Tools and Monte-Carlos
for the
New Physics

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OUTLINE

- What's a tool and what it takes to make one
- New Physics vs the Standard Model Physics

OUTLINE

- What's a tool and what it takes to make one: Structure of an event
- Components of a MC EG (Monte Carlo Event Generator)
- Integration and MC techniques
- PS: Parton Shower in a MC
- Matrix Element vs PS
- ME generation and ME generators
- Modular structure of codes, Les Houches Accords
- Tools for the New Physics

Further Reading and from where I borrowed

- Frank Krauss Bonn Lectures, 2006

  http://projects.hepforge.org/sherpa/dokuwiki/publications/presentations/index

- Fabio MALTONI HEPTOOLS School, Torino, 2008

  http://personalpages.to.infn.it/ maina/scuola08/Maltoni_Torino08.pdf

- Steve Mrenna CTEQSS, CTEQ05

- Peter Richardson CTEQ06 School, IPPP Durham, 2006

- Mike Seymour CERN Training Lectures 2003

  http://seymour.home.cern.ch/seymour/slides/CERNlecture1.ppt

- Torbjrn Sjostrand, 2006 European School of HEP, Aronsborg YETI06, IPPP Durham, see Pythia website http://www.thep.lu.se/ torbjorn

- Brian Webber 1st MCnet School, IPPP Durham 2007

- Les Houches Guidebook Les Houches Guidebook to Monte Carlo Generators for Hadron Collider Physics, hep-ph/0403045

- R.K. Ellis, W.J. Stirling and B.R. Webber QCD and Collider Physics Cambridge Monographs, Cambridge University Press

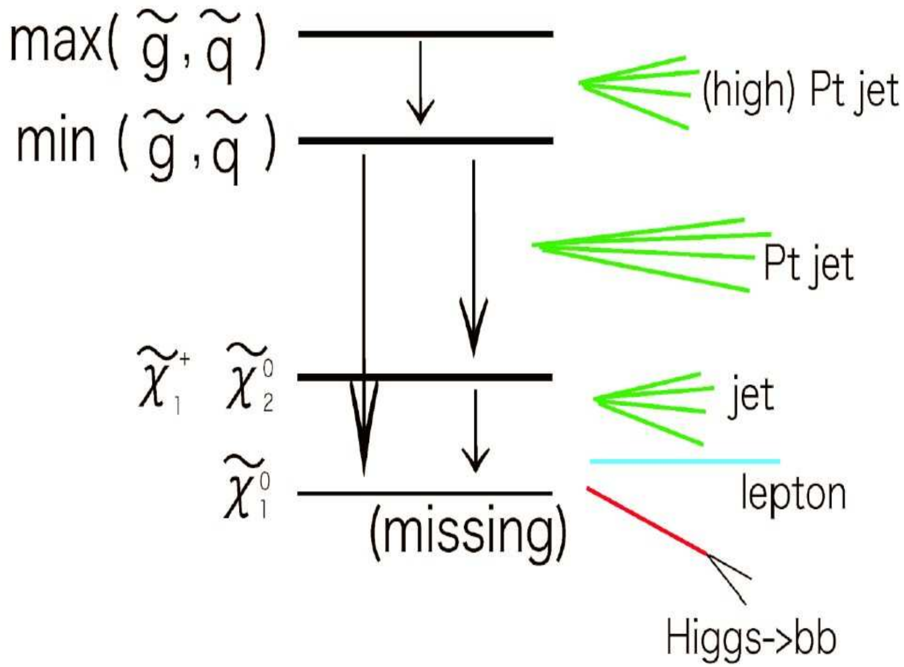
Great Idea: A New Physics Model

FINAL AIM

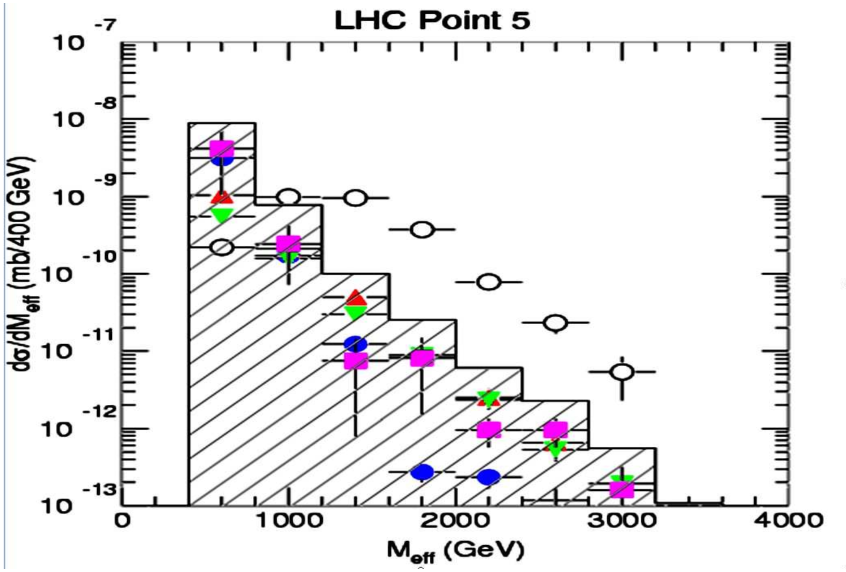
Nobel Prize if LHC validates!

Turn on the machine!

in 1998 we were told to expect an early SUSY discovery

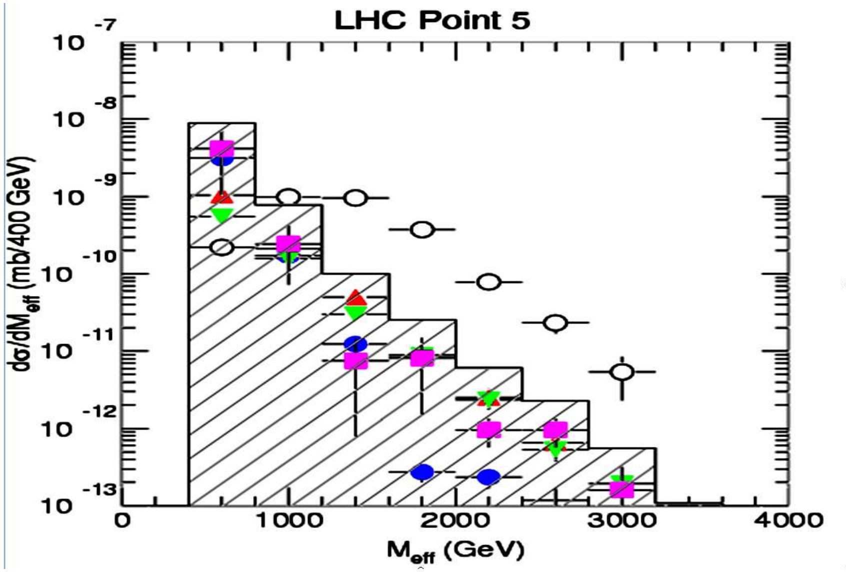


ATLAS TDR (same with CMS)



ATLAS TDR 98
(mSUGRA point, PreWMAP)

ATLAS TDR (same with CMS)



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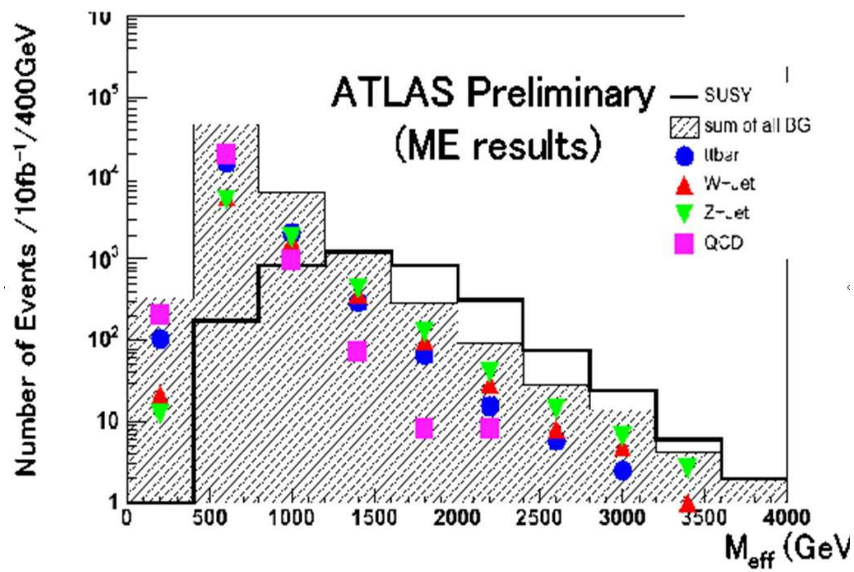

ATLAS 2006

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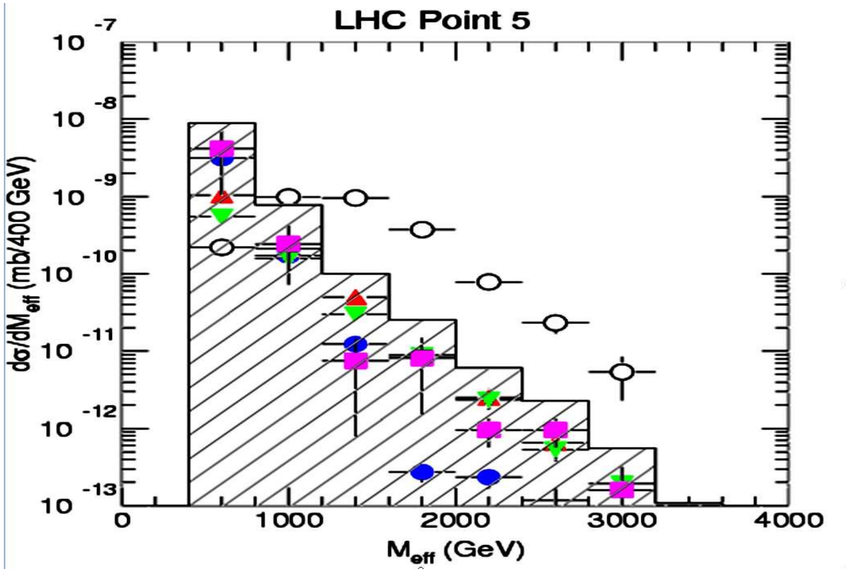

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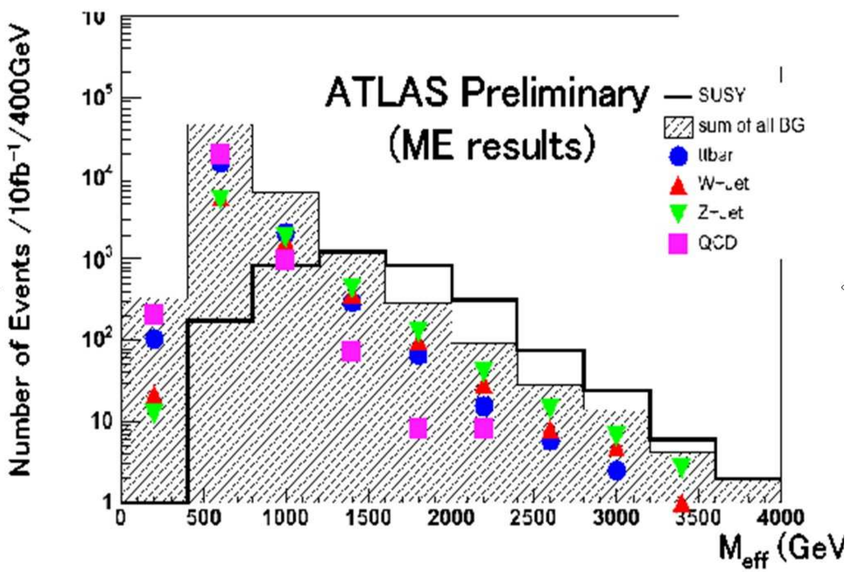


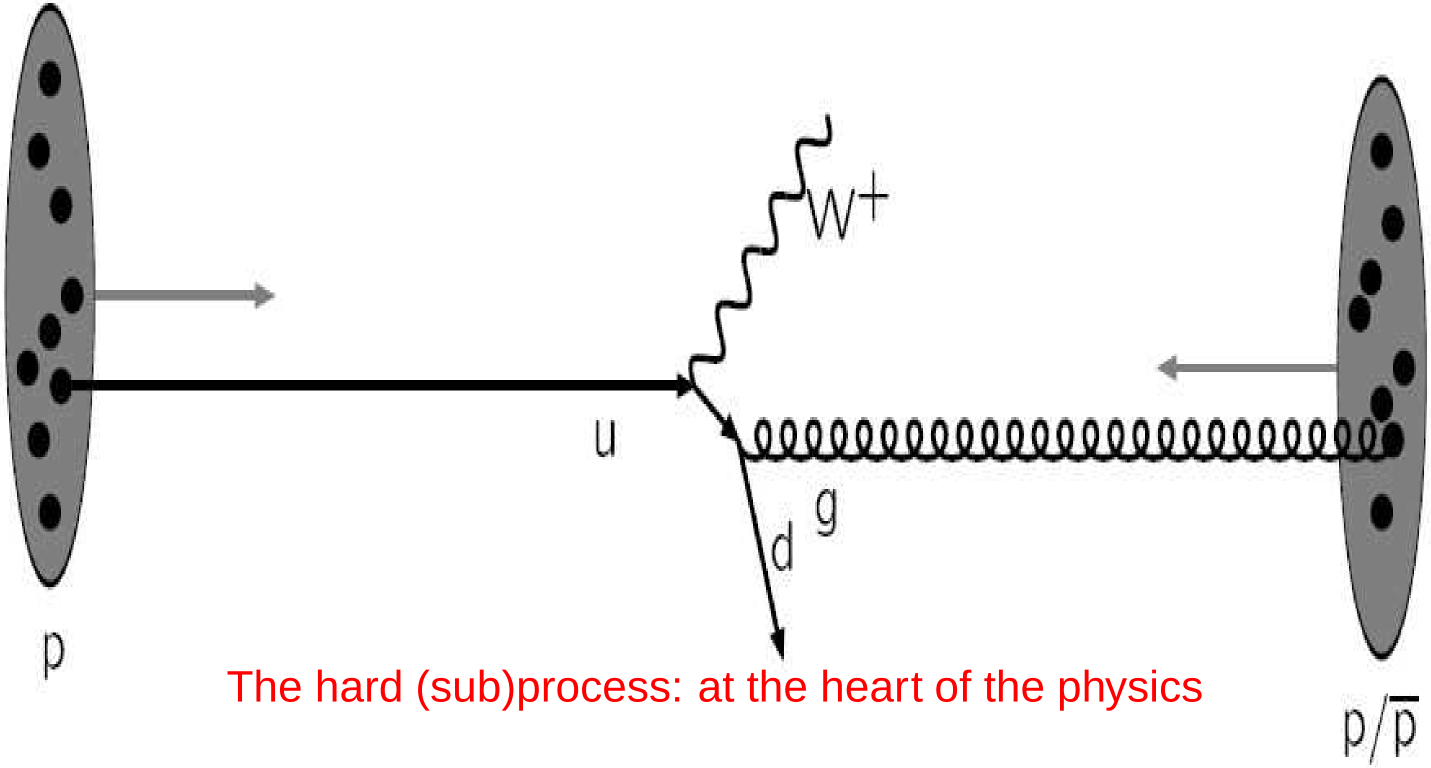
ATLAS 2006

What happened?

Let's turn the machine again, slow motion!

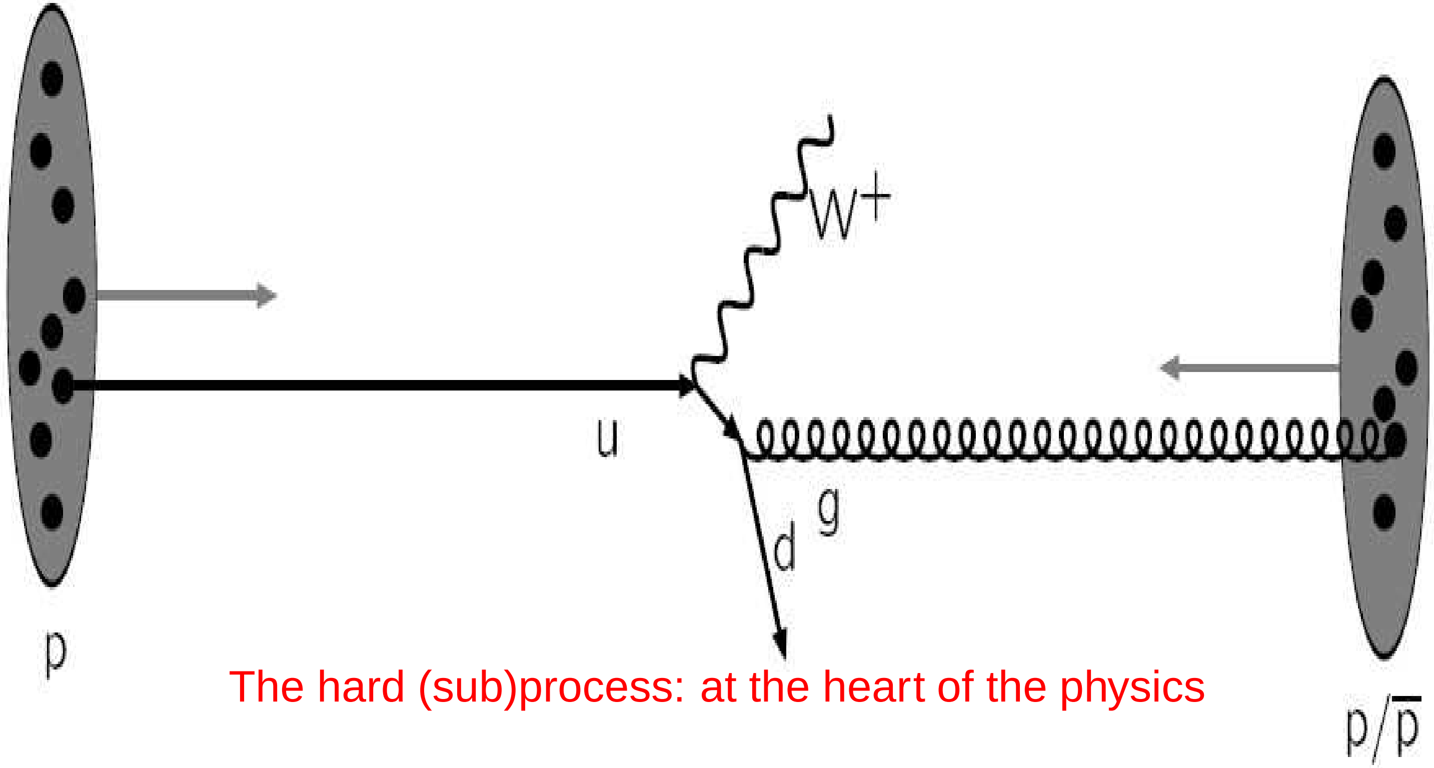
Movie: The structure of an event

Incoming beams: partons densities



The hard (sub)process: at the heart of the physics

- Hard process is well understood and well described: relies on a firm perturbative framework.



