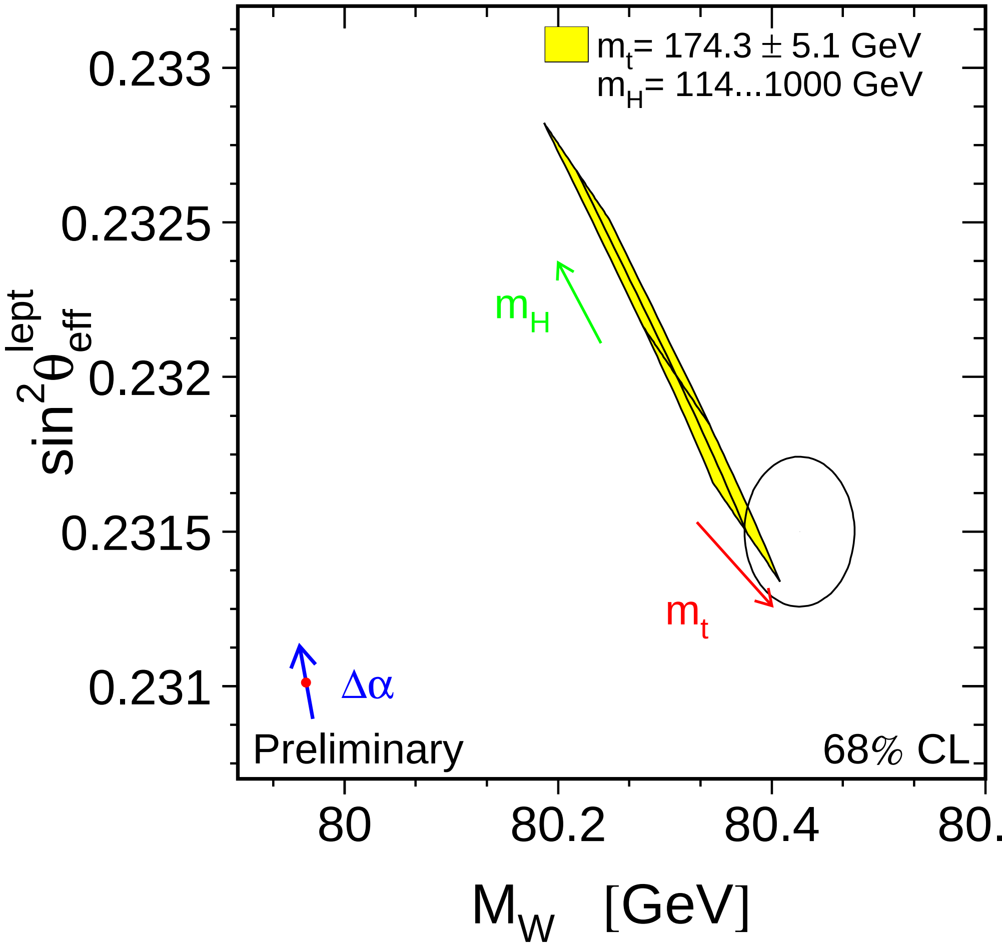
$\sin^2\theta$ in Z couplings

$$
\begin{aligned}
s_\star^2(k^2) &= s_Z^2\left\{1+(\Delta\alpha(k^2)-\Delta\alpha(M_Z^2)) + \frac{c_W^2}{c_W^2-s_W^2}(\Delta\alpha_{NP}(M_Z^2)-\Delta\rho+\delta r_{V+B})\right. \\
&+ \frac{e^2}{s_W^2}\frac{1}{c_W^2-s_W^2}\left(\Re e\Pi_{3Q}(M_Z^2)-\Re e\Pi_{33}(M_Z^2)\right)+\frac{e^2}{s_W^2}\left(\Pi_{3Q}(k^2)-\Re e\Pi_{3Q}(M_Z^2)\right) \\
s_\star^2(M_Z^2) &= s_Z^2\left\{1-\frac{c_W^2}{c_W^2-s_W^2}\left(\Delta\rho-\frac{\varepsilon_3}{c_W^2}-\Delta\alpha_{NP}(M_Z^2)-\delta r_{V+B}\right)\right\} \\
s_\star^2(M_Z^2) &= s_Z^2(1+\Delta\kappa')
\end{aligned}
$$

$$
\begin{aligned}
s_\star^2(M_Z^2) &= .23122 \quad m_t=174.3\text{GeV} \quad M_H=300\text{GeV} \\
s_\star^2(M_Z^2) &= .23018 \quad m_t=200\text{GeV} \quad M_H=300\text{GeV} \\
s_\star^2(M_Z^2) &= .23457 \quad m_t=50\text{GeV} \quad M_H=300\text{GeV}
\end{aligned}
$$

$sin^2\theta_{eff}$





Z couplings at the peak

$$\mathbf{Z(k^2)} = \frac{\mathbf{e_\star^2(k^2)}}{\mathbf{s_\star^2(k^2)c_\star^2(k^2)}}\left(\mathbf{1} + \frac{\mathbf{c_W^2 - s_W^2}}{\mathbf{s_W c_W}}\mathbf{\Pi_{\gamma Z}(k^2)} + \mathbf{\Pi_{\gamma\gamma}(k^2)}\right) \times \mathbf{D_Z}$$

$$\mathbf{D_Z} = \frac{\mathbf{1}}{\mathbf{k^2 - M_Z^2 + k^2\Pi_{ZZ}(k^2) - M_Z^2\Pi_{ZZ}(M_Z^2)}}$$

$$\mathbf{M_Z^{\star\,2}(k^2) = \{M_Z^2 - (k^2 - M_Z^2)(\Pi_{ZZ}(k^2) - \Pi_{ZZ}(M_Z^2))\}}$$

$$\mathbf{M_Z^{\star\,2}(M_Z^2) = M_Z^2} \quad \text{and} \quad \frac{\mathbf{dM_Z^{\star\,2}(k^2)}}{\mathbf{dk^2}}_{|\mathbf{k^2 = M_Z^2}} = \mathbf{0}$$

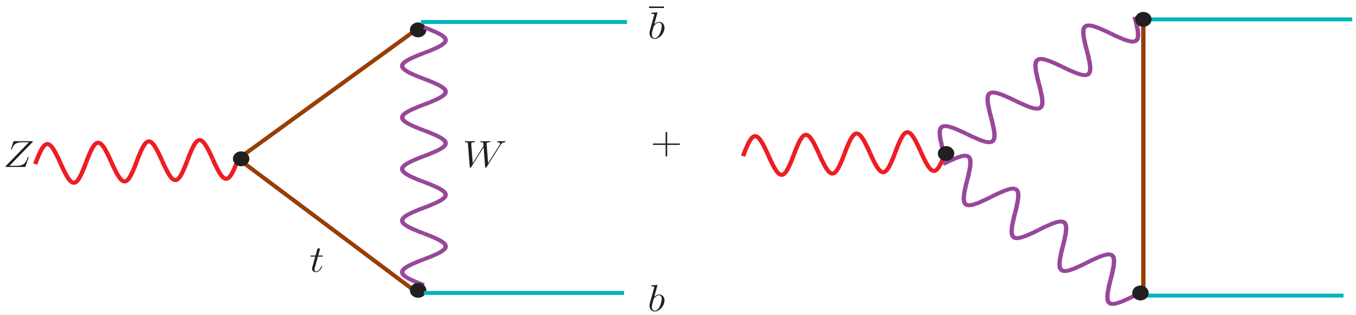
$$Z_\star(k^2) = 1 + \frac{c_W^2 - s_W^2}{s_W c_W}\Pi_{\gamma Z}(k^2) + \Pi_{\gamma\gamma}(k^2) - \Pi_{ZZ}(k^2) + (k^2 - 2M_Z^2)\tilde{\Pi}'_{ZZ}(k^2)$$

$$\Pi_{ZZ}(k^2) = \Pi_{ZZ}(M_Z^2) + (k^2 - M_Z^2)\tilde{\Pi}'_{ZZ}(k^2)$$

$$\frac{\mathbf{4\pi\alpha_\star(M_Z^2)}}{\mathbf{s_\star^2(M_Z^2)c_\star^2(M_Z^2)}}\mathbf{Z_\star(M_Z^2) = 4\sqrt{2}G_\mu M_Z^2}\left(\mathbf{1 + \Delta\rho - M_Z^2\tilde{\Pi}'_{ZZ}(M_Z^2)}\right) \simeq \mathbf{4\sqrt{2}G_\mu M_Z^2\,(1 + \Delta\rho}$$

$$\mathbf{g_A^{\text{lepton}}} = -\frac{\mathbf{1}}{\mathbf{2}}\sqrt{\mathbf{1 + \Delta\rho}} \quad , \quad \mathbf{g_V^{\text{lepton}} = g_A^{\text{lepton}}(1 - 4s^2_{\text{eff}\ ,\text{lepton}})}$$

$Z_{b\bar{b}}$: An important vertex correction



$$\mathcal{M}_{Zb_Lb} = -i\frac{e_\star^2}{c_\star^2 s_\star^2} Z_\mu \bar{b}\gamma_\mu b_L \left((\frac{1}{2} - \frac{1}{3}s_\star^2) - \delta v_b \right)$$

m_t^2 dependence in width almost washed away

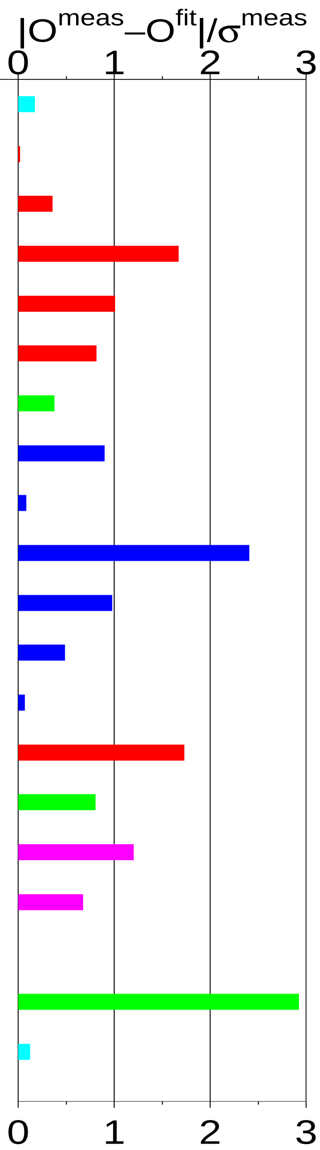
$$\Gamma_{Zb\bar{b}} \quad = \quad \Gamma^0_{Zb\bar{b}}(G_\mu, M_Z, \alpha(M_Z^2)) \left(1 + \frac{19}{13}(\Delta\rho + \delta v_b) \right)$$

$$\Delta\rho \quad \sim \quad +\frac{\alpha}{\pi}\frac{m_t^2}{M_Z^2} \text{ while } \delta v_b = -\frac{20\alpha}{19\pi}\left(\frac{m_t^2}{M_Z^2} + \frac{13}{6}\ln(m_t^2/M_Z^2) \right)$$

LEP Pulls

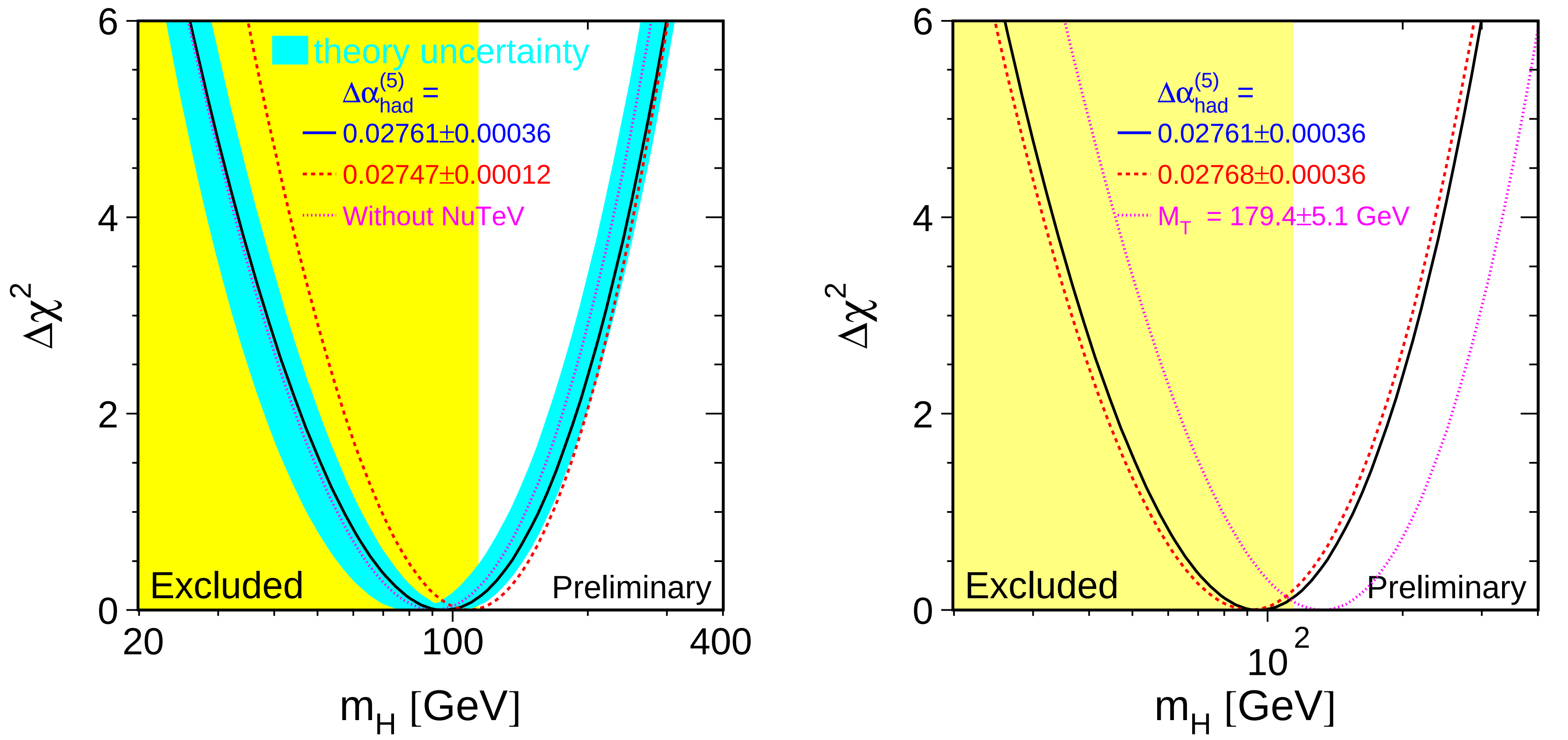
Summer 2003

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4960
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01636
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1477
R_b	0.21638 ± 0.00066	0.21579
R_c	0.1720 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0997 ± 0.0016	0.1036
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0740
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1477
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.426 ± 0.034	80.385
Γ_W [GeV]	2.139 ± 0.069	2.093
m_t [GeV]	174.3 ± 5.1	174.3
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	0.2229
Q_W(Cs)	-72.84 ± 0.46	-72.90



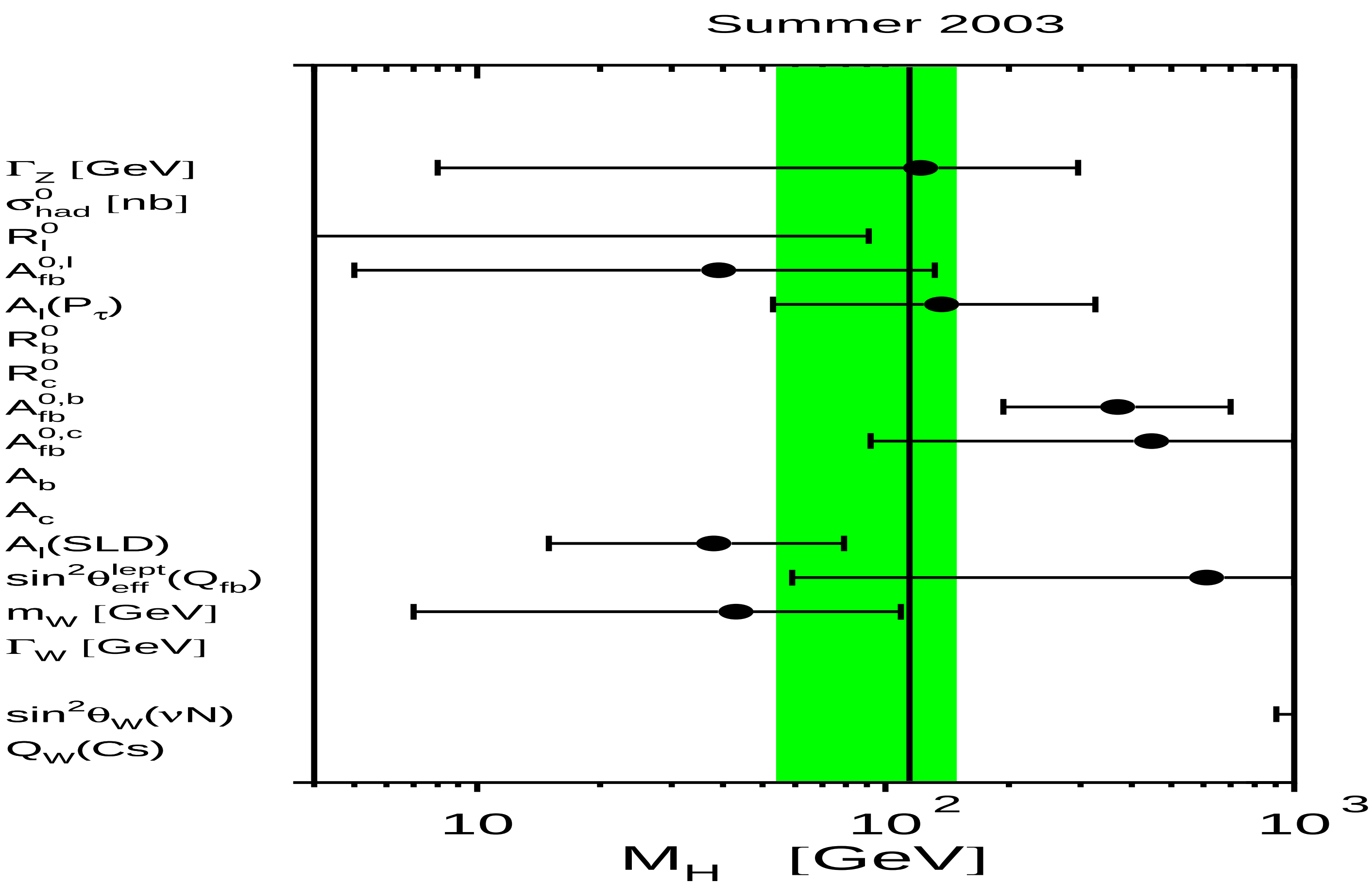
$\sigma^{meas.}$ is the error on $\mathcal{O}^{meas.}$

Higgs and the blue-band



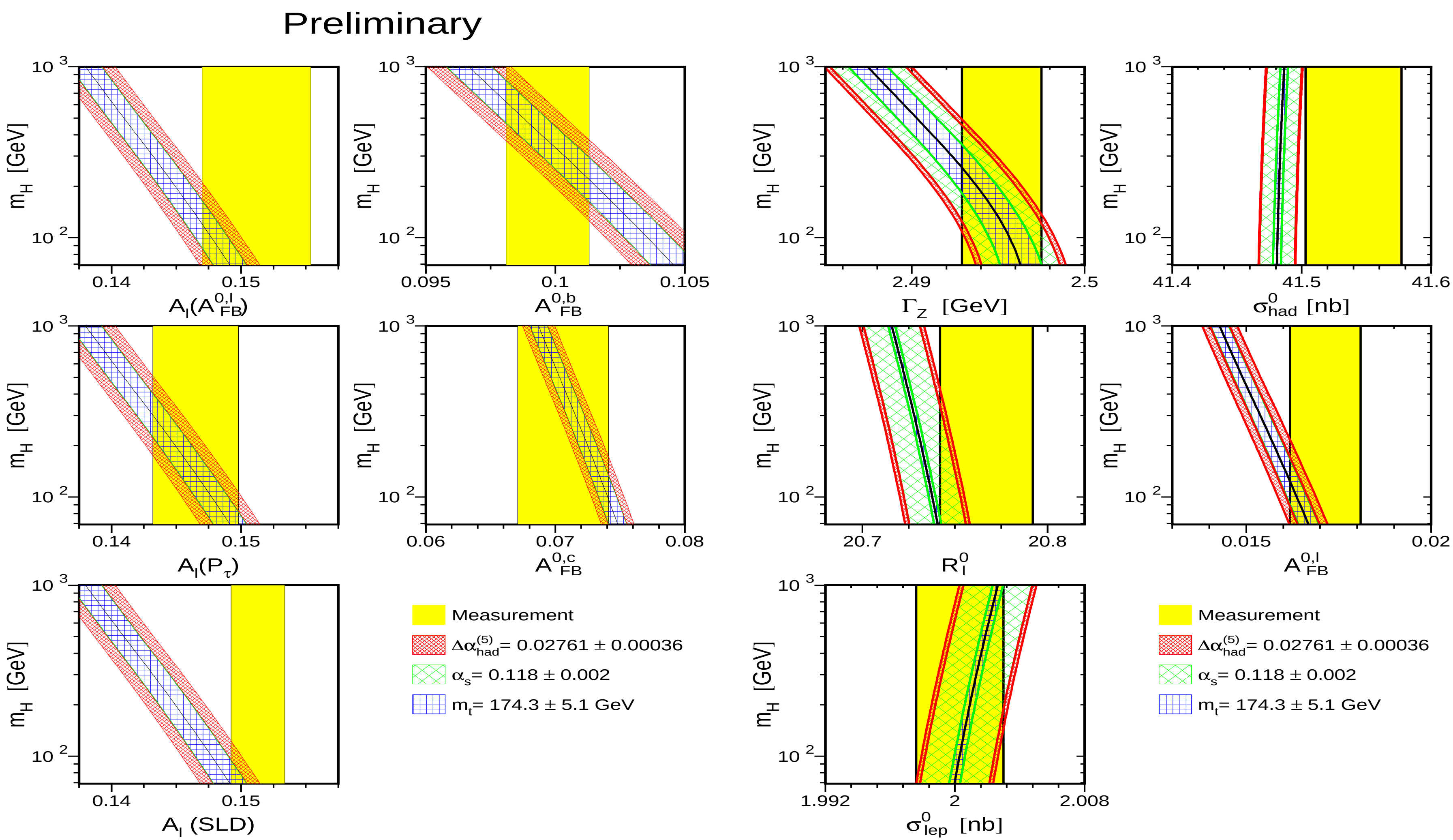
The famous blue band. This gives the limit on the Higgs mass from all precision data and also after excluding the anomalous NuTeV result. The area shaded in yellow is the mass excluded from a direct search. The blue band refers to the estimated theoretical uncertainty from missing higher order terms. The χ^2 is given for 2 values of $\Delta\alpha(M_Z^2)$. From LEP electroweak group. The second plot shows what happens if one increases the average value of the top mass within 1σ.

