The Standard Model in the LHC era

I: Histories and Symmetries

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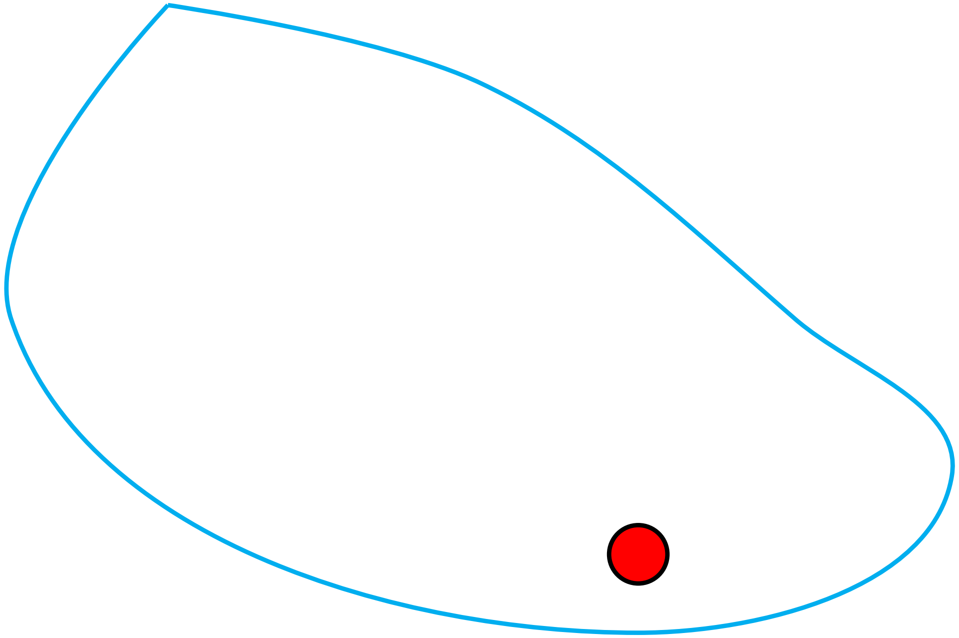
Introduction

EW theory is the combination of two fundamental principles

- Gauge Symmetry Principle
- Hidden symmetry or *Spontaneous symmetry breaking*

This allows
✓ a correct quantum description
✓ high degree of precision (LEP,SLC,Babar, ...LHC)

Charge conservation



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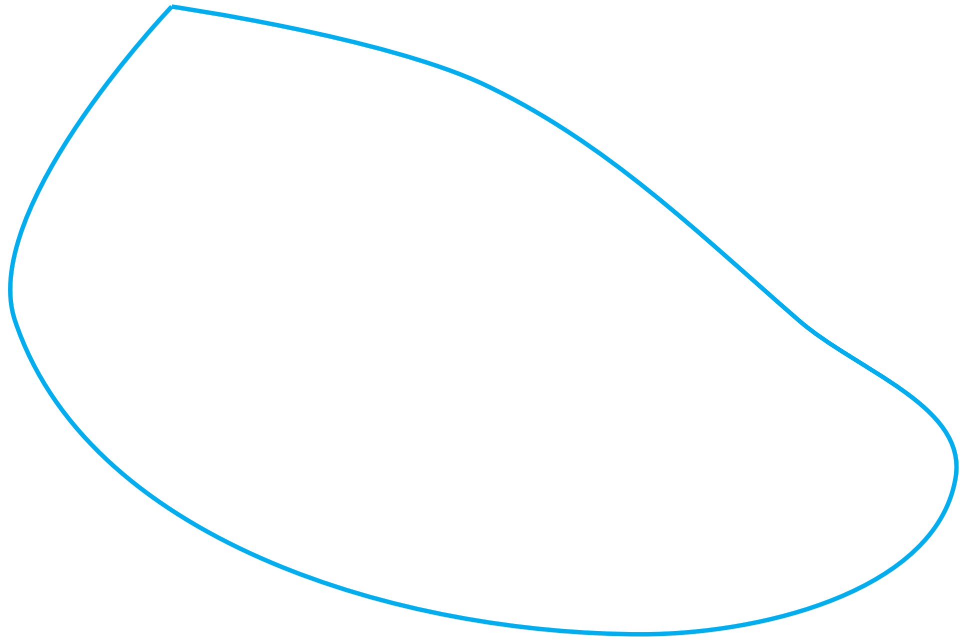
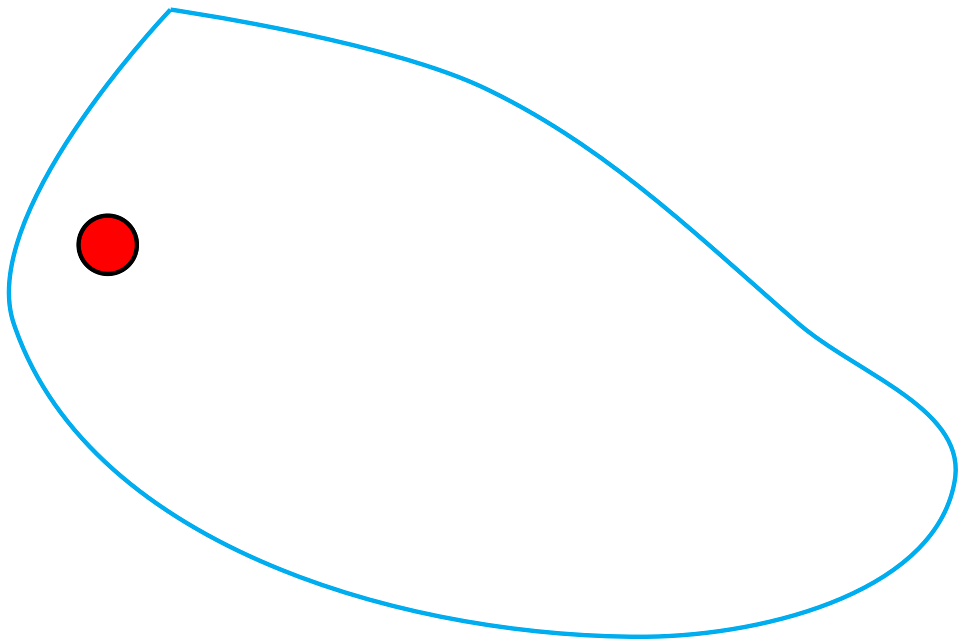


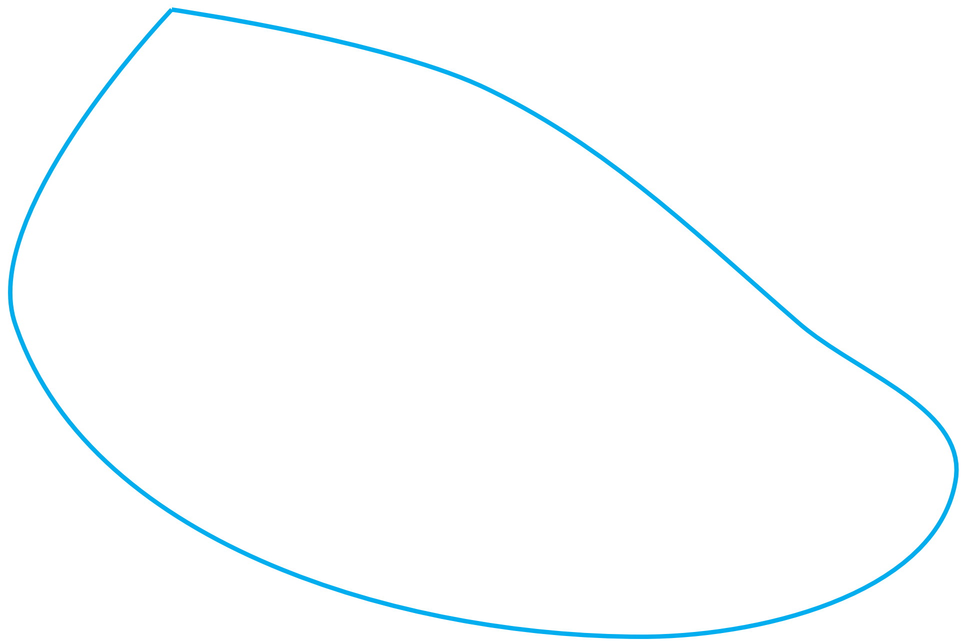
Charge can not just disappear like that!

Charge conservation



Total net charge is conserved, global charge conservation

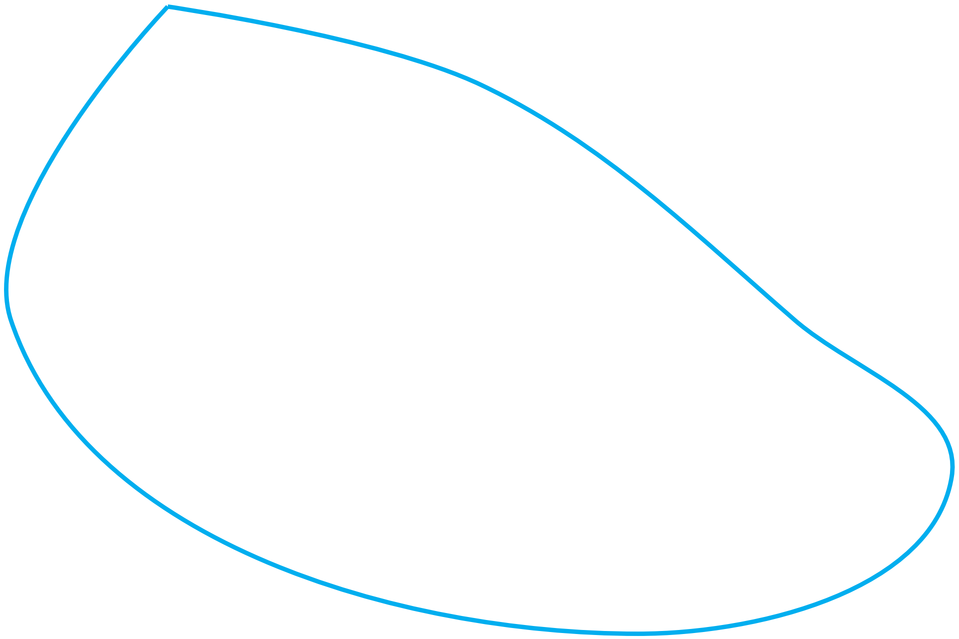
Charge conservation



but for a flicker of a second, as if charge was not conserved

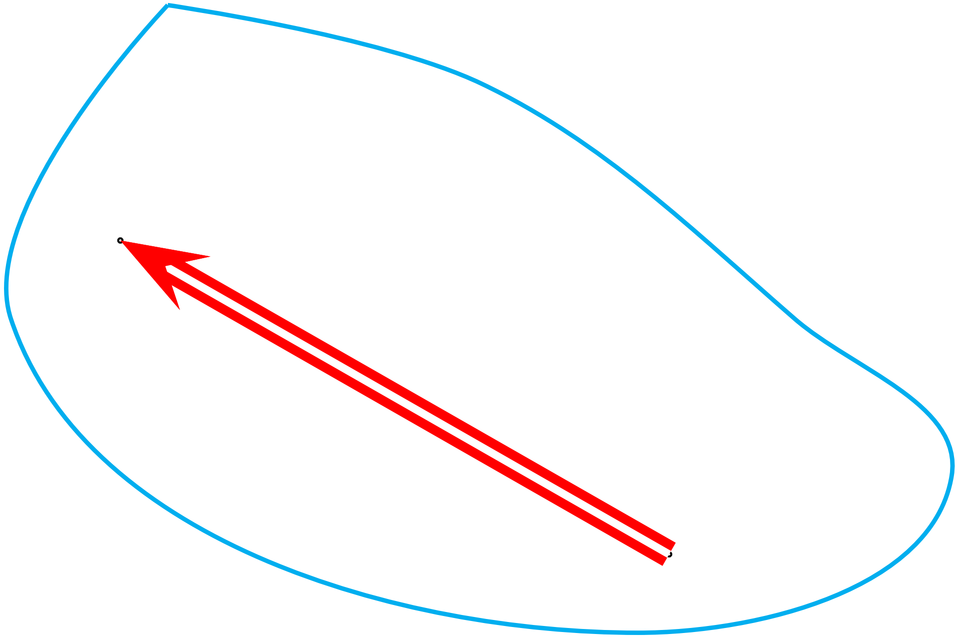
charge can not disappear at one point and reappear **instantaneously** at another point

Charge conservation



Must have Local charge conservation

Charge conservation



A change (in time) in the charge density is accompanied by a current flow

Local conservation of the electric charge

Charge must be conserved locally

continuity equation of the electric charge

$$\partial\rho/\partial t + \nabla . j = 0$$

$$\begin{aligned}
\mathbf{div}\,\overrightarrow{E} = \partial_i E_i &= \rho \quad \text{(Gauss)} \\
\mathbf{div}\,\overrightarrow{B} &= 0 \quad \text{(no magnetic charge)} \\
\mathbf{Curl}\,\overrightarrow{E} = \epsilon_{ijk}\partial_j E_k &= -\frac{\partial\,\overrightarrow{B}}{\partial t} \quad \text{(Faraday)} \\
\mathbf{Curl}\,\overrightarrow{B} &= \overrightarrow{j} \quad \text{(Ampere)}
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$$

Unfortunately the mathematics implies that

$$
\mathbf{div}(\mathbf{Curl}\overrightarrow{B}) = \nabla.(\nabla \times \mathbf{B}) = \mathbf{0} \Rightarrow \nabla.\mathbf{j} = \mathbf{divj} = \mathbf{0}
$$

$$\nabla.j = divj = 0$$

in conflict with the continuity equation, local conservation.

Maxwell equations: Unify $\overrightarrow{E}$ and $\overrightarrow{B}$

Local conservation of the electric charge

$$\partial j = 0 \qquad j^\mu = (\rho, \overrightarrow{j})$$

$$\mathbf{div}\,\overrightarrow{E} = \rho \qquad \mathbf{div}\,\overrightarrow{B} = \mathbf{0}$$

$$\mathbf{Curl}\,\overrightarrow{E} + \frac{\partial\,\overrightarrow{B}}{\partial \mathbf{t}} = \mathbf{0} \qquad \mathbf{Curl}\,\overrightarrow{B} - \frac{\partial\,\overrightarrow{E}}{\partial \mathbf{t}} = \overrightarrow{j}$$

(Gauge Invariance) electrostatic field (force) depends only on difference of potential

The quantum (for photons) is the **vector** potential $A^\mu(x) = (V, \overrightarrow{A})$,

$$\overrightarrow{B} = Curl\,\overrightarrow{A} \qquad \overrightarrow{E} = -GradV - \partial\,\overrightarrow{A}/\partial t.$$

Introduction: Electromagnetism as a prototype

Different A_μ lead to the same physical fields E, B.

Gauge invariance $A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Lambda(x)$.

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But Gauge invariance $F^{\mu\nu}(x) \to F^{\mu\nu}(x)$.

Introduction: Electromagnetism as a prototype

Different A_μ lead to the same physical fields E, B.

Gauge invariance $A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x)$.