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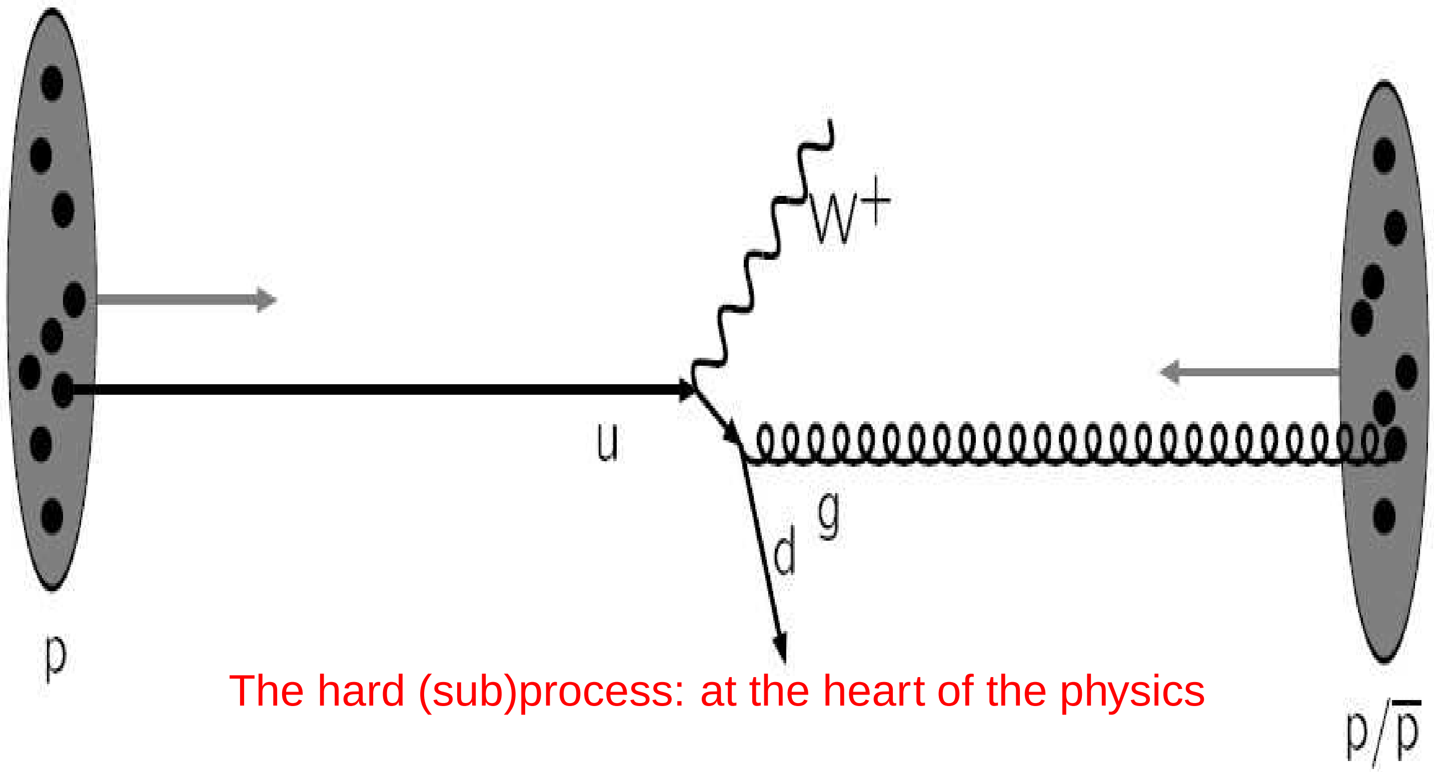
- described by Matrix Elements (ME)
  This does not mean that it is very well calculated

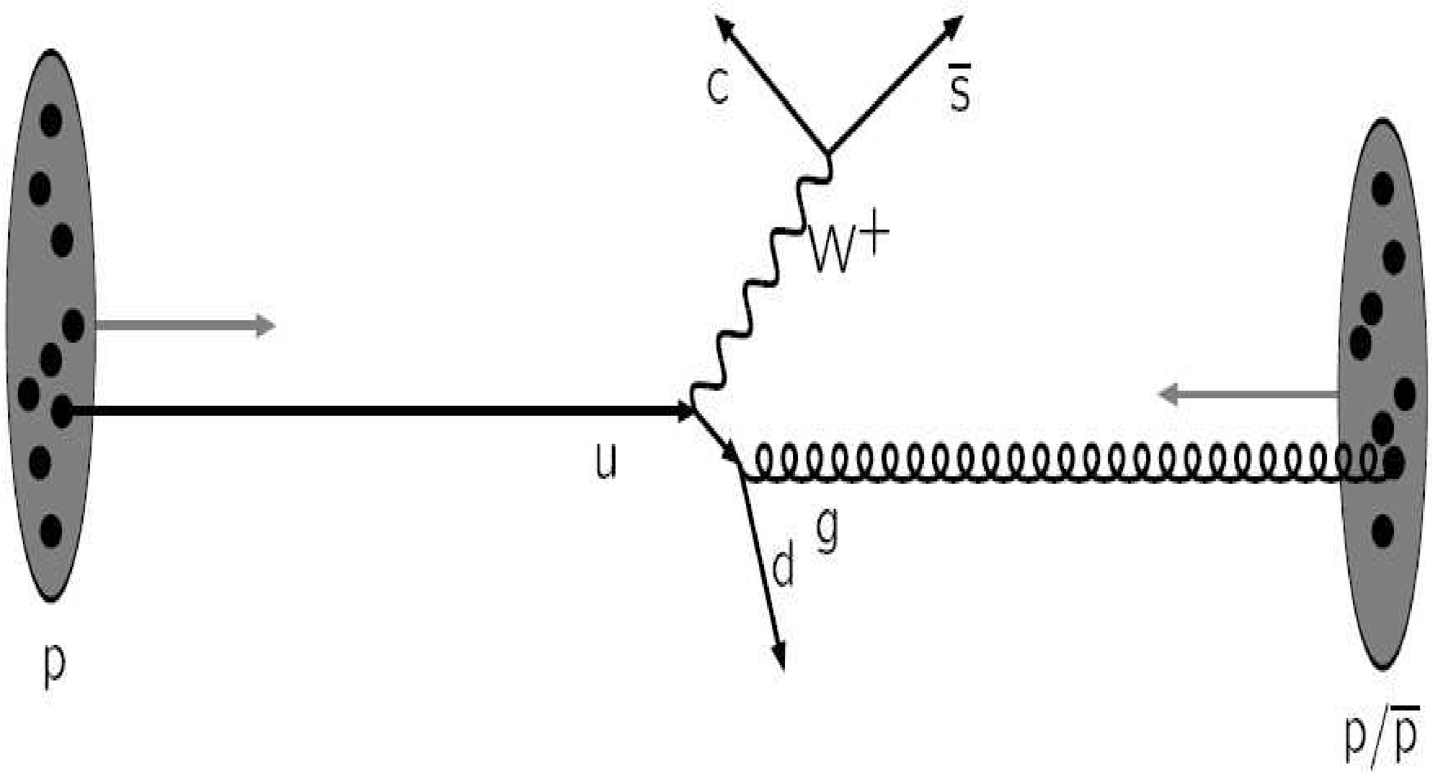
Movie: The structure of an event



The hard (sub)process: at the heart of the physics

- Hard process is well understood and well described: relies on a firm perturbative framework.
- described by Matrix Elements (ME)
  This does not mean that it is very well calculated
- issue of higher order (NLO), most calculations only LO say

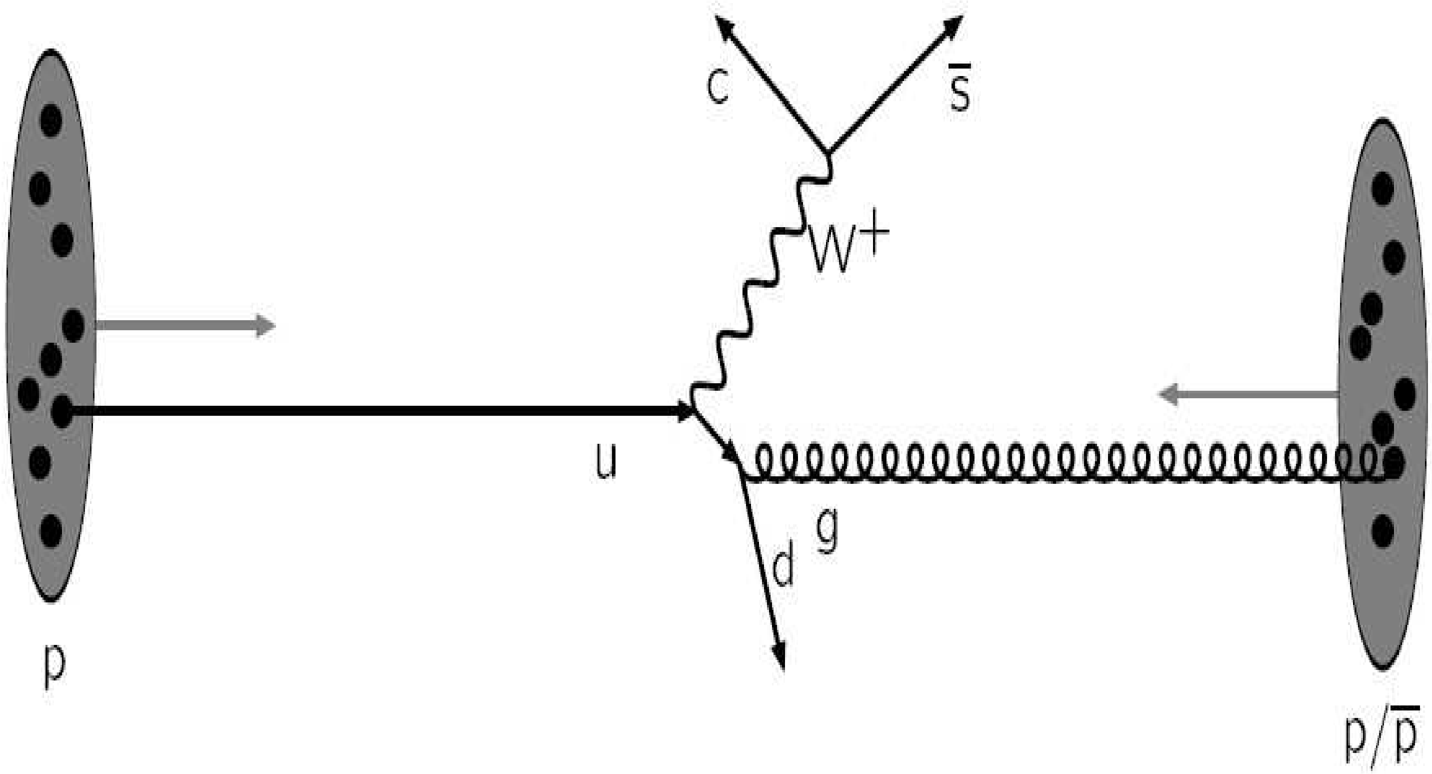
Movie: The structure of an event



Decays of resonances: correlated with hard process

- Approximation: W on-shell

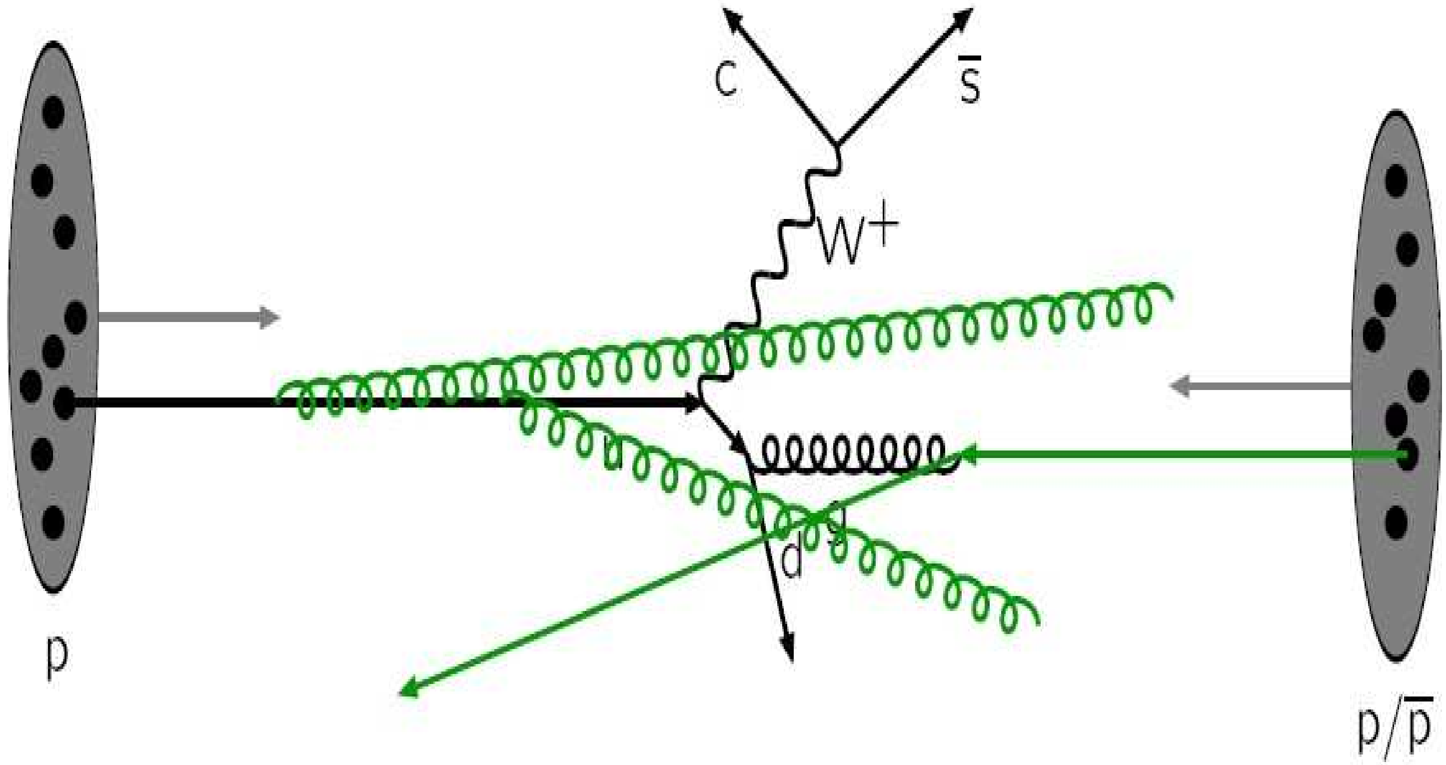
Movie: The structure of an event



Decays of resonances: correlated with hard process

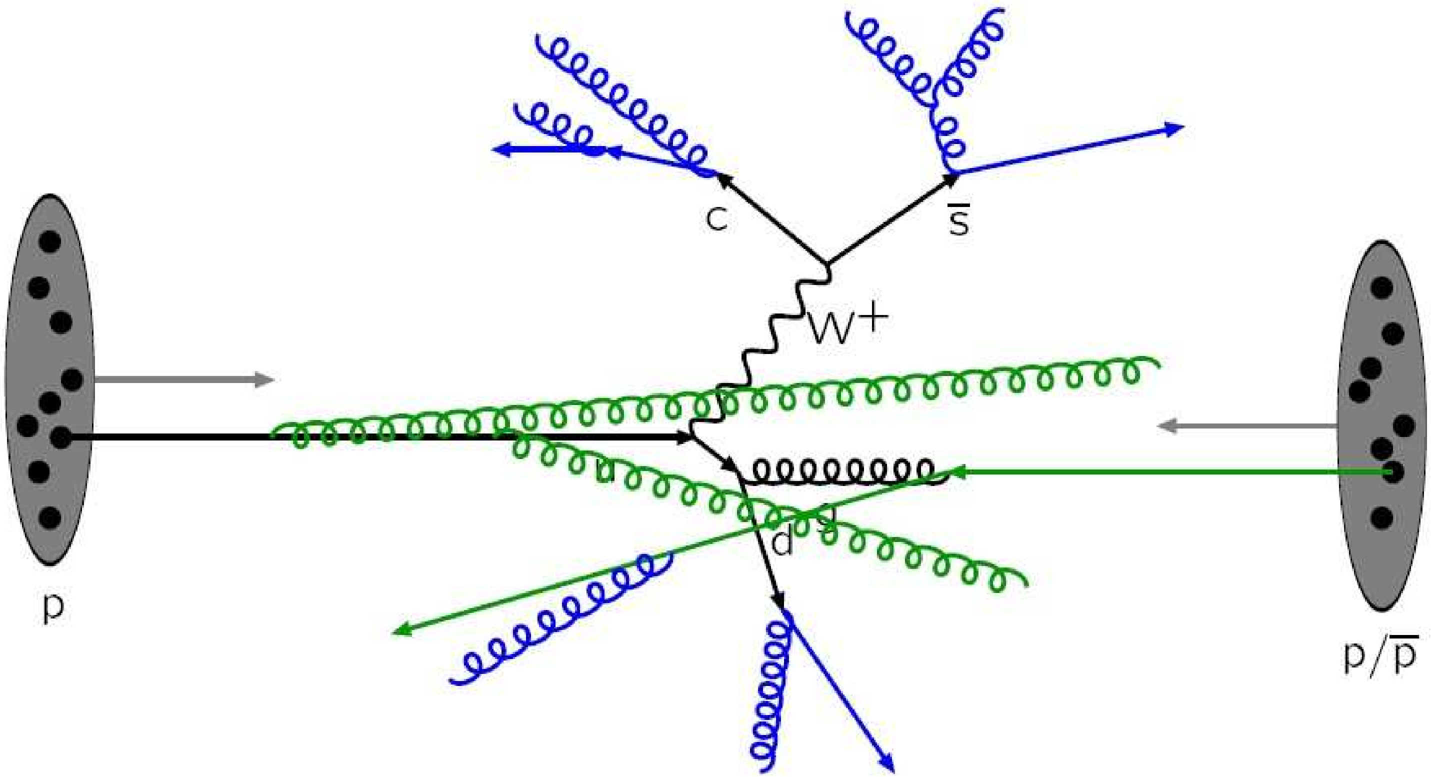
- Approximation: W on-shell
- Spin effect in decays?