M_H sensitivity (1)

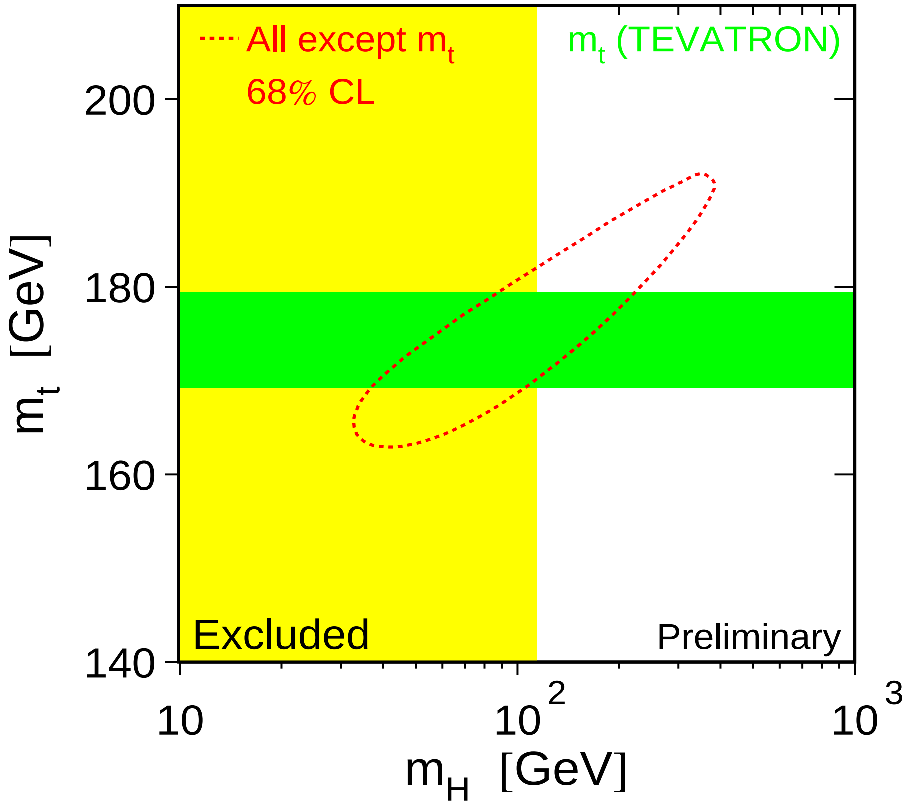


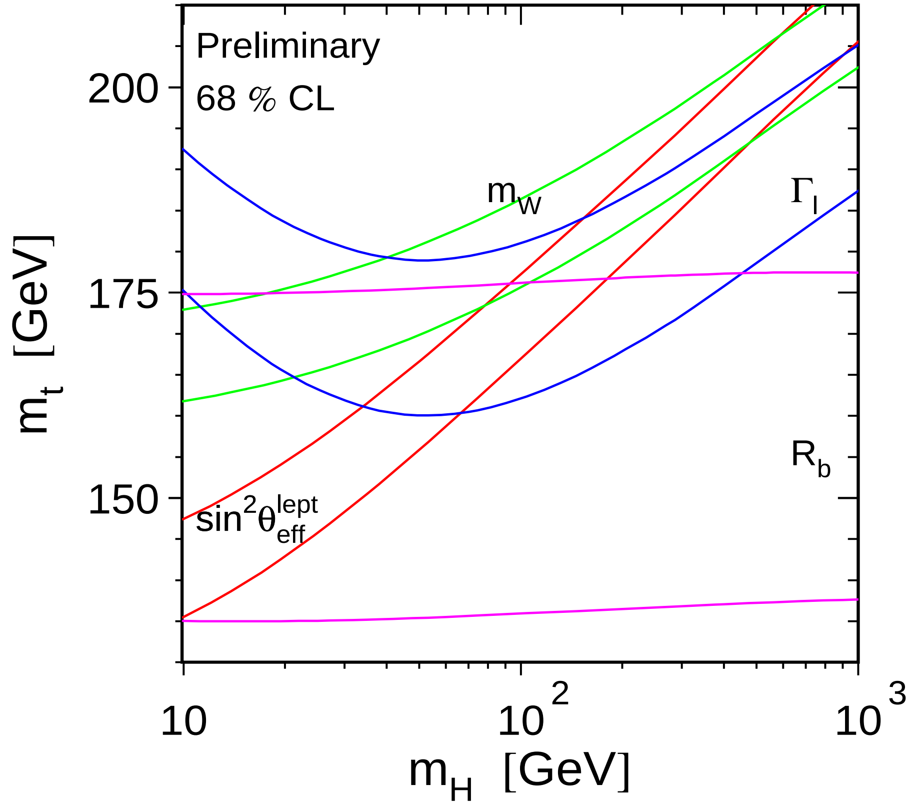
M_H extraction from different observables. Not all point to a low value of M_H. Some point to values much smaller than the direct limit.

M_H sensitivity (2)



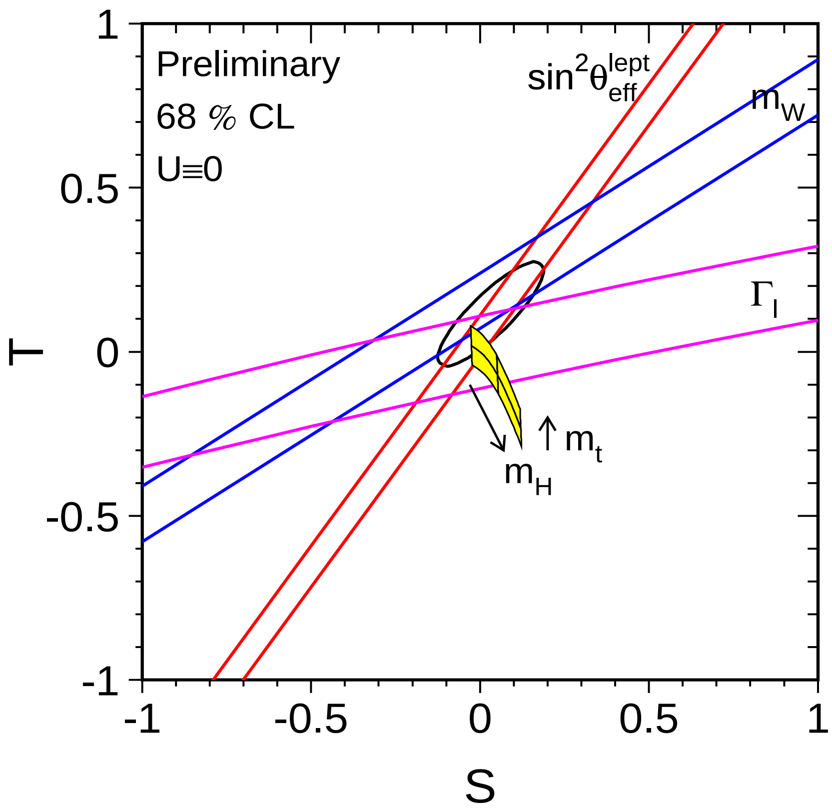
m_t vs M_H sensitivity

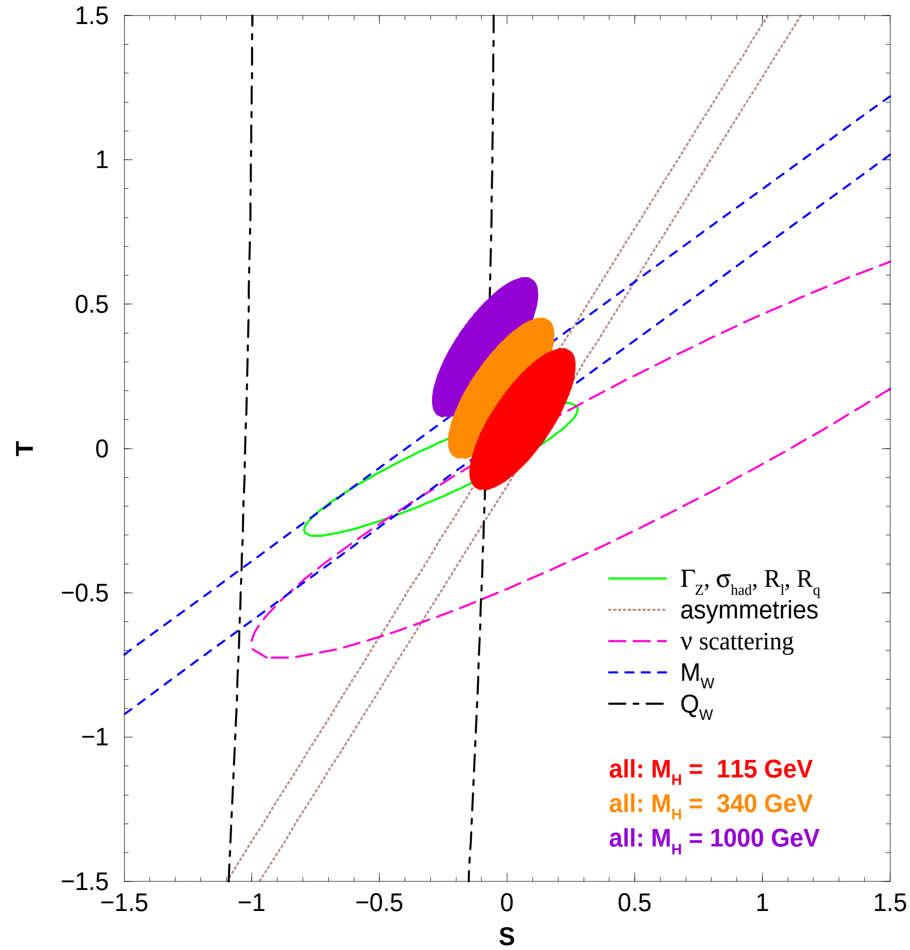




Prediction of the top mass from the precision measurement and how it compares with the direct limit on the top mass from the TEVATRON. The plot on the left shows how the different precision observables constraint in the M_H vs m_t plan.

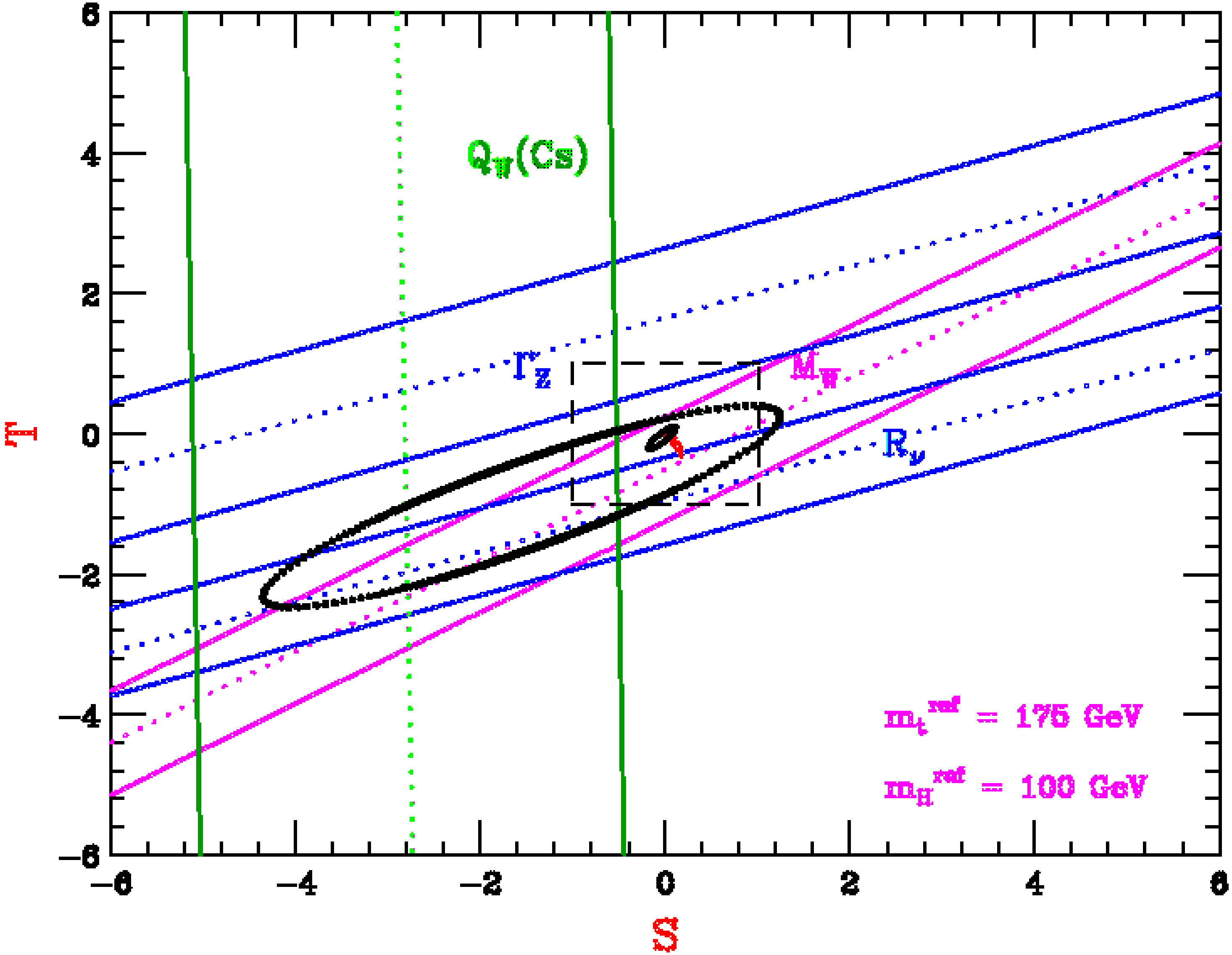
The S-T plane



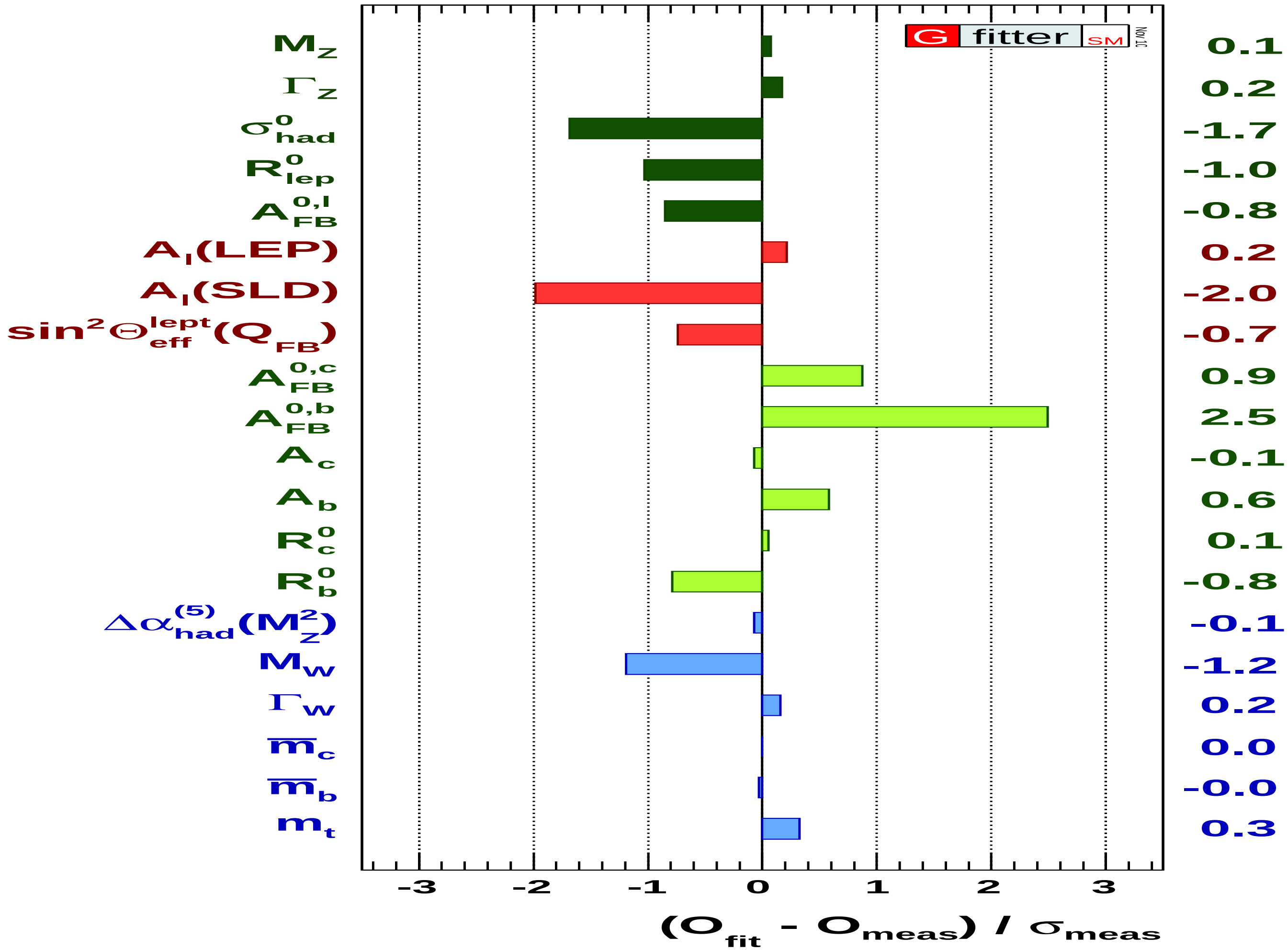


Constraint in on the S and T variables. The variation from m_t within its 1σ value and M_H is shown. The second graph, which fits for different values of M_H, shows that a high value of M_H is more comfortable with $S < 0$ but $T > 0$. T