With A_μ the equations can be written in a manifestly covariant form, with

$$\mathbf{F}^{\mu\nu} = \partial^\mu \mathbf{A}^\nu - \partial^\nu \mathbf{A}^\mu$$

$$\tilde{\mathbf{F}}^{\mu\nu} = \frac{\mathbf{1}}{\mathbf{2}} \varepsilon_{\mu\nu\rho\sigma} \partial^\nu \mathbf{F}^{\rho\sigma}$$

$$\partial_\mu \mathbf{F}^{\mu\nu} = \mathbf{j}^\nu \qquad \partial_\mu \tilde{\mathbf{F}}^{\mu\nu} = \mathbf{0}.$$

All of this can be derived from the Lagrangian

$$\mathcal{L}_{\rm em} = -\frac{\mathbf{1}}{\mathbf{4}} \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu} \equiv \frac{\mathbf{1}}{\mathbf{4}} \left((\vec{E} + \mathbf{i} \vec{B})^{\mathbf{2}} + (\vec{E} - \mathbf{i} \vec{B})^{\mathbf{2}} \right) .$$

only two Transverse polarisations/helicity states no longitudinal polarisation

Aside, Equation of motion:

The dynamics, the physics, is encoded in the action

$$S = \int d^4x \quad \mathcal{L}\left[\phi_i(x), \partial_\mu \phi_i(x)\right] \; .$$

The principle of least action: $\delta S = 0$ when varying $\delta\phi_i$ leads to the Euler–Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial^\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial^\mu \phi_i \right)} \right) = 0$$

Introduction: Electromagnetism as a prototype

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

Introduction: Electromagnetism as a prototype

The equation of motion for the free field, $j = 0$, lead to

$$\partial_\mu F^{\mu\nu} = 0 \Rightarrow \Box A^\nu - \partial^\nu \left(\partial . A\right) = 0$$

The freedom from GI allows us to take the gauge fixing

$$\partial A = 0$$

Out of the 4 degrees of freedom/components in A_μ, this condition freezes one of them

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Out of the 4 degrees of freedom/components in A_μ, this condition freezes one of them
Moreover there is still a lot freedom in $\partial A = 0$, there is still an invariance, overcounting, due to

$$A^\mu(x) + \partial^\mu \Lambda(x) \qquad \Box\Lambda = 0$$

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To remember: Working in a particular gauge, breaks gauge invariance, in fact it hides the gauge symmetry, the physics is independent of the gauge fixing

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The free field is then

$$\Box A^\nu = 0, \qquad k^2 A_\nu = 0$$

Photon is massless, it has 2 transverse polarisations

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$$\mathcal{L}_{\rm pem} = -\frac{\mathbf{1}}{\mathbf{4}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} \ -\mathbf{j}_\mu \mathbf{A}^\mu.$$

The Lagrangian is not invariant under the gauge transformation $A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Lambda(x)$ **. But the equation of motion requires** $\partial.j = 0$**.**

The action is gauge invariant but what is important is the action

$$\int \mathcal{L}_{\rm pem} = \int \left(-\frac{\mathbf{1}}{\mathbf{4}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} \ -\mathbf{j}_\mu \mathbf{A}^\mu \right) \Longrightarrow$$

$$\int \mathcal{L}_{\rm pem} = \int \left(-\frac{\mathbf{1}}{\mathbf{4}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} \ -\mathbf{j}_\mu \mathbf{A}^\mu \right) - \int \mathbf{\Lambda}\partial.\mathbf{j}$$

$$\mathcal{L}_{\rm pem} = -\frac{\mathbf{1}}{\mathbf{4}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m^2}\mathbf{A}_\mu\mathbf{A}^\mu - \mathbf{j}_\mu\mathbf{A}^\mu .$$

$$\begin{aligned} \Box A^\nu + m^2 A^\nu - \partial^\nu(\partial.A) &= j^\nu \Longrightarrow \\ m^2\partial.A &= \partial.j \end{aligned}$$

The Lagrangian and the action are not invariant under the gauge transformation
$A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu\Lambda(x)$.

But the equation of motion requires $\partial.A = 0$.

This is the spin-1 condition, no longer a gauge fixing.

Then 3 degrees of freedom. An extra Longitudinal degree of freedom

Quantization of the EM field

$$\boxed{[P, X] = i}$$

To quantize, find the conjugate momenta of each degree of freedom and impose equal time commutation

$$\Pi^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_\mu)} = F^{\mu 0} \Longrightarrow$$

$$\boxed{\Pi^0 = 0}$$

Fix a gauge for $\Pi^0 \neq 0$

$$\mathcal{L}_{\mathcal{G}_{\rm pem}} = -\frac{\mathbf{1}}{\mathbf{4}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} \; - \frac{\mathbf{1}}{\mathbf{2}\xi}\left(\partial.\mathbf{A}\right)^{\mathbf{2}}$$

Gauge Invariance and minimal subtraction, introducing interactions

Recall the Lorentz force acting on a moving particle in a electromagnetic field

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Gauge Invariance and minimal subtraction, introducing interactions

this is derived from the Hamiltonian

$$H = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + qV \quad \Leftrightarrow \quad (H - qV) = \frac{1}{2m}(\vec{p} - q\vec{A})^2$$

whereas for the corresponding free field

$$H = \frac{p^2}{2m}$$

to introduce the interaction one has made minimum substitution

$$P_\mu \longrightarrow P_\mu - qA_\mu$$

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in QM

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu = \partial_\mu + ieQA_\mu$$

D_μ is the covariant derivative.

Introduction: Gauge Invariance in Quantum Mechanics

Take Schrdinger's equation

$$(1/2m)(-i\overrightarrow{\nabla})^2\psi = i\partial\psi/\partial t$$

invariant under a *global* phase transformation

$$\psi \rightarrow \exp(i\lambda)\psi$$

what about invariance under *local* phase transformation?

$$\lambda \rightarrow q\Lambda(x = (t, \overrightarrow{x}))$$

Possible only if one introduces a *compensating* vector field which transforms exactly like A_μ ◀.

This prescription gives

$$(1/2m)\left(-i\overrightarrow{\nabla} + q\overrightarrow{A}\right)^2\psi = (i\partial/\partial t + qV)\,\psi\ .$$

This equation with interactions is invariant under gauge transformations consider free Dirac particle whose equation of motion is

$$\left(i\gamma_\mu \partial^\mu - m\right)\psi = \left(i\not{\partial} - m\right)\psi = 0$$

easily derived from the matter Lagrangian

Interaction of electrons

consider free Dirac particle whose equation of motion is

$$\Big(i\gamma_\mu \partial^\mu - m\Big)\psi = \Big(i\partial\!\!\!/ - m\Big)\psi = 0$$

easily derived from the matter Lagrangian

$$\mathcal{L}_{\rm M} = \overline{\psi}\Big(i\partial\!\!\!/ - m\Big)\psi$$

The interaction is obtained through the covariant derivative, leading to

$$\begin{aligned}
\mathcal{L} &= \overline{\psi}\Big(iD\!\!\!\!/ - m\Big)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \mathcal{L}_{\rm M} + \mathcal{L}_{\rm G} + \mathcal{L}_{\rm I} \\
&= \overline{\psi}\Big(i\partial\!\!\!/ - eQA\!\!\!/ - m\Big)\psi + \mathcal{L}_{\rm G} \\
\mathcal{L}_{\rm I} &= -eJ^Q_\mu A^\mu \qquad \frac{\partial \mathcal{L}}{\partial A_\mu} = -eJ^Q_\mu \quad \partial_\mu J^\mu = 0
\end{aligned}$$

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\end{aligned}$$

the electromagnetic spin-1/2 current

$$J^Q_\mu = Q\,\bar{\psi}\gamma_\mu\psi$$

$$\mathcal{L}_{\rm I} = -e J^Q_\mu A^\mu$$

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Interaction of electrons: Gauge Invariance

$$\mathcal{L}_{\mathrm{M}} = \overline{\psi}\Big(i\partial\!\!\!/ - m\Big)\psi$$

The free Dirac Lagrangian is invariant under a global symmetry : phase transformation

$$\psi \to U\psi \quad U = exp(i\lambda) \quad U^{\dagger}U = 1 \quad U \text{is unitary}$$

global means rigid, same for all x

Interaction of electrons: Gauge Invariance

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global means rigid, same for all x Promoting U to a local symmetry $\lambda \to \lambda(x)$ only possible by using covariant derivatives, compensating gauge field.

Interaction of electrons: Gauge Invariance

$$\mathcal{L}_{\rm M} \to \mathcal{L}_{\rm QED} \quad = \quad \overline{\psi}\Big(i\not{D} - m\Big)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

For $\lambda = \lambda(x)$

$$\overline{\psi}\not{\partial}\psi \to \overline{\psi}U^\dagger\not{\partial}\Big(U\psi\Big) \neq \overline{\psi}\not{\partial}\psi$$

must have

$$\overline{\psi}\not{D}\psi \to \overline{\psi}U^\dagger\not{D}'\Big(U\psi\Big) = \overline{\psi}\not{D}\psi$$

$$D' = UDU^\dagger \quad \Big(D\psi\Big)' = U\Big(D\psi\Big)$$

must require that

$$\psi \to U\psi \qquad A_\mu \to A_\mu + \partial_\mu \Lambda$$

$\lambda = -eQ\Lambda(x)$ Universality

QED U(1) Abelian theory

$F_{\mu\nu}$ as a covariant derivative

$$
\begin{aligned}
[D_\mu, D_\nu]\psi &= \left(\left(\partial_\mu + ieA_\mu\right)\left(\partial_\nu + ieA_\nu\right) - \mu \leftrightarrow \nu\right)\psi \\
&= ie\left(\partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]\right)\psi \\
&= ie\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\psi \qquad \text{Abelian}
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&= ie\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\psi \qquad \text{Abelian} \\
[D_\mu, D_\nu] &\equiv ieF_{\mu\nu} \\
F_{\mu\nu} &\rightarrow UF_{\mu\nu}U^\dagger \\
\mathrm{Tr}\left(F_{\mu\nu}F_{\mu\nu}\right) &\rightarrow \mathrm{Tr}\left(UF_{\mu\nu}F_{\mu\nu}U^\dagger\right) = \mathrm{Tr}F_{\mu\nu}F_{\mu\nu}
\end{aligned}
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charges: $\psi \rightarrow Q$, the antiparticle $\overline{\psi} \rightarrow -Q$ the photon $A_\mu \rightarrow Q = 0$ Lagrangian density has of course no charge whatsoever, is a true scalar in all respects.

Interaction of electrons: Gauge Invariance, Recap

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The mass term does not break charge (gauge) symmetry

Interaction of electrons: Gauge Invariance, Recap

ψ has 2 chirality states

$$\psi = \psi_L + \psi_R = P_L\psi + P_R\psi \quad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$$

$$\overline{\psi}_L = \overline{\psi} P_R \quad \overline{\psi}_R = \overline{\psi} P_L$$

The em current conserves chirality and from the point of view of the gauge interaction each component $\psi_{L,R}$ does not talk to each other

$$J_\mu^{\rm e.m} = Q_L \overline{\psi}_L \gamma_\mu \psi_L + Q_R \overline{\psi}_L \gamma_\mu \psi_L \qquad Q_L = Q_R = Q$$

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$\psi_L \rightleftharpoons \psi_R$ through the mass

$$m\left(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right) = m\overline{\psi}_R\psi_L + h.c \qquad m = m^\star$$

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in general ψ_L and ψ_R have different transformation properties. When they transform similarly, one has a vector theory, like in QED

Chiral limit

in the limit $m \longrightarrow 0$ there is another (global) symmetry

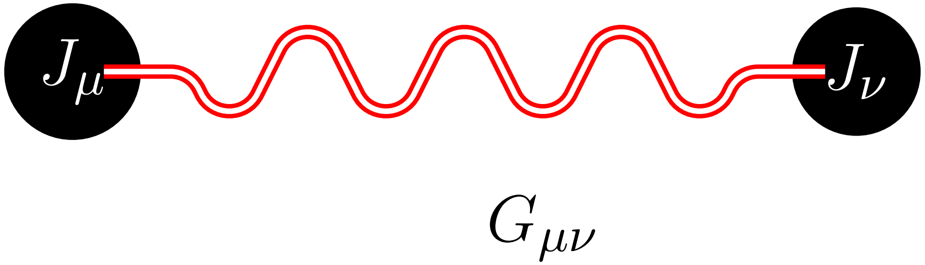
$$\psi \rightarrow exp(i\lambda_5\gamma_5)$$

$$\overline{\psi}_L\gamma_\mu\psi_L \rightarrow \overline{\psi}_L\gamma_\mu\psi_L \quad \overline{\psi}_R\gamma_\mu\psi_R \rightarrow \overline{\psi}_R\gamma_\mu\psi_R$$

but

$$m\overline{\psi}\psi \rightarrow \neq m\overline{\psi}\psi$$

Current-Current



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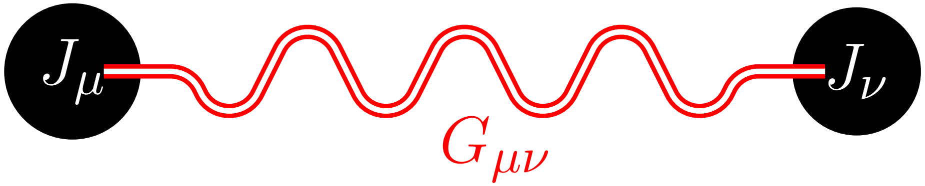

$G_{\mu\nu}$ is the Green's function or propagator

.

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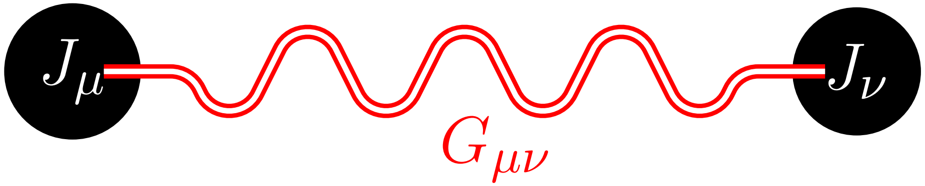

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$$\partial_\mu F^{\mu\nu} = J^\nu \Rightarrow A_\mu = G_{\mu\nu}J^\nu \quad \text{or}$$
$$G^{-1}_{\mu\nu}A_\mu = J^\nu \Rightarrow G^{-1}_{\mu\nu} = \Box g_{\mu\nu} - \partial_\mu\partial_\nu$$

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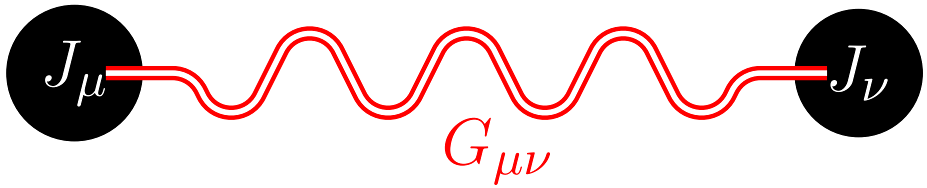

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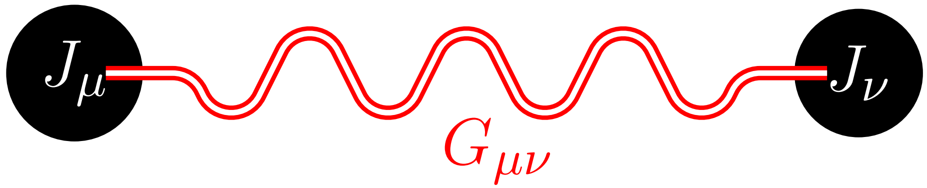
what is the inverse, G?

$$G^{-1}_{\mu\nu} G^{\nu\rho} = g^\rho_\mu \quad G^{\mu\rho} \text{does not exist !}$$



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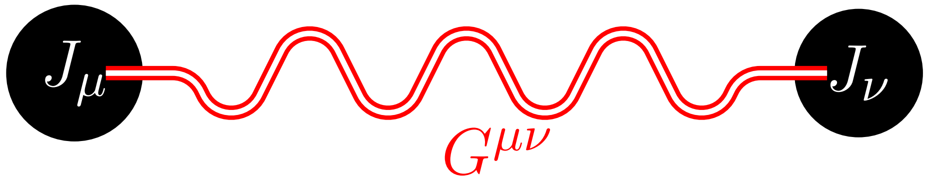
must include gauge fixing

$$\mathcal{L}_G = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial A)^2$$

$$G_{\mu\nu} = \frac{-i}{k^2}\left(g_{\mu\nu} - \underbrace{(1-\xi)\frac{k_\mu k_\nu}{k^2}}_{\text{unphysical longitudinal part}}\right) \qquad k.J = 0 \qquad \xi = 1 \ \text{Feynman Gauge}$$