Movie: The structure of an event



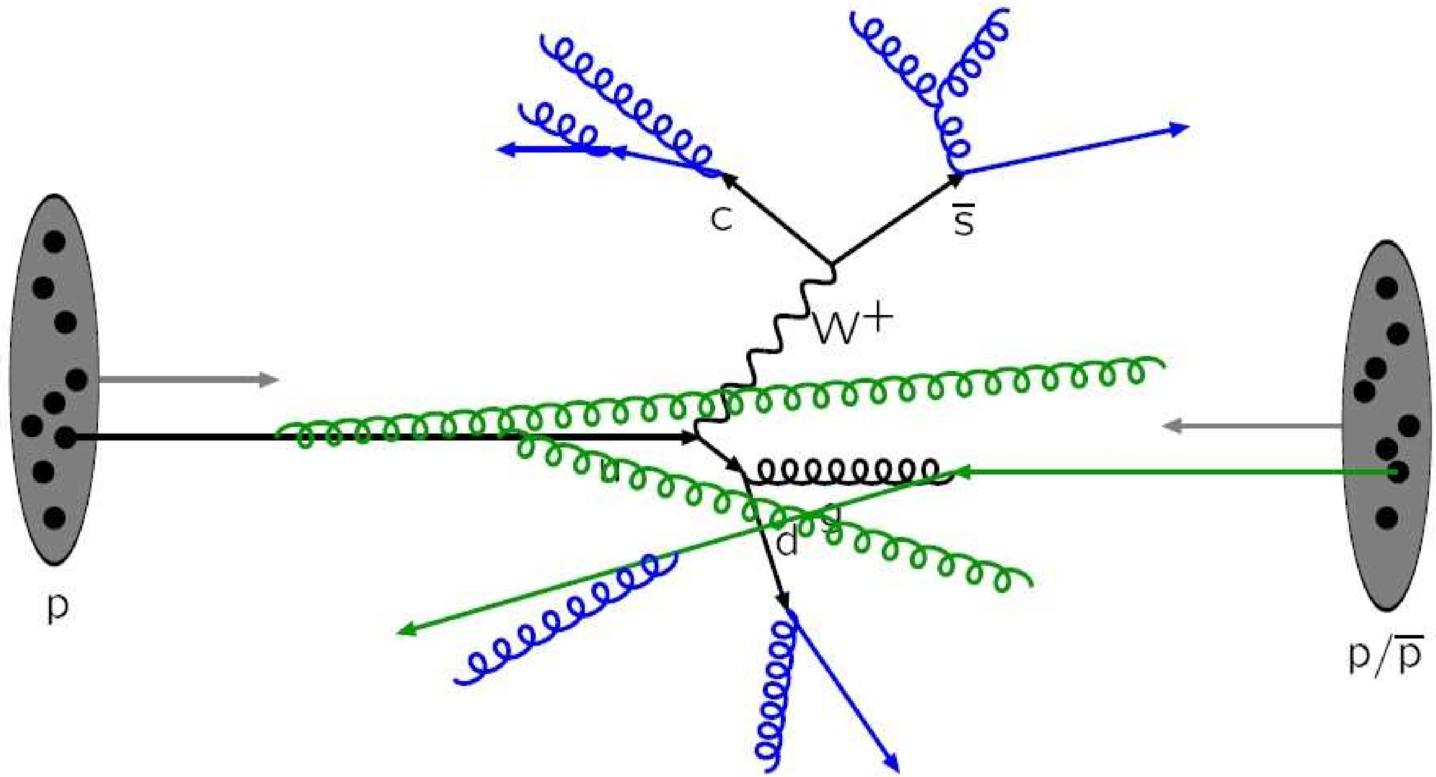
ISR: Initial State Radiation

Space-like parton showers (PS)



FSR: Final State Radiation

time-like parton showers (PS)



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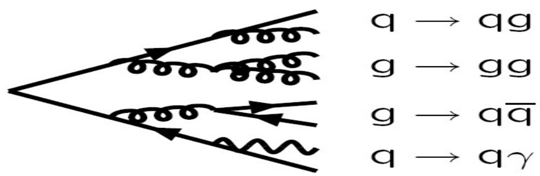
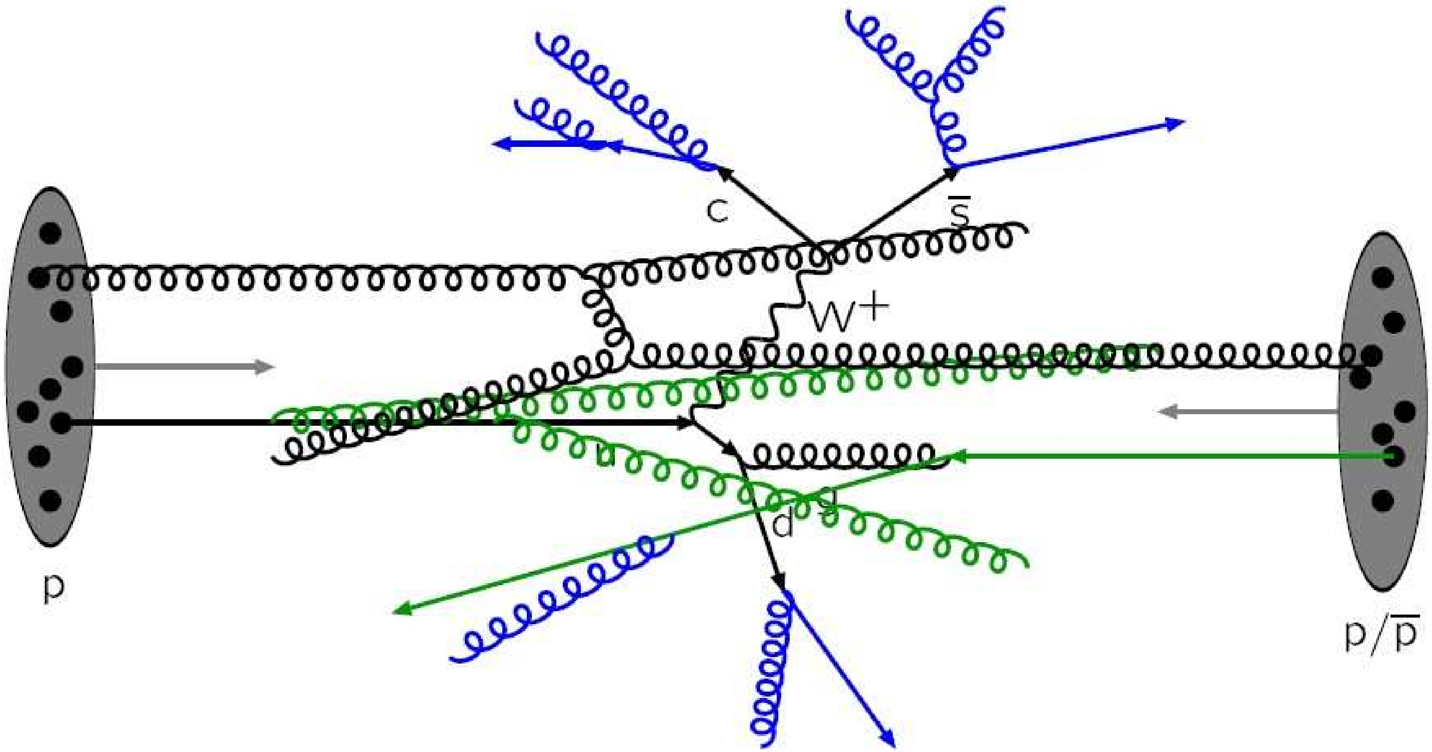
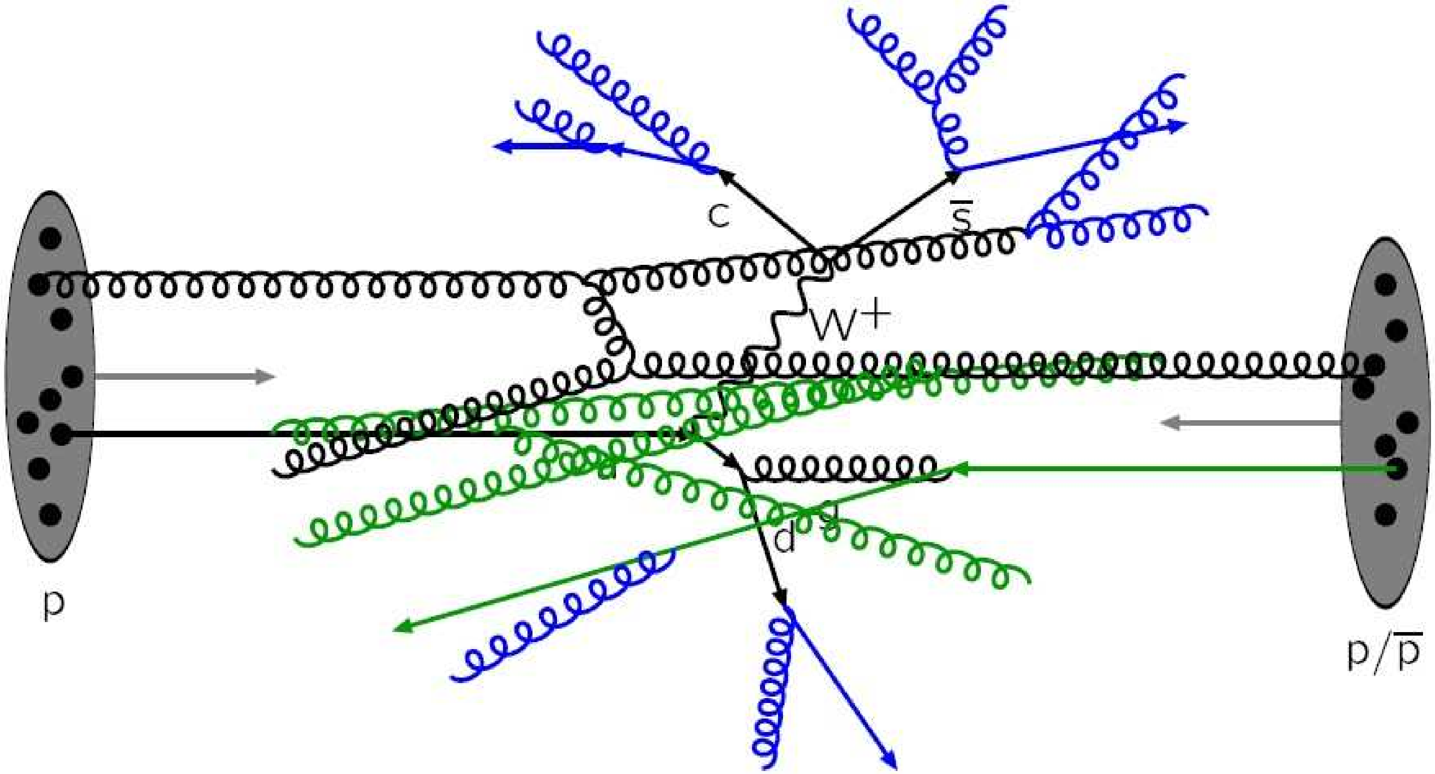


Movie: The structure of an event



Multiple parton-parton interactions (MPI)

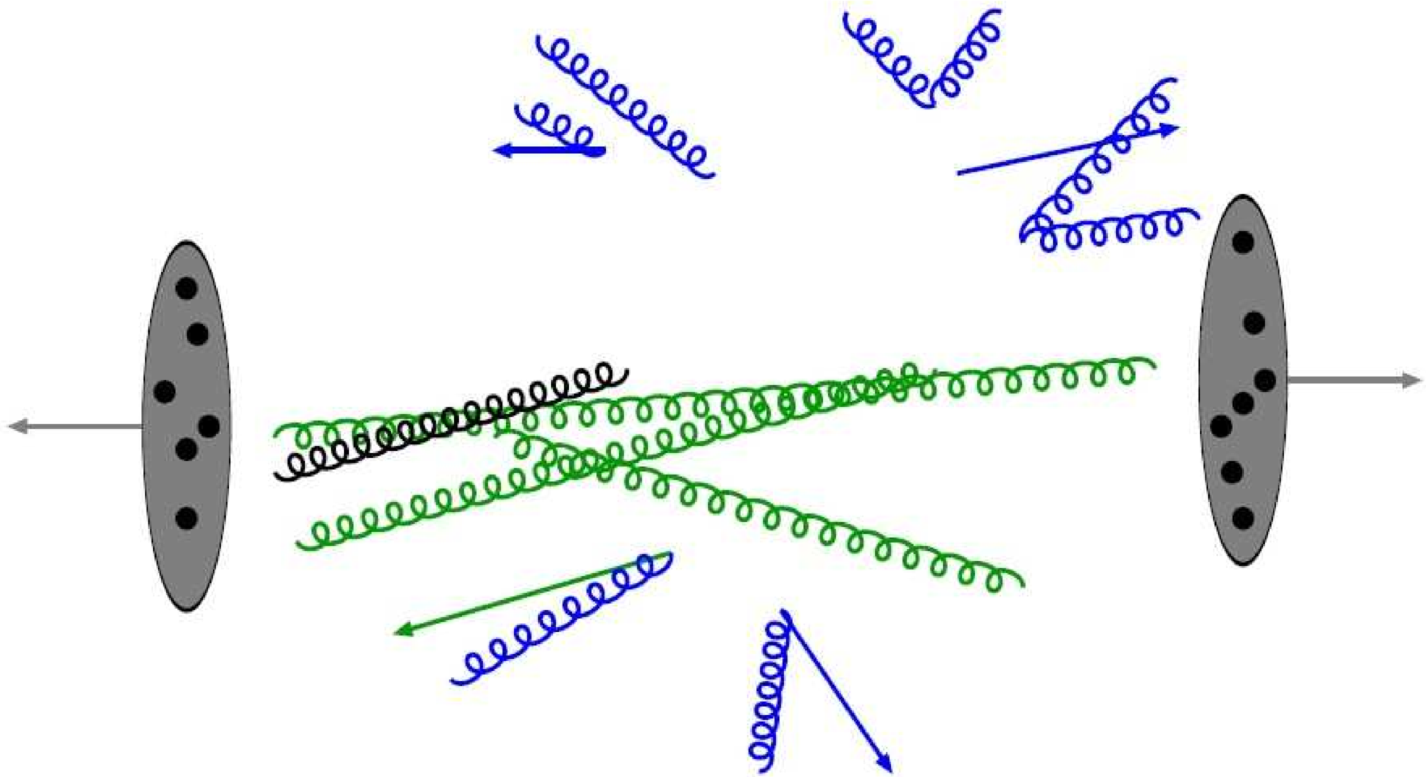
The muck



MPI with ISR and FSR !

The muck

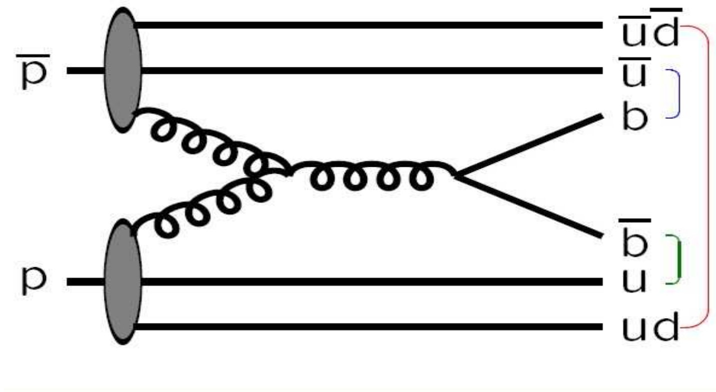
Movie: The structure of an event



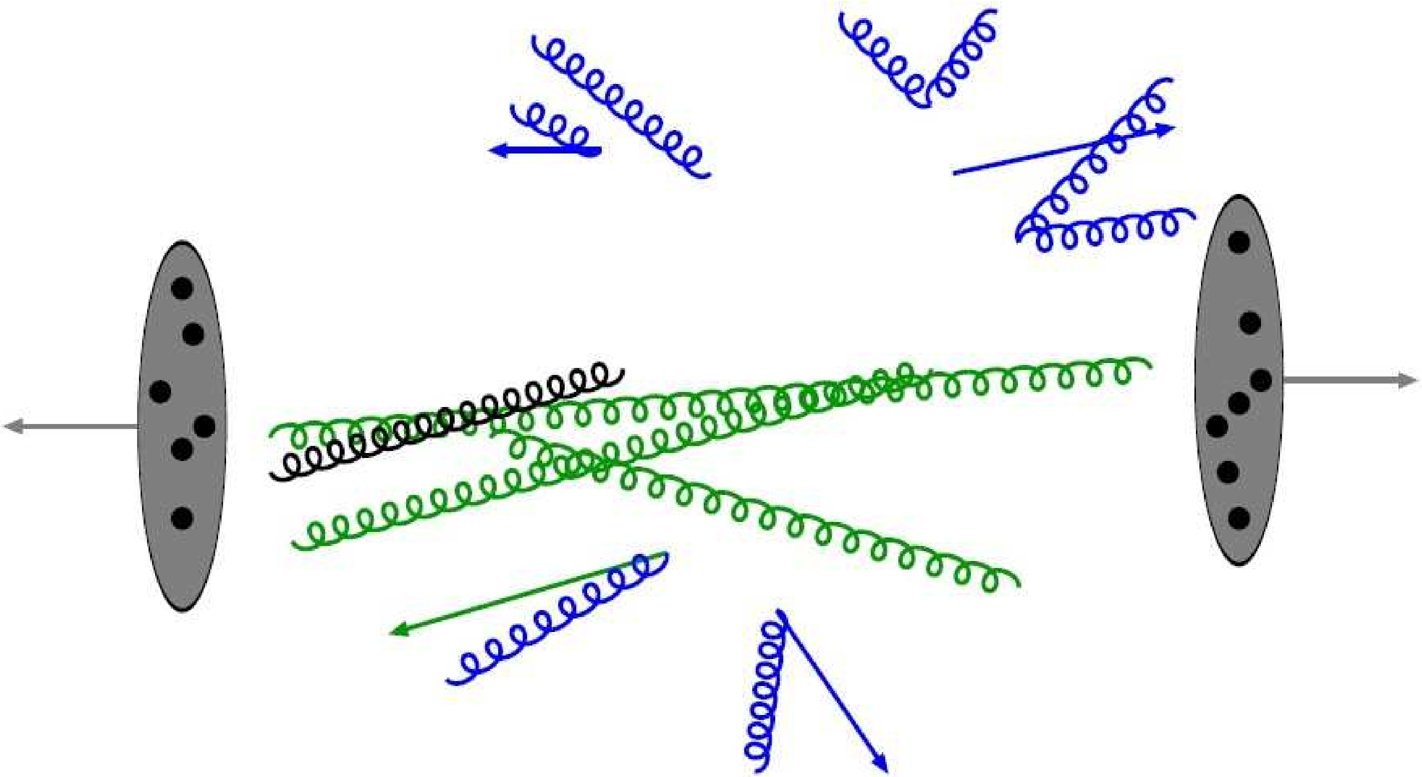
Beam remnants and other outgoing partons !

The muck

Movie: The structure of an event



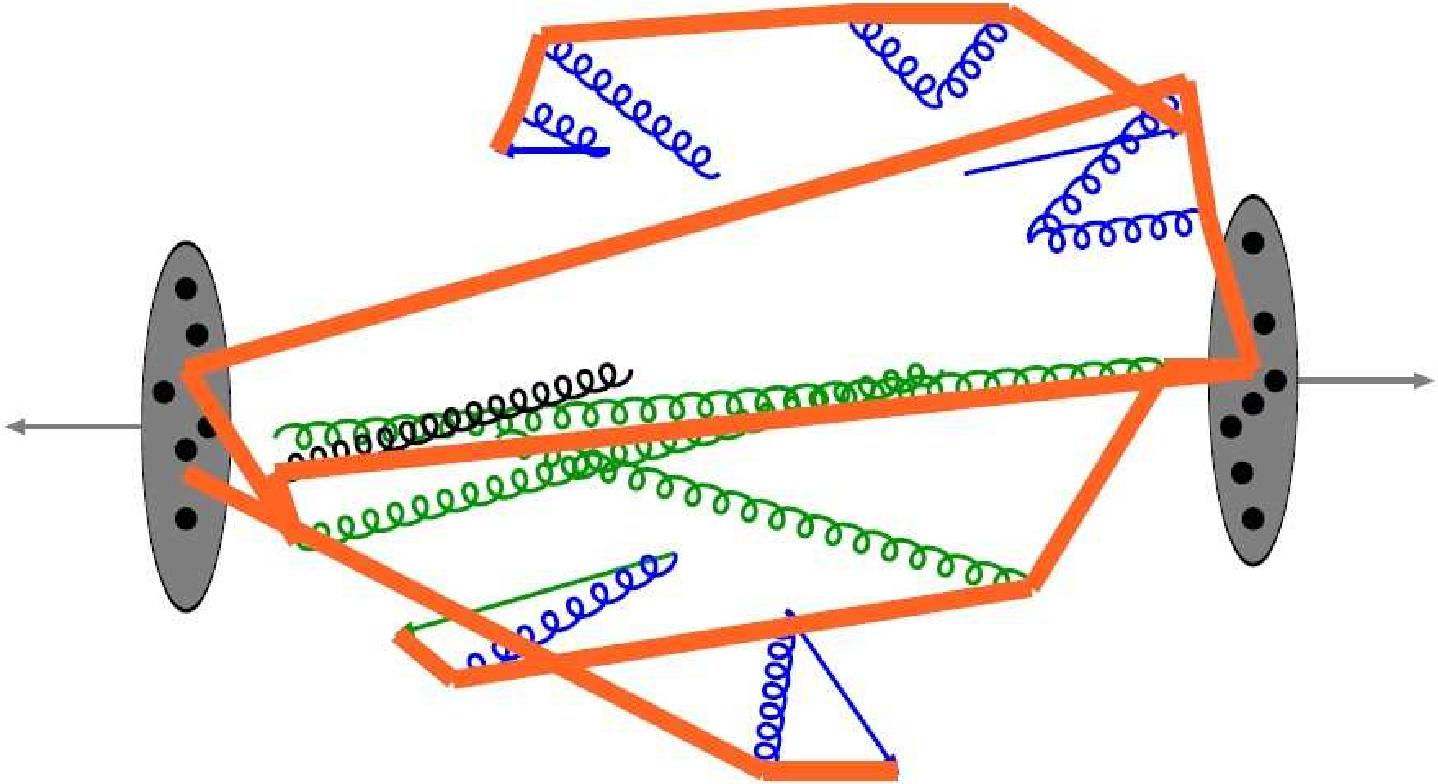
Beam remnants: coloured remains of the proton not taking part in the hard process, but they are colour connected to the hard process.