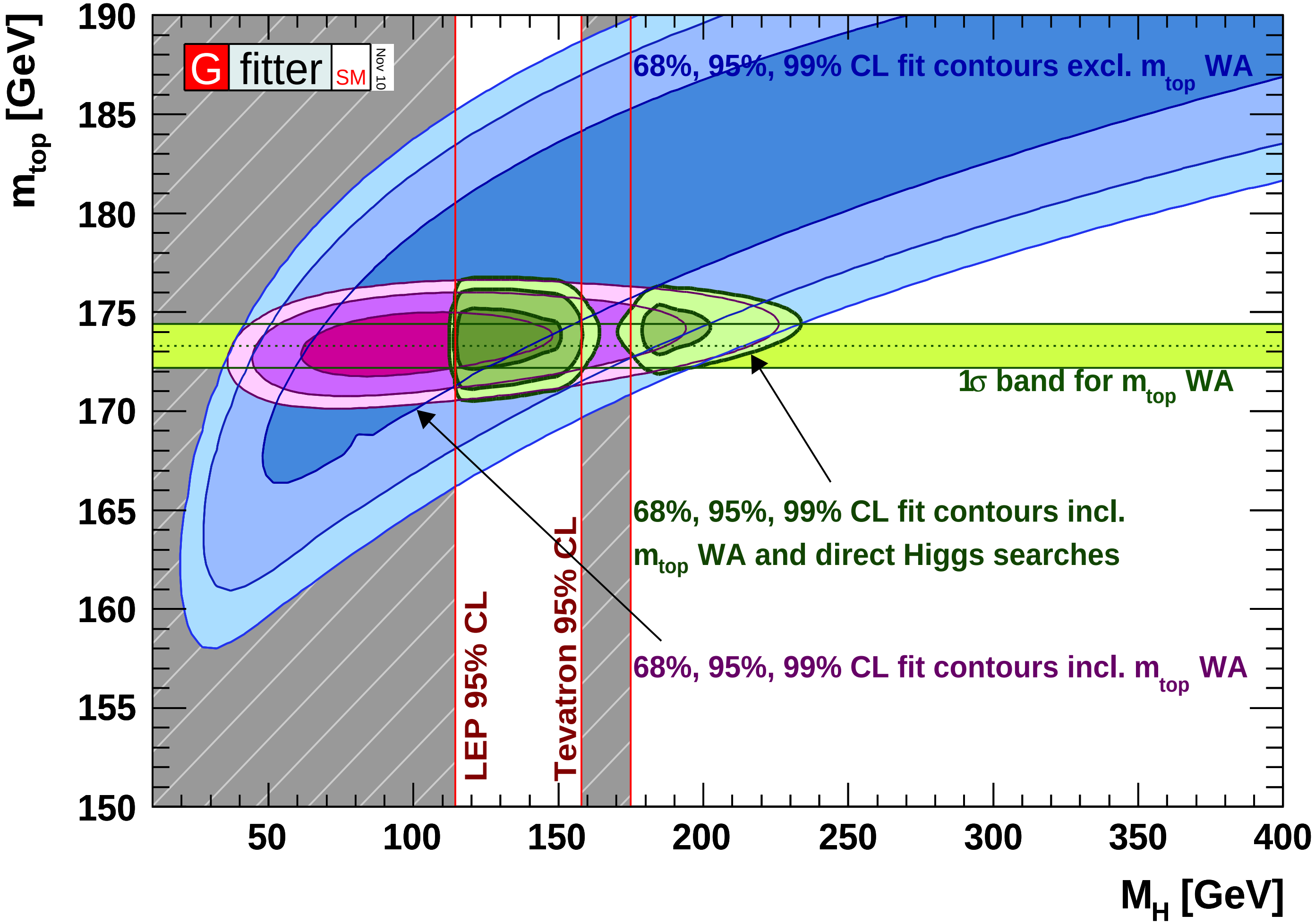
The S-T plane: 1989 *VS* 1999



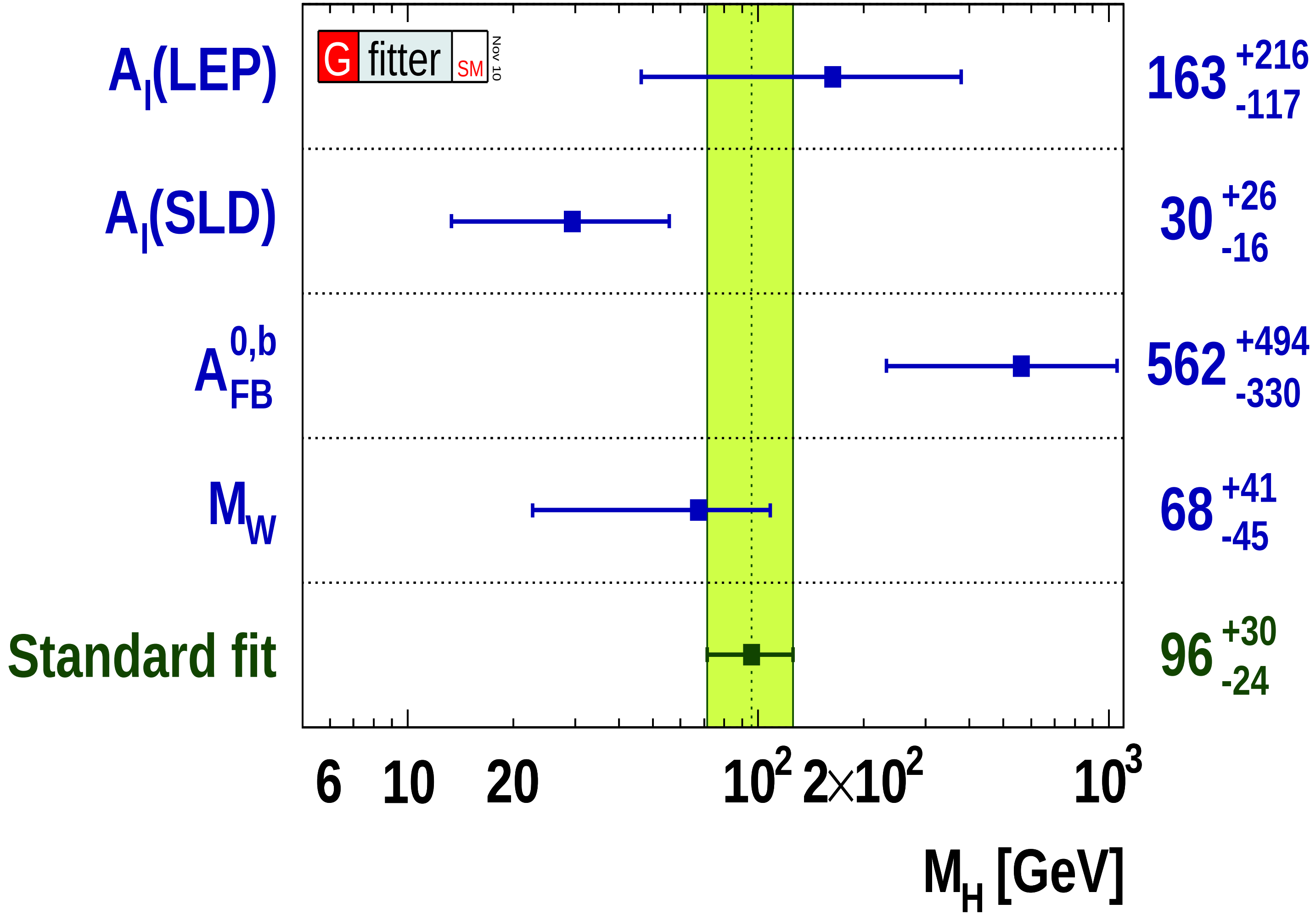
Gfitter, Pulls 2010-2011



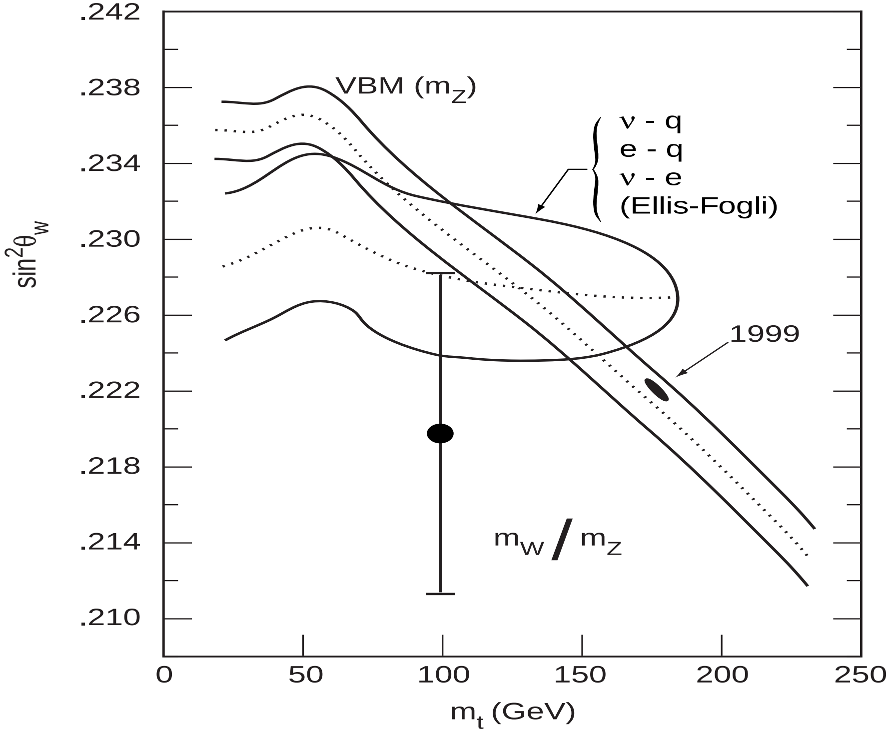
Gfitter 2010-2011

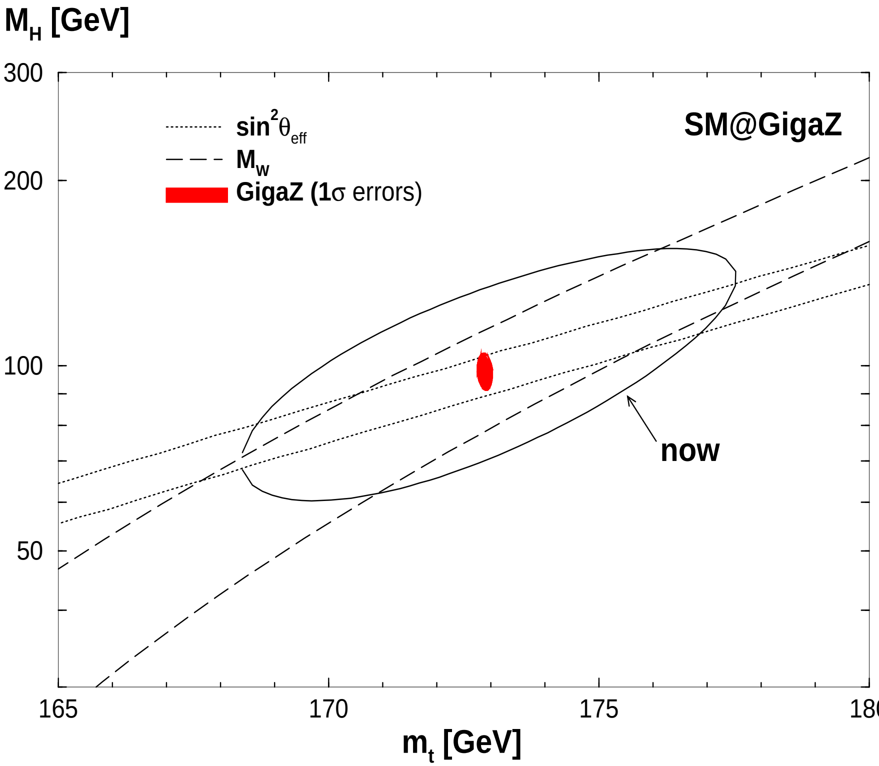


Gfitter 2010-2011, M_H **2010**



How far we've come and how far will go





A global fit on the mixing angle just before the start of LEP and how it compares with the situation in 1999!. If a linear collider is built and is run as a Z factory, the second plot shows how much improvement one expects.

Slow Process for NLO from 1 to 4

$pp \to W + 0jet$	1978	Altarelli, Ellis, Martinelli
$pp \to W + 1jet$	1989	Arnold, Ellis, Reno
$pp \to W + 2jets$	2002	Arnold, Ellis
$pp \to W + 3jets$	2009	BlackHat+Sherpa; Ellis, Melnikov, Zanderighi

Slow Process for NLO from 1 to 4

$pp \longrightarrow W + 0jet$	1978	Altarelli, Ellis, Martinelli
$pp \longrightarrow W + 1jet$	1989	Arnold, Ellis, Reno
$pp \longrightarrow W + 2jets$	2002	Arnold, Ellis
$pp \longrightarrow W + 3jets$	2009	BlackHat+Sherpa; Ellis, Melnikov, Zanderighi

$pp \longrightarrow t\bar{t}b\bar{b}$ Bredenstein, Denner, Dittmaier, Pozzorini

Bevilacqua, Czakon,

$pp \longrightarrow t\bar{t} + 2jets$ Bevilacqua, Czakon, Papadopoulos, Pittau, Worek

$pp \longrightarrow 4b$ Golem (Binoth, Guillet,Greiner, Guffanti,Reiter, Reuter..)

- End of 80's:

  Applications to LEP 1: 1-loop to $Z \to f\bar{f}$

  Labour of many years and many groups

  Essentially 2-point and 3-point vertex functions (some 4-points, 2-loop for self-energies)

  few ten's of diagrams

- Early 90's:

  Applications to LEP 2.

  3 years to achieve $e^+e^- \to W^+W^-$ (Leiden-Wurzburg)

- Year 95 (LEP2 WG):

  6 months to include the box needed for $b\bar{b}$ production!

- 2001: first full $2 \to 3$ NLO `GRACE-loop`

- up to 2009

  $e^+e^- \to \nu\nu HH$ Boudjema et al,.

  $e^+e^- \to 4f$ Denner et al,.

à la Feynman

The Feynmanians, Textbook

Draw all possible types of diagrams	topology
Figure out which particles can run on each type of diagram	combinatorics
Translate diagrams into expressions applying the Feynman rules	data-base look up
contract indices, take traces, (multiply add blocks)	algebra
Collect and write up the results as a computer code integrate over phase space	programming coding/computing
run the program to get numerical values	waiting!

Automation of LO calculations of (partonic) processes $2 \longrightarrow N$ are now automatised including integration, for (say) $N < 8$ based on different methods

- `ALPGEN`
- `CompHEP/CalcHEP`
- `Grace`
- `HELAS/PHEGAS`
- `MADGRAPH/MADEVENT`
- `O'Mega/WHIZARD`
- `SHERPA/Amegic`

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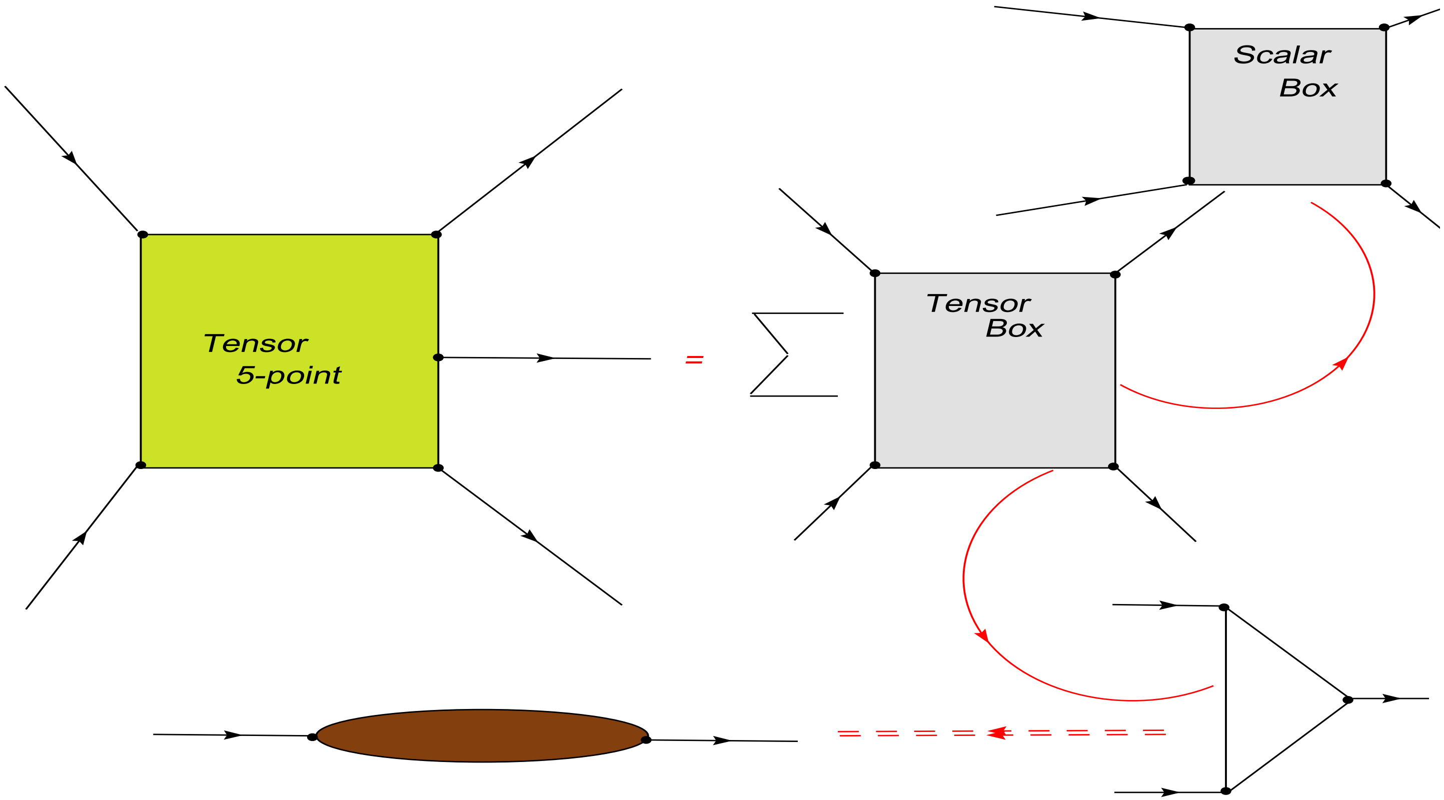
the more particles one deals with a (Feynman) diagrammatic calculations is costly as the number of diagrams grows N!

#gluons	2	3	4	5	6	7	8
#diagrams	4	25	220	2485	34300	0.5M	80M

$$\underbrace{T^{(N)}_{\mu\nu\cdots\rho}}_{M} = \int \frac{d^n l}{(2\pi)^n} \frac{l_\mu l_\nu \cdots l_\rho}{D_0 D_1 \cdots D_{N-1}}, \quad M \leq N$$



- Tensor integrals and scalar integrals with $N > 4$ reduced to scalars $N = 2, 3, 4$
- ex. rank 4 box need to solve a system of 15×15 equations. System involves, Gram determinants that may lead to severe instabilities

The Feynmanians, Textbook

$$C_\mu = \frac{(2\pi\mu)^{(4-n)}}{i\pi^2}\int d^n l \frac{l_\mu}{\left(l^2-m_1^2+i\varepsilon\right)\left((l+r_1)^2-m_2^2+i\varepsilon\right)\left((l+r_2)^2-m_3^2+i\varepsilon\right)}$$
$$= r_{1\mu}C_1 + r_{2\mu}C_2$$

use $2q.r = (q+r)^2-(q^2+r^2)$

with $f_i = r_i^2 - m_{i+1}^2 + m_1^2$

$$\begin{pmatrix} 2r_1^2 & 2r_1.r_2 \\ 2r_1.r_2 & 2r_2^2 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} B_0(r_2^2,m_1^2,m_2^2) - B_0((r_1-r_2)^2,m_2^2,m_3^2) - f_1 C_0 \\ B_0(r_1^2,m_1^2,m_2^2) - B_0((r_1-r_2)^2,m_2^2,m_3^2) - f_2 C_0 \end{pmatrix}$$

Inversion, solution in terms of scalar functions (B_0, C_0) involve the Gram determinant

$\det G_2 = det(2r_ir_j), i,j = 1,2$

If Gram Det small, numerical instability. Worse for higher rank tensors.

for Gram=0, use segmentation, FB 2005

CPU of the various N-point

Process	6-point	5-point	4-point	3-point	Others
$e^+e^- \rightarrow e^+e^- H$	-	33%	11%	47%	9%
		20	44	348	98
$e^+e^- \rightarrow \nu\bar{\nu}HH$	**67%**	13%	10%	8%	2%
	74	218	734	1804	586

Perhaps that Passarino Veltman no longer adequate for present day purposes. Many developments recently,...

SLOOPS

N. Baro, FB, G. Chalons, G. Drieu La Rochelle, S. Hao, Ninh Le Duc, A. Semenov, (D. Temes)

- Need for an automatic tool for susy calculations, for Colliders and Dark Matter, On-Shell scheme
- handles large numbers of diagrams both for tree-level
- and loop level
- able to compute loop diagrams at $v = 0$: dark matter, LSP, move at galactic velocities, $v = 10^{-3}$
- ability to check results: UV and IR finiteness but also gauge parameter independence for example
- ability to include different models easily and switch between different renormalisation schemes
- Used for SM one-loop multi-leg: new powerful loop libraries (with Ninh Le Duc)

Strategy: Exploiting and interfacing modules from different codes

Lagrangian of the model defined in LanHEP

- particle content
- interaction terms
- shifts in fields and parameters
- ghost terms constructed by BRST

$\downarrow$ $\downarrow$

Generic Model
-kinematical structures

Classes Model
-Feynman rules, including CT

$\Downarrow$

Evaluation via FeynArts-FormCalc

LoopTools modified!!
tensor reduction inappropriate for small relative velocities
(Zero Gram determinants)

$\Uparrow$

Renormalisation scheme

- definition of renorm. const. in the classes model

Non-Linear gauge-fixing constraints, gauge parameter dependence checks

From the Lagrangian to the Feynman Rules

```
vector
    A/A: (photon, gauge),
    Z/Z:('Z boson', mass MZ = 91.1875, gauge),
    'W+'/'W-': ('W boson', mass MW = MZ*CW, gauge).
scalar  H/H:(Higgs, mass MH = 115).



transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
    'W+'->'W+'*(1+dZW/2),'W-'->'W-'*(1+dZW/2).
transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
    'W+.f'->'W+.f'*(1+dZWf/2),'W-.f'->'W-.f'*(1+dZWf/2).

let pp = { -i*'W+.f',  (vev(2*MW/EE*SW)+H+i*'Z.f')/Sqrt2 },
PP=anti(pp).



lterm -2*lambda*(pp*anti(pp)-v**2/2)**2
     where
    lambda=(EE*MH/MW/SW)**2/16, v=2*MW*SW/EE .

let Dpp^mu^a = (deriv^mu+i*g1/2*B0^mu)*pp^a +
     i*g/2*taupm^a^b^c*WW^mu^c*pp^b.
let DPP^mu^a = (deriv^mu-i*g1/2*B0^mu)*PP^a
    -i*g/2*taupm^a^b^c*{'W-'^mu,W3^mu,'W+'^mu}^c*PP^b.
lterm DPP*Dpp.
```

Gauge fixing and BRS transformation

```
let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

lterm -'Z.C'*brst(G_Z).
```

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```

Output of Feynman Rules with Counterterms !!

```
M$CouplingMatrices = {

  (*------    H   H   ------*)
   C[ S[3], S[3] ] == - I *
{
 { 0 , dZH },
 { 0 , MH^2 dZH + dMHsq }
},
  (*------    W+.f  W-.f   ------*)
   C[ S[2], -S[2] ] == - I *
{
 { 0 , dZWf },
 { 0, 0 }
}, (*------    A  Z   ------*)
   C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
{
 { 0, 0 },
 { 0 , dZZA },
 { 0, 0 }
},

(*------    H  H  H  ------*)
   C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
{
 { 2 MH^2 , 3 MH^2 dZH -2 MH^2 / SW dSW - MH^2 / MW^2 dMWsq
},
  (*------    H  W+.f  W-.f   ------*)
   C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
{
 { 2 MH^2 , MH^2 dZH + 2 MH^2 dZWf -2 MH^2 / SW dSW - MH^2 /
},

  (*------    W-.C  A.c  W+  ------*)
   C[ -U[3], U[1], V[3] ] == - I EE *
{
 { 1 },
 { - nla }
},
```

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    A/A: (photon, gauge),
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    'W+'/'W-': ('W boson', mass MW = MZ*CW, gauge).
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transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
    'W+.f'->'W+.f'*(1+dZWf/2),'W-.f'->'W-.f'*(1+dZWf/2).

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PP=anti(pp).