Current-Current



The amplitude is

$$\mathcal{M} = e^2 J^{em}_\mu G^{\mu\nu} J^{em}_\nu = -i\frac{e^2}{k^2} J^{em}.J^{em} \hookrightarrow$$

$$\mathcal{L}_{\text{eff.}} = -\frac{e^2}{2}\frac{J^{em}_\mu J^{em}_\mu}{k^2}$$

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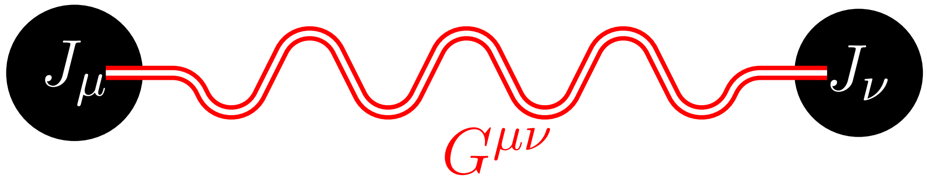

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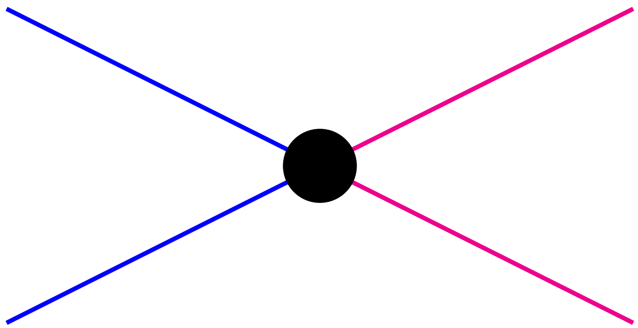
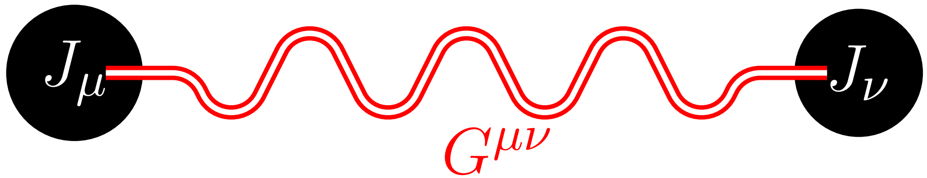
Contact interaction



Current-Current



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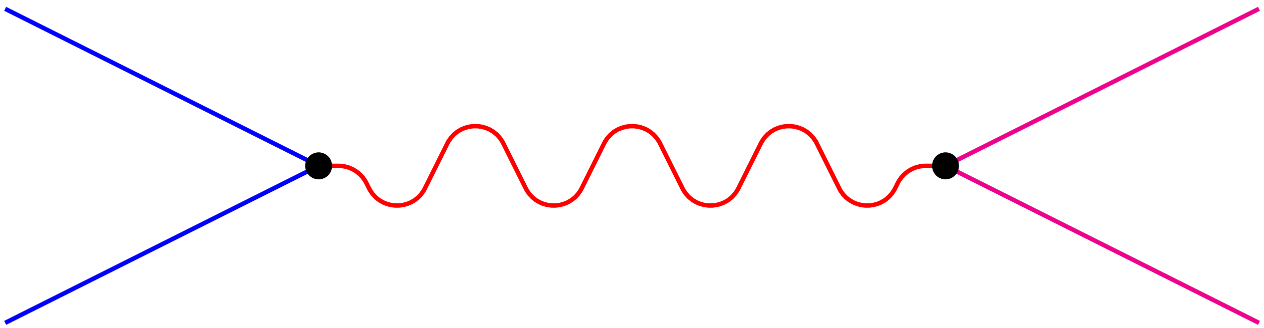
Fundamental interaction



Massive case

$$\mathcal{L}_G = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2A^2$$

$$G_{\mu\nu} = \frac{-i}{k^2 - M^2}\Bigg(g_{\mu\nu} - \underbrace{\frac{k_\mu k_\nu}{M^2}}_{\text{longitudinal part}}\Bigg)$$

$$\mathcal{L}_{\text{eff.}} = -\frac{e^2}{2}\frac{J_\mu^{em} J_\mu^{em}}{k^2 - M^2}$$

$$\mathcal{L}_{\text{eff.}} = \frac{e^2}{2}\frac{J_\mu^{em} J_\mu^{em}}{M^2} \equiv G_m J_\mu^{em} J_\mu^{em} \qquad k^2 \ll M^2$$

Weak Interactions and non Abelian theories

First time we became aware of a new type of interaction was at play was through the discovery of

β-decay: $n \to p + e^- + \bar{\nu}_e$ (Fermi 1933)

Difficult to accept and set up theoretically.

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Fermi's article to Nature rejected: "contains speculations too remote from reality to be of interest to the reader"...

It was also found that

- β-decay: $n \longrightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,
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- a change in the identity of the fermion
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Fermi postulated a current-current interaction

$$\mathcal{L}_{\rm Fermi} = -\frac{4G_F}{\sqrt{2}} J^+_\mu J^\mu - \qquad \frac{G_F}{\sqrt{2}} = 1.03510^{-5} M_P^{-2}$$

$$J_\mu = L_\mu^{\rm leptons} + H_\mu^{\rm hadrons}$$

Structure of the current was purely $(V-A)$: Parity violation

restrict myself to leptonic current

$$\begin{aligned} J^-_\mu &= \bar{e}\gamma_\mu \frac{(1-\gamma_5)}{2}\nu_e + \bar{\mu}\gamma_\mu \frac{(1-\gamma_5)}{2}\nu_\mu + \bar{\tau}\gamma_\mu \frac{(1-\gamma_5)}{2}\nu_\tau \\ &= \bar{e}\gamma_\mu \nu_{e_L} + \cdots \\ J^+_\mu &= (J^+_\mu)^\dagger = \bar{\nu}_{e\ L}\gamma_\mu e_L + \cdots \\ \text{analogy} \quad J^{em}_\mu &= \bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R \end{aligned}$$

for em same entity $e \leftrightarrows e$ here $e_L \leftrightarrows \nu_e$ looks like it is not the same entity, some charge not conserved. Nope.

Make it $E_L \leftrightarrows E_L$

$$J_\mu = \bar{e}\gamma_\mu \nu_{e_L} + \cdots$$

$$J_\mu = \bar{E}_L \;\; \gamma_\mu \;\; ? \;\; E_L$$

Define the doublet

$$E_L = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \qquad \bar{E}_L = \begin{pmatrix} \bar{\nu}_{e_L} & \bar{e}_L \end{pmatrix}$$

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$$\begin{aligned} J^+_\mu &= \bar{\nu}_{e_L}\gamma_\mu e_L = \begin{pmatrix} \bar{\nu}_{e_L} & \bar{e}_L \end{pmatrix} \gamma_\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \bar{E}_L \underbrace{\gamma_\mu}_{\text{spin}} \underbrace{\tau^+}_{\text{weak isospin}} E_L \\ &= \sqrt{2}\bar{E}_L \, \gamma_\mu T^+_L E_L \end{aligned}$$

This is the same maths are your spin/ 2-level system in QM

$$\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$$

QM of rotations:

$$O(3) \sim SU(2)$$

Pauli matrices, fundamental representation

Pauli matrices, 3 generators

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau^{\pm} = \frac{1}{\sqrt{2}}\left(\tau_1 \pm i\tau_2\right)$$

$$\left[\frac{\tau_i}{2}, \frac{\tau_j}{2}\right] = i f_{ijk}\frac{\tau_k}{2} = i\epsilon_{ijk}\frac{\tau_k}{2} \qquad f_{ijk}\text{structure constants}$$

$$\sum_i (t^i t^i)_{ab} = C_F \delta_{ab} = \frac{N^2-1}{2N}\delta_{ab} = \frac{3}{4}\delta_{ab}$$

$$\sum_{ij} f_{ijk} f_{ijl} = C_A \delta_{kl} = N\delta_{kl} = 2\delta_{kl}$$

where is τ_3??

We are forced to consider the group $SU(2)_{\rm W}$:

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\begin{aligned}
J_\mu^+ &= \bar{E}_L\,\gamma_\mu \frac{\tau^+}{\sqrt{2}} E_L \rightarrow \\
J_\mu^3 &= \bar{E}_L\,\gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2}\left(\bar{\nu}_{e_L}\gamma_\mu \nu_{e_L} - \bar{e}_L \gamma_\mu e_L\right)
\end{aligned}
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This is a neutral current but it is not the em current, this would have been too good

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However part of J^{em}_μ is contained in J^3_μ

The rest of the current and the hypercharge current

$$
\begin{aligned}
J_\mu^3 &= \bar{E}_L\, \gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2}\Big(\bar{\nu}_{e_L}\gamma_\mu \nu_{e_L} - \bar{e}_L \gamma_\mu e_L \Big) \\
J_\mu^{em}(Q) &= Q \Big(\bar{e}_L\gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R \Big) = - \Big(\bar{e}_L\gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R \Big) \\
&= - \Big(\bar{e}_R\gamma_\mu e_R \Big) + J_\mu^3 - \frac{1}{2}\Big(\bar{\nu}_{e_L}\gamma_\mu \nu_{e_L} + \bar{e}_L \gamma_\mu e_L \Big) \\
&= J_\mu^3 - \frac{1}{2} \bar{E}_L \gamma_\mu \mathbb{I} E_L - \Big(\bar{e}_R\gamma_\mu e_R \Big) = J_\mu^3 + Y_\mu
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Y_μ is the hypercharge current

The rest of the current and the hypercharge current

$$J^3_\mu = \bar{E}_L \gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2}\left(\bar{\nu}_{e_L}\gamma_\mu \nu_{e_L} - \bar{e}_L\gamma_\mu e_L\right)$$

$$J^{em}_\mu(Q) = Q\left(\bar{e}_L\gamma_\mu e_L + \bar{e}_R\gamma_\mu e_R\right) = -\left(\bar{e}_L\gamma_\mu e_L + \bar{e}_R\gamma_\mu e_R\right)$$

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$$Y_\mu = y_{e_R}(\bar{e}_R\gamma_\mu e_R) + y_{E_L}(\bar{E}_L\gamma_\mu \mathbb{I} E_L)$$

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\end{aligned}
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$$
Y_\mu = y_{e_R}(\bar{e}_R\gamma_\mu e_R) + y_{E_L}(\bar{E}_L\gamma_\mu \mathbb{I} E_L)
$$

This is indeed a $U(1)$ current

E_L is a doublet and there is a separate entity e_R which is a singlet (under $SU(2)$) each entity has its own hypercharge

$$
y_{e_R} = -1 \qquad\qquad y_{e_L} = -1/2
$$

$$
\begin{aligned}
Q &= T_3 + \frac{Y}{2} = \frac{\tau_3}{2} + y \\
Q_{e_R} &= 0 - 1 = -1 \qquad\qquad Q_{e_L} = -\frac{1}{2} - \frac{1}{2} = -1 \\
Q_{\nu_L} &= +\frac{1}{2} - \frac{1}{2} = -1
\end{aligned}
$$

For the effective QED operator and with $\alpha = e^2/4\pi$

$$\mathcal{L}_{\rm eff.} = -\frac{e^2}{2}\frac{J_\mu^{em}(e^+,e^-)J_\mu^{em}(\mu^+,\mu^-)}{k^2}$$

The cross section $e^+e^- \rightarrow \mu^+\mu^-$ behaves as

$$\sigma \propto \alpha^2/s$$

decreases as the energy decreases. Less probability, $\mathcal{P}$, to produce muons.

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For the Fermi interaction

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The cross section $e^-\bar{\nu}_e \rightarrow \mu^-\bar{\nu}_\mu$ behaves as

$$\sigma \propto G_F^2 \times s$$

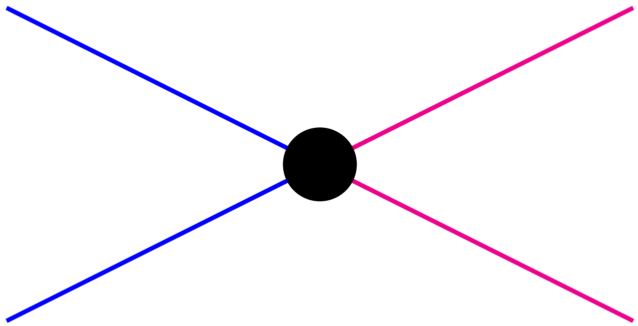
The probability $\mathcal{P}$ increases indefinitely.

But $\mathcal{P} < 1$, unitarity must be preserved.

This means something must happen at some energy to restore $\mathcal{P} < 1$

or theory not good!

Fermi Contact interaction



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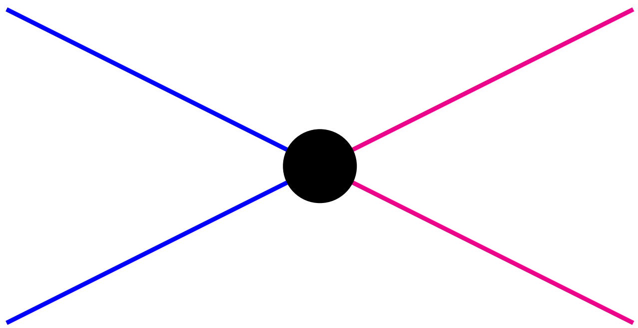
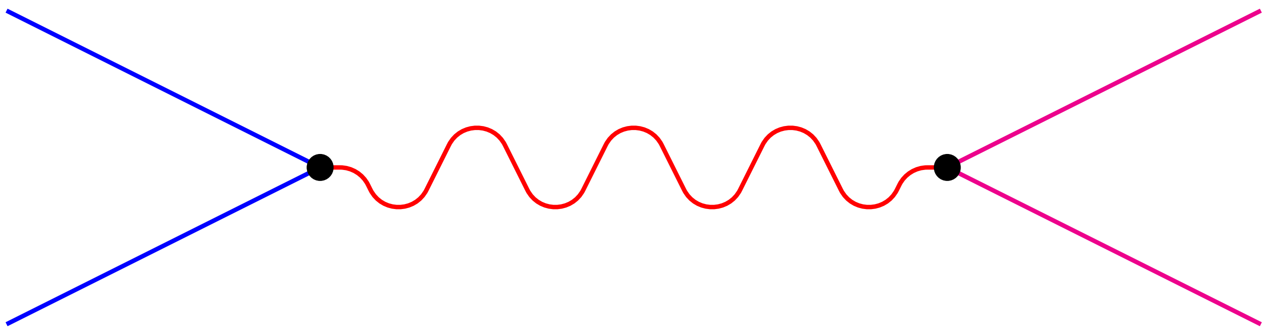


Where is the underlying fundamental interaction?