Beam remnants and other outgoing partons !

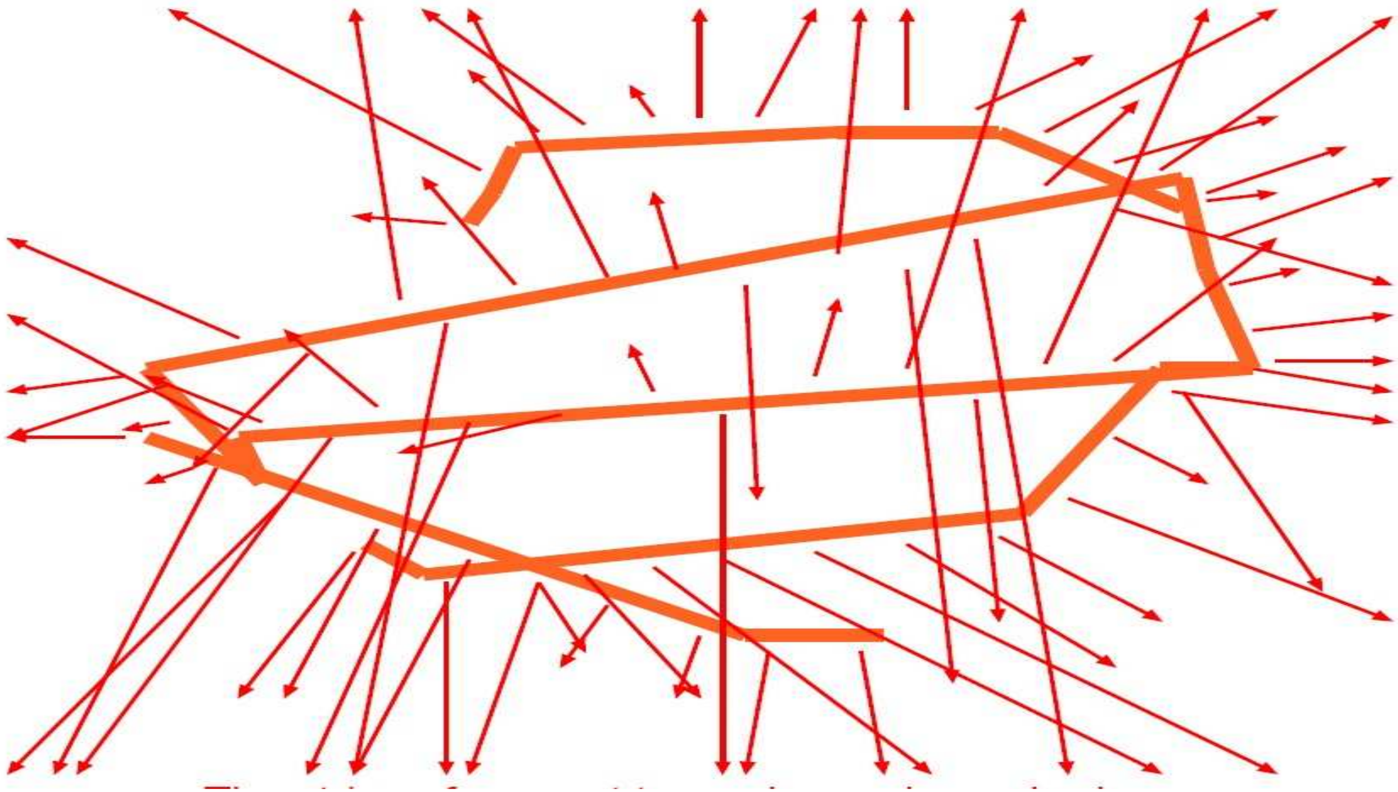
The muck: UE is not understood. It is modelled through some data that do not give strong constraints and hence extrapolation can be doubted.

Movie: The structure of an event

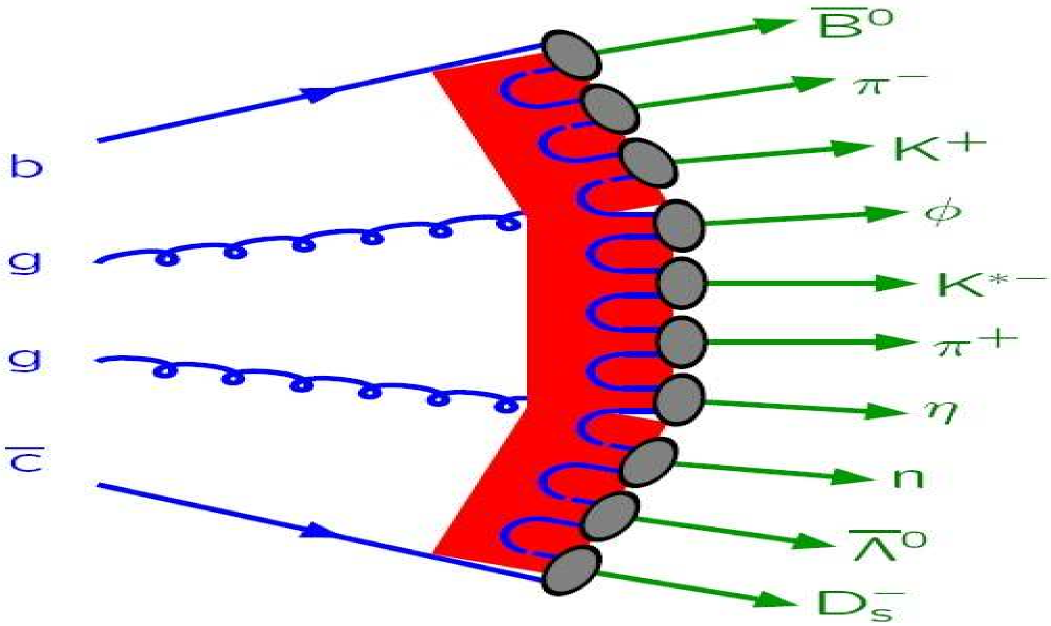


Everything is connected by colour confinement (here strings)

Movie: The structure of an event

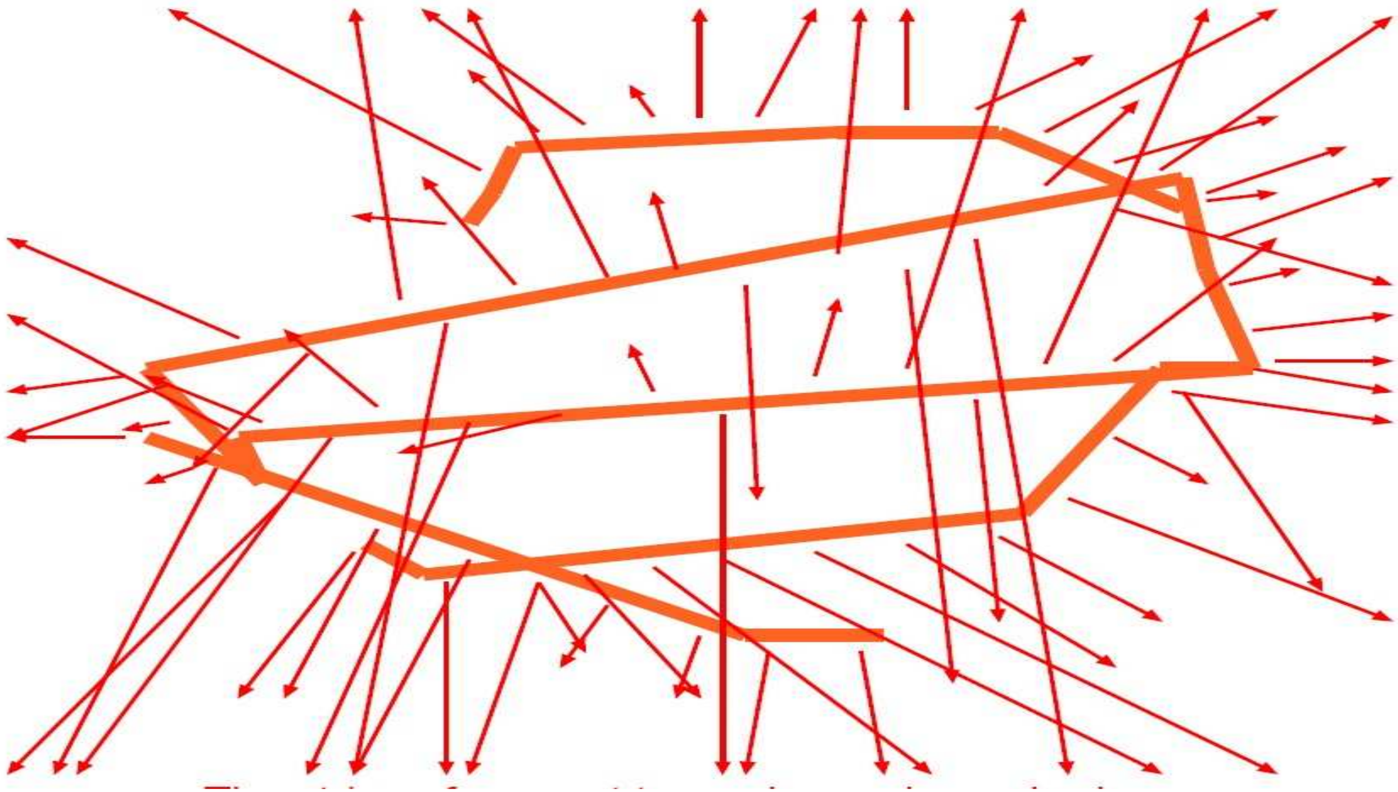


The strings fragments to produce hadrons



Hadronisation: Clusters to produce hadrons (Cluster Model)

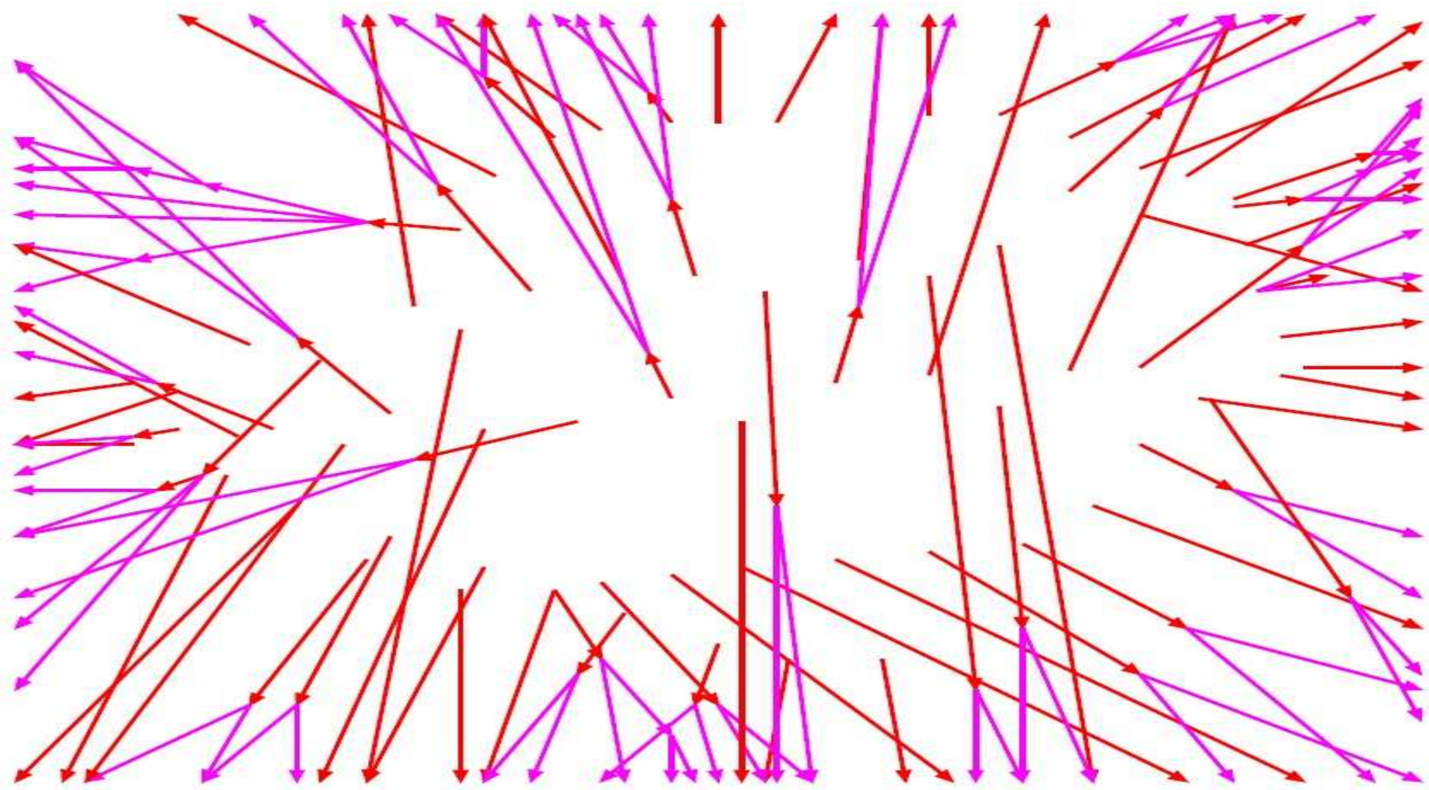
Movie: The structure of an event



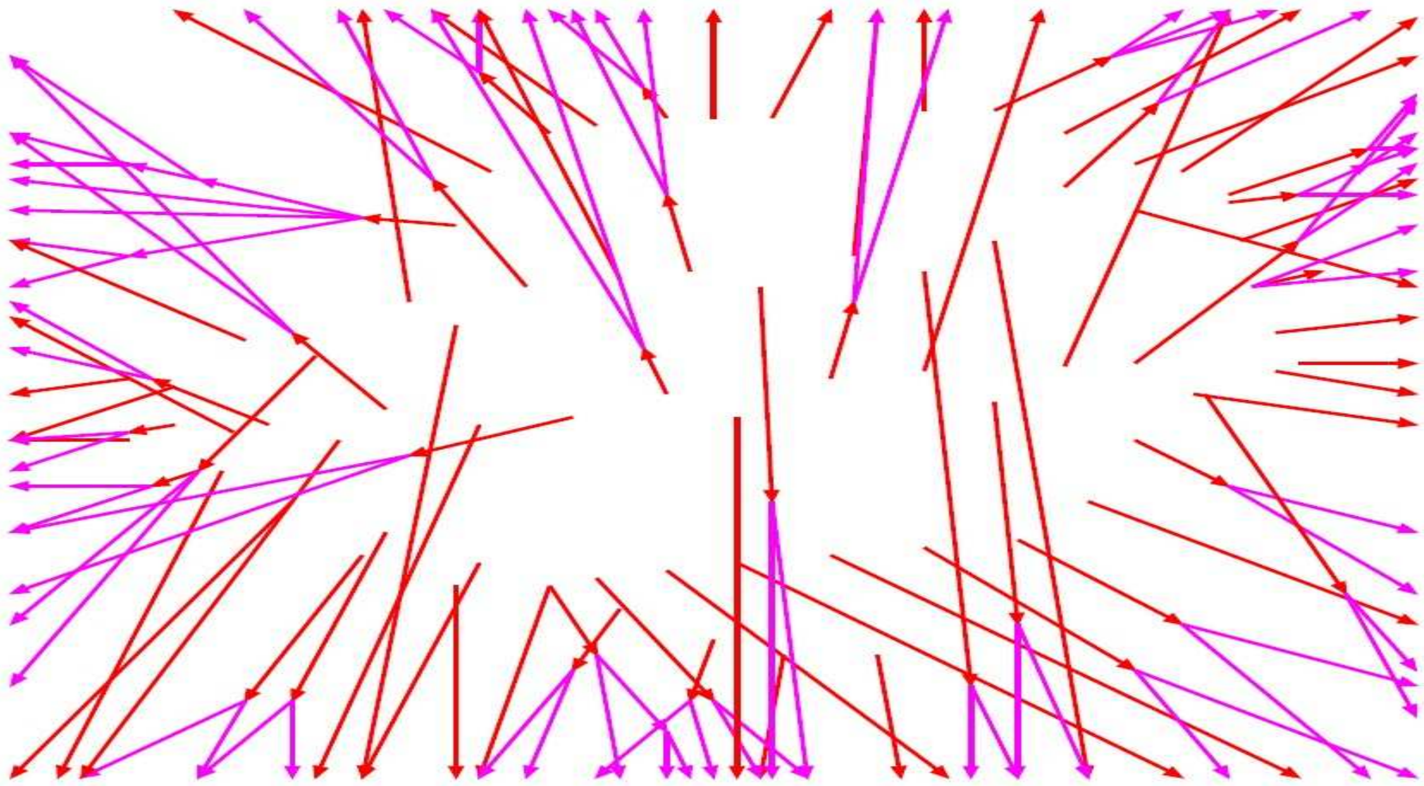
The strings fragments to produce hadrons (strings model)

Hadronisation is not really calculated from first principles, however it is modelled through various data and hence it is considered reliable

Movie: The structure of an event



Hadrons decay



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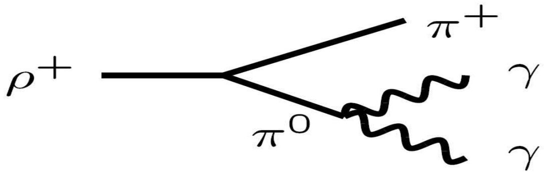

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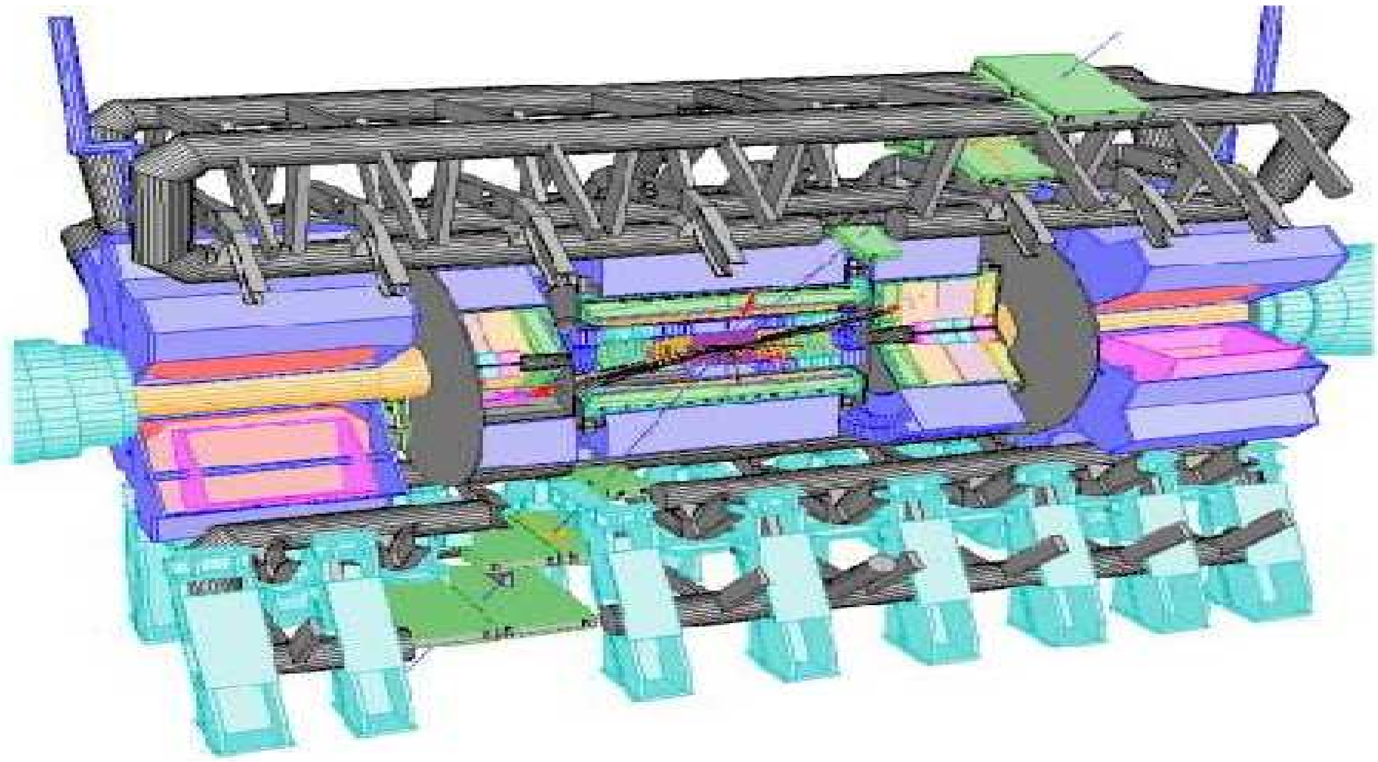

These are the particles that hit the detector

- Parton Shower is well understood , perturbation theory with a few approximations
- Hadronisation is not really calculated from first principles, however it is modelled through various data and hence it is considered reliable
- UE is not understood. It is modelled through some data that do not give strong constraints and hence extrapolation can be doubted.(minimum bias??)
- Important to have a “clear” picture of the physical situation

MC is probabilistic, divide and conquer

- generate events with as much details as possible:

  W will decay.