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```

```
RenConst[ dMHsq ] := ReTilde[SelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZH ] := -ReTilde[DSelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZZf ] := -ReTilde[DSelfEnergy[prt["Z.f"] -> prt["Z.f"],
MZ]] RenConst[ dZWf ] := -ReTilde[DSelfEnergy[prt["W+.f"] ->
prt["W+.f"], MW]]
```

Output of Feynman Rules with Counterterms !!

```
M$CouplingMatrices = {

  (*------    H   H   ------*)
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 { 0 , dZH },
 { 0 , MH^2 dZH + dMHsq }
},
  (*------    W+.f  W-.f  ------*)
   C[ S[2], -S[2] ] == - I *
{
 { 0 , dZWf },
 { 0, 0 }
}, (*------    A   Z   ------*)
   C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
{
 { 0, 0 },
 { 0 , dZZA },
 { 0, 0 }
},


(*------    H   H   H   ------*)
   C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
{
 { 2 MH^2 , 3 MH^2 dZH -2 MH^2 / SW dSW - MH^2 / MW^2 dMWsq
},
  (*------    H  W+.f  W-.f   ------*)
   C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
{
 { 2 MH^2 , MH^2 dZH + 2 MH^2 dZWf -2 MH^2 / SW dSW - MH^2 /
},


  (*------    W-.C  A.c  W+   ------*)
   C[ -U[3], U[1], V[3] ] == - I EE *
{
 { 1 },
 { - nla }
},
```

The Unitarians

THE ANALYTIC
S-MATRIX

BY

R. J. EDEN P. V. LANDSHOFF D. I. OLIVE
J. C. POLKINGHORNE

CAMBRIDGE
AT THE UNIVERSITY PRESS
1966

The Unitarians

Properties of the S-Matrix

The Unitarians

Properties of the S-Matrix

- **Analyticity** Scattering amplitudes are determined by their poles and branch-cuts
- **Unitarity** The residues at the poles and the branch-cut are products of subamplitudes that are simpler (less legs and less loops)

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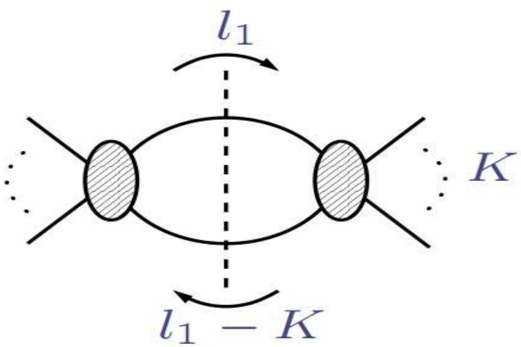
- **Unitarity** The residues at the poles and the branch-cut are products of subamplitudes that are simpler (less legs and less loops)

Unitarity and cutting rules (on-shell conditions)

- Ordinary Unitarity (optical theorem, Cutkosky's rule): put two particles on-shell, turn one-loop into products of trees $S = 1 + iT, SS^\dagger = 1 \Longrightarrow 2ImT = -i(T - T^\dagger) = T^\dagger T$



$$\frac{1}{(l_i^2 - m_i^2 + i0)} \rightarrow \delta(l_i^2 - m_i^2)$$

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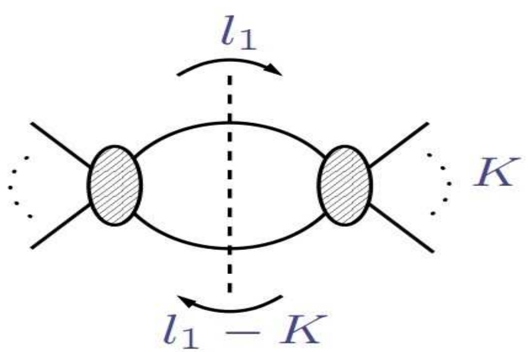


$$\frac{1}{(l_i^2 - m_i^2 + i0)} \to \delta(l_i^2 - m_i^2)$$

- **Generalised Unitarity:** put (1),3, 4 particles on-shell, multiple products of trees

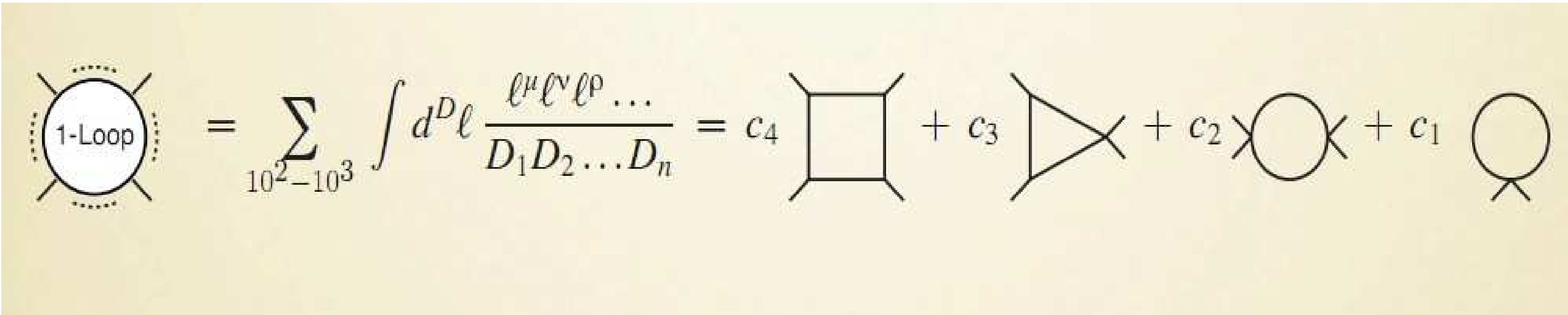
The Unitarians

Tree-amplitudes *vs* *Feynman diagrams*

amplitudes are gauge invariant on-shell objects

that corresponds to (large) sums of off-shell Feynman diagrams

Usual Passarino Veltman



in fact each tensor integral (there can be a few of these for each Feynman graph) is reduced to a set of scalar integrals, (from gauge theories to scalar field theory!)

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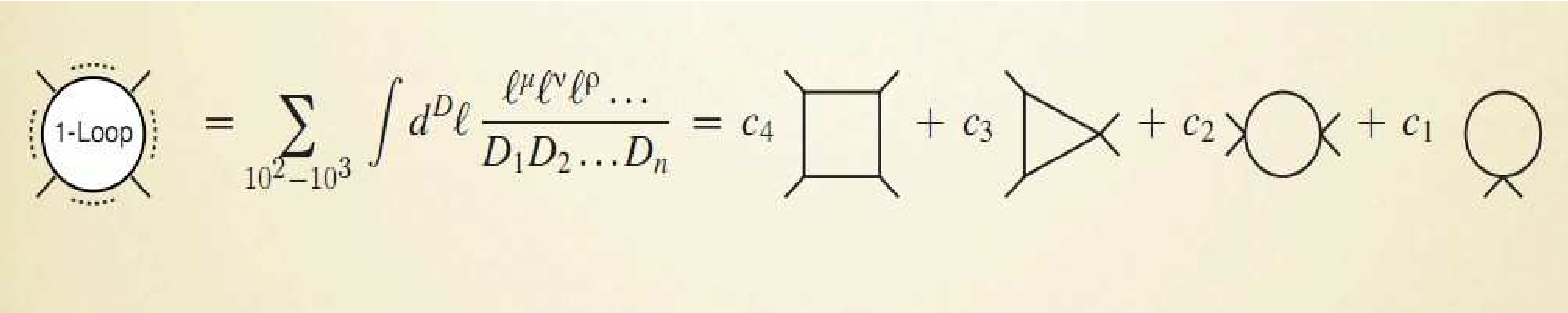

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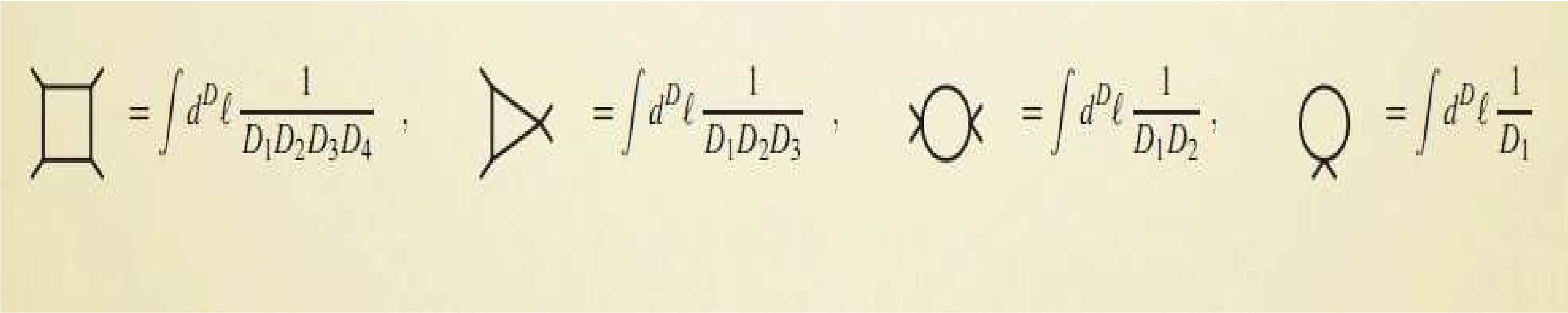
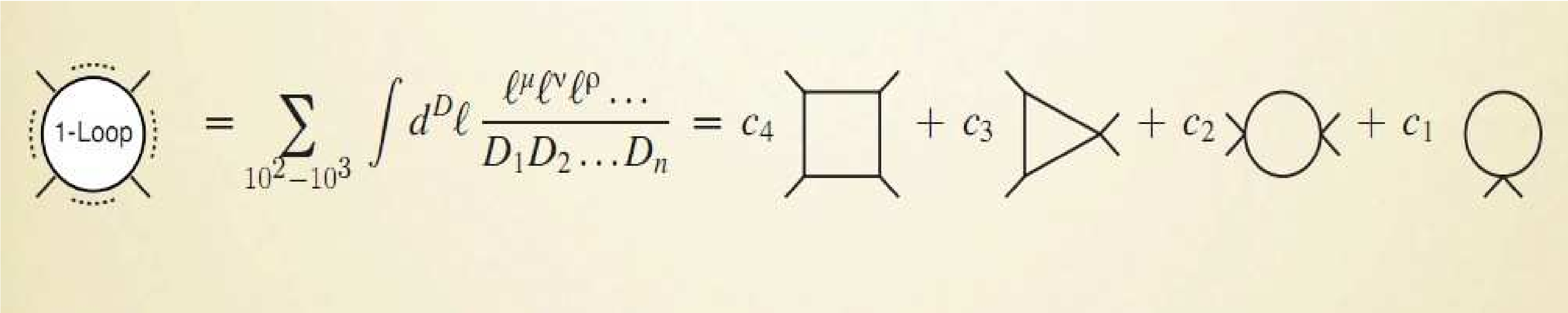
The Set or Basis is known (looked up): Master integrals





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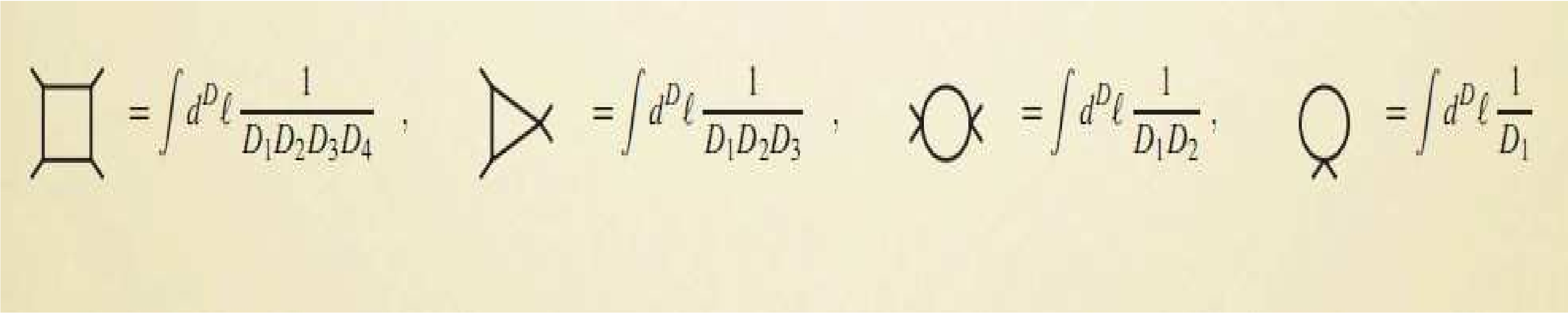
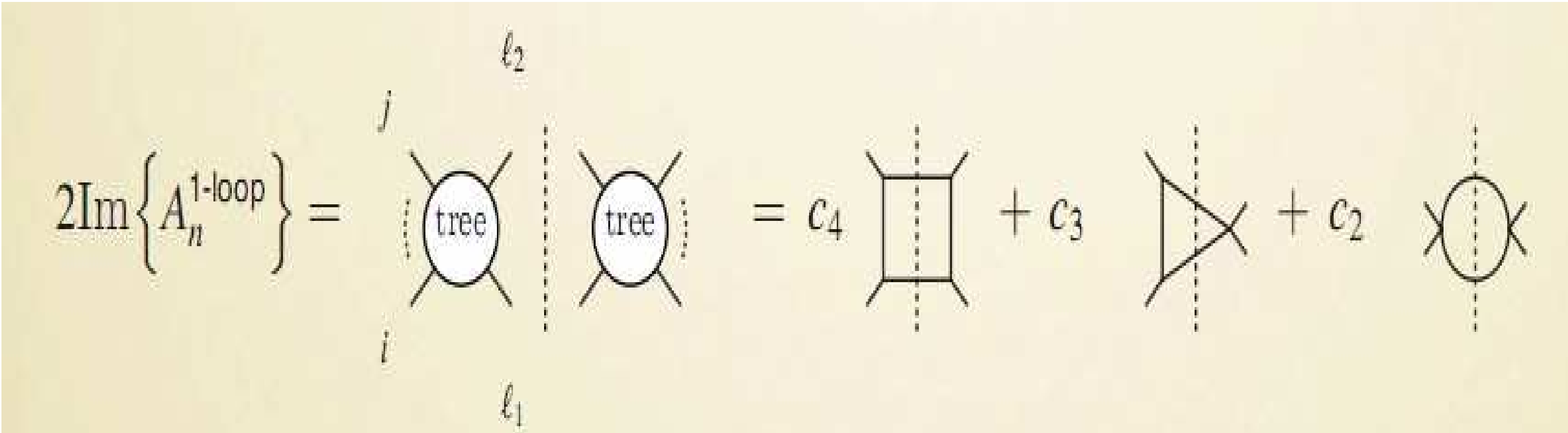


what is unknown are the c_i: rational functions of the external momenta (and internal masses)

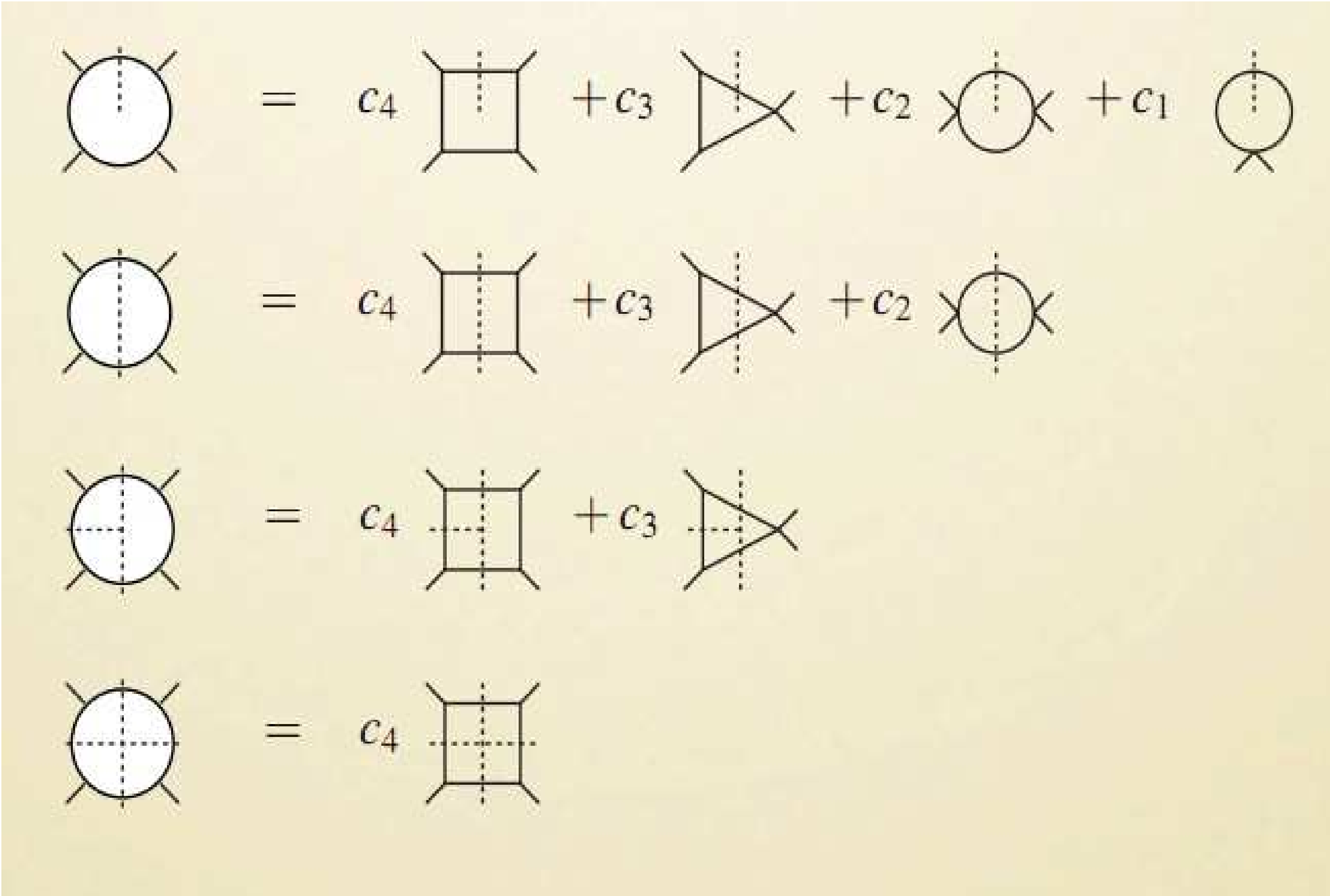
cuts and unitary approach

- Ordinary Cutkosky's cuts



▷ Match the cuts of any amplitude into the cuts of the master integrals, contribution from master integrals with different number of legs

Generalised unitarity



Ideally start with the quadruple cut as it isolates c_4

quadruple cut implies products of 4 trees (evaluated at complex momenta)

Make your way up

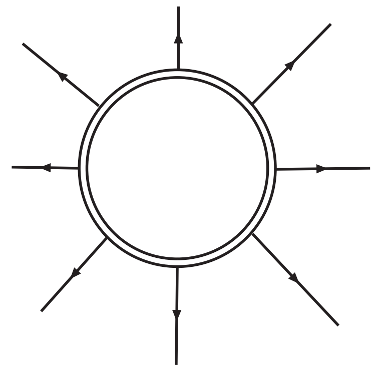
the more one cuts the simpler it gets but the more one looses also $c_1, ..$

Reduction at the integrand level

Take the m-point one-loop sub-amplitude calculated in DR

$$\mathcal{A}_m = \int [d^n \bar{l}] \bar{A}(\bar{l})$$

$$\bar{A}(\bar{l}) = \frac{\bar{N}(\bar{l})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}\,, \quad \bar{D}_i = (\bar{l} + p_i)^2 - m_i^2\,, \quad p_0 \neq 0\,.$$



bar to denote objects living in $n = 4 + \epsilon$ dimensions.
$\bar{l}^2 = l^2 + \tilde{l}^2$, where $\tilde{l}^2$ is ϵ-dimensional and $(\tilde{l} \cdot l) = 0$.

The numerator function $\bar{N}(\bar{l})$ can be also split into a 4-dimensional plus a ϵ-dimensional part
$\bar{N}(\bar{l}) = N(l) + \tilde{N}(\tilde{l}^2, l, \epsilon)\,.$

(mismatch between N and $\bar{N}$, D_i and $\bar{D}_i$ source of rational terms)

$$
\begin{aligned}
N(l) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(l; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(l; i_0 i_1 i_2)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(l; i_0 i_1)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(l; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i \\
&+ \tilde{P}(l) \prod_{i}^{m-1} D_i \,.
\end{aligned}
$$

$$
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+ \; & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(l; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
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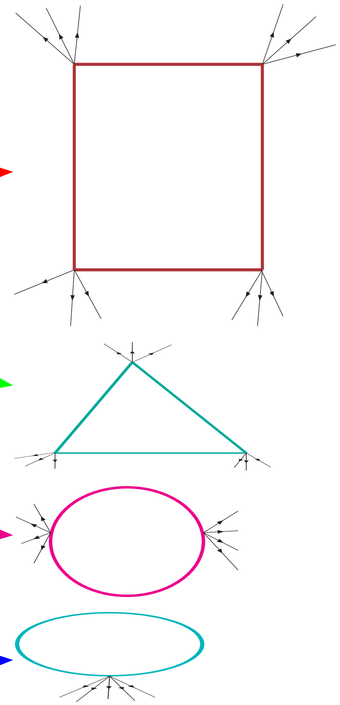

no pole

$$
\begin{aligned}
\int [d^n l] \bar{A}(\bar{l}) \quad &= \quad \int [d^n l] \frac{1}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \times \Bigg(\\
& \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(l; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
+ \quad & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(l; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
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+ \quad & \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(l; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i \\
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& \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
+ & \sum_{i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0 < i_1}^{m-1} b(i_0 i_1) \prod_{i \neq i_0, i_1}^{m-1} D_i \\
+ & \sum_{i_0}^{m-1} a(i_0) \prod_{i \neq i_0}^{m-1} D_i \Bigg).
\end{aligned}
$$

- The unknown coefficients d, c, b, a are computed from generating $N(l)$ a sufficient number of times (for different values of the loop momentum) and then inverting the system of equations.
- Need to be clever!, for $m = 6$ need a system of 56 equations! Not advisable to invert a 56×56 matrix
- Unitarity way: single out particular values of l so that $4, 3, 2, 1 D_i$ vanishes simultaneously.
- l such that $D_i(l) = 0$ for $i = 0, \cdots, M_1$
- Start with $4D_i, (M = 4)$ (quadruple cut), etc...
- Algorithm can be carried at amplitude level. $N(l)$ generated from trees.

Dual to gravity/string theory on $AdS_5 \times S^5$. Very similar in IR to QCD (Magnea,..)

Planar (large N_c) theory is integrable, Yangian (Beisert, Drummond)

Strong-coupling limit and Wilson loops (Alday, Maldacena)

Planar amplitudes possess dual conformal invariance (Drummond,Henn, Korchemsky, Sokatchev)

Some planar amplitudes *known* to all orders in coupling (Bern, Dixon, Smirnov,...)

Grassmannian technology fss (Arkani-Hamed et al.)