To each current associate a vector particle, spin-1, a gauge particle

QED recap

$$J_\mu^{em} \longrightarrow eJ_\mu(Q)A^\mu \qquad \mathcal{L} = -eJ_\mu(Q)A^\mu$$

Turn the derivatives of the free Lagrangian into covariant derivatives to get the interaction with the gauge field

$$\mathcal{L}_{int,QED} = i\bar{\psi}_e\gamma^\mu\left(\partial_\mu + ieQA_\mu\right)\psi_e$$

Universality

$$e \leftrightarrow A_\mu$$

To each current associate a vector particle, spin-1, a gauge particle

$$J_\mu^{i=\pm,3} \longrightarrow W_\mu^{i=\pm,3} \longrightarrow g$$
$$Y_\mu \longrightarrow B_\mu \longrightarrow g'$$

$g \neq g'$ (partial unification)

To each current associate a vector particle, spin-1, a gauge particle

$$
\begin{aligned}
\mathcal{L} \quad = \quad & i\bar{E}_L\gamma_\mu\left(\partial_\mu \mathbb{I} + ig\left(\frac{\tau^i}{2}\right)W^i_\mu + ig'(y_{E_L})\mathbb{I}B_\mu\right)E_L \\
+ \quad & i\bar{e}_R\gamma_\mu\left(\partial_\mu \qquad\qquad\qquad\qquad + ig'(y_{e_R})B_\mu\right)e_R
\end{aligned}
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$$\begin{aligned}\mathcal{L} &= i\bar{E}_L\gamma_\mu\left(\partial_\mu \mathbb{I} + ig\left(\frac{\tau^i}{2}\right)W^i_\mu + ig'(y_{E_L})\mathbb{I}B_\mu\right)E_L \\ &+ i\bar{e}_R\gamma_\mu\left(\partial_\mu \qquad\qquad\qquad\qquad + ig'(y_{e_R})B_\mu\right)e_R\end{aligned}$$

$$\begin{aligned}\tau^i W^i &= \tau^3 W^3 + \tau^1 W^1 + \tau^2 W^2 = \tau^3 W^3 + \tau^+ W^+ + \tau^- W^- \\ W^\pm_\mu &= \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}\end{aligned}$$

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$$\tau^iW^i = \tau^3W^3 + \tau^1W^1 + \tau^2W^2 = \tau^3W^3 + \tau^+W^+ + \tau^-W^-$$
$$W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}$$

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\left(J^+_\mu W^+ + J^-_\mu W^{-,\mu}\right)$$

$W^\pm$ have electric charge, they should couple to the photon

Counting the gauge fields one has 4 3 of the triplet W and B.
The photon must emerge as the physical field A_μ of $W^3 - B$. The other orthogonal physical field is the Z_μ boson. Two requirements, i) A_μ couples to the em current ii) with strength e.

$$\mathcal{L}_{NC} = -\left(gJ_\mu^3 W^3 + g' Y_\mu B^\mu\right) = -eJ_\mu(Q)A^\mu - g_Z J_\mu^Z Z^\mu$$

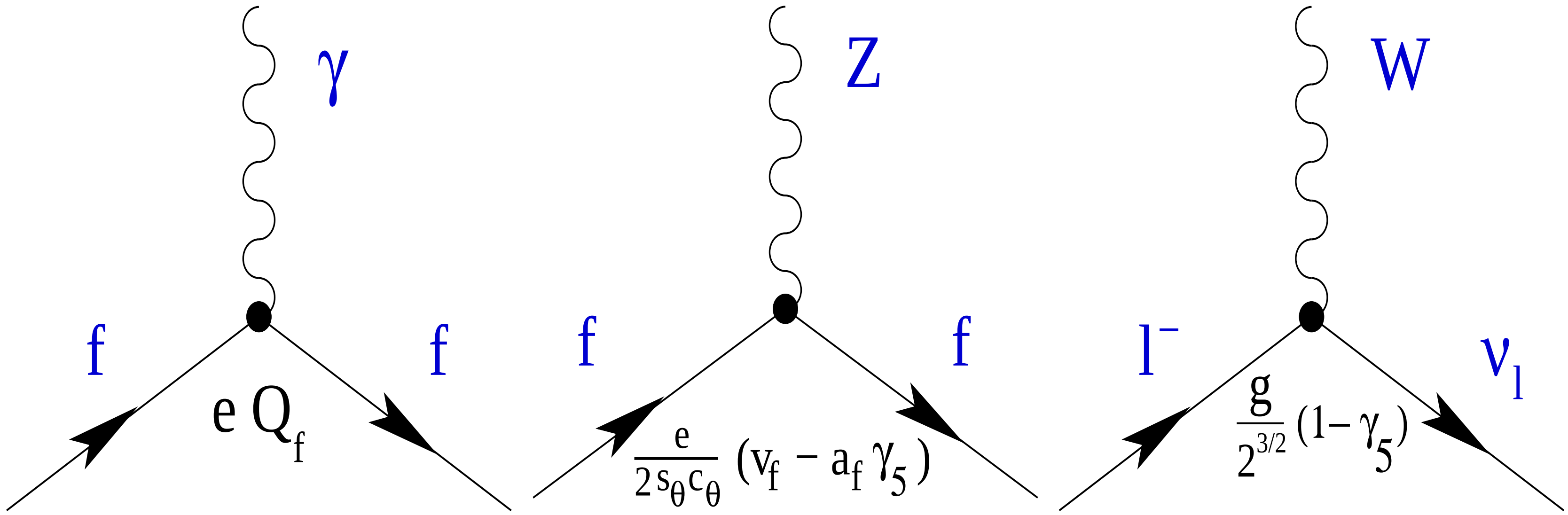
$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} .$$

using $J_\mu^Q = J_\mu^3 + Y_\mu$

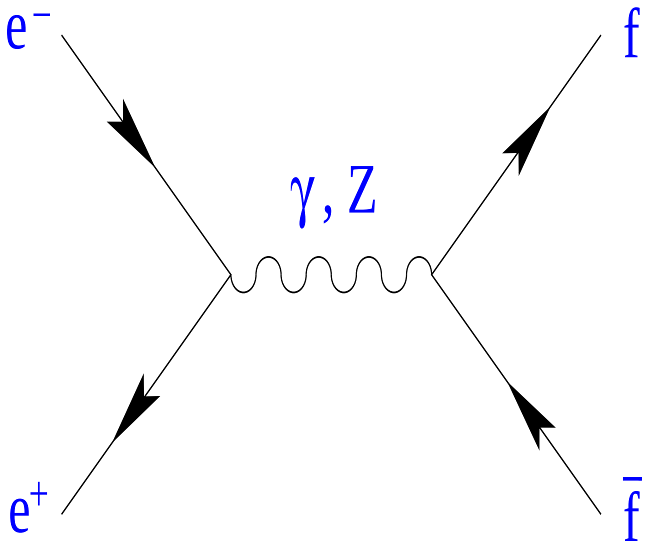
$$g \sin\theta_W = g' \cos\theta_W = e$$

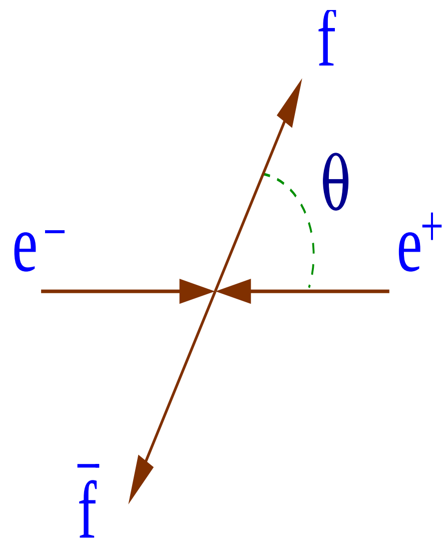
$$J_\mu^Z = J_\mu^3 - \sin^2\theta_W J_\mu^Q \quad ; \qquad g_Z = \frac{g}{\cos\theta_W} = \frac{g}{\cos\theta_W \sin\theta_W}$$

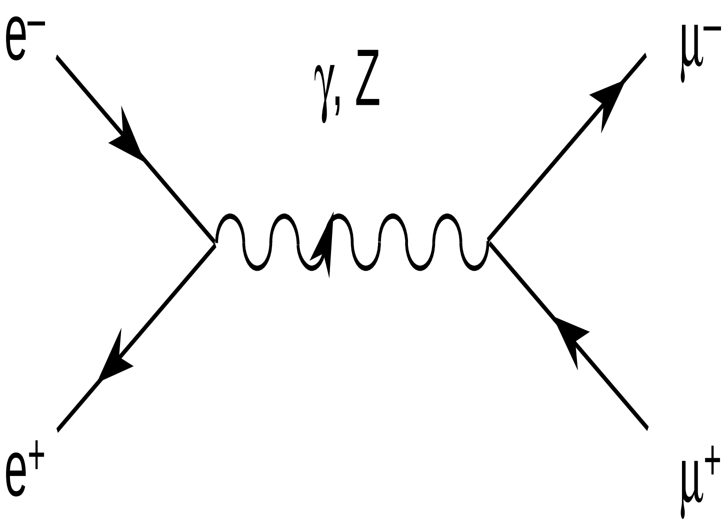
Feynman Rules, coupling g, s_W unspecified as yet

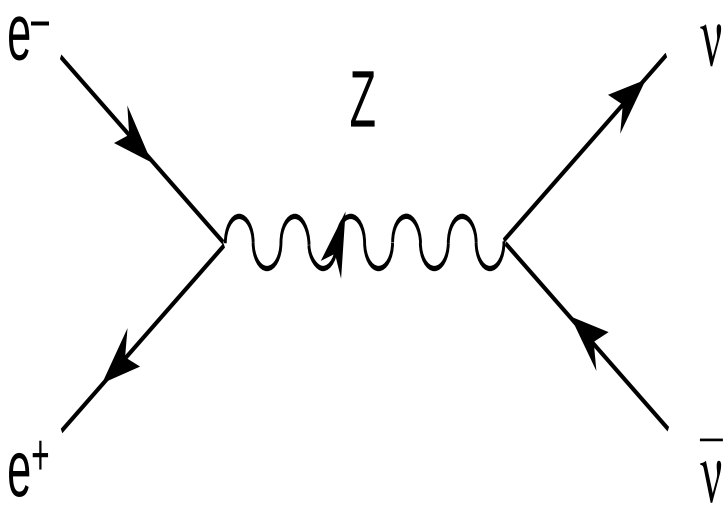


Cross sections









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- β-decay: $n \longrightarrow p \; + \; e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \longrightarrow \nu_\mu \; + \; e^- + \bar{\nu}_e$ leptonic decay

were of the same nature and have the same strength.

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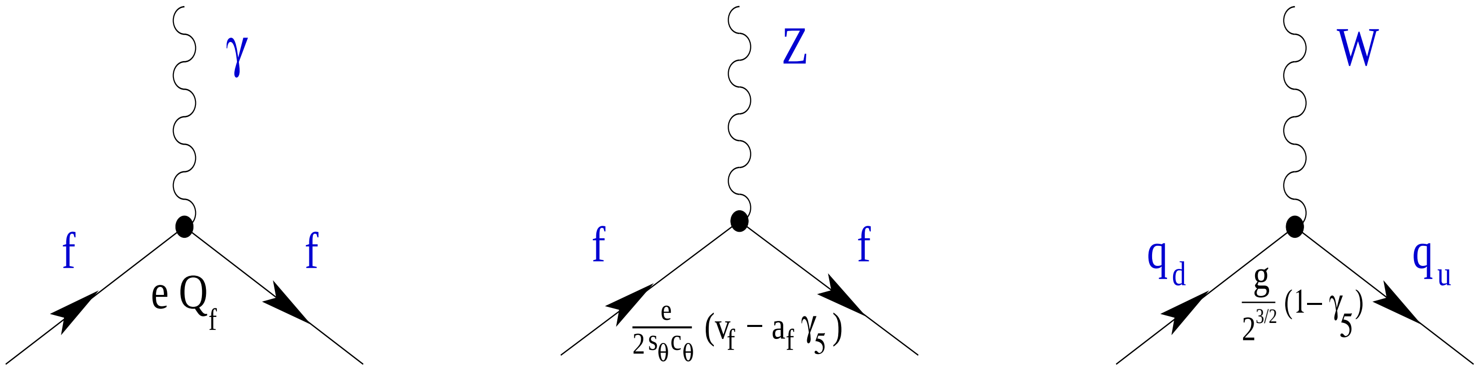
$Q_u = 2/3 \quad Q_d = -1/3 \quad \begin{pmatrix} u \\ d \end{pmatrix}_L$ and singlets $\quad u_R; d_R$ (both $_R$ have hypercharge)

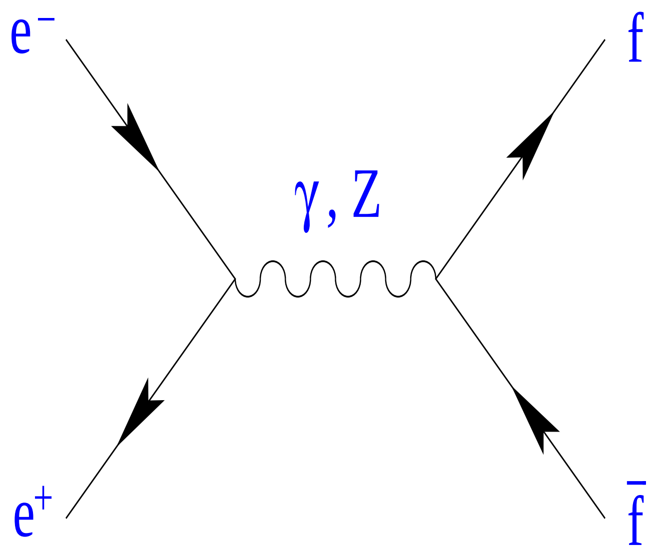
Feynman Rules, coupling g, s_W unspecified as yet

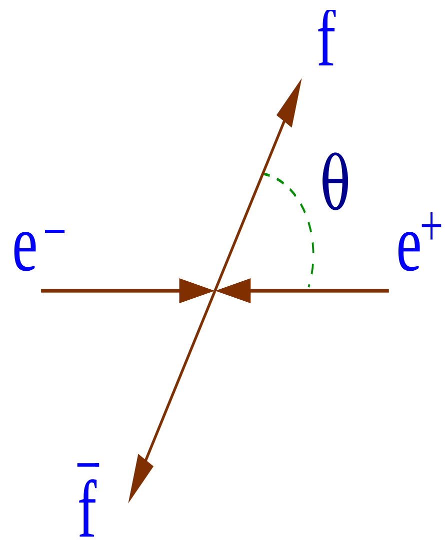


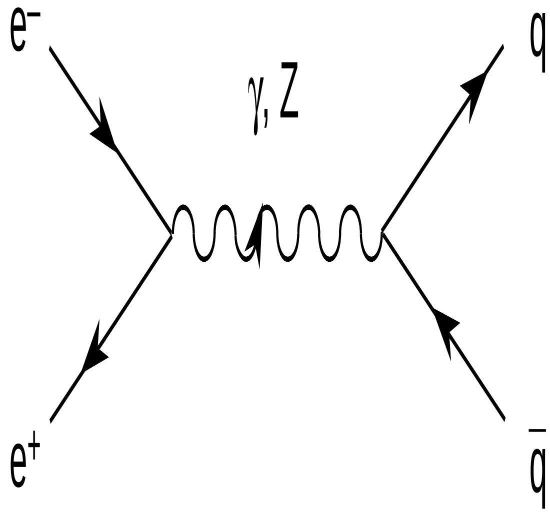
$$\mathcal{L}_{\rm NC}^{Z} = -\frac{e}{2\sin\theta_W\cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu(v_f - a_f\gamma_5) f\,, \quad a_f = T_3^f \quad v_f = T_3^f\,(1-4|Q_f|\sin^2\theta_W)$$

	u	d	ν_e	e
$2\,v_f$	$1-\frac{8}{3}\sin^2\theta_W$	$-1+\frac{4}{3}\sin^2\theta_W$	1	$-1+4\sin^2\theta_W$
$2\,a_f$	1	-1	1	-1

Cross sections







When the precision got better (1960) it was in fact found that the Fermi constant in β decay was 3% smaller than that measured in muon decay!

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Moreover at that time we knew of strange particles, $\Lambda \equiv (uds)$ for example.
Λ seemed to decay like the neutron but the associated effective coupling was measured much smaller: strangeness suppression!