  To τ?, τ will decay,

  there is no quark,

  only hadrons,...

  production comes with non negligible radiation

- $\sigma_{\rm final\ state} = \sigma_{\rm hard\ process}\ \mathcal{P}_{\rm tot}$

- $\mathcal{P}_{\rm tot} = \mathcal{P}_{\rm decay}\mathcal{P}_{\rm ISR}\mathcal{P}_{\rm FSR}\mathcal{P}_{\rm remnants}\mathcal{P}_{\rm hadronise}\mathcal{P}_{\rm ord.\ dec.}$

- **Divide and Conquer : each $\mathcal{P}_{\rm i}$ handled in turn**

- an event with n particles involve about 10n random choices (flavour, mass, momentum,spin,...). At the LHC expect about 100 charged and 200 neutral particles, thus totalling a few thousand choices

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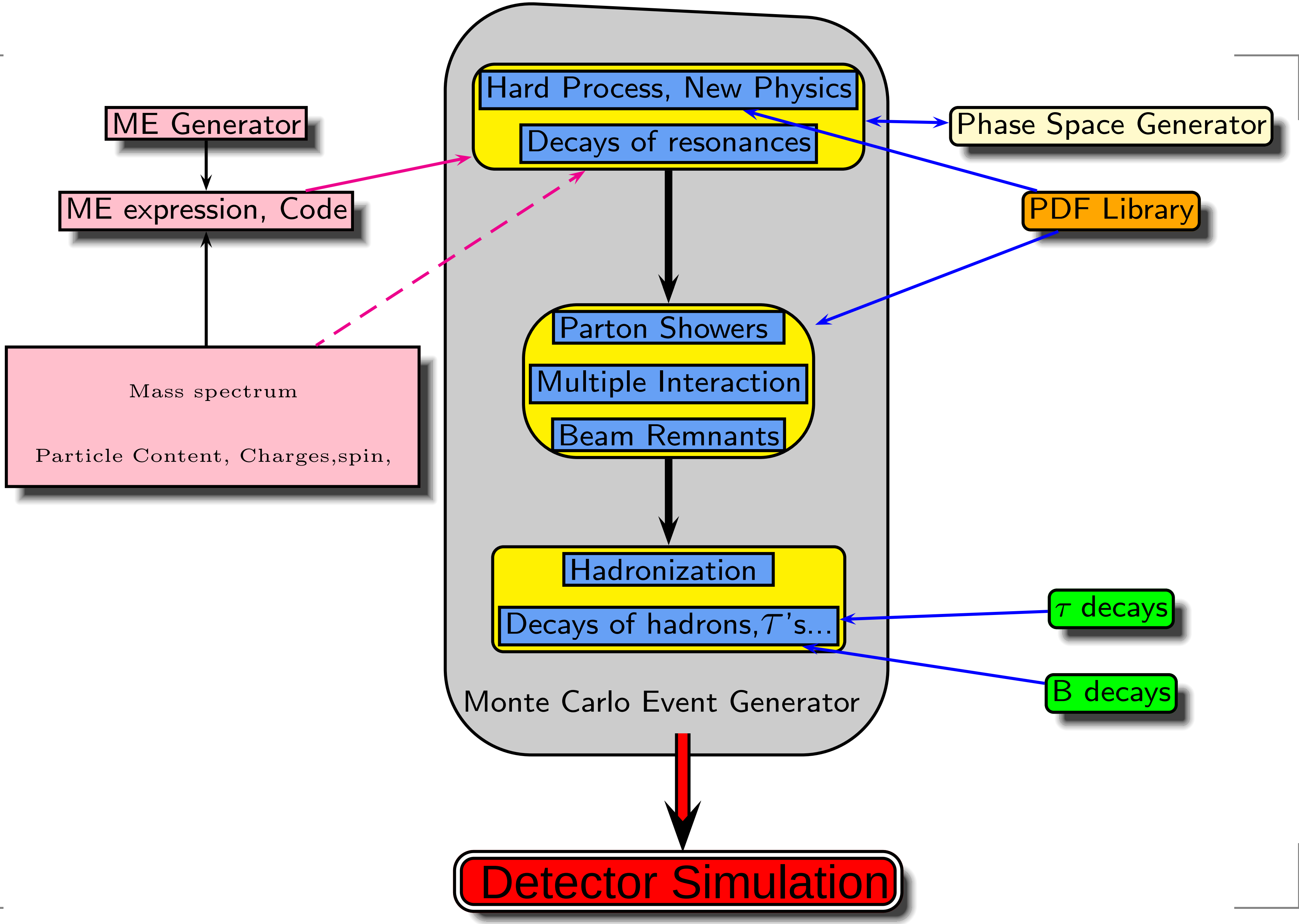
Divide and Conquer : each $\mathcal{P}_{\text{i}}$ handled in turn $\rightarrow$ Modular Structure

Putting all together



MEG vs General purpose MC EG

ME Generators

Hard Process and Decay

Many

....

(see later)

General Purpose MC EG

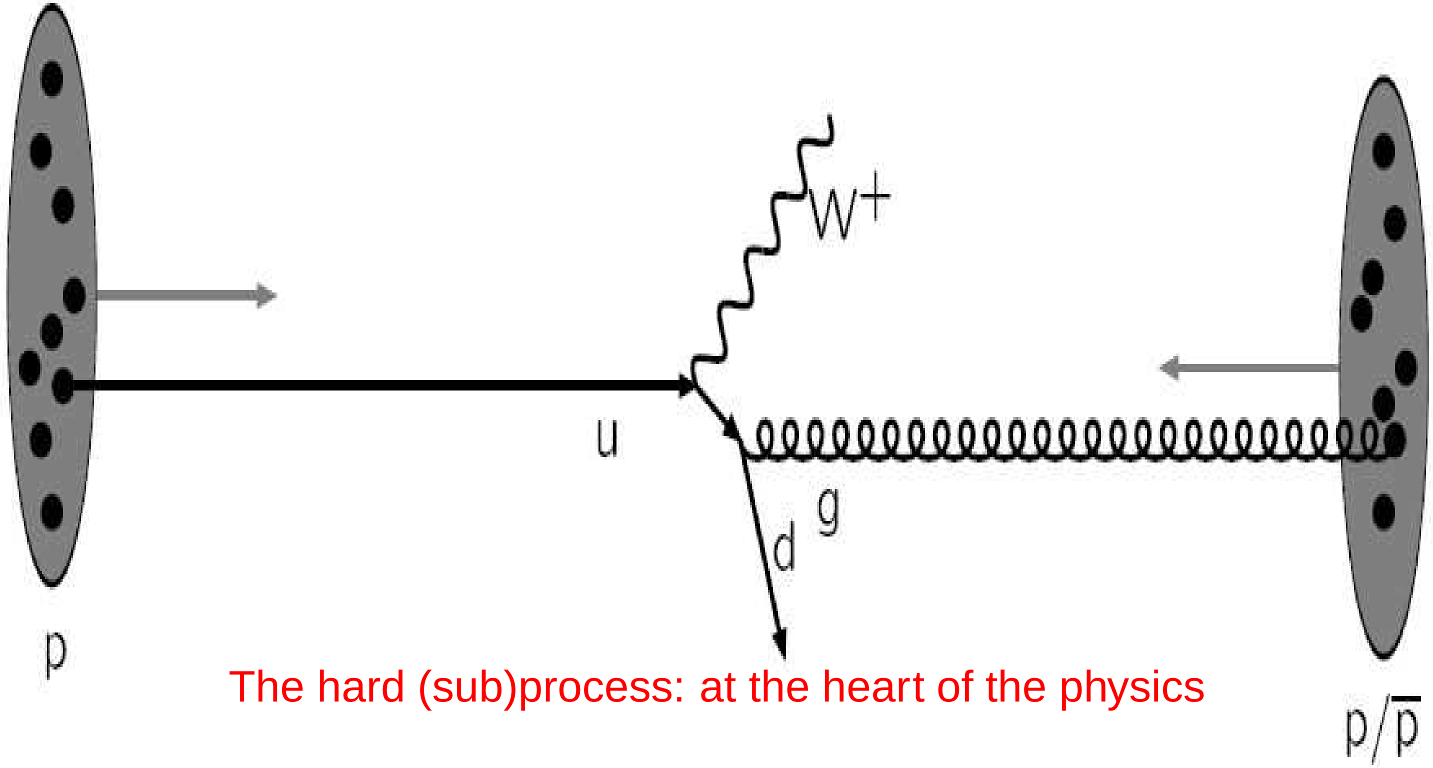
HERWIG++

PYTHIA

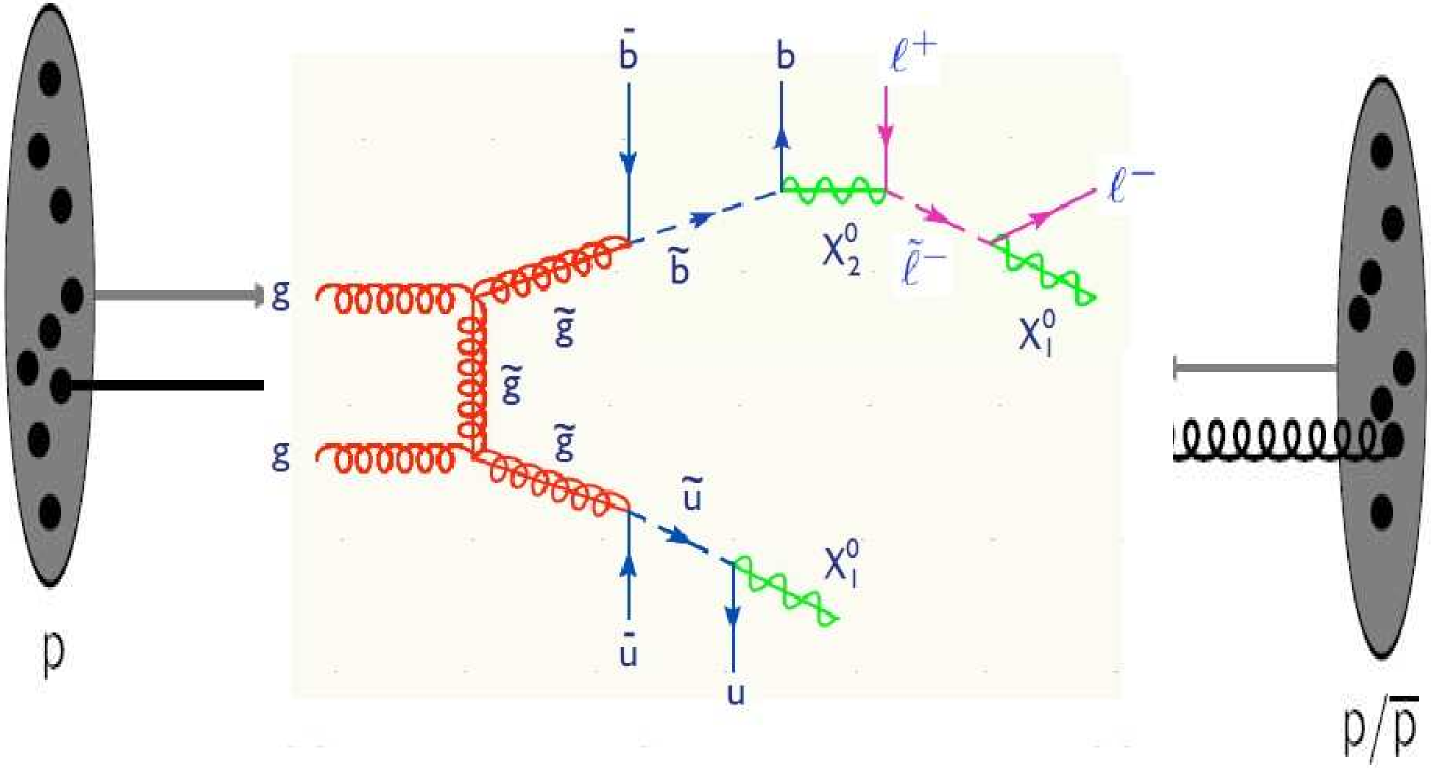
SHERPA

(ISAJET)

Integration: PDF and Cross sections



The hard (sub)process: at the heart of the physics



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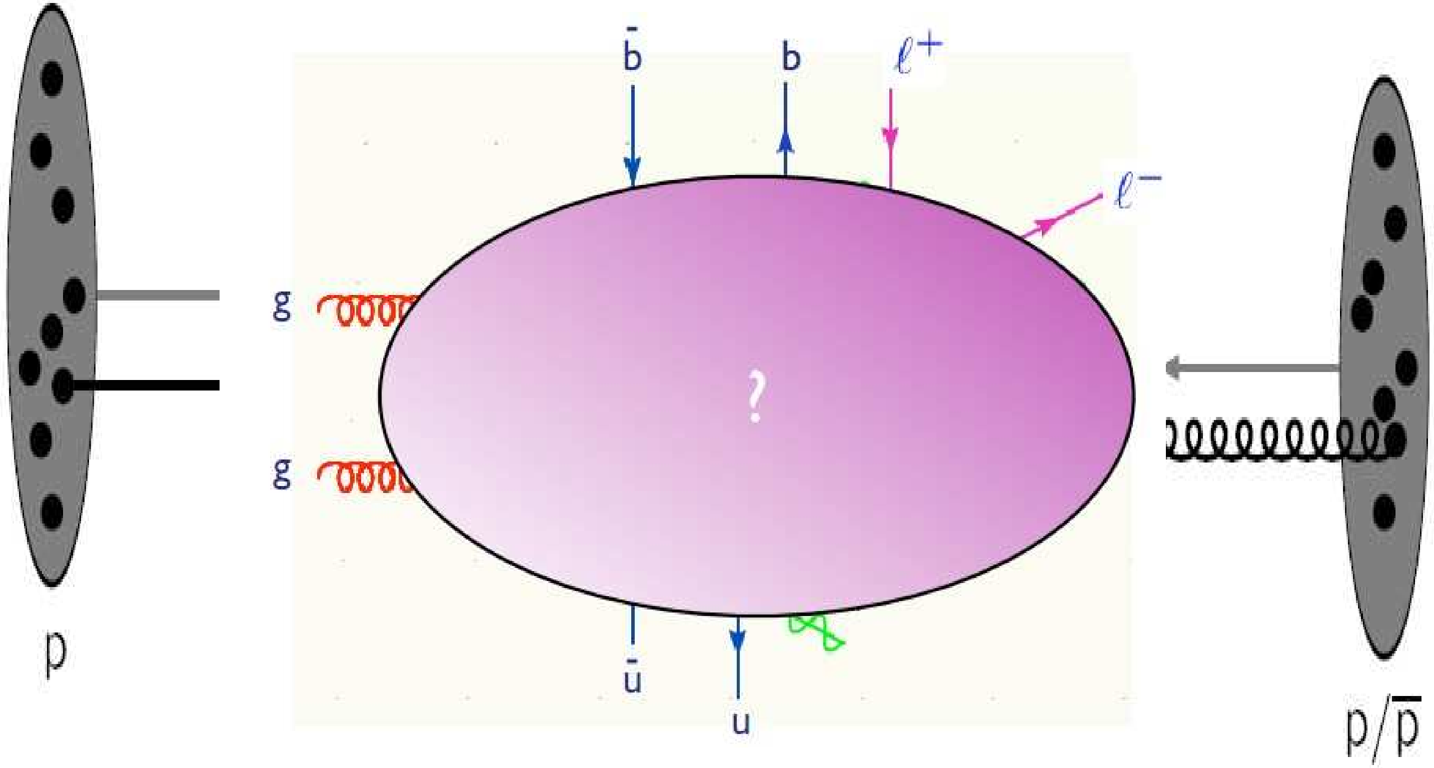


The hard (sub)process: at the heart of the physics

Factorisation and Parton Distribution Functions

$$\sigma_{pp\to X} = \sum_{a,b=q,\bar{q},g} \int \mathrm{d}x_1 \mathrm{d}x_2 f_a(x_1,\mu^2) f(x_2,\mu^2) \hat{\sigma}_{ab\to X}(\hat{s},\mu^2)$$

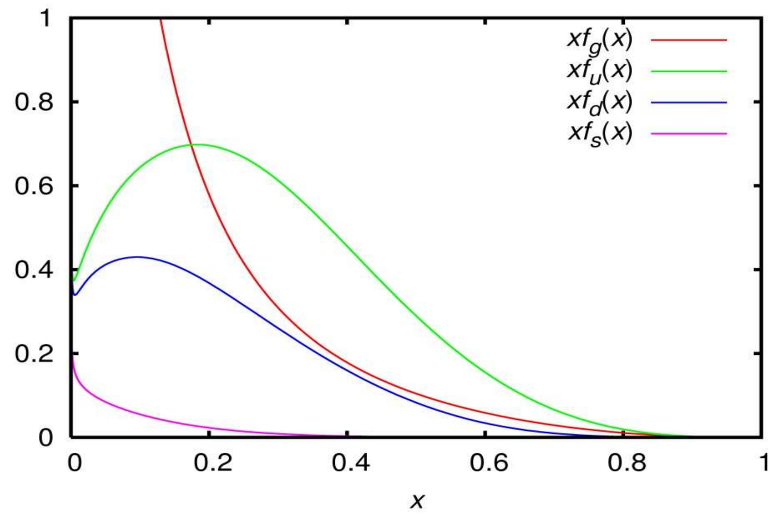
$f_i(x,\mu^2)$ is the **P**arton **D**istributions **F**unction

μ^2 is the factorisation scale !