Ideal framework for testing on-shell and related methods

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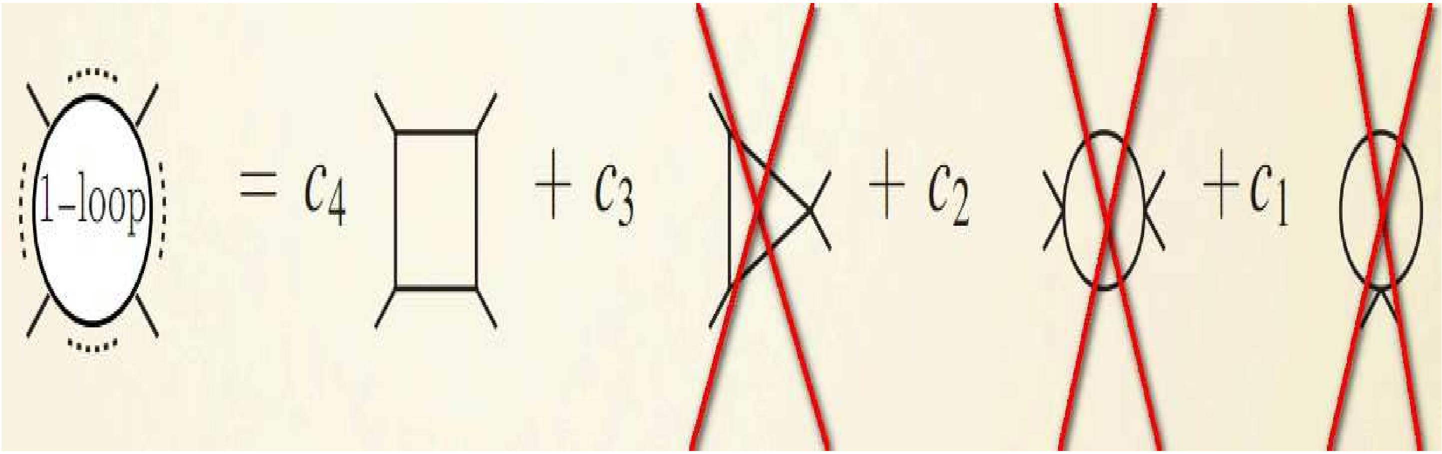
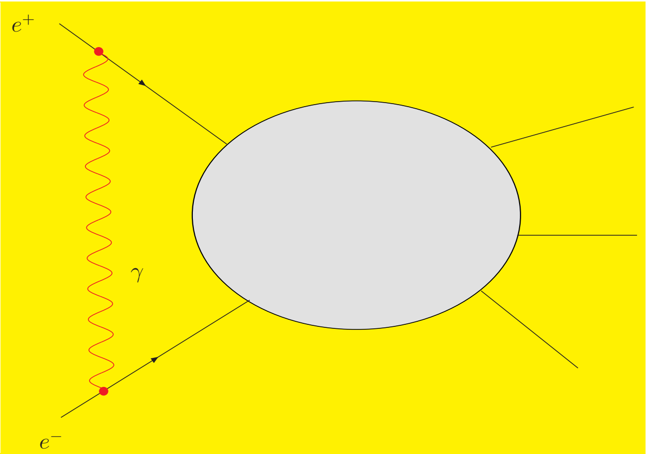
Ideal framework for testing on-shell and related methods

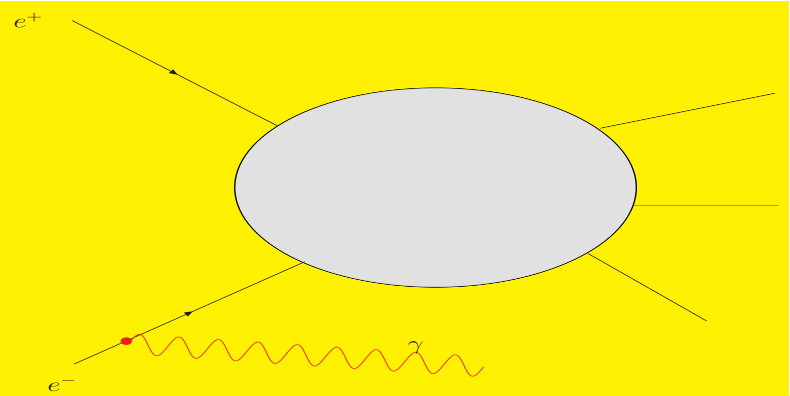
Yuri Dokhsitzer: "virtual SUSY" is helping QCD , QCD will pay back discovering "real" SUSY

New ideas from SYM, $N = 4$



INFRARED/COLLINEAR DIVERGENCES





infrared divergent needs photon mass λ

$d\sigma_V(\lambda)$

collinear sing. need $m_f = m_e, \ldots$.

must include bremsstrahlung

$d\sigma_s(\lambda, E_\gamma < k_c) + d\sigma_H(\lambda, E_\gamma > k_c)$

$d\sigma_s \rightarrow$ analytical: factorisation (automatised) ;

$d\sigma_H \rightarrow$ adaptive MC

$$\sigma_{\mathcal{O}(\alpha)} = \underbrace{\int d\sigma_0 \left(1 + \delta_V^{EW}\right)}_{\sigma_0(1+\delta_W)} + \underbrace{\int d\sigma_0 \left(\delta_V^{QED}(\lambda) + \delta_S(\lambda, k_c)\right)}_{\sigma_{V+S}^{QED}(k_c)} + \underbrace{\int d\sigma_H(k_c)}_{\sigma_H(k_c)} .$$

strong cancellation, CPU time consuming for collinear parts in $\sigma_H(k_c)$ and $\sigma_{V+S}^{QED}(k_c)$

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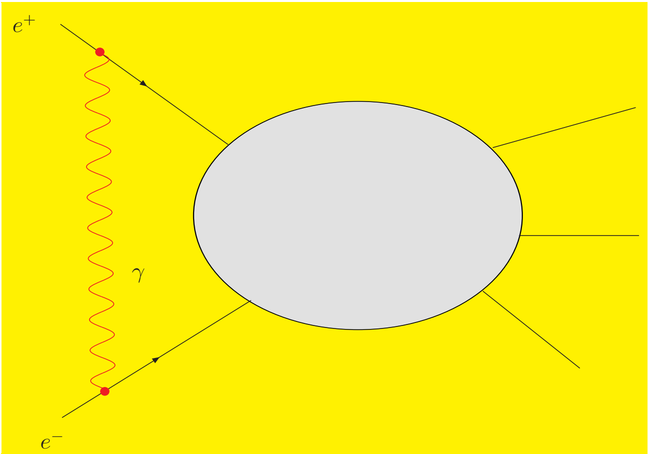

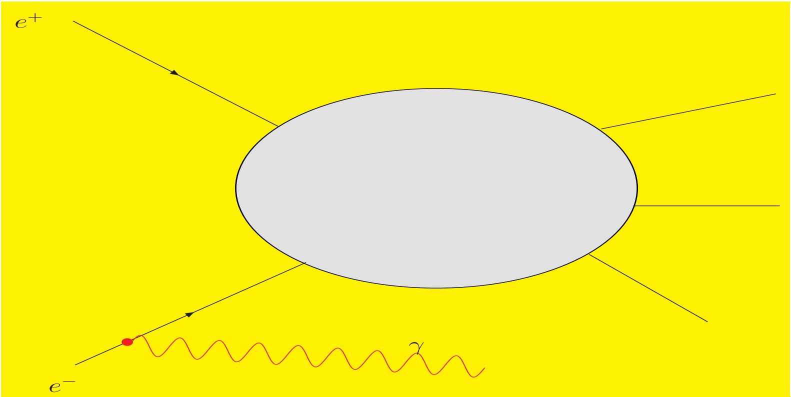

infrared divergent needs photon mass λ

$d\sigma_V(\lambda)$

collinear sing. need $m_f = m_e, \ldots$.

Dim Reg: $\lambda, m_e \to 1/\epsilon^2, 1/\epsilon$.

must include bremsstrahlung

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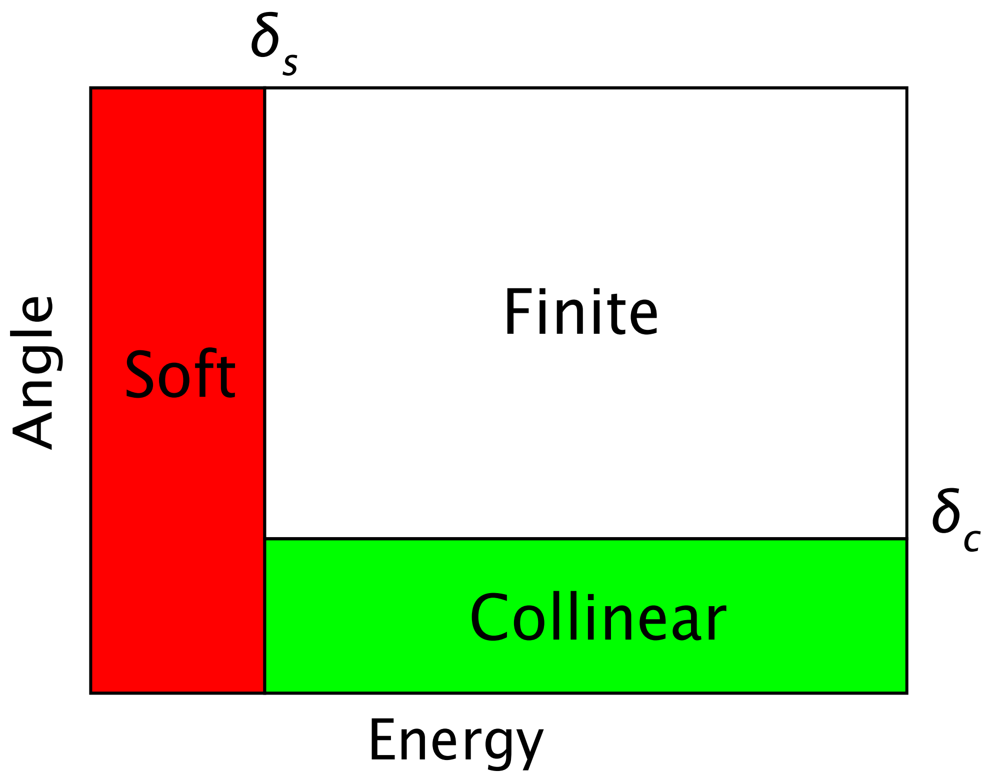
$d\sigma_s \to$ analytical: factorisation (automatised) ;

$d\sigma_H \to$ adaptive MC

Dim Reg: $d\sigma_s$ part of $d\sigma_H \to$ Integrate in d-dim.

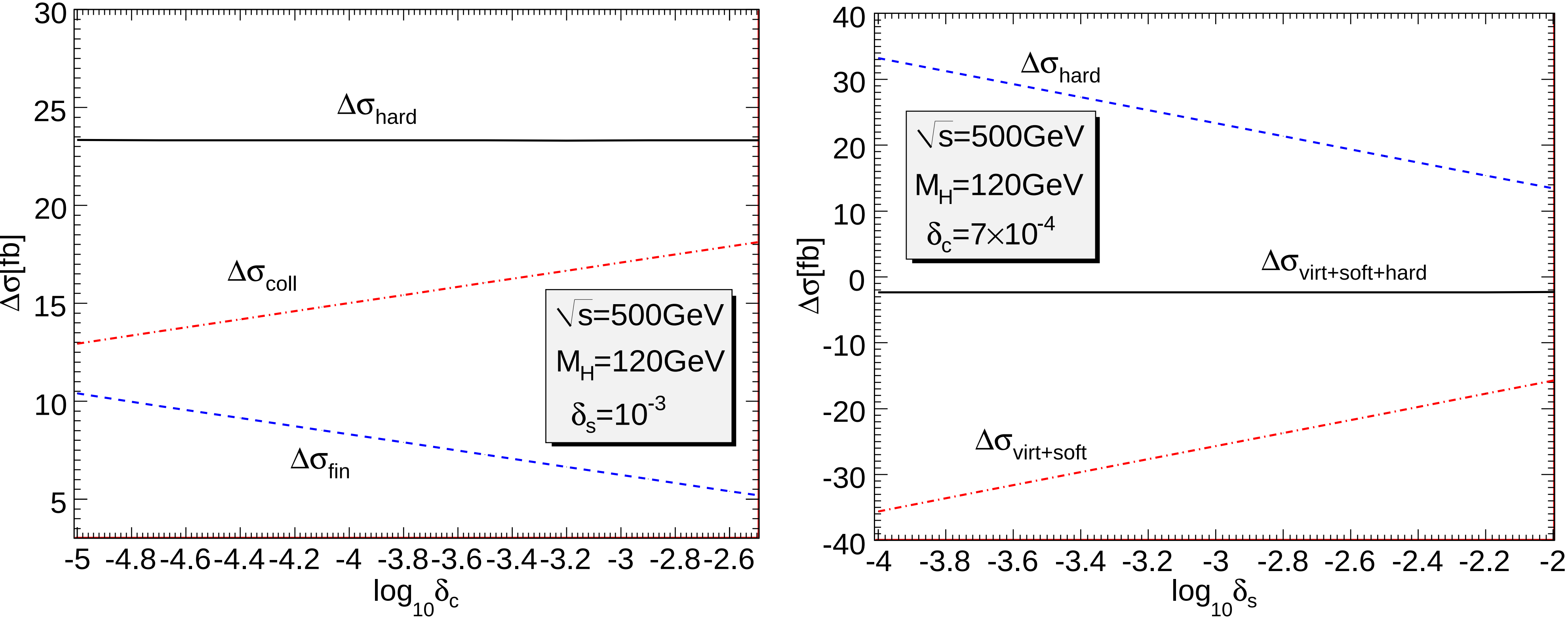
$$\sigma_{\mathcal{O}(\alpha)} \quad = \quad \underbrace{\int d\sigma_0 \left(1+\delta_V^{EW}\right)}_{\sigma_0(1+\delta_W)} + \underbrace{\int d\sigma_0 \left(\delta_V^{QED}(\lambda) + \delta_S(\lambda, k_c)\right)}_{\sigma_{V+S}^{QED}(k_c)} + \underbrace{\int d\sigma_H(k_c)}_{\sigma_H(k_c)} .$$

Subtraction based on factorisation of collinear sing., more involved: dipoles, antennas,..but much more efficient (QCD) recent example: NLO to $e^+e^- \to W^+W^-Z$ (Boudjema, Ninh Le Duc, Sun Hao, M. Weber, 2009)

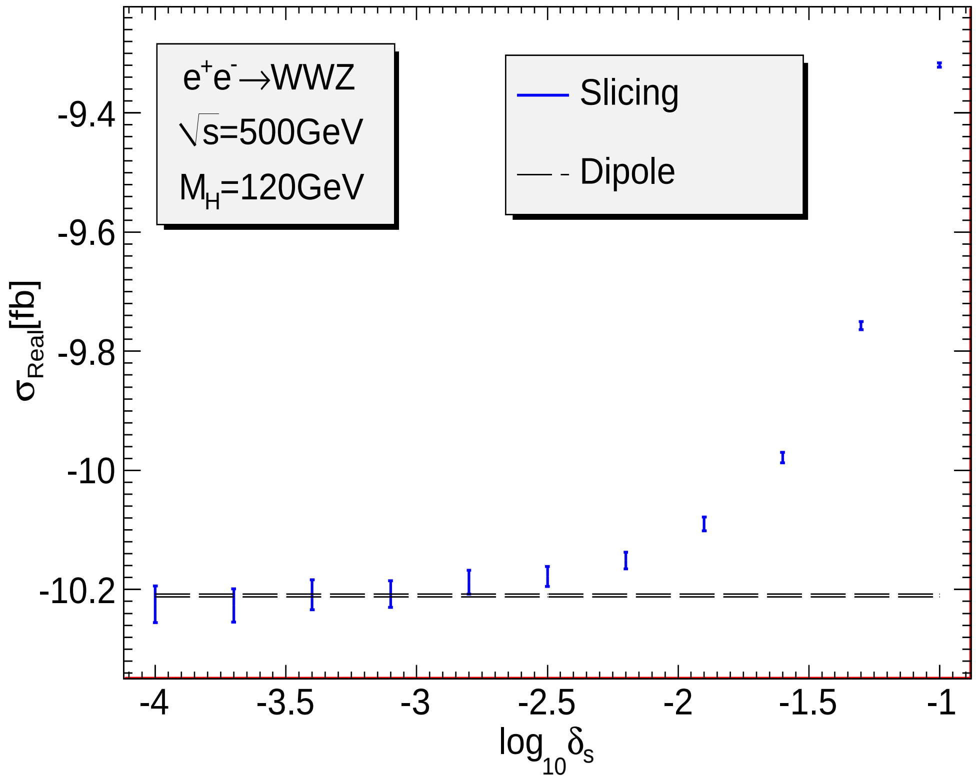


$$
\begin{aligned}
d\sigma^{e^+e^-\to W^+W^-Z} &= d\sigma_{virt}^{e^+e^-\to W^+W^-Z} + d\sigma_{real}^{e^+e^-\to W^+W^-Z\gamma}, \\
d\sigma_{real}^{e^+e^-\to W^+W^-Z\gamma} &= d\sigma_{soft}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) + d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s), \\
d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) &= d\sigma_{coll}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c) + d\sigma_{fin}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c),
\end{aligned}
$$

The Old not so good slicing method: careful choice of matching/cuts



Slicing vs Dipole in $e^+e^- \rightarrow W^+W^-Z$

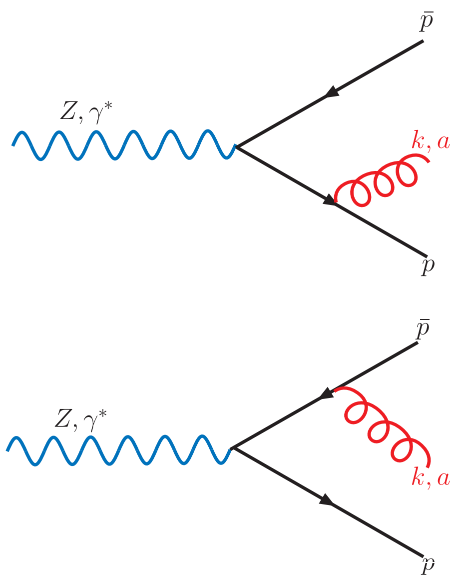


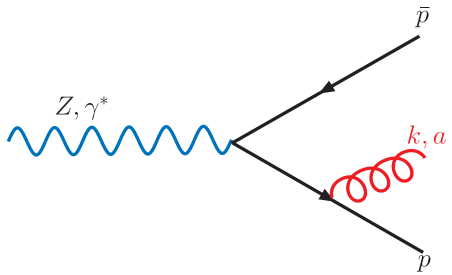
error in the dipole, thickness of line

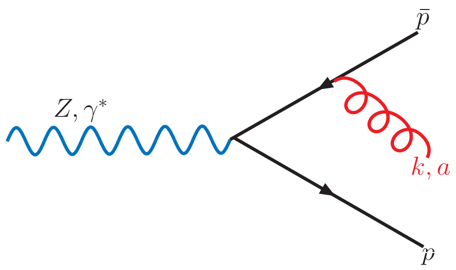
slicing 10-30 times slower for the same precision.

but even dipole takes longer than (optimised) virtual corrections (factor 2-3).