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e \equiv s \rightarrow u + e^- + \bar{\nu}_e$$

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breakdown of universality? gauge principle ???

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breakdown of universality? gauge principle ???

Universal coupling but apparent non universality due to mixing, cf Z, W

Inclusion of hadrons

$$
\begin{aligned}
J^-_\mu &= \cos\theta_c \bar{u}\gamma_\mu P_L d + \sin\theta_c \overbrace{\bar{u}\gamma_\mu P_L s}^{\Delta s \neq 0} \\
&= \bar{u}\gamma_\mu P_L \underbrace{\left(\cos\theta_c d + \sin\theta_c s\right)}_{d'}
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\end{aligned}
$$

$$\cos\theta_c = V_{ud} = 0.97$$

$$\sin\theta_c = V_{us} = 0.24$$

$$J^-_\mu = \cos\theta_c \bar{u}\gamma_\mu P_L d + \sin\theta_c \overbrace{\bar{u}\gamma_\mu P_L s}^{\Delta s \neq 0}$$
$$= \bar{u}\gamma_\mu P_L \underbrace{\left(\cos\theta_c d + \sin\theta_c s\right)}_{d'}$$

with d' universality reinstated

use weak current eigenstate

$$U_L = \begin{pmatrix} u \\ d' \end{pmatrix}_L$$

what to do with the orthogonal state to d'?

$$s'_L = -\sin\theta_c d + \cos\theta_c s$$

$$J_\mu^\pm = \bar{U}_L \gamma_\mu \tau^+ U_L$$

$$\Downarrow$$

$$J_\mu^3 = \frac{1}{2}\left(\bar{u}_L \gamma_\mu u_L - \bar{d}'_L \gamma_\mu d'_L\right)$$

$$= \frac{1}{2}\left(\bar{u}_L \gamma_\mu u_L - \cos^2\theta_c \bar{d}_L \gamma_\mu d_L - \sin^2\theta_c \bar{s}_L \gamma_\mu s_L - \sin\theta_c \cos\theta_c \left(\underbrace{\bar{d}_L \gamma_\mu s_L \bar{s}_L \gamma_\mu d_L}_{\Delta S=1\ ,\ FCNC}\right)\right)$$

of course one requires the em current to be diagonal

$$J_\mu(Q) = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\left(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s\right)$$

$$= J_\mu^3 + Y_\mu$$

ex. find all quantum numbers of s including ...s_R

$\Delta S = 1$ lead to FCNC $K_0(d\bar{s}) \to \mu^+\mu^-$ occurs at tree-level and leads to a large rate for this decay! But experimentally

$$B_{K\mu} = \frac{\Gamma(K^0 \to \mu^+\mu^-)}{K^+ \to \mu^+\nu_\mu} \sim 10^{-8}$$

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The sdZ coupling must be eliminated!

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GIM=Glashow Illiopoulos Maiani postulate that a cousin of the u exists, the c quark that should form a doublet with s'_L

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$$J^+_\mu = \underbrace{\overline{\begin{pmatrix} u, & c \end{pmatrix}_L}}_{\text{family space}} \; \gamma_\mu \; \tau^+ V_{\text{Cabbibo}} \begin{pmatrix} d \\ s \end{pmatrix} \Big\} \text{family space}$$

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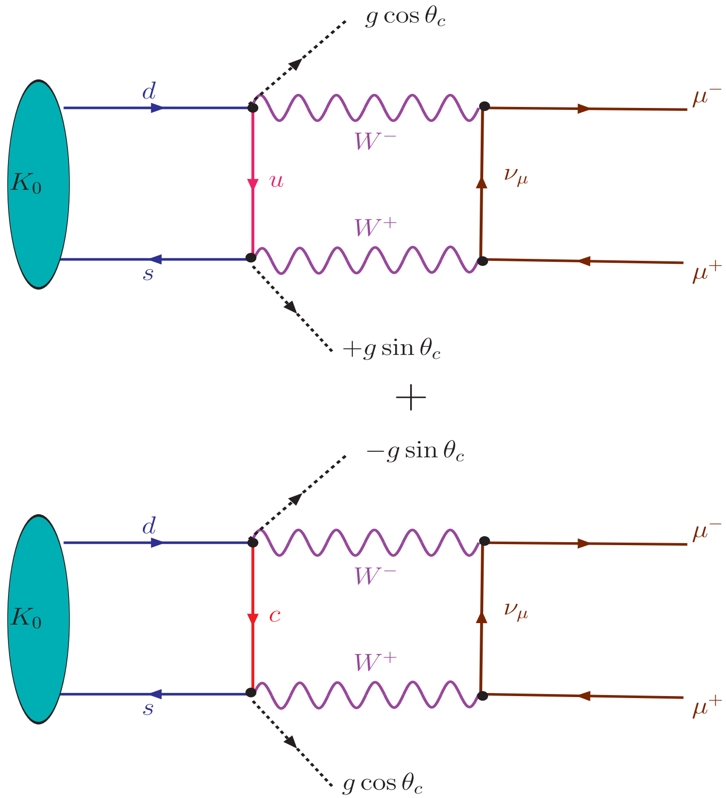
With a bonus!

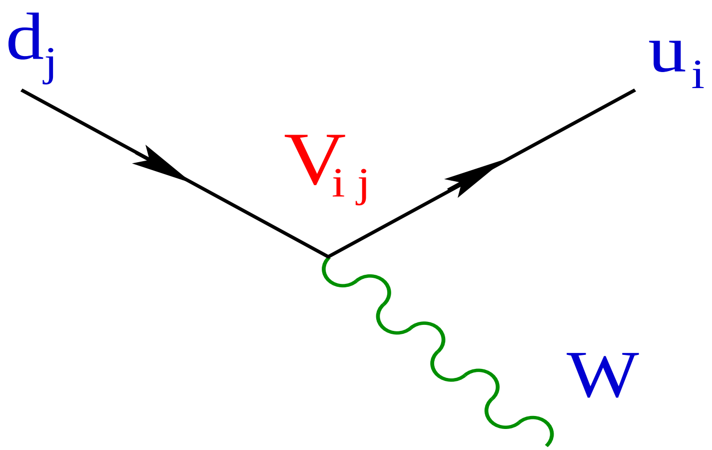


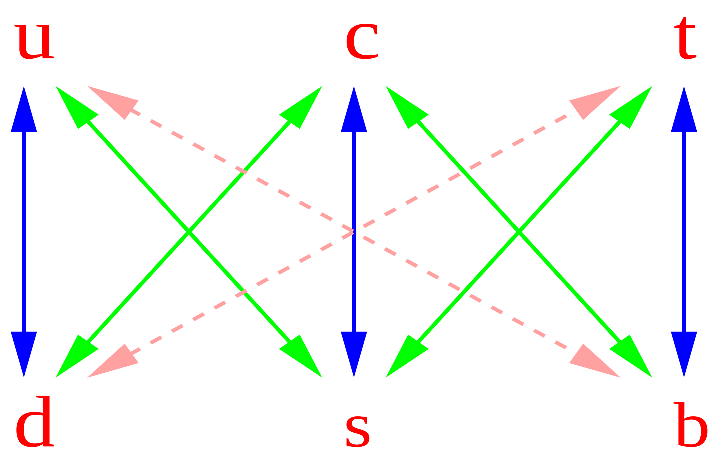
if $m_u = m_c$ total cancellation, GIM suppression even at one-loop.

if $m_c \gg mu_u$ rate still too large. J. Ellis and M.K. Gaillard, predicted m_c in the range $1-3$GeV to account for the experimental rate.

Loop calculations and masses!

$$J_\mu^+ = \underbrace{\overline{\begin{pmatrix} u & c & t \end{pmatrix}_L}}_{\text{gauge eigenstates}} \gamma_\mu\, \tau^+ V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \Bigg\} \text{family space}$$





$$\mathbf{G = SU(2)_L \otimes U(1)_Y} \text{ theory}$$

$$\psi_1(x) = \begin{pmatrix} u \\ d \end{pmatrix}_L , \qquad \psi_2(x) = u_R , \qquad \psi_3(x) = d_R .$$

$$\psi_1(x) = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \qquad \psi_2(x) = \nu_{eR} , \qquad \psi_3(x) = e^-_R .$$

As in the QED start from the free Lagrangian (no masses)

$$\mathcal{L}_0 = \sum_{j=1}^{3} i\,\overline{\psi}_j(x)\,\gamma^\mu\,\partial_\mu\psi_j(x)$$

$\mathcal{L}_0$ is invariant under global G transformations in flavour space:

$$\psi_1(x) \xrightarrow{G} \psi_1'(x) \equiv \exp\{iy_1\beta\}\, U_L\, \psi_1(x), \qquad U_L \equiv \exp\left\{i\,\frac{\tau_i}{2}\,\alpha^i\right\}\ (i=1,2,3)$$

$$\psi_2(x) \xrightarrow{G} \psi_2'(x) \equiv \exp\{iy_2\beta\}\, \psi_2(x)$$

$$\psi_3(x) \xrightarrow{G} \psi_3'(x) \equiv \exp\{iy_3\beta\}\, \psi_3(x) ,$$

Requiring local gauge transformation, $\alpha^i = \alpha^i(x)$ and $\beta = \beta(x)$ we must make $\partial_\mu \to D_\mu$

$$D_\mu \psi_1(x) \equiv \left[\partial_\mu - i\, g\, \widetilde{W}_\mu(x) - i\, g'\, y_1\, B_\mu(x)\right] \psi_1(x)\,, \qquad \widetilde{W}_\mu(x) \equiv \frac{\tau_i}{2} W^i_\mu(x)$$

$$D_\mu \psi_2(x) \equiv \left[\partial_\mu - i\, g'\, y_2\, B_\mu(x)\right] \psi_2(x)\,,$$

$$D_\mu \psi_3(x) \equiv \left[\partial_\mu - i\, g'\, y_3\, B_\mu(x)\right] \psi_3(x)\,,$$

$D_\mu \psi_j(x)$ transforms (covariantly) like $\psi_j(x) \Rightarrow$

$$B_\mu(x) \xrightarrow{G} B'_\mu(x) \equiv B_\mu(x) + \frac{1}{g'} \partial_\mu \beta(x),$$

$$\widetilde{W}_\mu \xrightarrow{G} \widetilde{W}'_\mu \equiv U_L(x)\, \widetilde{W}_\mu\, U_L^\dagger(x) - \frac{i}{g} U_L(x)\, \partial_\mu U_L^\dagger(x).$$

$$W^i_\mu \xrightarrow{G} W{i'}_\mu \equiv W^i_\mu - \frac{1}{g}\alpha^i - (\vec{\alpha} \times \vec{W})^i \quad (\text{infinitesimal })$$

This fixes the gauge-matter interaction

$$\mathcal{L} = \sum_{j=1}^{3} i\, \overline{\psi}_j(x)\, \gamma^\mu\, D_\mu \psi_j(x)$$

• for the $U(1)_Y$ field do the same as with the electromagnetic field.
The field strength

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \qquad B_{\mu\nu} \xrightarrow{G} B_{\mu\nu} \, ,$$

$$\mathcal{L}_{\rm Kin,B} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

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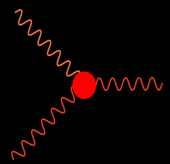
We can not just use $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu$ Note the right gauge transformation.
W^i is isospin charged and the normal derivative should be turned into a covariant derivative

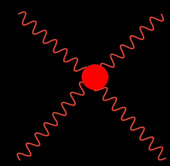
$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\epsilon^{ijk} W^j_\mu W^k_\nu$$

$$\widetilde{W}_{\mu\nu} \xrightarrow{G} U_L \widetilde{W}_{\mu\nu} U_L^\dagger$$

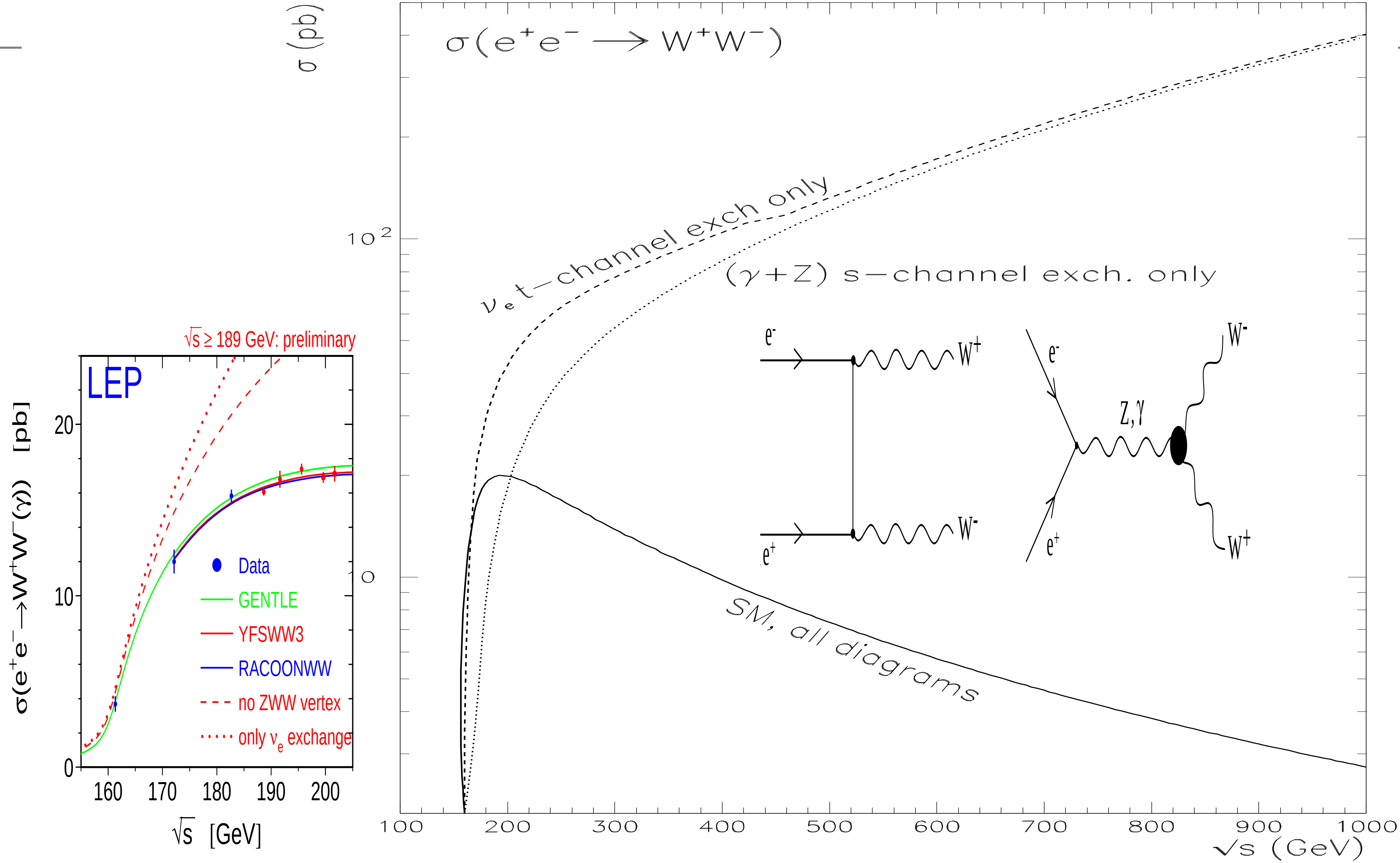
$$\mathcal{L}_{\rm Kin} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \,\mathrm{Tr}\left[\widetilde{W}_{\mu\nu} \widetilde{W}^{\mu\nu}\right] = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W_i^{\mu\nu} .$$

Trilinear and Quartic Gauge Boson Couplings





Gauge Invariance: $g_{ffV} = g_{VVV}$



- LEP legacy: We know that WWV can not deviate too much (10%) from SM gauge value.
- But slightest deviations are revealed at higher energies (LHC?)