Many libraries exist (CTEQ, MRSx)

reliable in the range

$10^{-3} < x < 0.8 \quad (2\mathrm{GeV})^2 < \mu^2 < (1TeV)^2$



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$f_i(x,\mu^2)$ is the Parton Distributions Function

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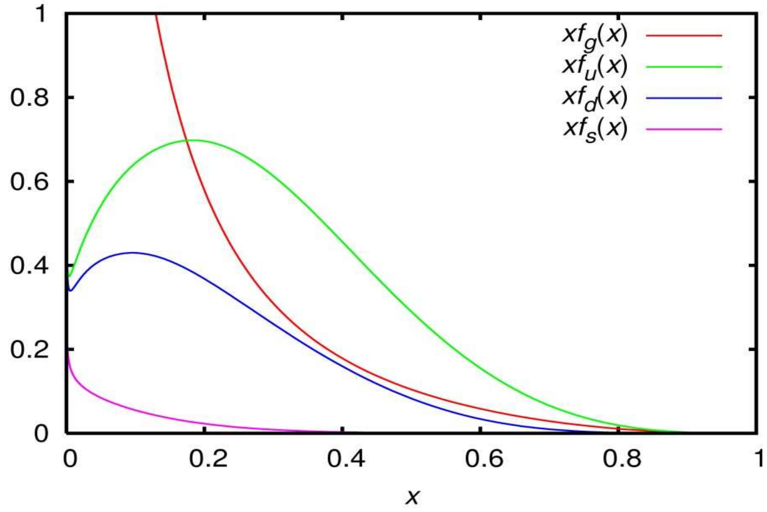

Phase Space

$$\hat{\sigma}_{ab\to X} = \frac{1}{2\hat{s}} \sum_{spin,..} \int_{\Phi_N} |\mathcal{M}|^2 \mathrm{d}\Phi_N$$

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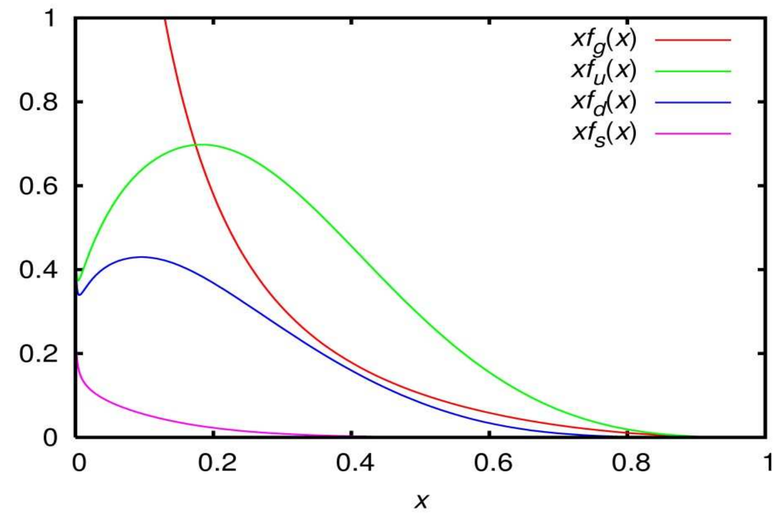

Phase Space

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Integrals $\longrightarrow \int$

At the heart of the ME is the hard process, that is where the physics lies and that is what gives the probability of a particular event
For the hard process

- amplitude $\mathcal{M} \longrightarrow |\mathcal{M}|^2$
- $N_{\rm evt,cuts} \propto \int d\sigma = \int |\mathcal{M}|^2 {\rm d}\Phi(n)$
- Integration over a phase space with of large number n of dimensions, each particle $\rightarrow 3$ variables (momenta)
- $Dim[d\Phi(n)] \sim 3n$

$$
{\rm d}\Phi(n) = \left(\prod_i n \frac{{\rm d}^2 p_i}{(2\pi)^3 (2E_i)} \right) (2\pi)^4 \delta\left(P_{in} - \sum_i^n p_i \right)
$$

- MC is a numerical method for calculating/estimating an integral based on a random evaluation of the integrand
- Particularly useful because one deals with a large number of (integration) variables (momenta of particles)
- Limits of integration (cuts) are often complicated
- Integrand is a convolution of different functions

$$I = \int_{x_1}^{x_2} f(x)dx = (x_2 - x_1) < f(x) > \quad \text{(usually } \ x_1 = 0, x_2 = 1)$$

The average can be calculated by selecting N values *randomly* $x_i, i = 1, \cdots N$ from *uniform distribution*, calculate $f(x_i)$

$$I = I_N = \frac{1}{N}(x_2 - x_1)\sum_{i=1}^{i=N} f(x_i) = \frac{1}{N}\sum_{i=1}^{i=N} W(x_i) \quad W(x_i) = \text{weight}$$

- Sum is invariant under reordering (randomize)
- Obviously approximation better if number of points N is larger
- Error given by the Central Limit Theorem

One dimension, example. The variance

$$I \simeq I_N \pm \sqrt{\frac{V_N}{N}} = I_N \pm \sqrt{\frac{< f^2 > - < f >^2}{N}}$$

- MC converges as $1/\sqrt{N}$

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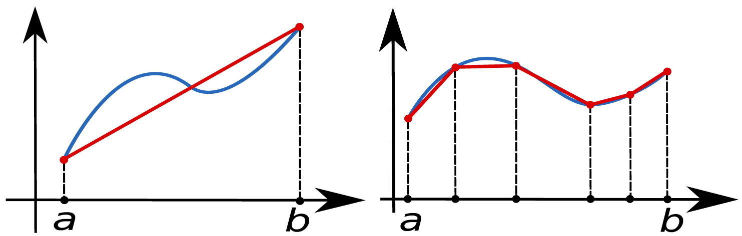
- compare to trapezium rule convergence $\propto 1/N^2$ (if derivative exists)



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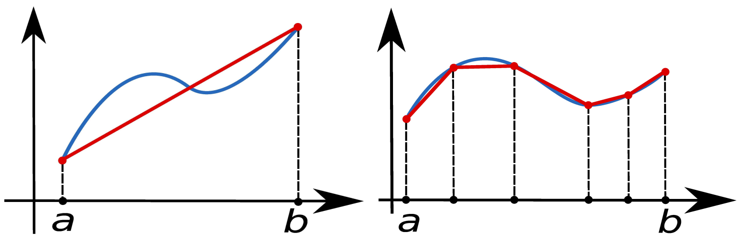
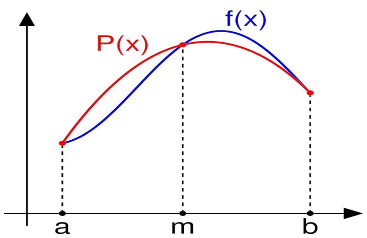


- Simpson (quadratic interpolation) $\propto 1/N^4$ (if derivative exists)



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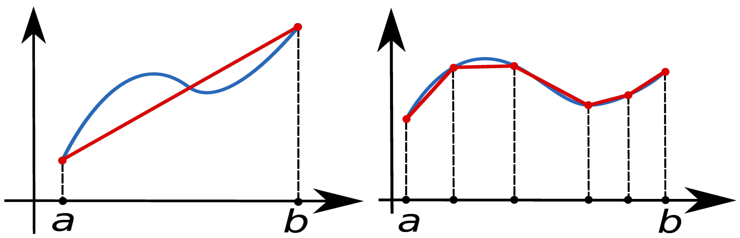

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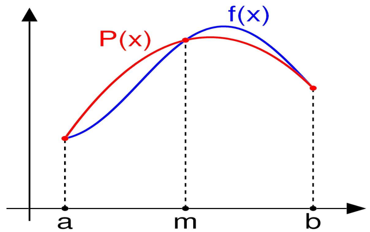


- but this is only in one dimension!

- Convergence may seem slow $\sqrt{1/N}$, but it can be estimated easily
- MC error does not depend on $\#$ of dimensions, d, $\propto 1/\sqrt{N}$

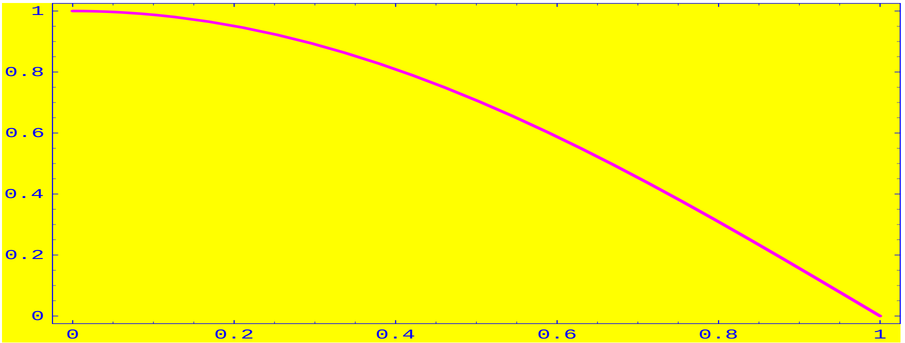
  Trapeze $\propto \rightarrow 1/N^{2/d}$

  Simpson $\propto \rightarrow 1/N^{4/d}$
- in MC one can improve convergence by minimising V_N while keeping the same number of points N
- **Importance Sampling**: non uniform sampling more efficient
- Convergence improved by putting more samples in regions where function is largest (where variance is largest)
- Hint: observe that if $f(x) = cste$ then $V_N = 0 \longrightarrow$ **make f as a close to a constant as possible!**

$$I \simeq I_N \pm \sqrt{\frac{V_N}{N}} = I_N \pm \sqrt{\frac{< f^2 > - < f >^2}{N}}$$

Example: Importance Sampling

Take $f(x) = cos \pi x/2$ then
$I = 2/\pi = 0.637$
MC, $I_N = 0.637 \pm 0.308/\sqrt{N}$
$(0.308 = \sqrt{V_N} = \sqrt{1/2 - (2/\pi)^2})$



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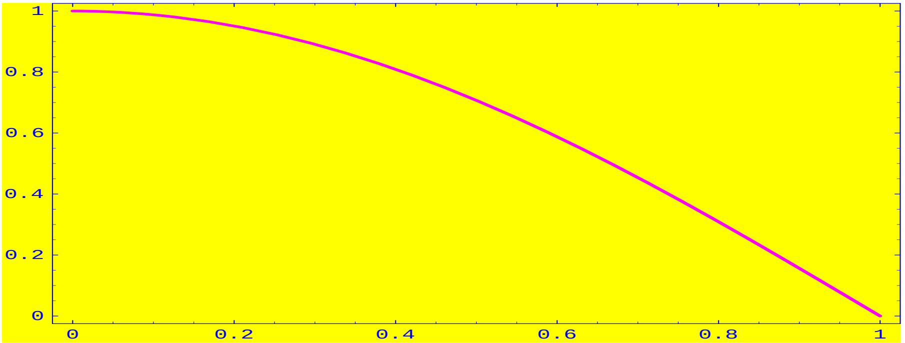

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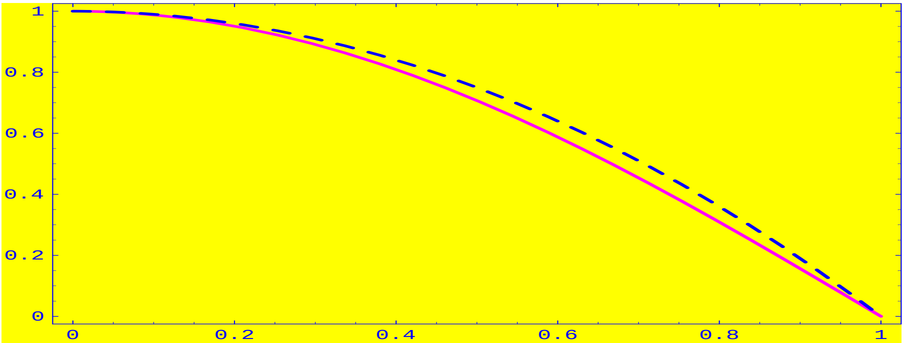
$$= \int_{y_1}^{y_2} dy \frac{cos\pi x[y]/2}{1-x[y]^2}$$



MC, $I_N = 0.637 \pm 0.031/\sqrt{N}$

Example: Importance Sampling

Take $f(x) = cos\pi x/2$ then
$I = 2/\pi = 0.637$
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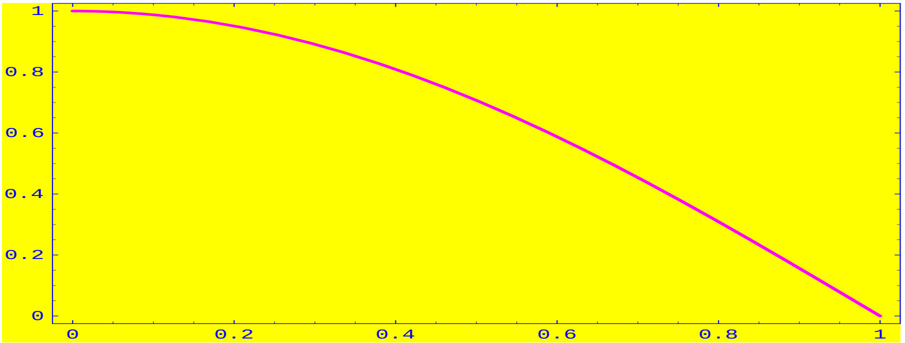

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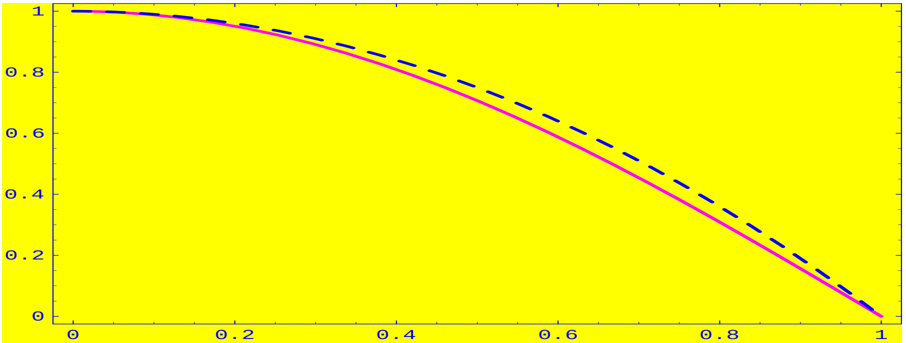

MC, $I_N = 0.637 \pm 0.031/\sqrt{N}$

- For the same accuracy $N \to N/100$ events
- We have in fact made a change of variables
- Note however that change of variables may be not so trivial and requires that one knows the function, here is relatively ok
  $y = x - x^3/3$!

Over a Breit-Wigner distribution

in HEP many sharp peaks from resonances, apart from peaks due to forward scattering,..

$$I = \int_{m^2_{min}}^{m^2_{max}} \mathrm{d}m^2 \frac{1}{(m^2 - M^2)^2 + M^2\Gamma^2}, \quad \Gamma/M \ll 1$$

$$= \frac{1}{M^2} \int_{x_{min}}^{x_{max}} \mathrm{d}x \frac{1}{(x-1)^2 + \epsilon^2}, \quad x = m^2/M^2, \quad \epsilon = \Gamma/M$$

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$$
\begin{aligned}
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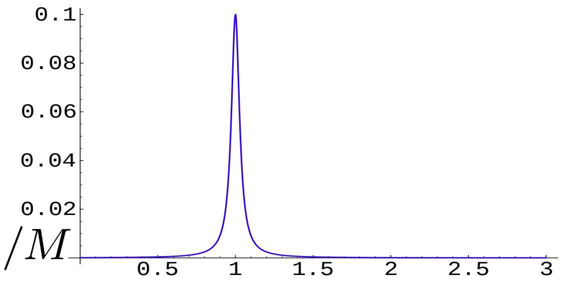
$$



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change of variable $x = \varepsilon \tan\theta + 1, dx = \varepsilon(1 + \tan^2\theta)dt$

$$I = \frac{1}{M\Gamma} \int_{\theta_{min}}^{\theta_{max}} d\theta$$

The peak has been smoothed out completely.

Flat distribution, the error has been reduced to 0

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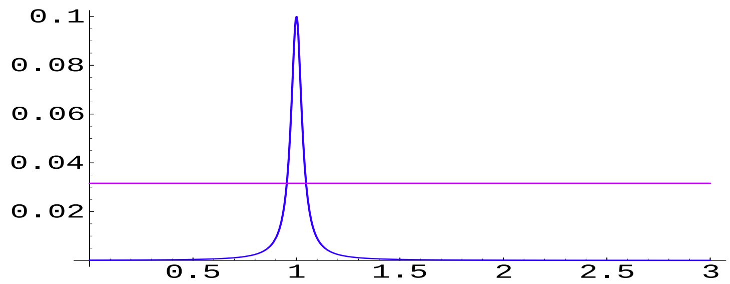

Unfortunately we can not always do the Jacobian trick efficiently, we do not always know $f(x)$

However, as we have seen, finding a simple function, $p(x)$, that approximate $f(x)$ reduces the error drastically

(up to normalisation) take

$$p(x), \int_{x_1}^{x_2} p(x) = 1, \qquad \rightarrow I = \int_{x_1}^{x_2} dx p(x) \frac{f(x)}{p(x)}$$

$$I = \langle \frac{f}{p} \rangle \pm \frac{1}{\sqrt{N}} \sqrt{\langle \frac{f^2}{p^2} \rangle - \langle \frac{f}{p} \rangle^2}$$

Sample according to $p(x)$ and make f/p as small as possible.