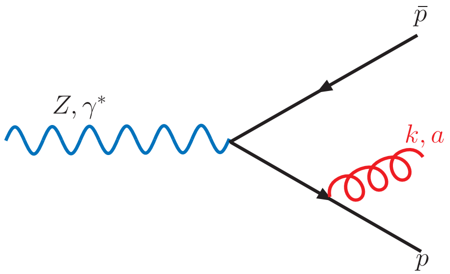
Origin and justification of PS: soft and collinear divergencies

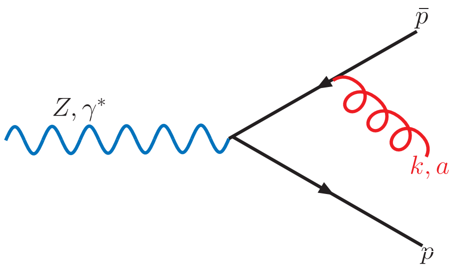






$$
\begin{aligned}
\mathcal{A}_\mu &= \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \quad m_q = 0 \\
&+ \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon} v(\bar{p}) \\
&= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k}\right)t_a \\
2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \to 0 \text{ for } E_g \to 0, \quad \theta_{pk} \to 0
\end{aligned}
$$





$$\mathcal{A}_\mu = \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \quad m_q = 0$$

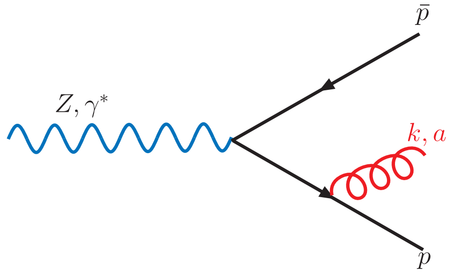
$$+ \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon} v(\bar{p})$$

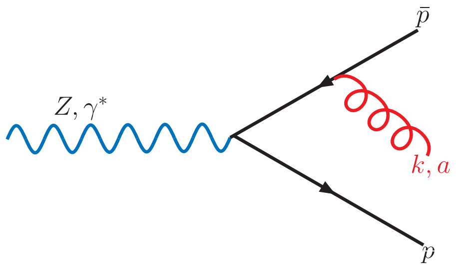
$$= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k}\right)t_a$$

$$2p.k = 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \longrightarrow 0 \text{ for } E_g \to 0, \quad \theta_{pk} \to 0$$

$$\mathcal{A}_{\text{soft}}(k \to 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_0 \quad \text{diverges } k \to 0$$

Origin and justification of PS: soft and collinear divergencies





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&= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k}\right)t_a \\
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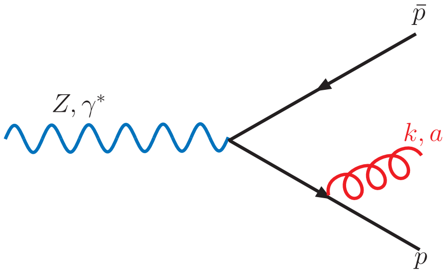
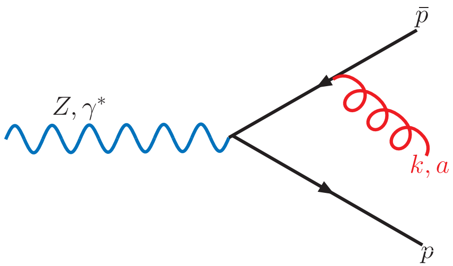
\end{aligned}$$

$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_{0g}$$





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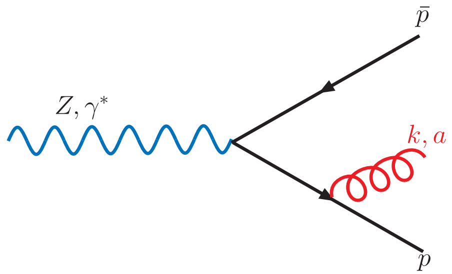
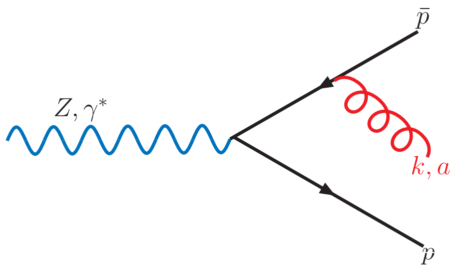
Universal Radiator Factor

We have **factorisation** of the soft emission (long distance) from the short distance *i.e.* the hard process

Squaring soft/collinear





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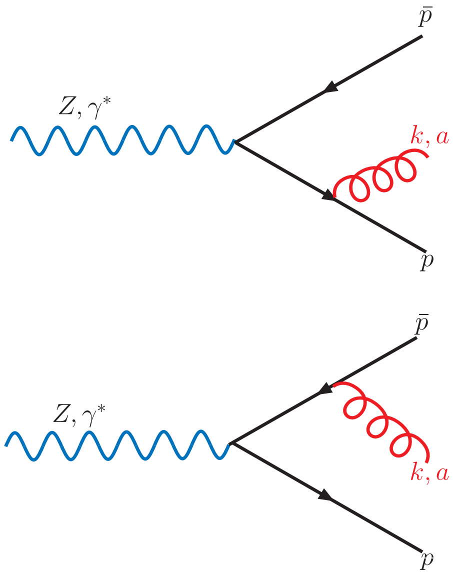

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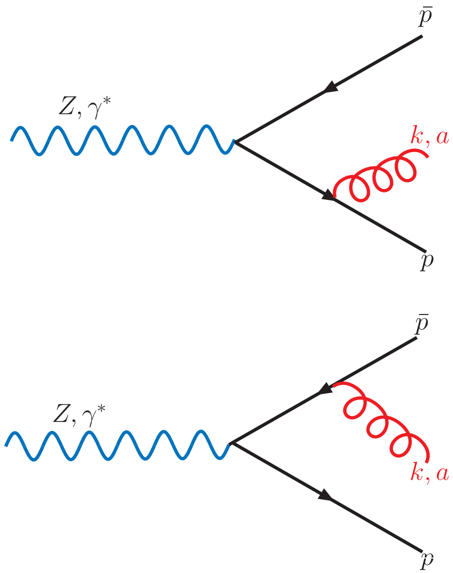
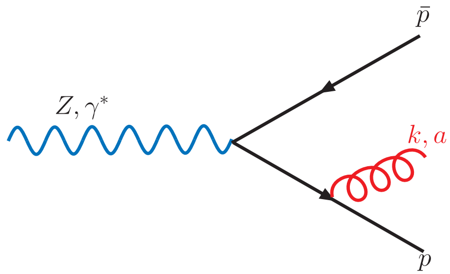


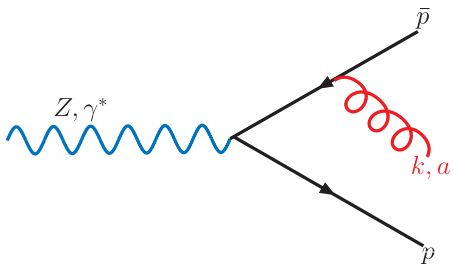
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right) \mathcal{A}_{0g}$$

$$|\mathcal{M}_{1g}|^2 = \sum_{a,pol.(\epsilon)} |\mathcal{A}_{1g}(k \to 0)|^2 = C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k} |\mathcal{M}_{0g}|^2$$

Squaring soft/collinear



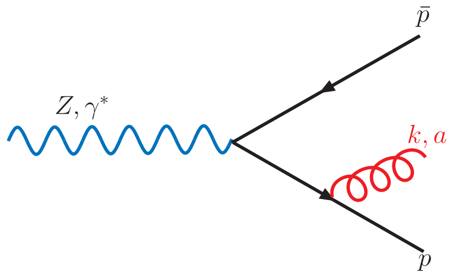
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

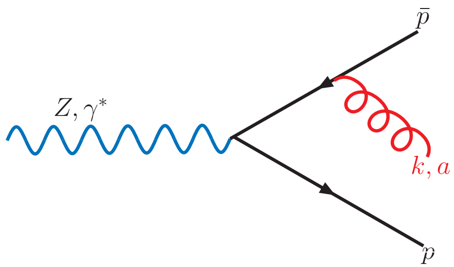


Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}} \right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3 \vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

Squaring soft/collinear





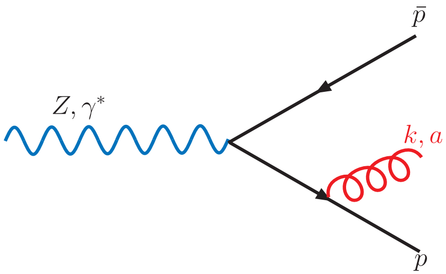
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

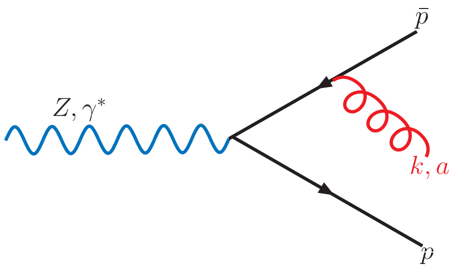
Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}} \right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k \; \bar{p}.k}$$

$$\theta = \theta_{\angle pk} \;, \; \phi = \text{azimuth}$$

$$\mathrm{d}\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$





$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}} \right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k \; \bar{p}.k}$$

$$\theta = \theta_{\angle pk} \ , \ \phi = \text{azimuth}$$

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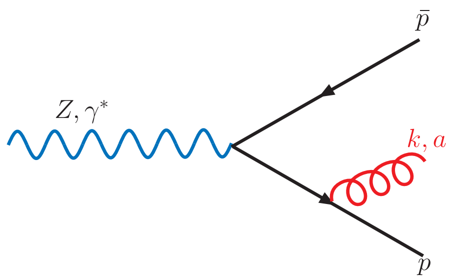
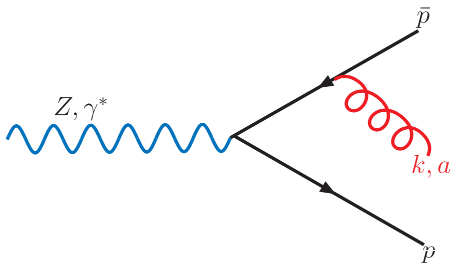
- $\mathrm{d}\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$, collinear divergence

Squaring soft/collinear

$$\mathcal{A}_{1g}(k\to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_{0g}$$





Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}}\right)\mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k(2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

$$\theta = \theta_{\angle pk}\ ,\ \phi = \text{azimuth}$$

$$\mathrm{d}\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi}\frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}\theta}{\sin\theta}\frac{\mathrm{d}\phi}{2\pi}$$

$$x_i = 2E_i/E_{\text{tot}} \quad p \to 1, \quad k \to 3$$

$$\mathrm{d}\mathcal{S}_\phi \quad = \quad \frac{\alpha_s C_F}{2\pi}\mathrm{d}x_1 \mathrm{d}x_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$= \quad \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3} - x_3\right)\mathrm{d}\cos\theta \mathrm{d}x_3$$

- $\mathrm{d}\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$, collinear divergence

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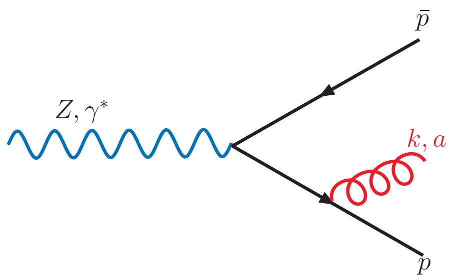

Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}}\right)\mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k(2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

$$\theta = \theta_{\angle pk}\ ,\ \phi = \text{azimuth}$$

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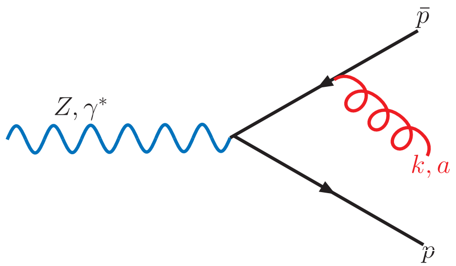

$$x_i = 2E_i/E_{\text{tot}} \quad p \to 1, \quad k \to 3$$

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$$= \quad \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3} - x_3\right)\mathrm{d}\cos\theta \mathrm{d}x_3$$

- $\mathrm{d}\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$, collinear divergence
- collinear divergence for $x_1 \longrightarrow 1$ or $x_2 \longrightarrow 1$ and Infrared divergence for $x_3 \longrightarrow 0$

Splitting

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta}=\frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}}\sim\frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2} \quad \text{for } \theta,\bar{\theta}\sim 0 \ll 1$$

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar\theta}{1-\cos\bar\theta} \sim \frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar\theta^2}{\bar\theta^2} \quad \text{for} \quad \theta,\bar\theta \sim 0 \ll 1$$

q and $\bar q$ as independent emitters, notion of splitting as a probability

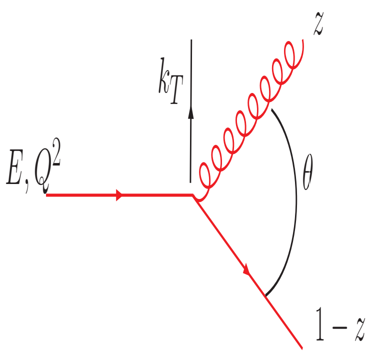
$$\mathrm{d}\sigma_{1g} \sim \mathrm{d}\sigma_{0g}\sum_{q\to qg}^{\bar q\to\bar q g} C_F\frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\frac{1+(1-z)^2}{z}\mathrm{d}z \qquad (z\equiv x_3)$$

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\,\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar\theta}{1-\cos\bar\theta} \sim \frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar\theta^2}{\bar\theta^2} \quad \text{for} \;\; \theta,\bar\theta \sim 0 \ll 1$$

q and $\bar q$ as independent emitters, notion of splitting as a probability

$$\mathrm{d}\sigma_{1g} \sim \mathrm{d}\sigma_{0g} \sum_{q\to qg}^{\bar q\to\bar q g} C_F\,\frac{\alpha_s}{2\pi}\,\frac{\mathrm{d}\theta^2}{\theta^2}\,\frac{1+(1-z)^2}{z}\,\mathrm{d}z \qquad (z\equiv x_3)$$

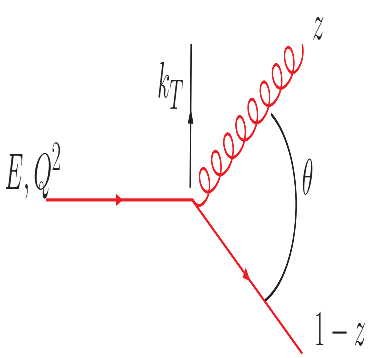


$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}} \sim \frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2} \quad \text{for} \quad \theta,\bar{\theta}\sim 0 \ll$$

q and $\bar{q}$ as independent emitters, notion of splitting as a probability

$$\mathrm{d}\sigma_{1g} \sim \mathrm{d}\sigma_{0g}\sum_{q\to qg}^{\bar{q}\to\bar{q}g} C_F\,\frac{\alpha_s}{2\pi}\,\frac{\mathrm{d}\theta^2}{\theta^2}\,\frac{1+(1-z)^2}{z}\mathrm{d}z \qquad (z\equiv x_3)$$



different choices of the evolution variables, equivalent in the collinear limit (diff. in practice/different codes)

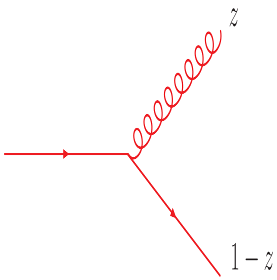
$$Q^2 = E^2 z(1-z)\theta^2 \qquad k_T^2 = E^2 z^2(1-z)^2\theta^2$$

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}Q^2}{Q^2} = \frac{\mathrm{d}k_T^2}{k_T^2}$$

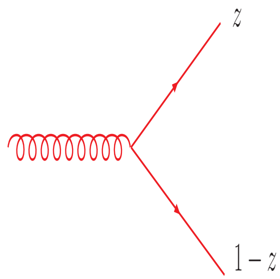
DGLAP

This generalises to different parton branching (gluon, quarks)

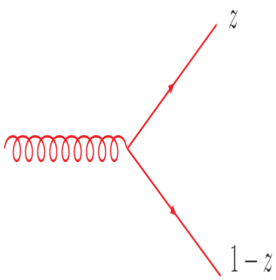
$$d\sigma_{bc} \sim d\sigma_a \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} P_{a\to bc}(z) dz$$



$$P_{gq}(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

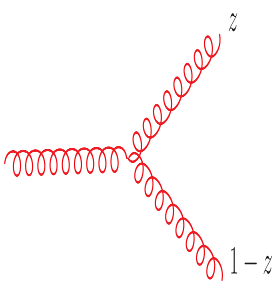


$$P_{qg}(z) = C_F \left(\frac{1+(1-z)^2}{z} \right)$$



$$P_{qg}(z) = T_R \left(z^2 + (1-z)^2 \right) \qquad T_R = \frac{n_f}{2}$$

(divergences at $z = 0, 1$ dealt with soft/virtual corr.)



$$P_{gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right) \qquad C_A = 3 \qquad (C_F = 4/3)$$

Gluons radiate the most

$P(z,\phi)$ can be defined for polarisation effects

- The singular parts of the QCD/QED matrix elements for real emmission can be singled out in a general way through the factorisation of soft and collinear radiation
- Improved general purpose factorisation formulae have been derived: dipole factorisation formula.

Strategy

$$\sigma_{ab}^{LO} = \int_m d\sigma_{ab}^{B},$$

$$\sigma_{ab}^{NLO} = \underbrace{\underbrace{\int_{m+1} d^{(d)}\sigma_{ab}^{R}}_{div\ if\ d=4} + \underbrace{\int_m d^{(d)}\sigma_{ab}^{V}}_{div\ if\ d=4}}_{sum\ is\ finite}.$$

for $2 \longrightarrow m$ (Born, V) and $2 \longrightarrow m+1$ (real)

$$\sigma_{ab}^{NLO} = \underbrace{\underbrace{\int_{m+1}\left(d^{(d)}\sigma_{ab}^{R} - d^{(d)}\sigma_{ab}^{A}\right)}_{finite} + \underbrace{\int_{m}\left(d^{(d)}\sigma_{ab}^{V} + d^{(d)}\sigma_{ab}^{A}\right)}_{finite}}_{sum\ is\ finite}.$$

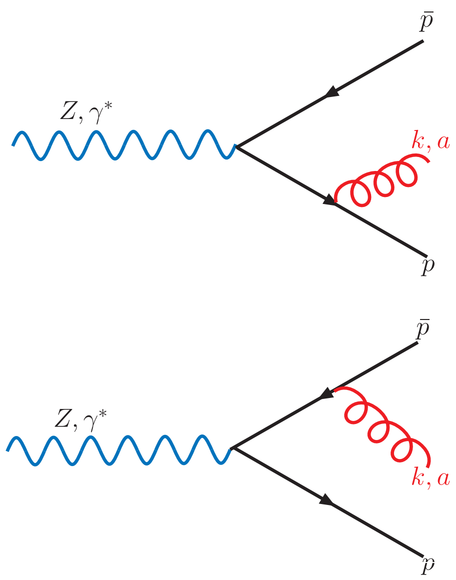
$$\sigma_{ab}^{NLO} = \int_{m+1}\left(d^{(4)}\sigma_{ab}^{R} - d^{(4)}\sigma_{ab}^{A}\right)$$

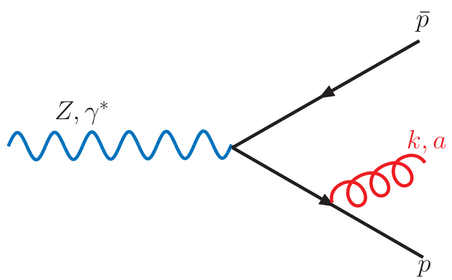
$$+ \underbrace{\int_{m}\left(\int_{loop} d^{(d)}\sigma_{ab}^{V} + \int_{1} d^{(d)}\sigma_{ab}^{A}\right)_{\varepsilon=0}}_{finite}.$$

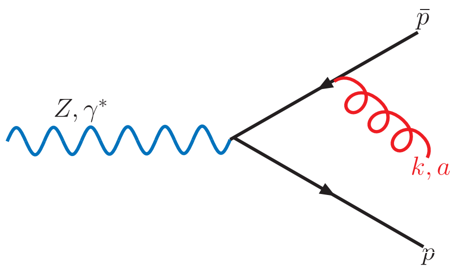
- For a given process $d\sigma^A$ independent of the particular jet observable
- $d\sigma^A$ has to match the singularities of $d\sigma^R$ in d dimension
- form convenient for Monte Carlo integration routines
- has to be analytically integrable in d-dimension over the single parton PS

$$d^{(d)}\sigma^A_{ab} = \sum_{\text{dipoles}} d^{(4)}\sigma^B_{ab} \times d^{(d)}V_{\text{dipole}}.$$

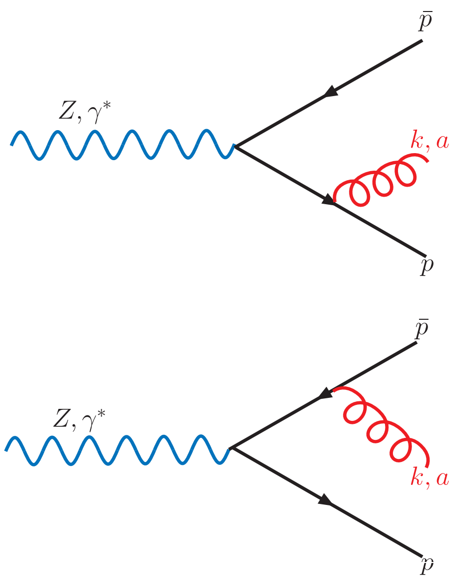
Origin and justification of PS: soft and collinear divergencies







$$\begin{aligned}
\mathcal{A}_\mu &= \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \qquad m_q = 0 \\
&+ \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon} v(\bar{p}) \\
&= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k}\right)t_a \\
2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \;\longrightarrow\; 0 \text{ for } \; E_g \to 0, \;\; \theta_{pk} \to 0
\end{aligned}$$



$$
\begin{aligned}
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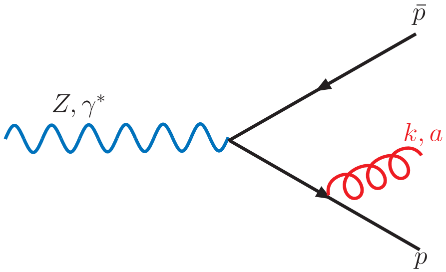
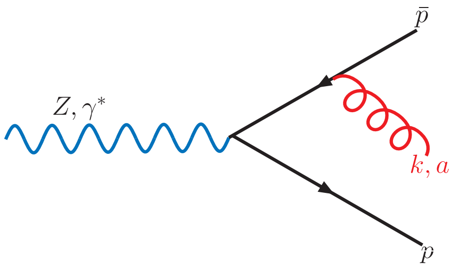
&= -g_s \left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k} \right) t_a \\
2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \quad \longrightarrow \; 0 \text{ for } \; E_g \to 0, \quad \theta_{pk} \to 0
\end{aligned}
$$

$$\mathcal{A}_{\rm soft}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_0 \quad \text{diverges} \;\; k \to 0$$





$$\begin{aligned}
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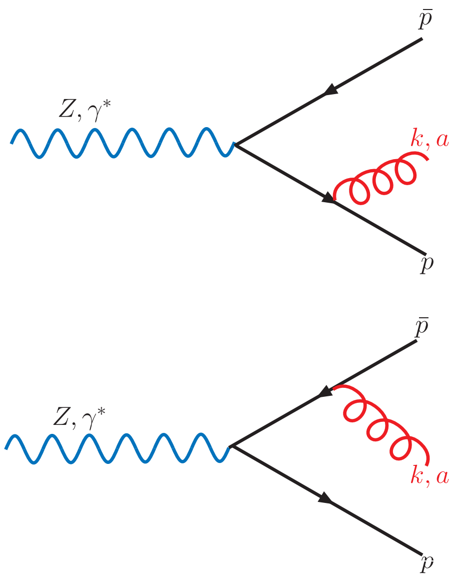
&+ \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon}v(\bar{p}) \\
&= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon}v(\bar{p})}{2\bar{p}.k}\right)t_a \\
2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \quad \rightarrow 0 \text{ for } E_g \rightarrow 0, \quad \theta_{pk} \rightarrow 0
\end{aligned}$$