Origin of VVVV and VVV: Gauge Invariance

$$\mathcal{L}_{\rm Gauge} = -\frac{1}{2}\left[\mathrm{Tr}(\boldsymbol{W}_{\mu\nu}\boldsymbol{W}^{\mu\nu}) + \mathrm{Tr}(\boldsymbol{B}_{\mu\nu}\boldsymbol{B}^{\mu\nu})\right] \text{ GI kinetic term}$$

$$\boldsymbol{W}_{\mu\nu} = \frac{1}{2}\left(\partial_\mu \boldsymbol{W}_\nu - \partial_\nu \boldsymbol{W}_\mu + \frac{i}{2}\boxed{g[\boldsymbol{W}_\mu, \boldsymbol{W}_\nu]}\right) = \frac{\tau^i}{2}\left(\partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\nu\right)$$

$$\boldsymbol{B}_{\mu\nu} = \frac{1}{2}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right) \qquad \boldsymbol{B}_\mu = B_\mu$$

$$\mathcal{L}_{WWV} = -ie\Big\{\left[A_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + F_{\mu\nu}W^{+\mu}W^{-\nu}\right]$$

$$+\frac{c_W}{s_W}\left[Z_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + Z_{\mu\nu}W^{+\mu}W^{-\nu}\right]\Big\} \; \boxed{\text{No}} \; ZZZ, ZZ\gamma, Z\gamma\gamma$$

tri-linear couplings

$$\mathcal{L}_{WWV} = -ie\Bigg\{\Bigg[A_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + \overbrace{(1+\Delta\kappa_\gamma)}^{\kappa_\gamma}F_{\mu\nu}W^{+\mu}W^{-\nu}\Bigg]$$

$$+\frac{c_W}{s_W}\Bigg[\overbrace{(1+\Delta g_1^Z)}^{g_1^Z}Z_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + \overbrace{(1+\Delta\kappa_Z)}^{\kappa_Z}Z_{\mu\nu}W^{+\mu}W^{-\nu}\Bigg]$$

$$+\frac{1}{M_W^2}\left(\lambda_\gamma\, F^{\nu\lambda} + \lambda_Z\,\frac{c_W}{s_W}Z^{\nu\lambda}\right)W^+_{\lambda\mu}W^{-\mu}{}_\nu\Bigg\}$$

No $ZZZ, ZZ\gamma, Z\gamma\gamma$

$$\mathcal{L}_{WWV_1V_2} =$$

$$
\begin{aligned}
- \quad & e^2 \Big\{ \left(A_\mu A^\mu W^+_\nu W^{-\nu} - A^\mu A^\nu W^+_\mu W^-_\nu\right) \\
+ \quad & 2\frac{c_W}{s_W}\left(A_\mu Z^\mu W^+_\nu W^{-\nu} - \frac{1}{2}A^\mu Z^\nu (W^+_\mu W^-_\nu + W^+_\nu W^-_\mu)\right) \\
+ \quad & \frac{c_W^2}{s_W^2}\left(Z_\mu Z^\mu W^+_\nu W^{-\nu} - Z^\mu Z^\nu W^+_\mu W^-_\nu\right) \\
+ \quad & \frac{1}{2s_W^2}\left(W^{+\mu} W^-_\mu W^{+\nu} W^-_\nu - W^{+\mu} W^+_\mu W^{-\nu} W^-_\nu\right) \Big\}
\end{aligned}
$$

Aspects of Colour
QCD

Make sense of the hadron spectrum economically

The Particle Data Book lists a plethora, that might seem as a zoo, of particles in the hadron family, between the mesons and the baryons. But !

everything fits neatly with much fewer fundamental constitutents of spin-1/2

Quarks that come in 6 flavours

The whole spectrum of hadrons can be constructed out of

- Mesons $M = q\bar{q}$
- Baryons $B = qqq$

This picture faces a major problem

$$\Delta^{++}_{J_Z=+3/2, J=3/2} = u^{\uparrow}u^{\uparrow}u^{\uparrow}$$

The wave function is symmetric for a fermion! and therefore does not obey spin-statistics.

For such a state to exist, the three u can not be the same, they must carry (at least) **3** different quantum numbers.

$$\Delta^{++} = u^{\uparrow}u^{\uparrow}u^{\uparrow}$$

Simplicity and minimality suggests number of colours is

$$N_c = 3$$

then each quark is $q^\alpha, \alpha = 1, 2, 3 =$red,green blue.

Then $\Delta^{++} \sim \epsilon^{\alpha\beta\gamma} \, |u_\alpha^\uparrow u_\beta^\uparrow u_\gamma^\uparrow\rangle$.

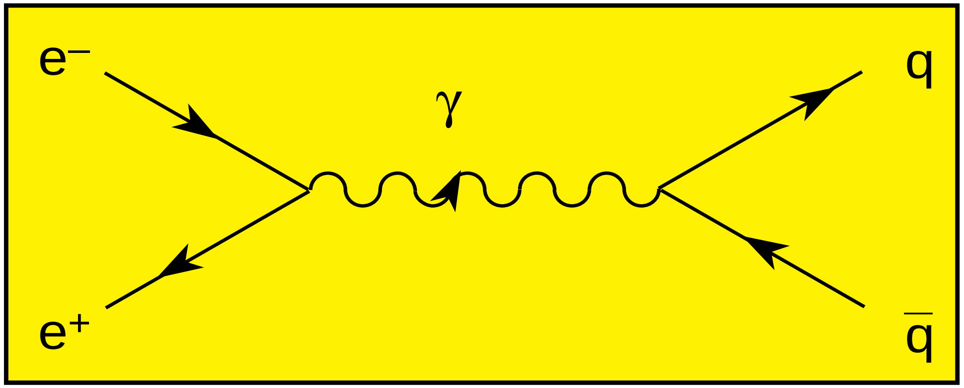
Baryons and mesons are described by colour singlet construct

$$B = \frac{1}{\sqrt{6}} \epsilon^{\alpha\beta\gamma} \, |q_\alpha q_\beta q_\gamma\rangle \,, \qquad M = \frac{1}{\sqrt{3}} \delta^{\alpha\beta} \, |q_\alpha \bar{q}_\beta\rangle$$

We observe no free quarks and moreover we do not see any combination of states/particles that carry colour: confinement

$$R_{e^+e^-} \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} .$$

Probability for quarks to hadronize is one and for $s \ll M_Z^2$



$$R_{e^+e^-} \approx N_C \sum_{f=1}^{N_f} Q_f^2 = \begin{cases} \frac{2}{3} N_C = 2\,, & (N_f = 3 \;:\; u, d, s) \\ \frac{10}{9} N_C = \frac{10}{3}\,, & (N_f = 4 \;:\; u, d, s, c) \\ \frac{11}{9} N_C = \frac{11}{3}\,, & (N_f = 5 \;:\; u, d, s, c, b) \end{cases} .$$

Colour, more evidence

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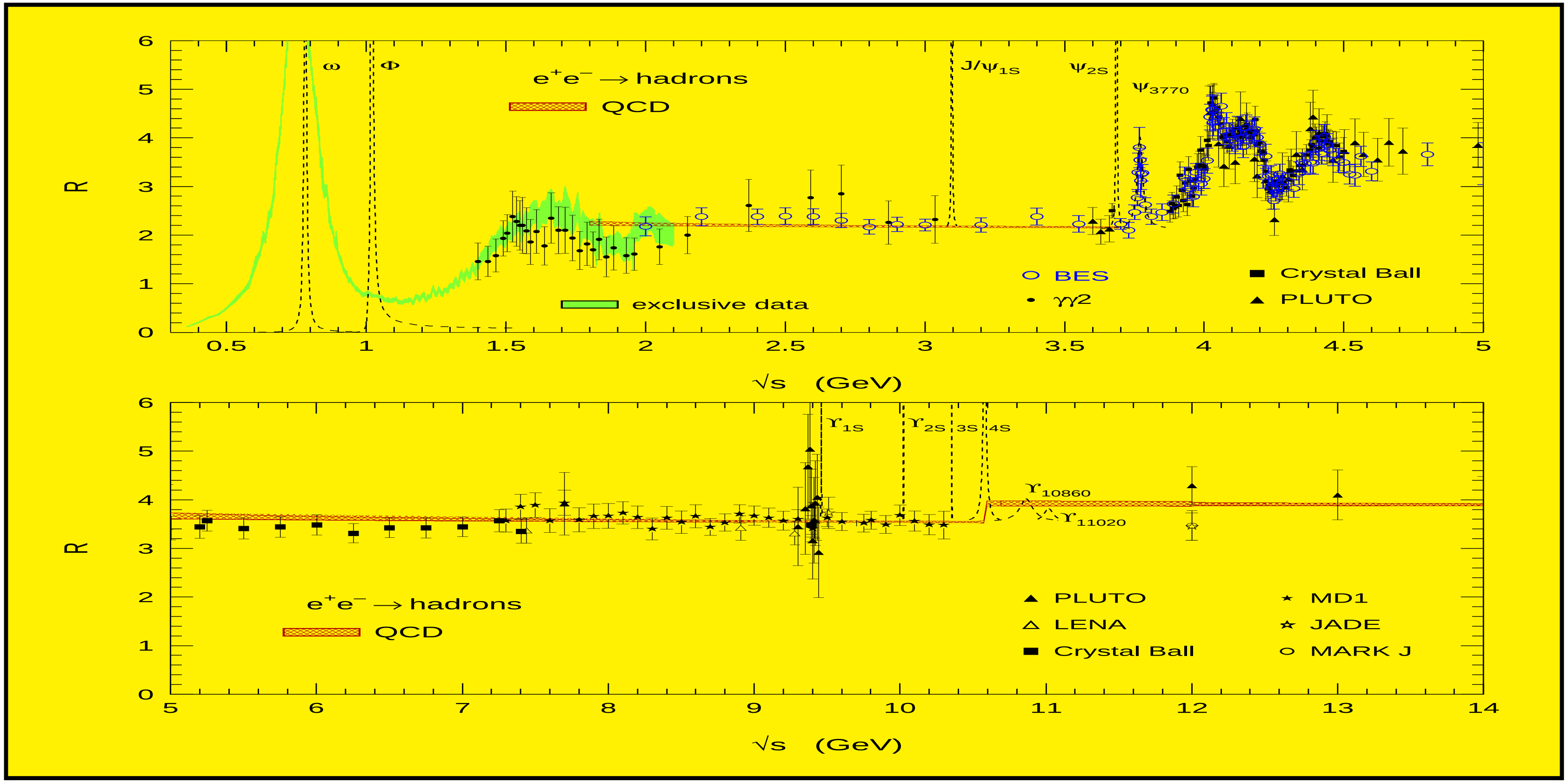

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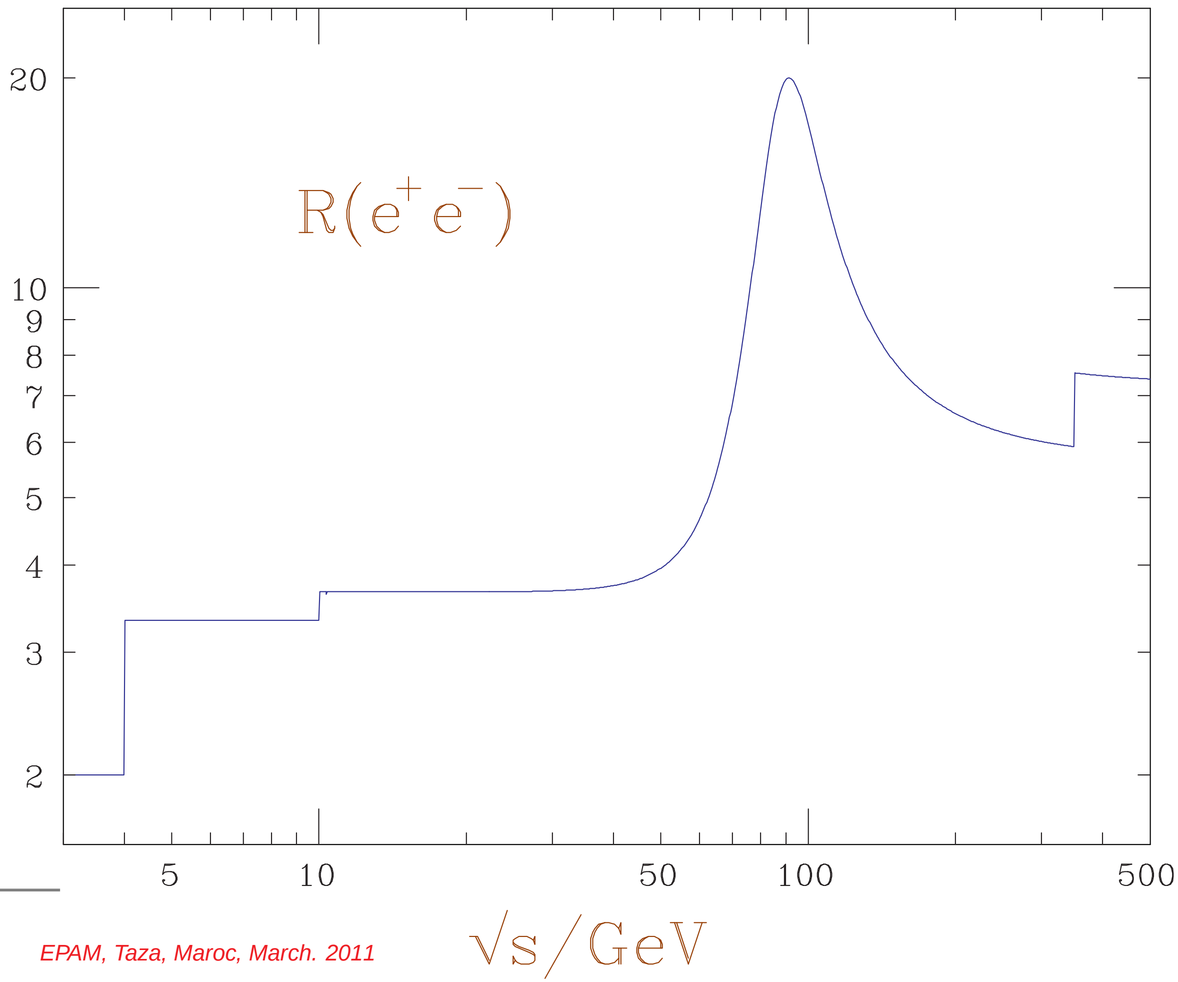
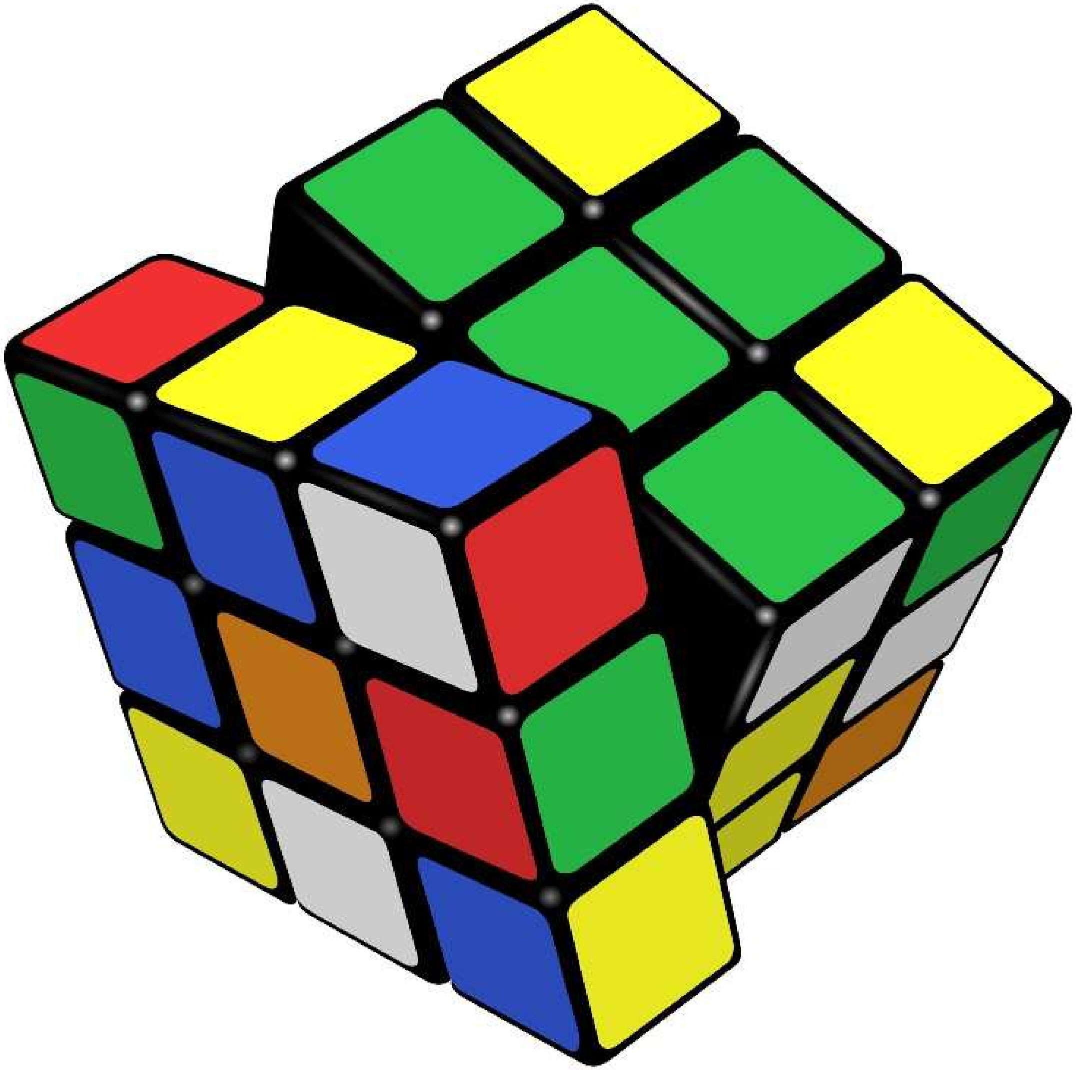