Unfortunately we usually do not know much about $f(x)$

But as we sample we can know more, reconstruct $p(x)$ piecemeal, with step function

$$p(x) = \frac{1}{N_b \Delta x_i} \quad \text{for} \quad x_i - \Delta x_i \leq x \leq x_i$$

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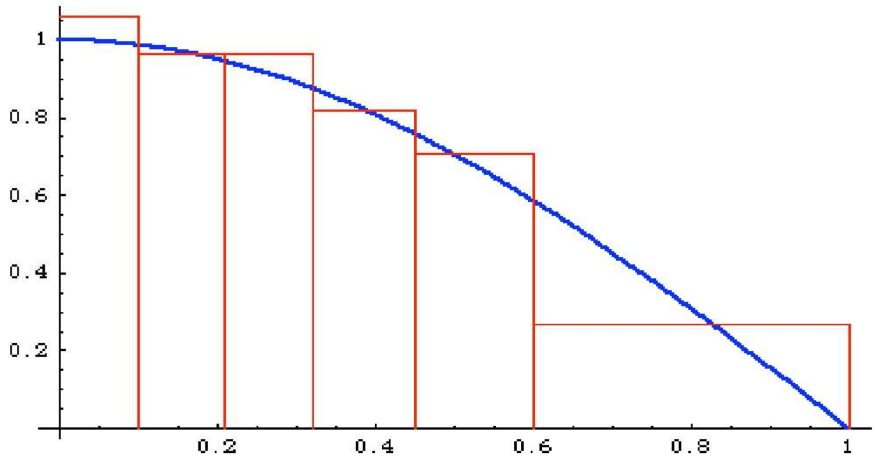

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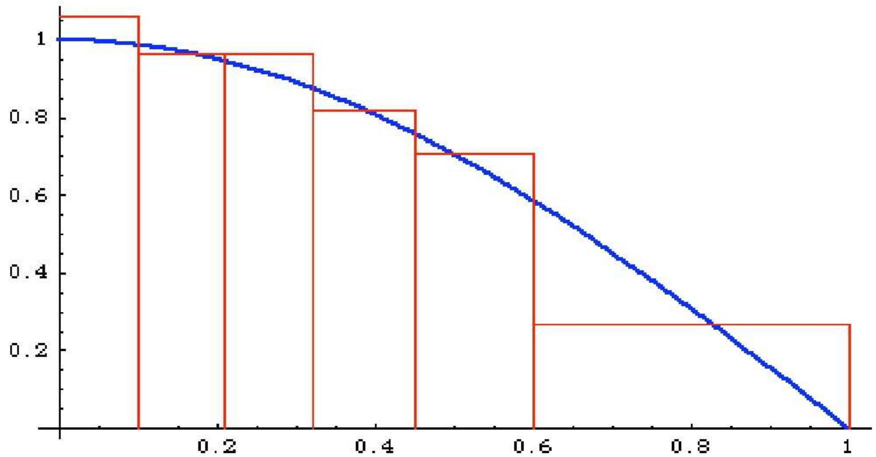

- Improve the fit by generating more points where $f(x)$ is large, *i.e* where the variance is large
- Adjust the bin size so that each bin has the same area

Iterative algorithm: VEGAS

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Many variables, VEGAS bis

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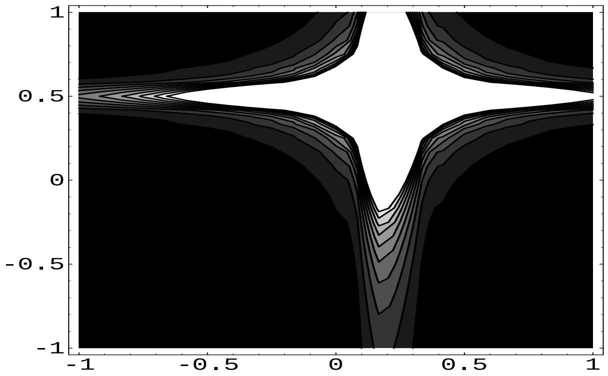
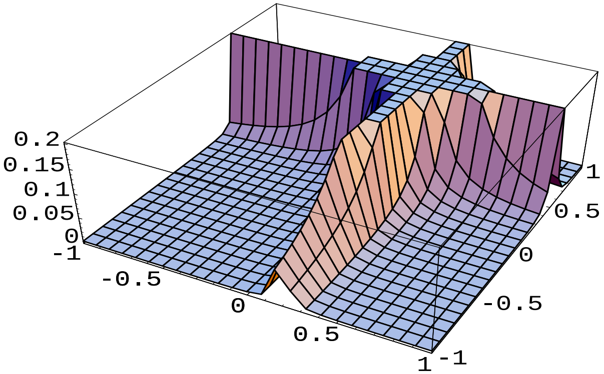
- this means 2^n possible kinematical invariants
- A scattering amplitude may have many peaks each aligned on a different invariant

VEGAS and alignment



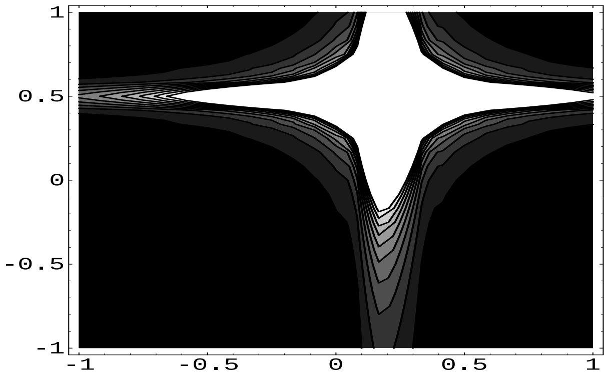


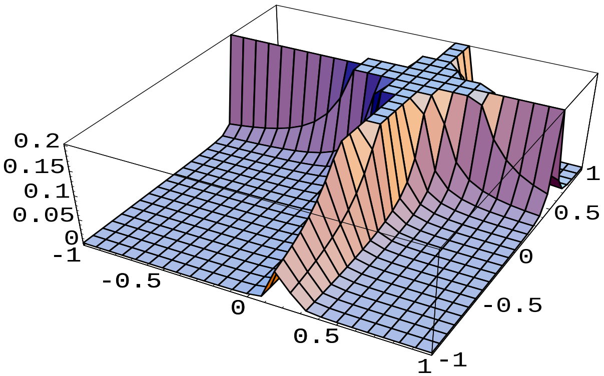
$$\left(((x-0.2)^2+\epsilon)((y-0.5)^2+\epsilon)\right)^{-1}$$

$$\epsilon = 0.002$$

Ok for VEGAS, peaks aligned along the axes

VEGAS and alignment





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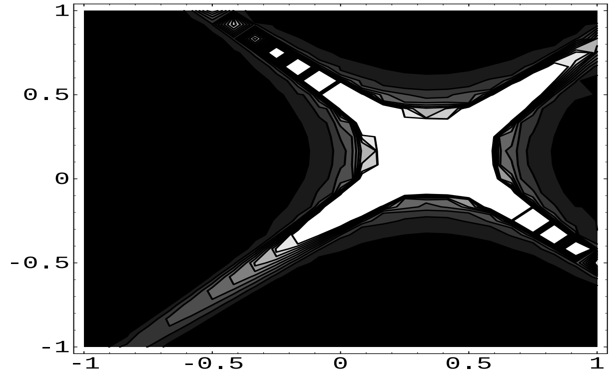

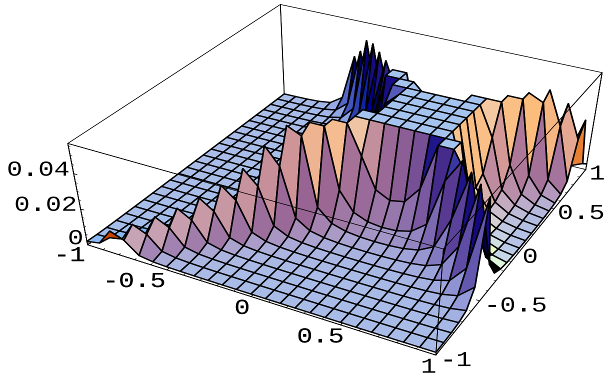

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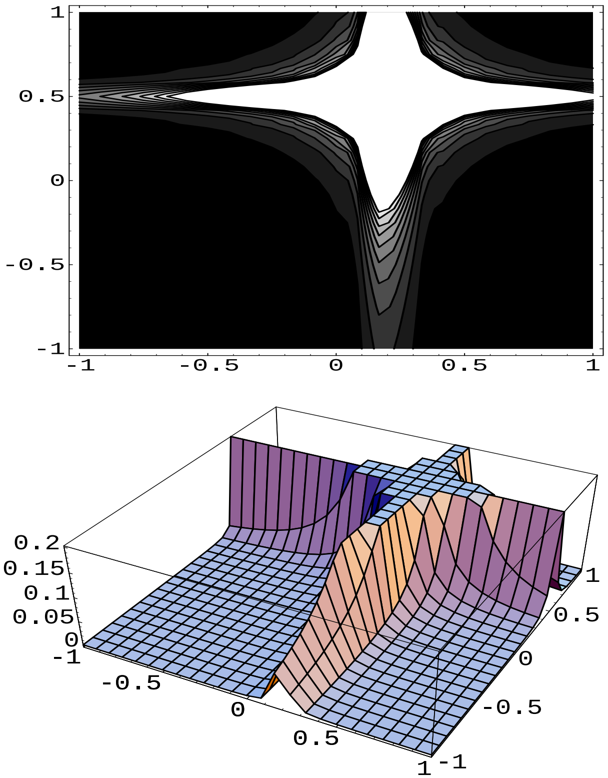
NOT Ok for VEGAS, here rotate the axes

VEGAS and alignment



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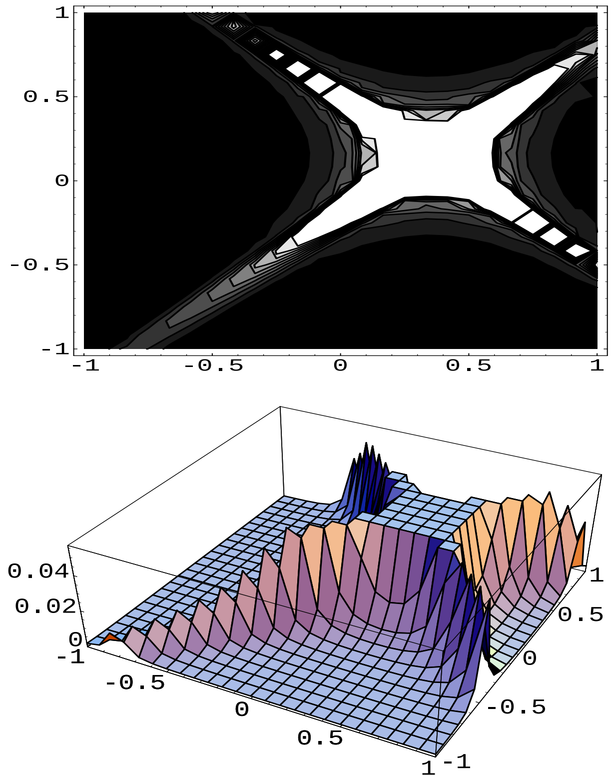

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For physical processes we usually know where the peaks are

Suppose instead of having one peak we have an integral with lots of peaks, say from the inclusion of excited ρ resonances in some process.

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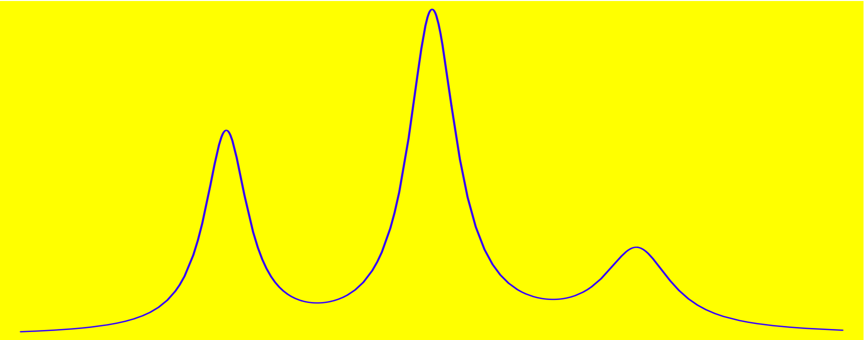

Multichannel d=1

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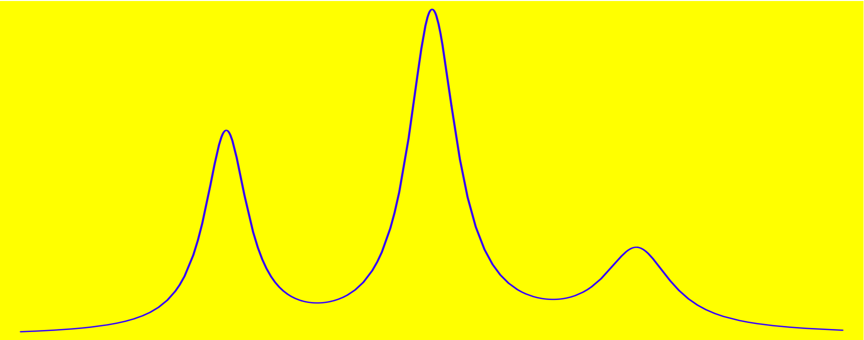

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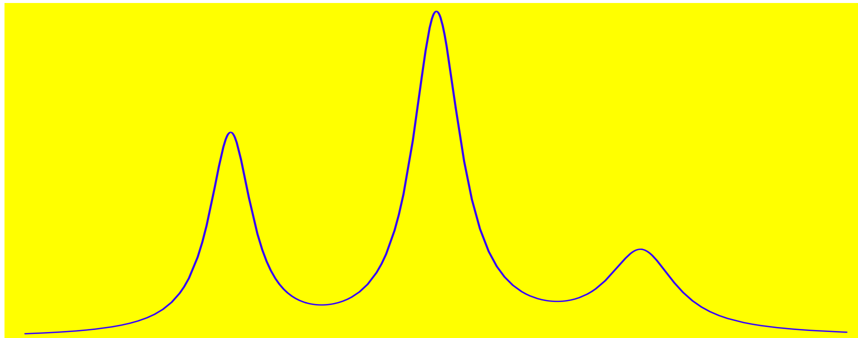

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$$h(m^2) = \sum_i \alpha_i g_i(m_i) = \sum_i \alpha_i \frac{1}{(m^2 - M_i^2)^2 + M_i^2\Gamma_i^2}, \quad \alpha_i = \text{weight} \quad \sum_i \alpha_i = 1$$

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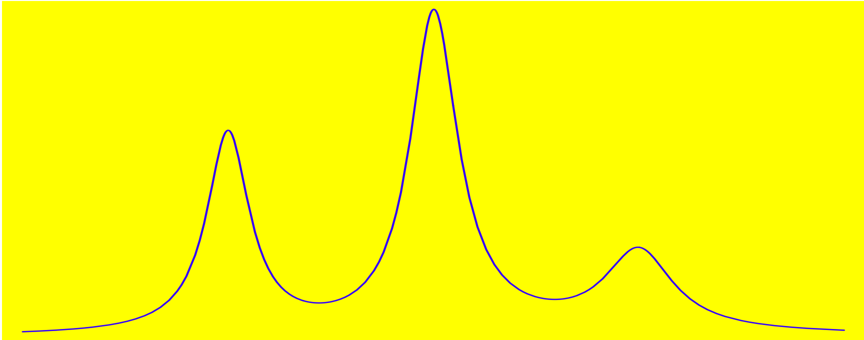

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$$= \sum_i \alpha_i \int_{\theta_{min}}^{\theta_{max}} d\theta_i^2 \frac{f(m^2}{h(m^2)}$$

Pick one of the integrals (channels) with prob α_i then calc. weight

α_i can be automatised. $\int$ does not depend on α_i but V_N does

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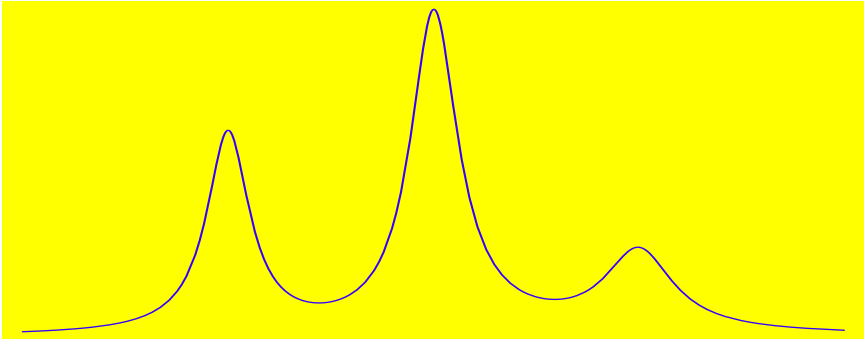

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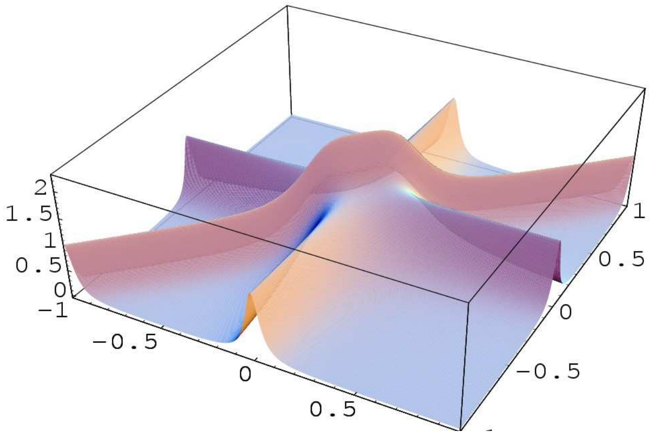
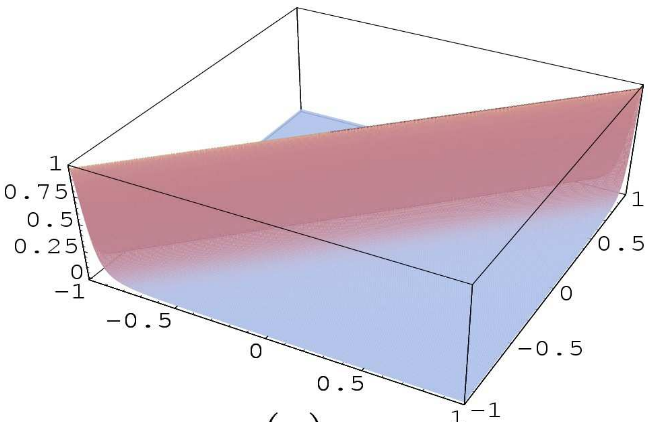
In general not each channel is invertible $\rightarrow g_i$ (peaking may be more complicated)
N coupled equations for α_i, so best when number of channels small.

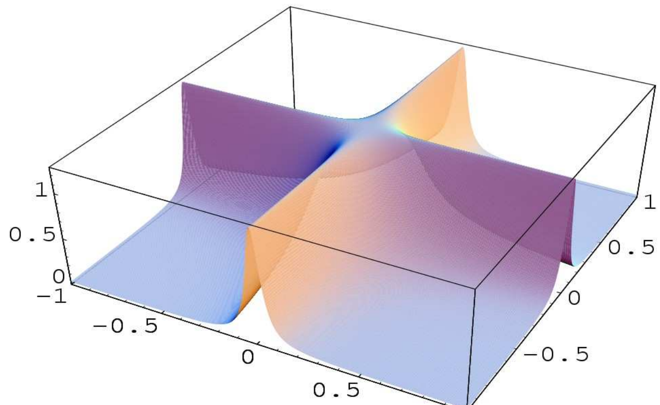
This is the method (multi-channel) used in the most sophisticated codes.



- what to do here?
- decompose into different channels
- $\alpha_1 p_1(x) + \alpha_2 p_2(x)$



$p_1(x)$



$p_2(x)$

For physical processes we usually know where the peaks are

cross section integrator vs event generator

$$\mathrm{d}\sigma(u\bar{u} \to Z^0 \to d\bar{d}) = \frac{1}{\hat{s}}|\mathcal{M}|^2 \frac{\mathrm{d}\cos\theta \mathrm{d}\phi}{8(2\pi)^2}$$

- sample the phase space (2-dim) $-1 < \cos\theta < 1,\, 0 < \phi < 2\pi$
- choosing $\cos\theta, \phi$ variables using uniformly distributed random number generator defines a candidate event
- $\mathrm{d}\sigma$ is the event weight (probability of the event)
- $< \mathrm{d}\sigma > \sim \int \mathrm{d}\sigma$ converges to the cross section
- at this point candidate events $\theta\phi$ are distributed flat and carry no physics

cross section integrator vs event generator

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If function to be integrated is a probability density (positive definite, $f(x) > 0$) one can convert it to arrive at a simulation of physical processes or Event Generator

- In addition to calculating the integral we often also want to select values of x (momenta,..) *at random according to* $f(x)$. This is easy provided that we know the maximum value of the function in the region we are integrating over.
- Then we randomly generate values of x in the integration region and keep them with probability

$$\mathcal{P} = \frac{f(x)}{f_{\max}} \leq R$$

- which is easy to implement by generating a random number between 0 and 1 and keeping the value of x if the random number R is less than the probability.
- This is called unweighting.