$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_{0g}$$



$$
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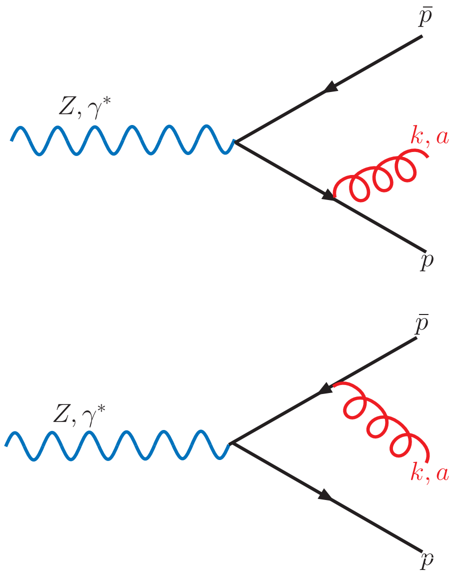
&= -g_s \left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k} \right) t_a \\
2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \quad \rightarrow \; 0 \text{ for } \; E_g \rightarrow 0, \quad \theta_{pk} \rightarrow 0
\end{aligned}
$$

$$
\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}
$$

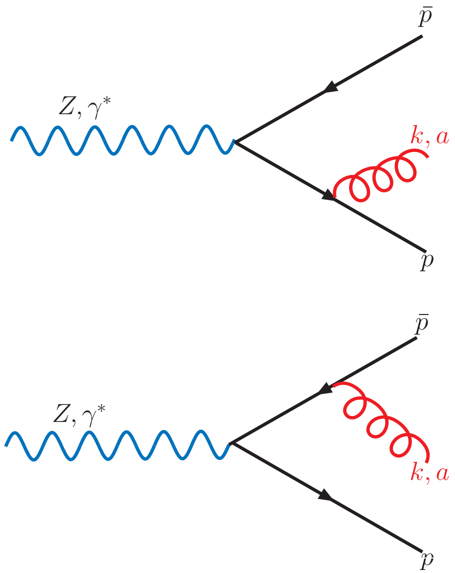
Universal Radiator Factor

We have **factorisation** of the soft emission (long distance) from the short distance *i.e.* the hard process

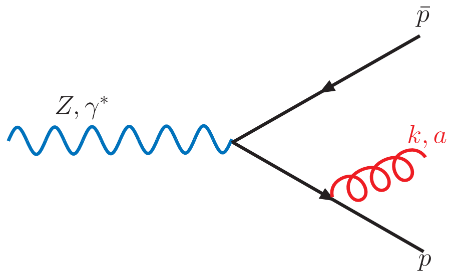
Squaring soft/collinear

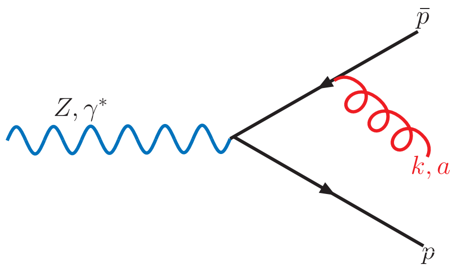


Squaring soft/collinear



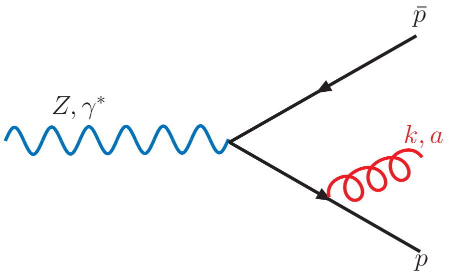
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$



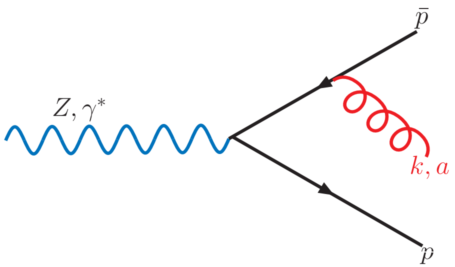


$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

$$|\mathcal{M}_{1g}|^2 = \sum_{a, pol.(\epsilon)} |\mathcal{A}_{1g}(k \to 0)|^2 = C_F g_s^2 \frac{2p.\bar{p}}{p.k \; \bar{p}.k} |\mathcal{M}_{0g}|^2$$



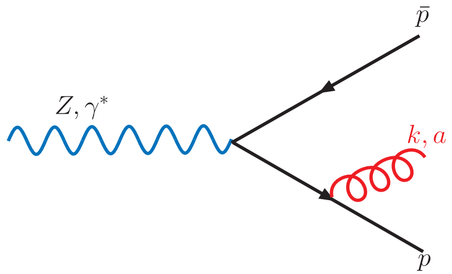
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

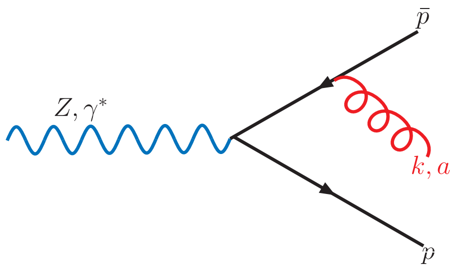


Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}} \right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3 \vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k \ \bar{p}.k}$$

Squaring soft/collinear





$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

Phase Space

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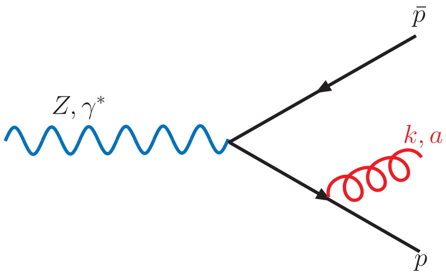
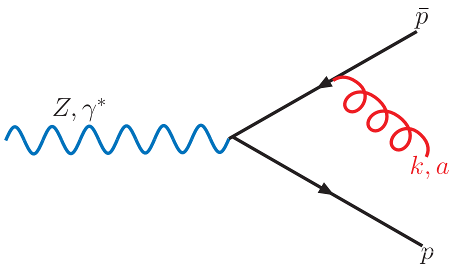
$$\theta = \theta_{\angle pk} \ , \ \phi = \text{azimuth}$$

$$\mathrm{d}\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$





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Phase Space

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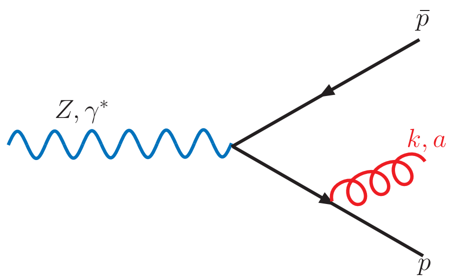
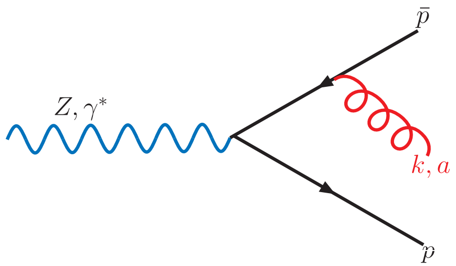
- $\mathrm{d}\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$, collinear divergence

$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right) \mathcal{A}_{0g}$$





Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}}\right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3\vec{k}}{2\omega_k(2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

$$\theta = \theta_{\angle pk}\ ,\ \phi = \text{azimuth}$$

$$d\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$$x_i = 2E_i/E_{\text{tot}} \quad p \to 1, \quad k \to 3$$

$$d\mathcal{S}_\phi \quad = \quad \frac{\alpha_s C_F}{2\pi} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$= \quad \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3\right) d\cos\theta\, dx_3$$

- $d\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
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Squaring soft/collinear

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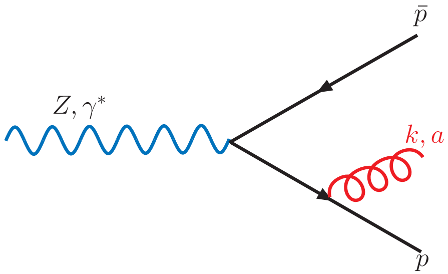

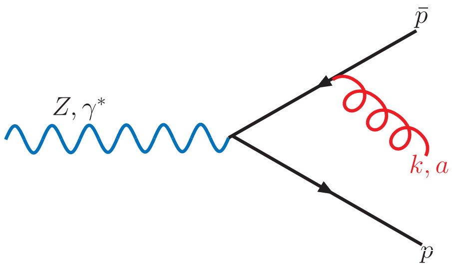

Phase Space

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$$\mathrm{d}\mathcal{S} \simeq \frac{2\alpha_s C_F}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$

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$$\mathrm{d}\mathcal{S}_\phi = \frac{\alpha_s C_F}{2\pi} \mathrm{d}x_1 \mathrm{d}x_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$= \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3\right) \mathrm{d}\cos\theta \mathrm{d}x_3$$

- $\mathrm{d}\mathcal{S}$ diverges for $\omega \to 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \to 0$ and $\theta \to \pi$, collinear divergence
- collinear divergence for $x_1 \to 1$ or $x_2 \to 1$ and Infrared divergence for $x_3 \to 0$

$$
\begin{aligned}
\mathrm{d}\mathcal{S}_\phi &\simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3 \\
\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta}=\frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}}\sim\frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2} \quad \text{for}\ \ \theta,\bar{\theta}\sim 0 \ll 1
\end{aligned}
$$

Splitting

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar\theta}{1-\cos\bar\theta} \sim \frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar\theta^2}{\bar\theta^2} \quad \text{for } \theta,\bar\theta \sim 0 \ll 1$$

q and $\bar q$ as independent emitters, notion of splitting as a probability

$$\mathrm{d}\sigma_{1g} \sim \mathrm{d}\sigma_{0g}\sum_{q\to qg}^{\bar q\to\bar q g} C_F\frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\frac{1+(1-z)^2}{z}\mathrm{d}z \qquad (z\equiv x_3)$$

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar\theta}{1-\cos\bar\theta} \sim \frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar\theta^2}{\bar\theta^2} \quad \text{for } \theta,\bar\theta \sim 0 \ll 1$$

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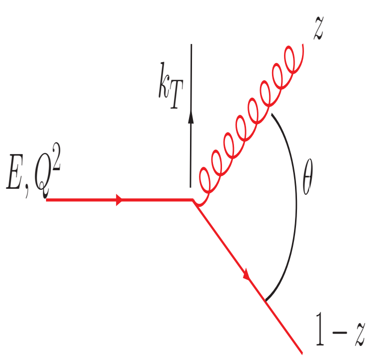
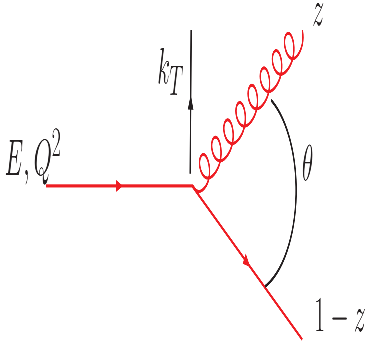


$$
\begin{aligned}
\mathrm{d}\mathcal{S}_\phi &\simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3 \\
\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar\theta}{1-\cos\bar\theta} \sim \frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar\theta^2}{\bar\theta^2} \quad \text{for} \;\; \theta,\bar\theta \sim 0 \ll 1
\end{aligned}
$$

q and $\bar q$ as independent emitters, notion of splitting as a probability

$$
\mathrm{d}\sigma_{1g} \sim \mathrm{d}\sigma_{0g} \sum_{q\to qg}^{\bar q\to \bar q g} C_F \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\frac{1+(1-z)^2}{z}\mathrm{d}z \qquad (z\equiv x_3)
$$

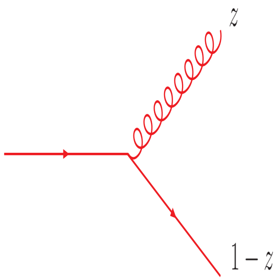


different choices of the evolution variables, equivalent in the collinear limit (diff. in practice/different codes)

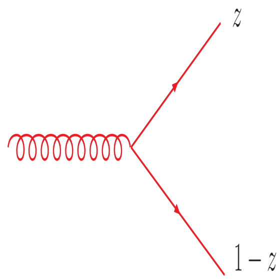
$$
\begin{aligned}
Q^2 &= E^2 z(1-z)\theta^2 \qquad k_T^2 = E^2 z^2 (1-z)^2\theta^2 \\
\frac{\mathrm{d}\theta^2}{\theta^2} &= \frac{\mathrm{d}Q^2}{Q^2} = \frac{\mathrm{d}k_T^2}{k_T^2}
\end{aligned}
$$

This generalises to different parton branching (gluon, quarks)

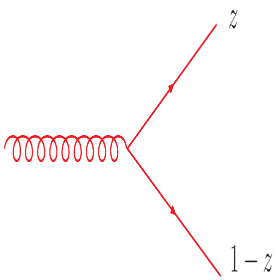
$$\mathrm{d}\sigma_{bc} \sim \mathrm{d}\sigma_a \frac{\alpha_s}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2} P_{a\to bc}(z)\mathrm{d}z$$



$$P_{gq}(z) = C_F\left(\frac{1+z^2}{1-z}\right)$$

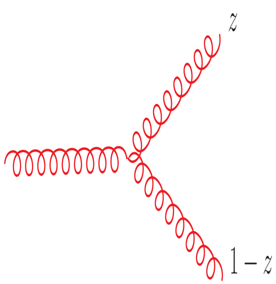


$$P_{qg}(z) = C_F\left(\frac{1+(1-z)^2}{z}\right)$$



$$P_{qg}(z) = T_R\left(z^2+(1-z)^2\right) \qquad T_R = \frac{n_f}{2}$$

(divergences at $z = 0, 1$ dealt with soft/virtual corr.)

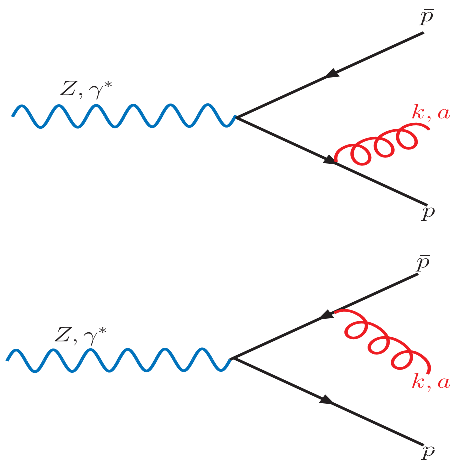


$$P_{gg}(z) = C_A\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right) \qquad C_A = 3 \qquad (C_F = 4/3)$$

Gluons radiate the most

$P(z,\phi)$ can be defined for polarisation effects

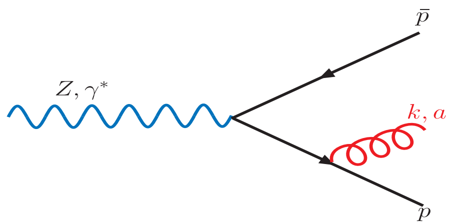
Compensation



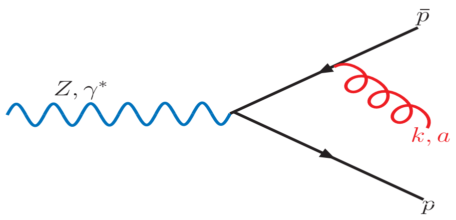
$$\sigma_{\mathrm{real}} \sim \frac{C_F \alpha_s}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon)\right)\sigma_{\mathrm{LO}}$$

For $1^\star \to 2$, analytical result, int. easy.

Compensation

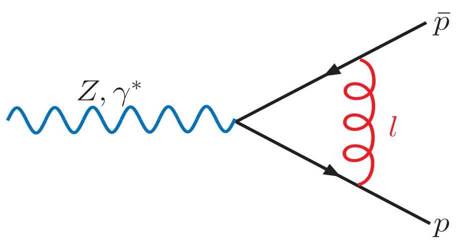


$$\sigma_{\mathrm{real}} \sim \frac{C_F \alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right) \sigma_{\mathrm{LO}}$$



For $1^\star \to 2$, analytical result, int. easy.

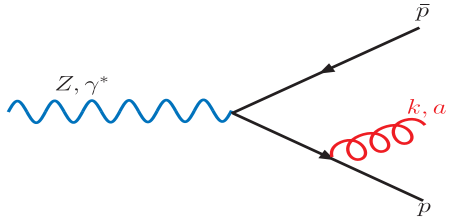
$$\int \frac{d^d l}{(2\pi)^d} \frac{N}{l^2 (l-p)^2 (l+\bar{p})^2}$$



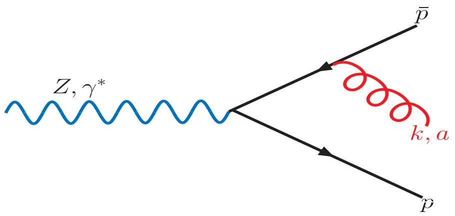
$l \to 0$ IR div, $l \to \infty$ UV
$l = xp, y\bar{p}$ Coll Div for any $x, y \to 0$ For $1^\star \to 2$, analytical result, int. easy.

$$\sigma_{\mathrm{virt}} \sim \frac{C_F \alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right) \sigma_{\mathrm{LO}}$$

Compensation

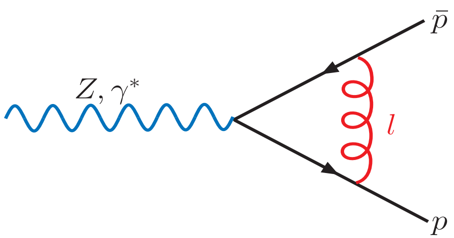


$$\sigma_{\text{real}} \sim \frac{C_F \alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right) \sigma_{\text{LO}}$$



For $1^\star \to 2$, analytical result, int. easy.

$$\int \frac{d^d l}{(2\pi)^d} \frac{N}{l^2 (l-p)^2 (l+\bar{p})^2}$$



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$$\sigma_{\text{virt}} \sim \frac{C_F \alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right) \sigma_{\text{LO}}$$

$$\sigma_{NLO} = \left(1 + \frac{\alpha_s}{\pi} \right) \sigma_{LO}$$

- The singularities in the real emission, either soft or collinear factorise and are universal
- *i.e* Process Independent
- these universal terms are known, if we subtract their contribution from the full real emission terms, the obtained contribution has no singularity and could therefore be integrated numerically over all of phase space
- the singularities in the real emission compensate those in the virtual emission, this assumes we are using the same regularisation scheme

When dealing with multiparticle final states,

integration over phase space can only be performed numerically

these observation are very important

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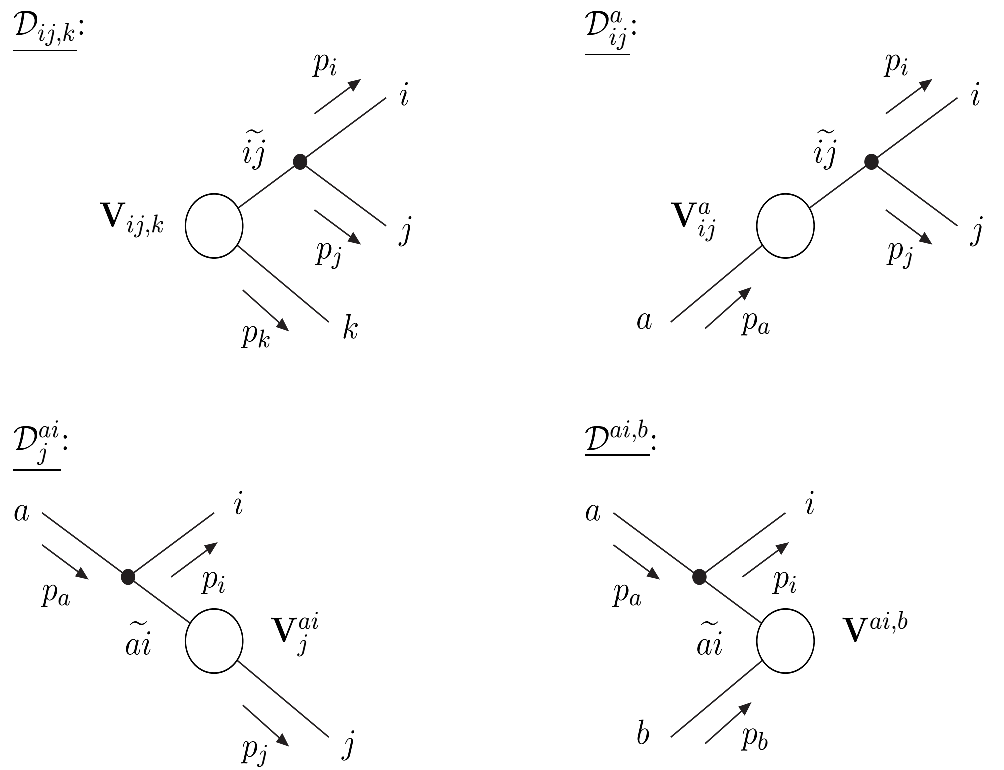
integration over phase space can only be performed numerically

these observation are very important

- Calculate the real terms and subtract the soft/collinear counterterms
- One can then integrate in 4-dimension numerically
- add these counterterms *analytically* to the virtual contribution (properly UV renormalised) to obtain a diff cross section that is soft/coll finite and that can be integrated numerically
- some massaging to do, can be automated: $dPS_{LO+1} \rightarrow dPS_{LO} \times dPS_{\mathrm{gluon}}$ (boosts,...)
- there can be a lot of emitters/dipoles!

Dipoles la Catani-Seymour



Dipoles: final-state emitter with final-state spectator ($\mathcal{D}_{ij,k}$), final-state emitter with initial-state spectator ($\mathcal{D}_{ij}^{a}$), initial-state emitter with final-state spectator ($\mathcal{D}_{j}^{ai}$) and initial-state emitter with initial-state spectator ($\mathcal{D}^{ai,b}$).

Physics!!!

(Gleisberg, Hoeche, Krauss, Schoenher
Schumann, Siegert, Winter)

NLO with **BlackHat+Sherpa**

$$\sigma^{\mathrm{NLO}} = \int_{m+1}\left[\mathrm{d}^{(4)}\sigma^{\mathrm{R}} - \mathrm{d}^{(4)}\sigma^{\mathrm{A}}\right] + \int_{m}\left[\int_{\mathrm{loop}}\mathrm{d}^{(d)}\sigma^{\mathrm{V}} + \int_{1}\mathrm{d}^{(d)}\sigma^{\mathrm{A}}\right]_{\epsilon=0}$$

(S. Catani, M.H. Seymour,1997)

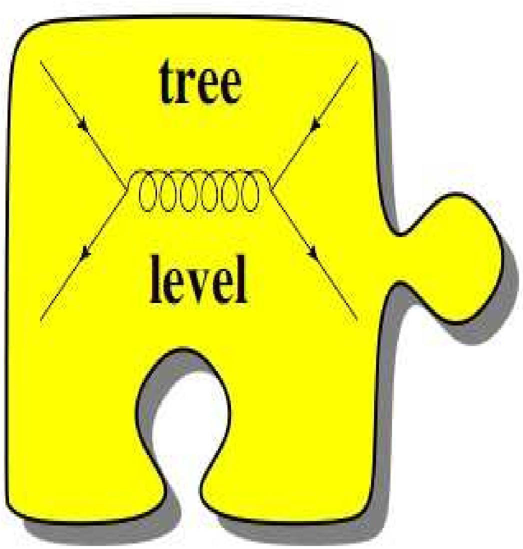
(T. Gleisberg, F. Krauss, 2007)

+

(a glance to NLO automation!)