$$R^Z_{had} = N_c \frac{\sum_q v_q^2 + a_q^2}{v_\mu^2 + a_\mu^2} = 20.095.$$

- Exact colour symmetry Symm_3
- Colour is what binds hadrons together.
- $N_C = 3$. Quarks belong to the triplet representation $\underline{3}$ of Symm_3.
- Quarks and antiquarks are different states. Therefore, $\underline{3}^* \neq \underline{3}$.
- Confinement hypothesis: hadronic states are colour singlets.
- The interaction must be quite strong $\rho \to 2\pi \quad \tau \sim 10^{-22} s$ compared to $\mu \to e\bar{\nu}_e\nu_\mu \quad 10^{-6} s$

$$\mathrm{Symm}_3 = SU(3)_C$$

$$\begin{aligned}
q\bar{q} : &\qquad \underline{3} \otimes \underline{3}^* = \underline{1} \oplus \underline{8}\,, \\
qqq : &\qquad \underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10}\,, \\
qq : &\qquad \underline{3} \otimes \underline{3} = \underline{3}^* \oplus \underline{6}\,, \\
qqqq : &\qquad \underline{3} \otimes \underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{3} \oplus \underline{3} \oplus \underline{3} \oplus \underline{6}^* \oplus \underline{6}^* \oplus \underline{15} \oplus \underline{15} \oplus \underline{15} \oplus \underline{15'}\,,
\end{aligned}$$



$SU(2)_{\rm weak\ isospin}$

- Matter fields in doublets, L_a, a=1,2
- Generators of the group, τ^A Pauli matrices, $A = 1...(N^2 - 1) = 3$
- $N^2 - 1 = 3$ gauge bosons
- Invariance under $U = exp(i\tau^A \alpha^A)$

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$SU(3)_C$

- Matter fields, q_a, a=1,2,3
- Generators of the group, t^A, $A = 1...(N^2-1) = 8$
- $N^2 - 1 = 8$ gauge bosons
- $U = exp(i\tau^A \theta^A)$

Recap $SU(2)$ set up $SU(3)$, $SU(N)$

$SU(2)_{\text{weak isospin}}$

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- Generators of the group, τ^A Pauli matrices, $A = 1...(N^2-1) = 3$
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$$T_R = \frac{1}{2},$$
$$C_F = \frac{4}{3},$$
$$C_A = 3,$$

The fundamental representation $T^a = \lambda^a/2$ is N–dimensional. For $N = 2$, λ^a are the usual Pauli matrices, while for $N = 3$, they correspond to the eight Gell-Mann matrices:

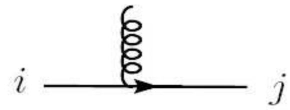
$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Fundamental representation 3:

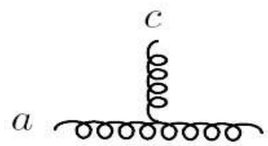
$= \delta_{ij}$



$= t^a_{ij}$

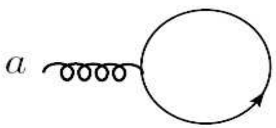
Adjoint representation 8:

$= \delta_{ab}$



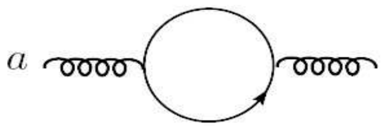
$= i f_{abc}$

Trace identities:



$= 0$

$\mathrm{Tr}(t^a) = 0$

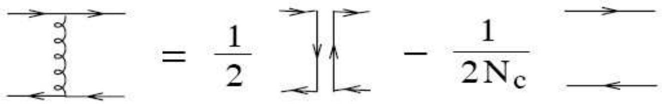


$= T_R$

$\mathrm{Tr}(t^a\, t^b) = \mathrm{T_R}\delta^{ab}$

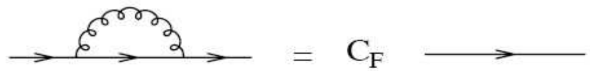
Fierz identity:

$$(t^a)^i_k (t^a)^l_j = \frac{1}{2}\delta^i_j \delta^l_k - \frac{1}{2N_c}\delta^i_k \delta^l_j$$



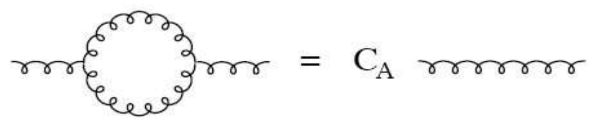
Fundamental representation 3:

$$\sum_a (t^a_{ij})(t^a_{kj}) = C_F \delta_{ij} \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$



Adjoint representation 8:

$$\sum_{cd} f^{acd} f^{bdc} = C_A \delta^{ab} \qquad C_A = N_c$$



QCD Lagrangian

From the free Lagrangian

$$\mathcal{L}_{\rm quarks} = \sum_i^{n_f} \bar{q}_i^a (i\partial\!\!\!/ - m_i)_{ab} q_i^b ,$$

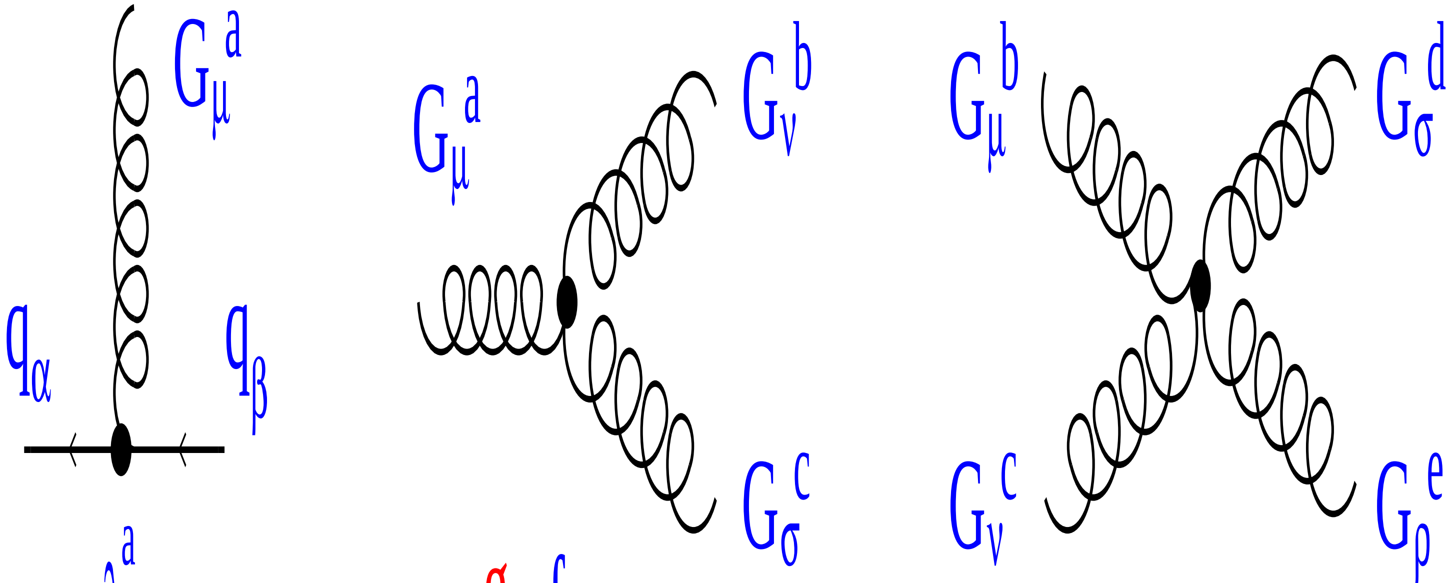
turn to the covariant derivative

$$D_{\mu,ab} = \partial_\mu 1_{ab} + i g_s (t\cdot A_\mu)_{ab},$$

A_μ^a are coloured vector fields, gluons

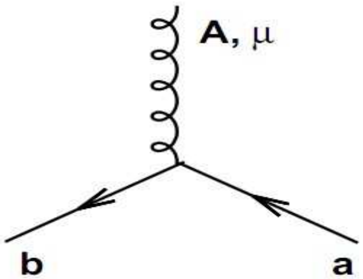
$$\begin{aligned} \mathcal{L}_{\rm kin} &= -\frac{1}{4} F^A_{\mu\nu} F_A^{\mu\nu} , \\ F^A_{\mu\nu} &= \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g_s f^{ABC} A^B_\mu A^C_\nu , \end{aligned}$$

$$\boxed{\mathcal{L}_{\rm QCD} = -\tfrac{1}{4} F^A_{\mu\nu} F_A^{\mu\nu} + \textstyle\sum_i^n \bar{q}_i^a (iD\!\!\!\!/ - m_i)_{ab} q_i^b - \tfrac{1}{2\lambda}\left(\partial^\mu A^A_\mu\right)^2 + \mathcal{L}_{\rm ghost} .}$$

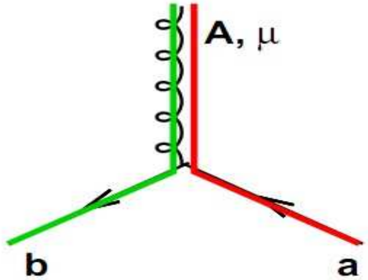


$$g_s \frac{\lambda^a_{\alpha\beta}}{2} \gamma_\mu \qquad g_s\, f_{abc} \qquad g_s^2\, f_{abc}\, f_{ade}$$

QCD Feynman rules



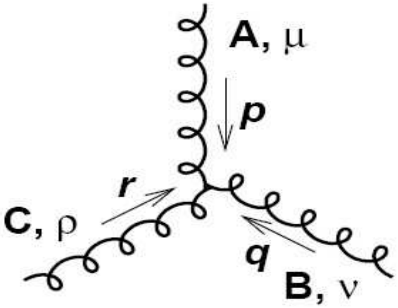
$\bar{\psi}_b(-ig_s t^A_{ba}\gamma^\mu)\psi_a$



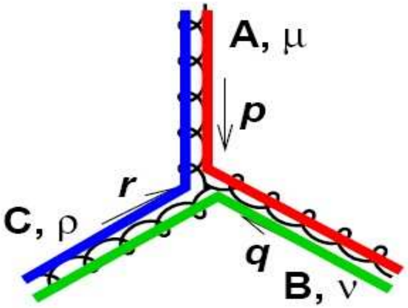
$$\underbrace{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}}_{\bar{\psi}_b} \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{t^1_{ab}} \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\psi_a}$$

A gluon emission **repaints** the quark colour.
A gluon itself carries colour and anti-colour.

QCD Feynman rules



$$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$$



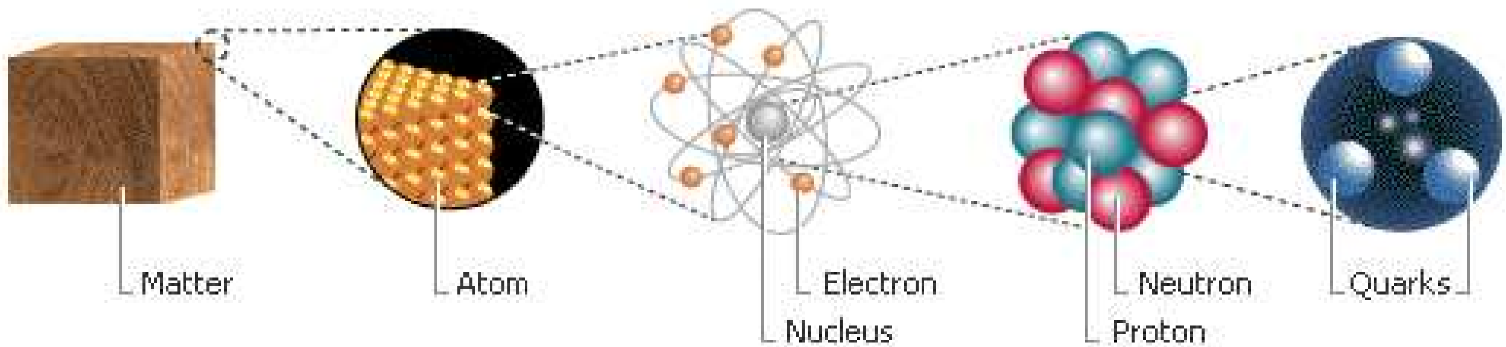
A gluon emission also repaints the gluon colours.

Because a gluon carries colour + anti-colour, it emits $\sim$ twice as strongly as a quark (just has colour)

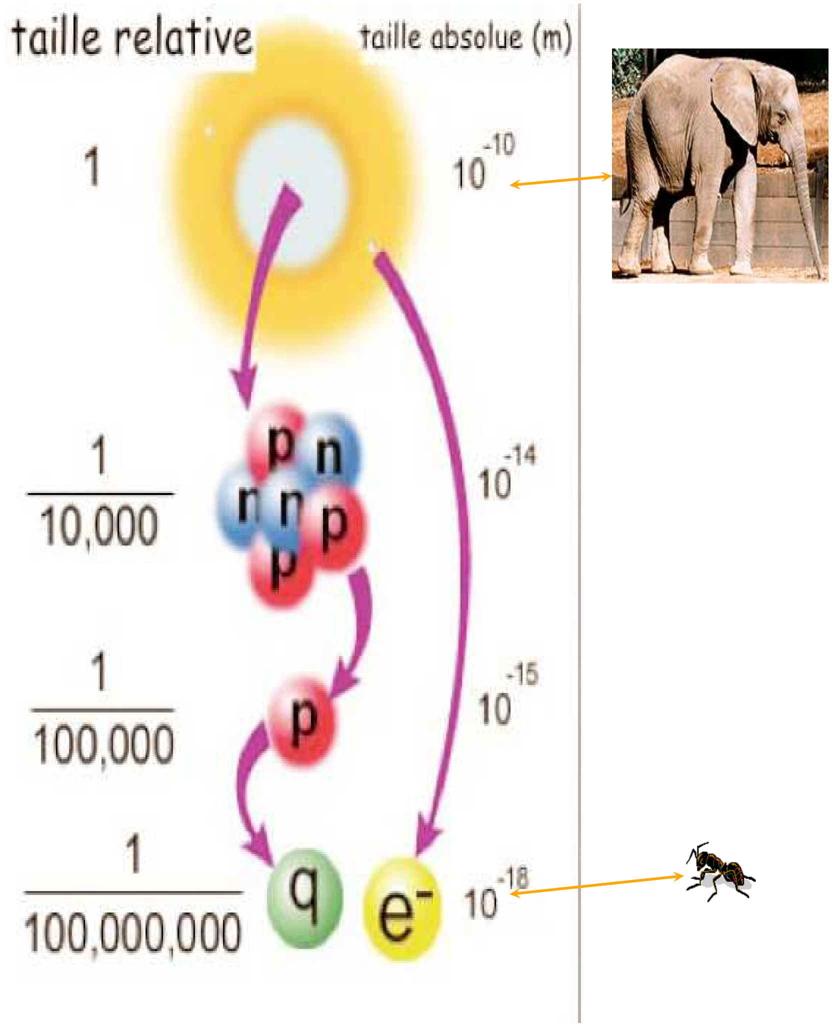
▶

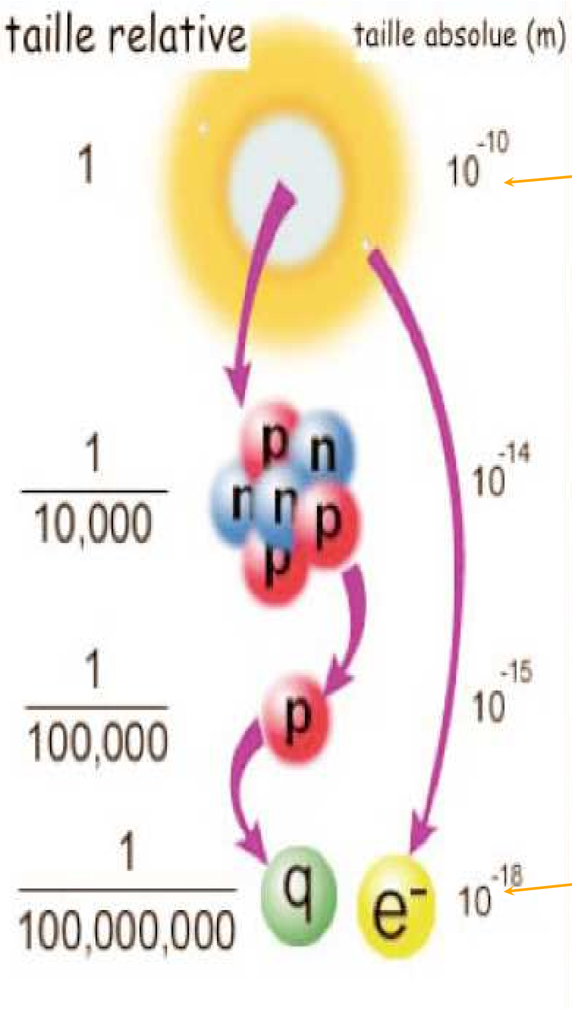
	SU(3)	SU(2)$_L$	U(1)$_Y$	$Q = T_3 + Y$
$Q = (u_L, d_L)$	3	2	$\frac{1}{6}$	$(\frac{2}{3}, -\frac{1}{3})$
u_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L = (\nu_L, e_L)$	1	2	$-\frac{1}{2}$	$(0, -1)$
e_R	1	1	-1	-1
ν_R	1	1	0	0

De quoi sommes nous faits ▶ 1



De quoi sommes nous faits ▶ 1





interaction lectromagnetique (QED)

interaction forte (QCD)

Interaction faible

vers 1880, D'ou vient l'energie du soleil?

Kelvin, Helmholtz, etc...: "contraction gravitationnelle du nuage solaire..."

age de l'Univers (soleil): qq millions d'annees

Darwin (evolution, erosion,...): milliards d'annees pour la terre!

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desintegration β

$n \longrightarrow p e^- \nu_e$

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desintegration β

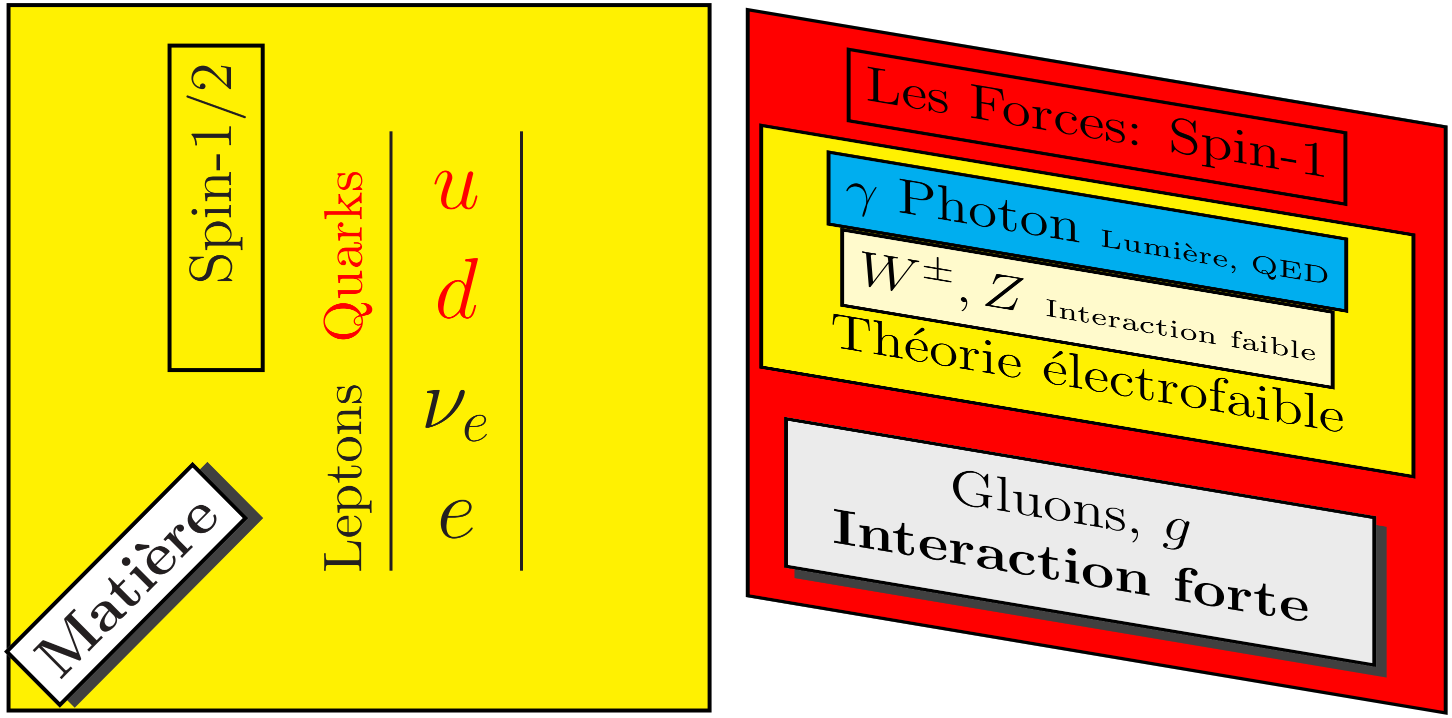
$n \longrightarrow p e^- \nu_e$

reaction nucleaire: fusion

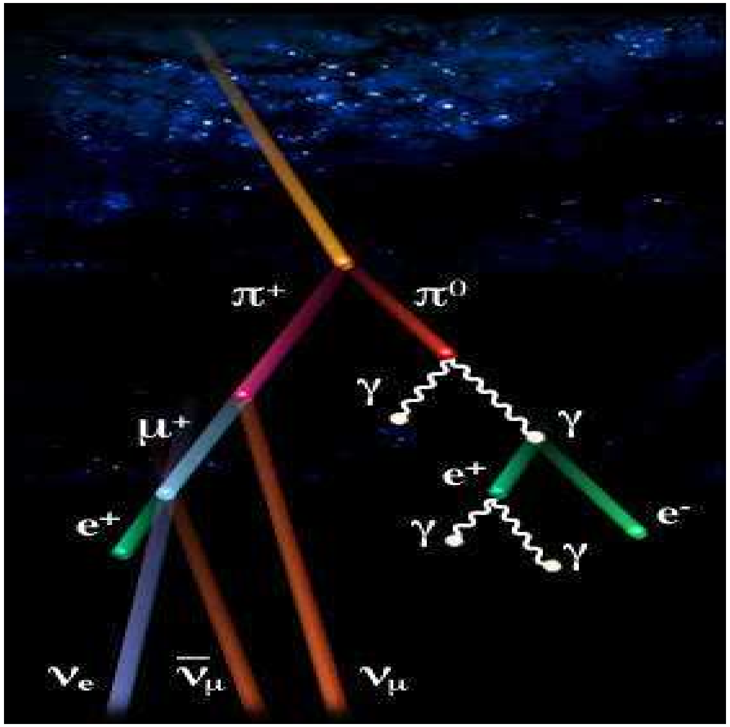
bruler de l'hydrogene:

$$4\,{}^1H + 2e \rightarrow {}^4He + 2\nu + 6\gamma + \text{Energie}$$

$$(CH_4 + 2O_2 \rightarrow 2H_2O + \text{chaleur})$$


Spin-1/2
Quarks
Leptons
u
d
ν_e
e
Matière
Les Forces: Spin-1
γ Photon Lumière, QED
$W^\pm, Z$ Interaction faible
Théorie électrofaible
Gluons, g
Interaction forte

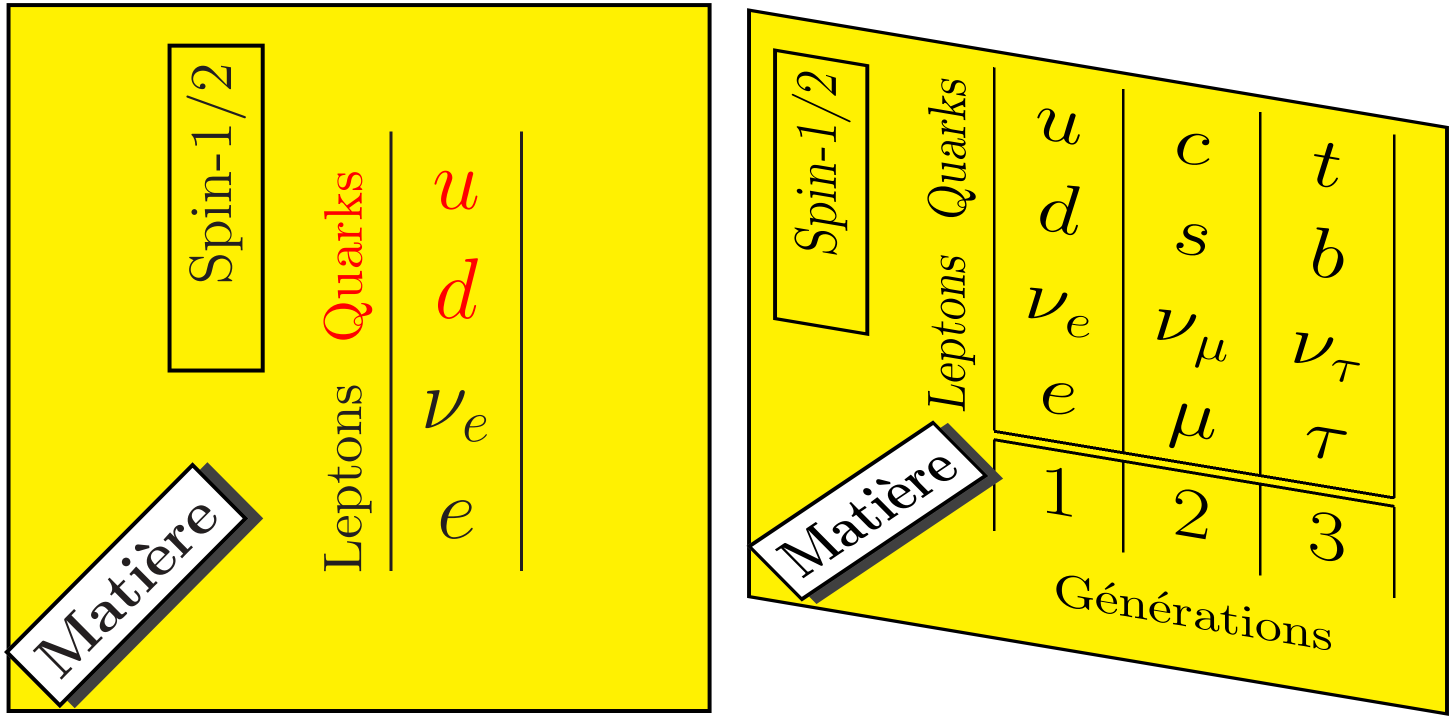
Rayons Cosmiques



μ: muon a part la masse, en tout point comme l'electron

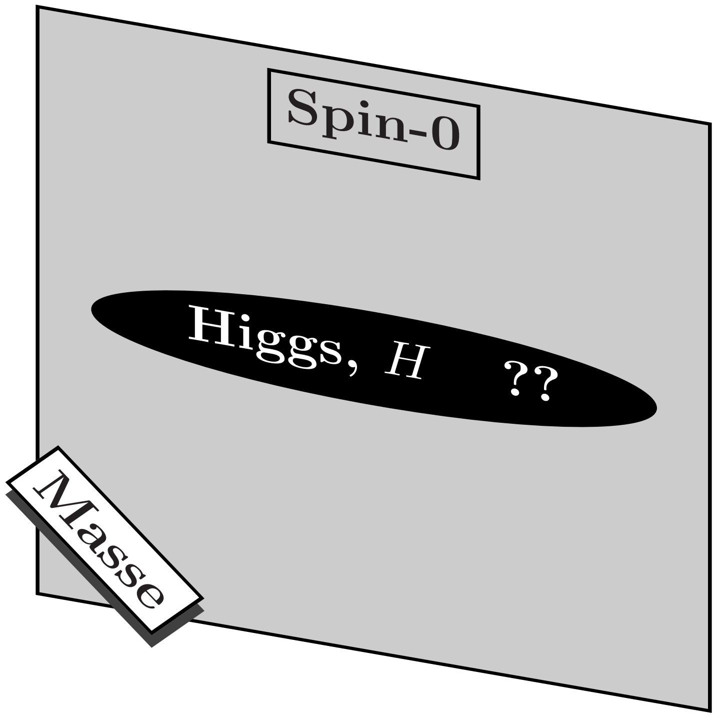
accelerateurs pour produire ces nouvelles particules ou d'autres pour mieux les etudier

Et!



Spin-1/2
Leptons Quarks
u
d
ν_e
e
Matière
Spin-1/2
Leptons Quarks
u c t
d s b
ν_e ν_μ ν_τ
e μ τ
1 2 3
Générations
Matière

Spin-0???



Le Modèle Standard