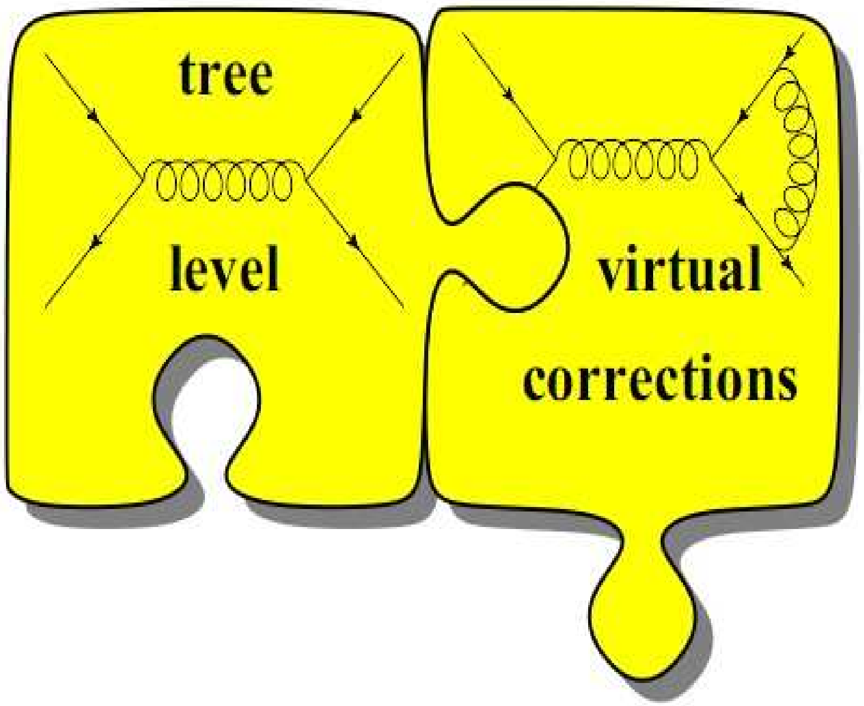
High level of automatisation, public

- ALPGEN (Mangano et al.)
- CalcHEP (Pukhov,Belyaev,Christensen)
- CompHEP (Boos et al.)
- Grace (Yuasa et al.)
- HELAS/PHEGAS (Papadopoulos et al.)
- MADGRAPH/MADEVENT (Maltoni,Stelzer)
- O'Mega/WHIZARD (Kilian,Moretti,Ohl,Reuter)
- SHERPA/Amegic (Krauss,Kuhn)

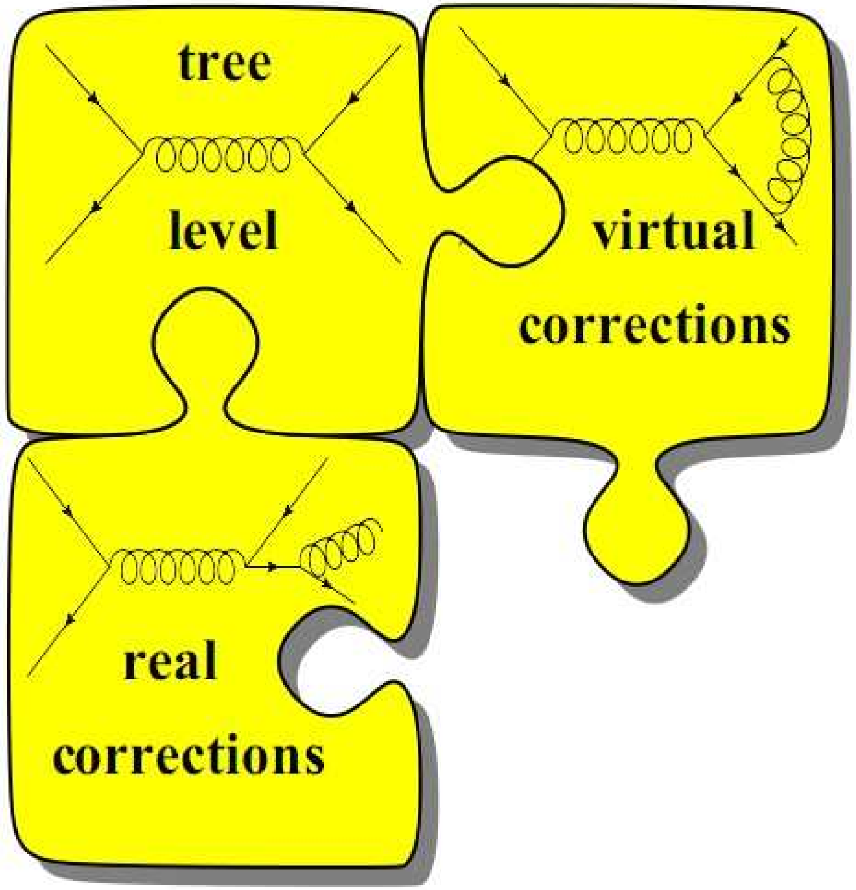


virtual Corrections:

- Feynmanians
    FeynArts/FormCalc (
    Hahn,Perez-Victoria,v.Oldenborgh
    Grace-loop (Shimzu et al.
    Golem (Binoth et al,)
    SloopS (Boudjema et al.)
    Many Process Specific
    few)
- Unitaritarians/cuts (at amplitude leve
    so)
    HELAC-1LOOP+CutTools
    (v.Hameren,Ossola,Papadopoulos
    BlackHat (Berger et al.)
    Rocket (Ellis, Melnikov,
    Zanderighi)
    Generalized Color-Dres
    Unitarity (Giele,Kunszt,Winter



Radiation, in principle same as tree-level



- Catani-Seymour dipoles

  `AutoDipole` (Hasegawa, Moch, Uwer)

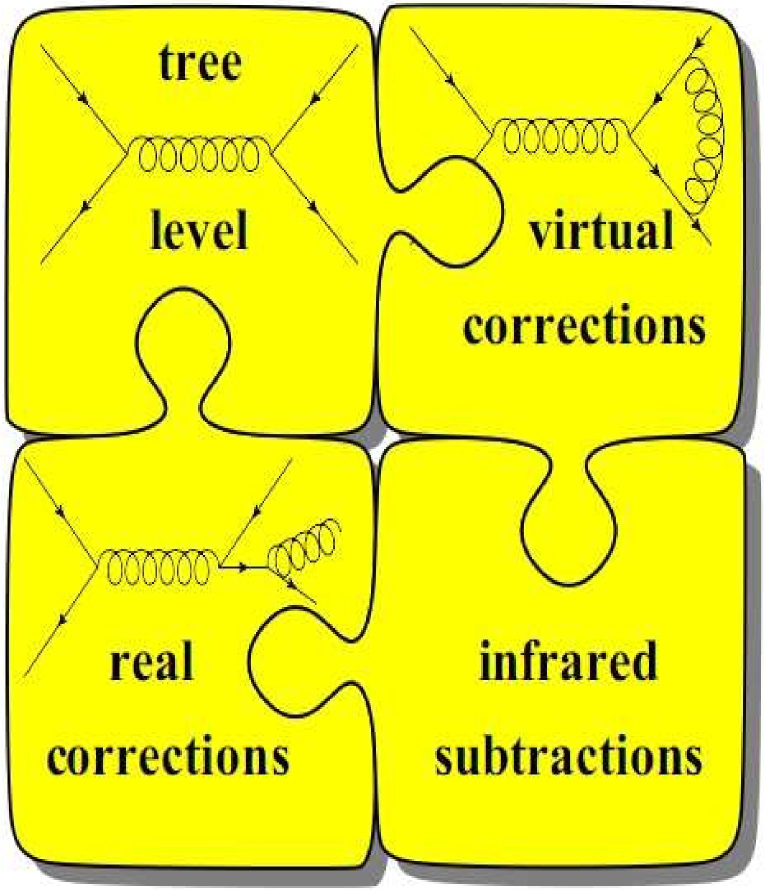
  `HELAC-DPOLE` (Czakon, Papadopoulos, Worek)

  `MadDipole` (Frederix, Gehrmann, Greiner)

  `Sherpa` (Gleisberg, Krauss)

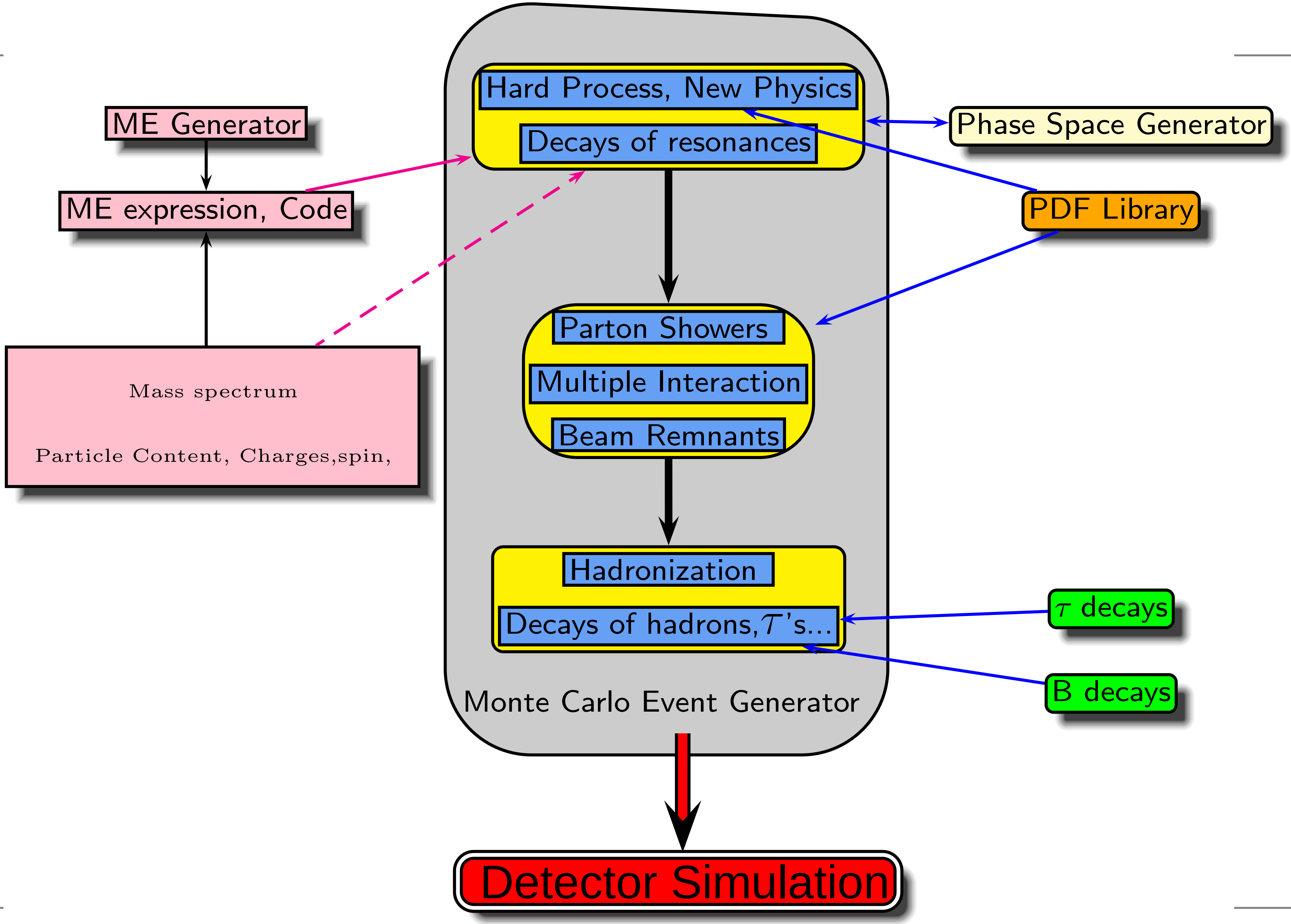
  `TevJet` (Seymour, Tevlin)

- `MadFKS` (Frederix, Frixione, Maltoni, Stelzer)

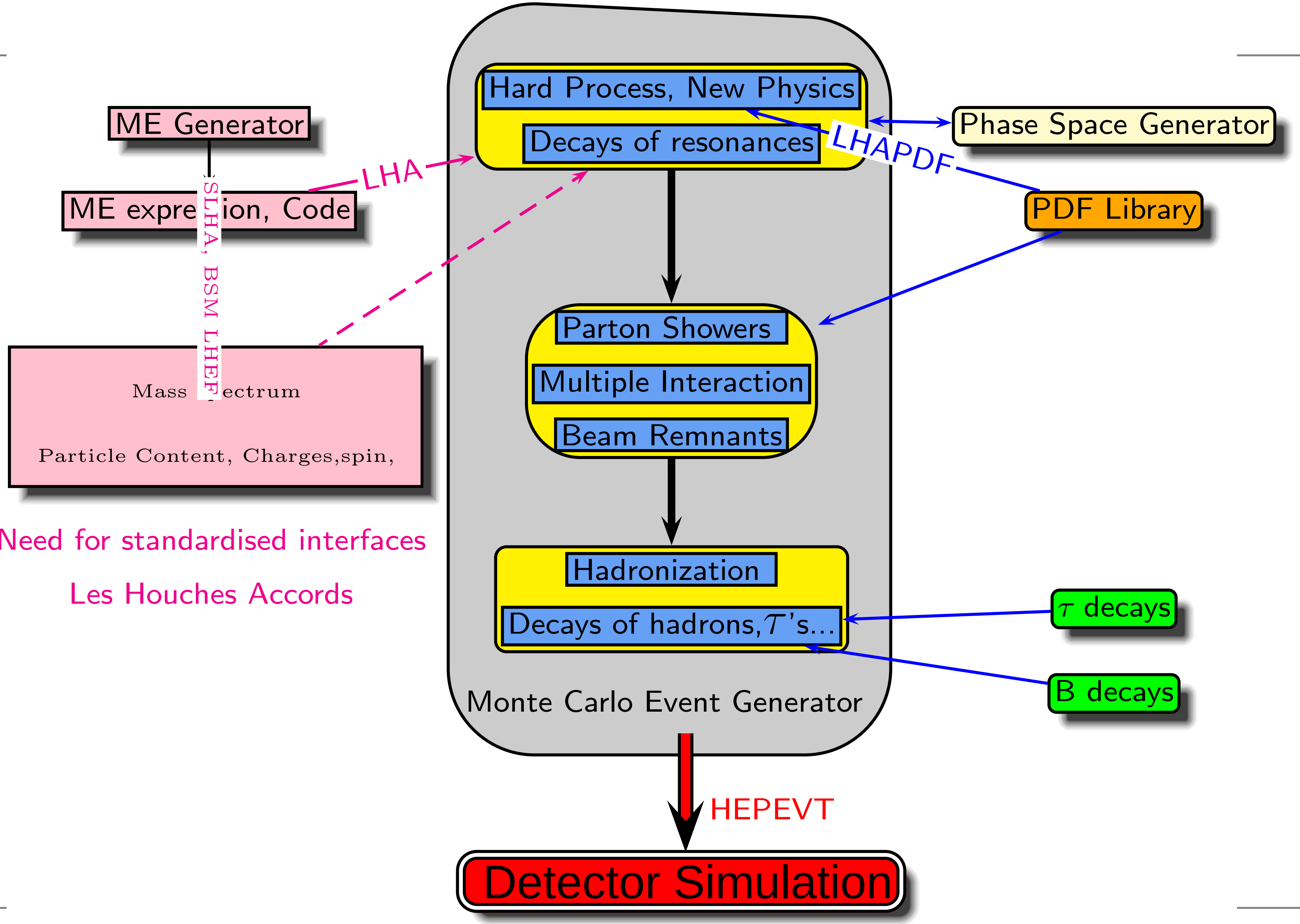



2 Higgs phenomenology
3. New NLO calculations/ wishlist
4. NLO techniques Standardization/automation
5. NLO + parton shower
09.06.2009

Putting all together



Putting all together, Les Houches Accords



- hadronic cross section and partonic subprocesses

$$\sigma_{had}(p_1,p_2) = \sum_{a,b}\int dx_1\, f_{a/H_1}(x_1,\mu_{\rm F}^2)\int dx_2\, f_{b/H_2}(x_2,\mu_{\rm F}^2) \times\Big[d\sigma_{ab}^{\rm LO}(x_1p_1,x_2p_2;\mu_{\rm R}^2) + d\sigma_{ab}^{\rm NLO}(x_1p_1,x_2p_2;\mu_{\rm R}^2,\mu_{\rm F}^2)\Big],$$

-

$$\sigma_{ab}^{\rm LO} = \int_m d\sigma_{ab}^B\,,$$
$$\sigma_{ab}^{\rm NLO} = \int_{m+1} d\sigma_{ab}^R + \int_m d\sigma_{ab}^V + \int_m d\sigma_{ab}^C(\mu_{\rm F}^2,{\rm F.S.})\,.$$

- for $2\to m$ (Born, V) and $2\to m+1$ (real)

$$d\sigma_{ab}^V = d\,\mathsf{LIPS}(\{k_j\})\,\mathcal{I}(\{k_j\})\,.$$

- Take DR after renormalisation

$$\mathcal{I}(\{k_j\},{\rm R.S.},\mu_{\rm R}^2,\alpha_{\rm S}(\mu_{\rm R}^2),\alpha,\ldots) = C(\epsilon)\left(\frac{A_2}{\epsilon^2}+\frac{A_1}{\epsilon}+A_0\right).$$

Binoth LHA

The goal of the interface is to facilitate the transfer of information between

one-loop programs, OLP

and programs which provide

tree amplitude information and incorporate methods to

perform the integration over the phase space:

Monte Carlo tool (MC).

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tree amplitude information and incorporate methods to

perform the integration over the phase space:

Monte Carlo tool (MC).

Interaction works in 2 phases

- `initialisation` exchange of basic information: availability of sub-processes, input parameters, schemes,..
- `Run-time` MC asks OLP for one-loop contributions at points in PS. Finite part may be split (more efficient integration, sampling)

OLP	Input	Output
Initialisation	Model parameters: $\alpha(0),\, \alpha_S(M_Z), \ldots,\, m_t,\, m_b, \ldots,$ CKM values	confirm values
	Schemes: UV-renormalisation / IR-factorisation	confirm schemes
	Operational information: colour/helicity treatment, approximations, etc.	confirm options

OLP	Input	Output
Initialisation	Model parameters: $\alpha(0),\ \alpha_S(M_Z),\ldots,\ m_t,\ m_b,\ldots,$ CKM values	`confirm values`
	Schemes: UV-renormalisation / IR-factorisation	`confirm schemes`
	Operational information: colour/helicity treatment, approximations, etc.	`confirm options`
Run-time	Events: $(E, p_x, p_y, p_z, M)_{j=1,\ldots,m+2},\ \mu,\ \alpha_S(\mu_R)$	$\left(A_2, A_1, A_0, \lvert\text{Born}\rvert^2\right)$ `optional information`


INITIALIZATION PHASE
LOOP
ME
FILE
PHASE
COMPLEX-MASS SCHEME
DECAY
't Hooft Veltman
12.06.2009

Example: Here is an example of an order file for the partonic $2 \to 3$ processes, $gg \to t\bar{t}g$, $q\bar{q} \to t\bar{t}g$ and $qg \to t\bar{t}q$, needed for the evaluation of $pp \to t\bar{t}$ + jet

```
# example order file

MatrixElementSquareType   CHsummed
IRregularisation          CDR
OperationMode             LeadingColour
ModelFile                 ModelInLHFormat.slh
SubdivideSubprocess       yes
AlphasPower               3
CorrectionType            QCD

# g g   -> t tbar g
 21 21 -> 6 -6 21
# u ubar -> t tbar g
  2 -2 -> 6 -6 21
# u g   -> t tbar u
  2 21 -> 6 -6  2
```

Example:

```
# example contract file
# contract produced by OLP, OLP authors, citation policy

MatrixElementSquareType CHsummed                | OK
IRregularisation        CDR                     | OK
OperationMode           LeadingColour           | OK
ModelFile               ModelFileInLHFormat.slh | OK
SubdivideSubprocess     yes                     | OK
CorrectionType          QCD                     | OK

# g g -> t tbar g
21 21 -> 6 -6 21       | 2 13 35 # 2 channels: cut-constructable,&
                                         & rational part
# u ubar -> t tbar g
 2 -2 -> 6 -6 21       | 1 29
# u g -> t tbar u
 2 21 -> 6 -6  2       | 3 8 23 57 # 3 channels: leading,&
                         & subleading, subsubleading colour
```

The contract file

Example:

```
# example contract file
# contract produced by OLP, OLP authors, citation policy

MatrixElementSquareType CHsummed | Error: unsupported flag
# CHaveraged is supported
IRregularisation        DRED     | Error: unsupported flag
# CDR, tHV are supported
OperationMode      LeadingColour | Error: unsupported flag
# see OLP Documentation
ModelFile     FavouriteModel.slh | Error: file not found
# Modelfile is called: SM.slh
SubdivideSubprocess yes          | Error: unsupported flag
# no is supported
CorrectionType           EW      | Error: unsupported flag
# QCD is supported
MyWayOfDoingThings true          | Error: unknown option

# g g -> t tbar g
21 21 -> 6 -6 21    |   Error: massive quarks not supported
# u ubar -> t tbar g
 2 -2 -> 6 -6 21    |   Error: process not available
# u g -> t tbar u
 2 21 ->> 6 -6  2   |   Error: check syntax
```

In Memoriam

Dedicated to Thomas Binoth

(Les Houches 2009)

Warning ! Used by Thomas often at LH09

" ....NLO tools are not DAUs..." (Stefan Dittmaier)

DAU= dümmst anzumehmender user = most imaginable ignorant user

Warning ! Used by Thomas often at LH09

" ....NLO tools are not DAUs..." (Stefan Dittmaier)

DAU= dümmst anzumehmender user = most imaginable ignorant user

A Dahu, quoi..! as I told him for a multi-leg alpine (?) animal...

Chuss Thomas!

Les Houches 2011