Selection of x according to $f(x)$, in a random probabilistic way, event as they occur in Nature

$$\int_{x_{min}}^{x} f(z)dz = R\int_{x_{min}}^{x_{max}} f(z)dz = RI$$

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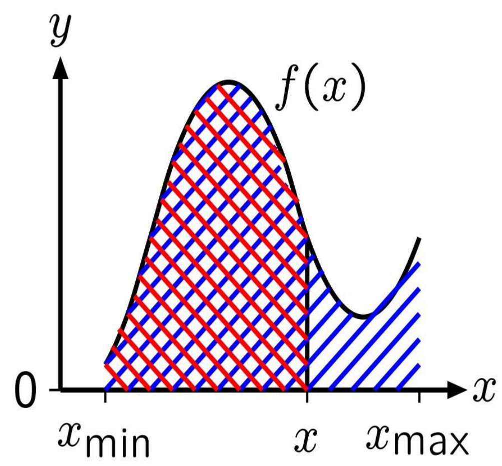

- **Analytical (assumes primitive and its inverse known)**

$x = F^{-1}(F(x_{min}) + R\ I)$

Selection of x according to $f(x)$, in a random probabilistic way, event as they occur in Nature

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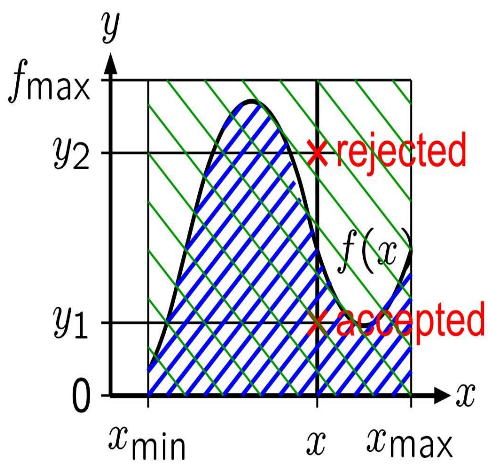
●● **Hit and miss: assumes f_{max} known**



$$I = \int_{x_{min}}^{x_{max}} f(x)dx = f_{max}(x_{max} - x_{min}) \frac{N_{acc}}{N_{tries}}$$

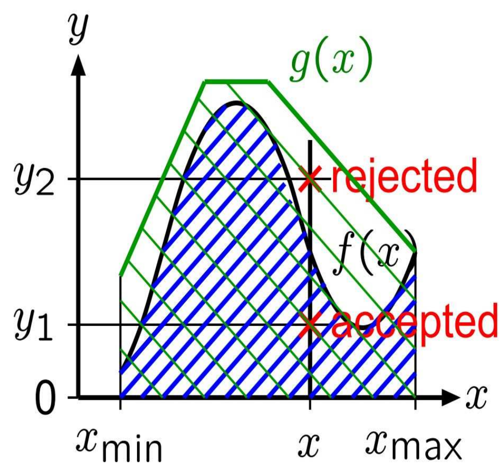
$$\frac{N_{acc}}{N_{tries}} = \text{efficiency}$$

MC $\rightarrow$ Event Generator, involves acceptance/rejection

Selection of x according to $f(x)$, in a random probabilistic way, event as they occur in Nature

$$\int_{x_{min}}^{x} f(z)dz = R\int_{x_{min}}^{x_{max}} f(z)dz = RI$$

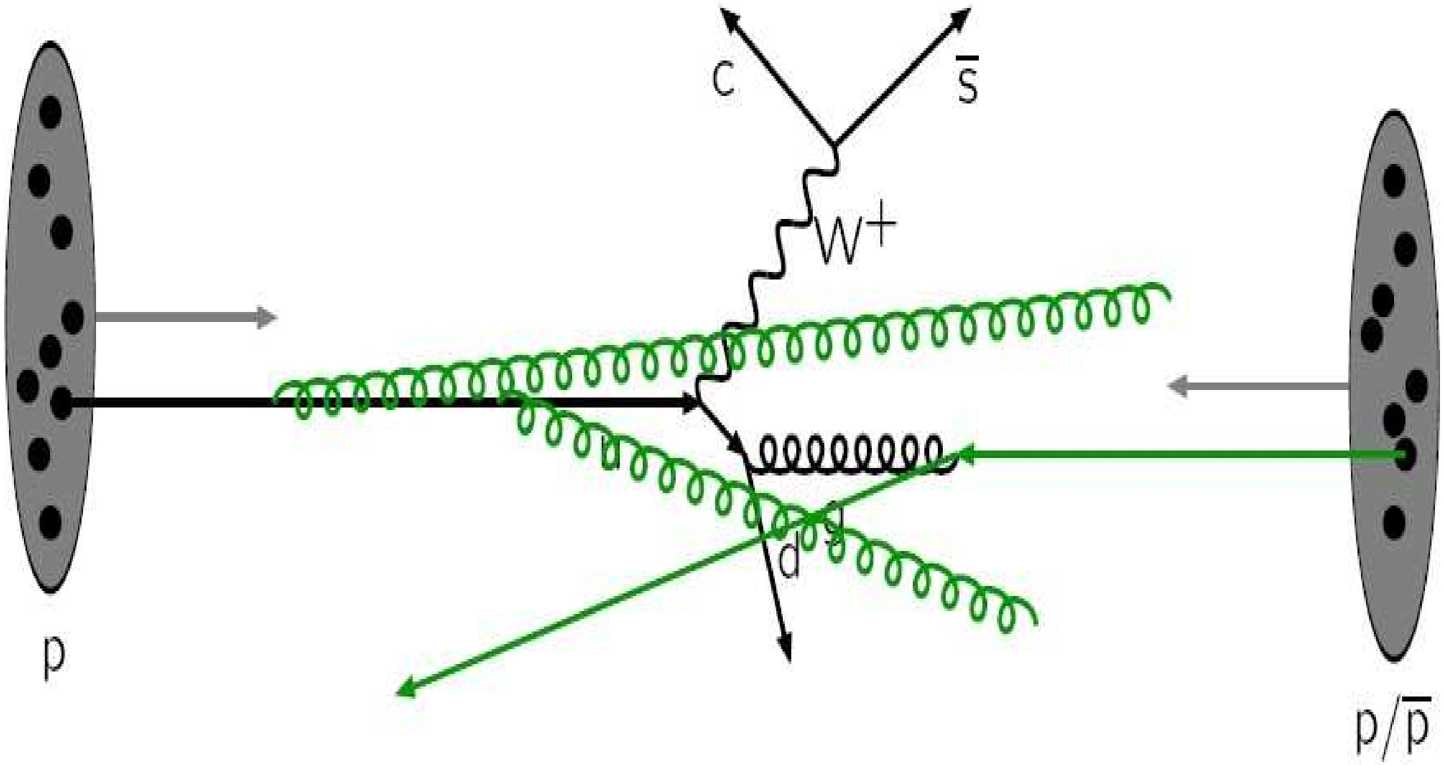
• • •**Importance Sampling: take $f(x) < g(x)$ where $G(x)$ and G^{-1} simple**



1. select x according to $g(x)$ 2. select $y = Rg(x)$ (new R)
   if $y > f(x)$ go back to 1

- Advantages of Monte Carlo Fast convergence in many dimensions
  - Arbitrarily complex integration regions
  - Few points needed to get first estimate
  - Each additional point improves the accuracy
  - Easy error estimate
  - More than one quantity can be evaluated at once.
- Disadvantages of Monte Carlo Slow convergence in few dimensions, but that is hardly the case in particle physics
- MC is well suited for particle physics where phase space integration involves a lot of variables with a complicated often not smooth function representing the cross section

- With an integrand that is positive definite, which is the case for MC at LO, one deals with a probability. This lends itself to an event generator
- Allows a fully exclusive treatment exactly like *real data*
- At the most basic level a Monte Carlo event generator is a program which simulates particle physics events with the same probability as they occur in nature.
- In essence it performs a large number of integrals and then unweights to give the momenta of the particles which interact with the detector

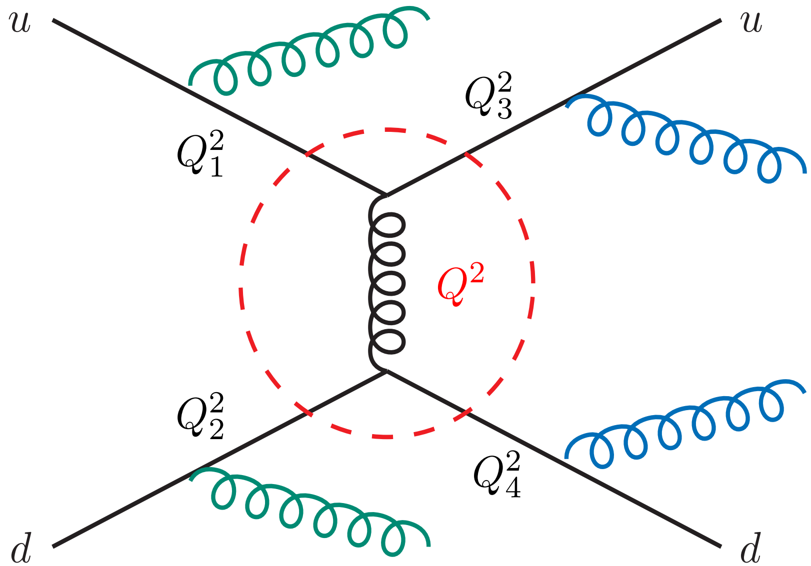


ISR: Initial State Radiation

Parton Shower Approach

$\mathcal{P}_{\rm ISR/FSR}$ Accelerated charged particles radiate

$$2 \to n = (2 \to 2)_{\rm On\ Shell} + \text{ISR} + \text{FSR}$$



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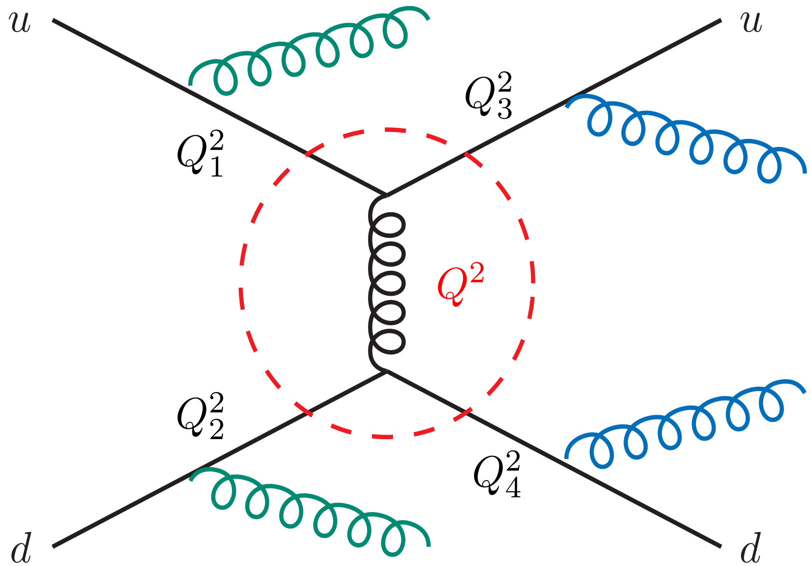

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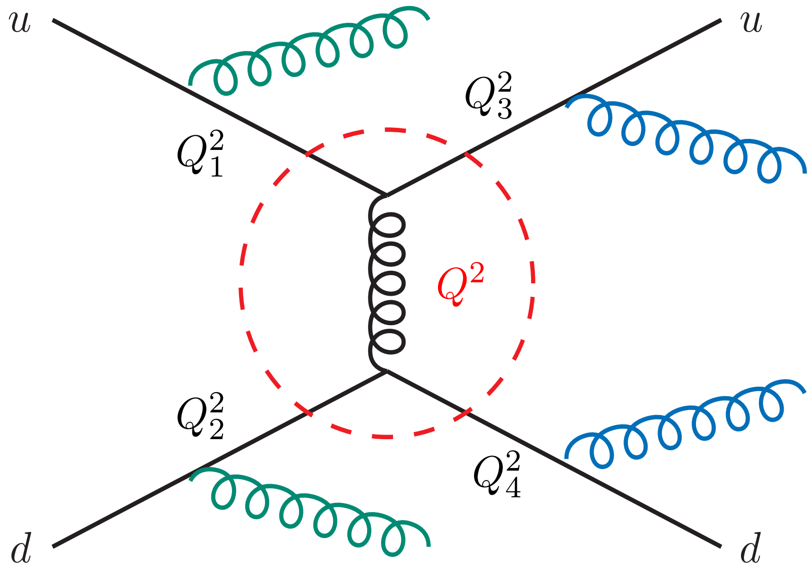
- ISR is space-like shower $Q_i^2 < 0$ increasing, physics complicated

- Shower is viewed as a probabilistic process which occurs with unit probability
- The (total) cross section is not affected but indirectly it is since the event shape is changed
- Obviously it is an approximation to the full process

Parton Shower Approach

$\mathcal{P}_{\mathrm{ISR/FSR}}$ Accelerated charged particles radiate

$2 \rightarrow n = (2 \rightarrow 2)_{\mathrm{On\ Shell}}$ + ISR + FSR

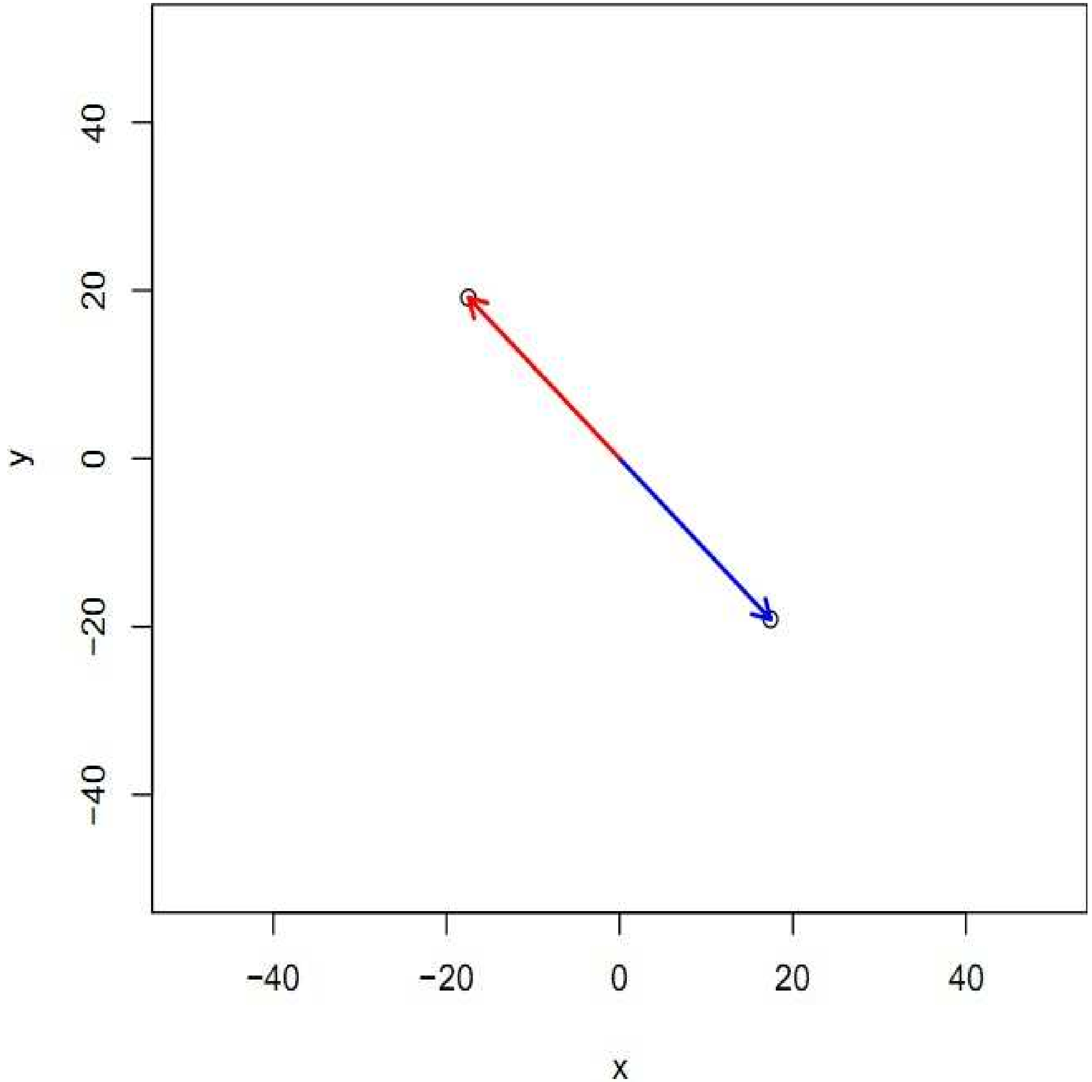


- FSR is time-like shower $Q_i^2 > 0$ decreasing, relatively simple
- ISR is space-like shower $Q_i^2 < 0$ increasing, physics complicated

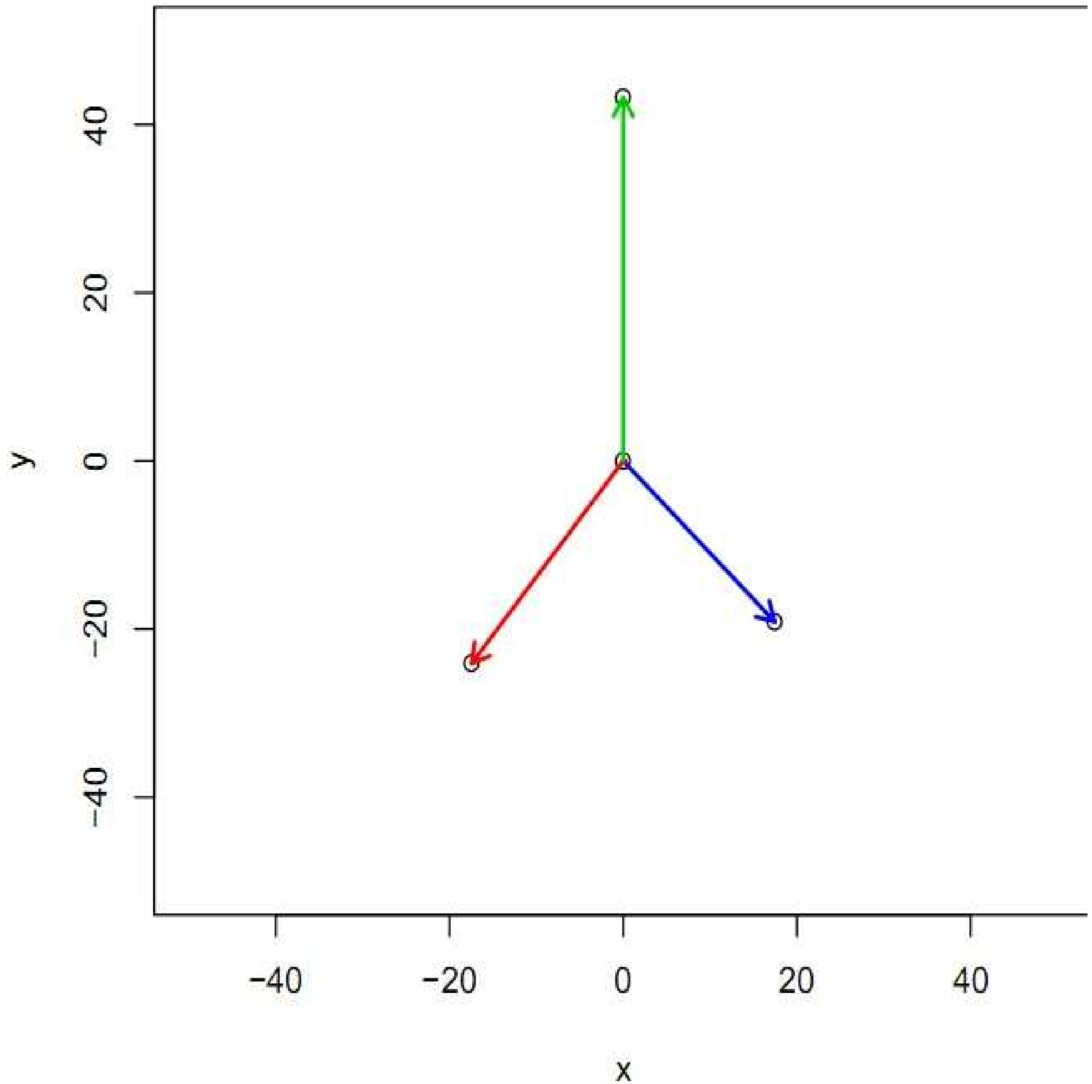
- Shower is viewed as a probabilistic process which occurs with unit probability
- The (total) cross section is not affected but indirectly it is since the event shape is changed
- Obviously it is an approximation to the full process

watch

Parton Shower movie



Parton Shower movie



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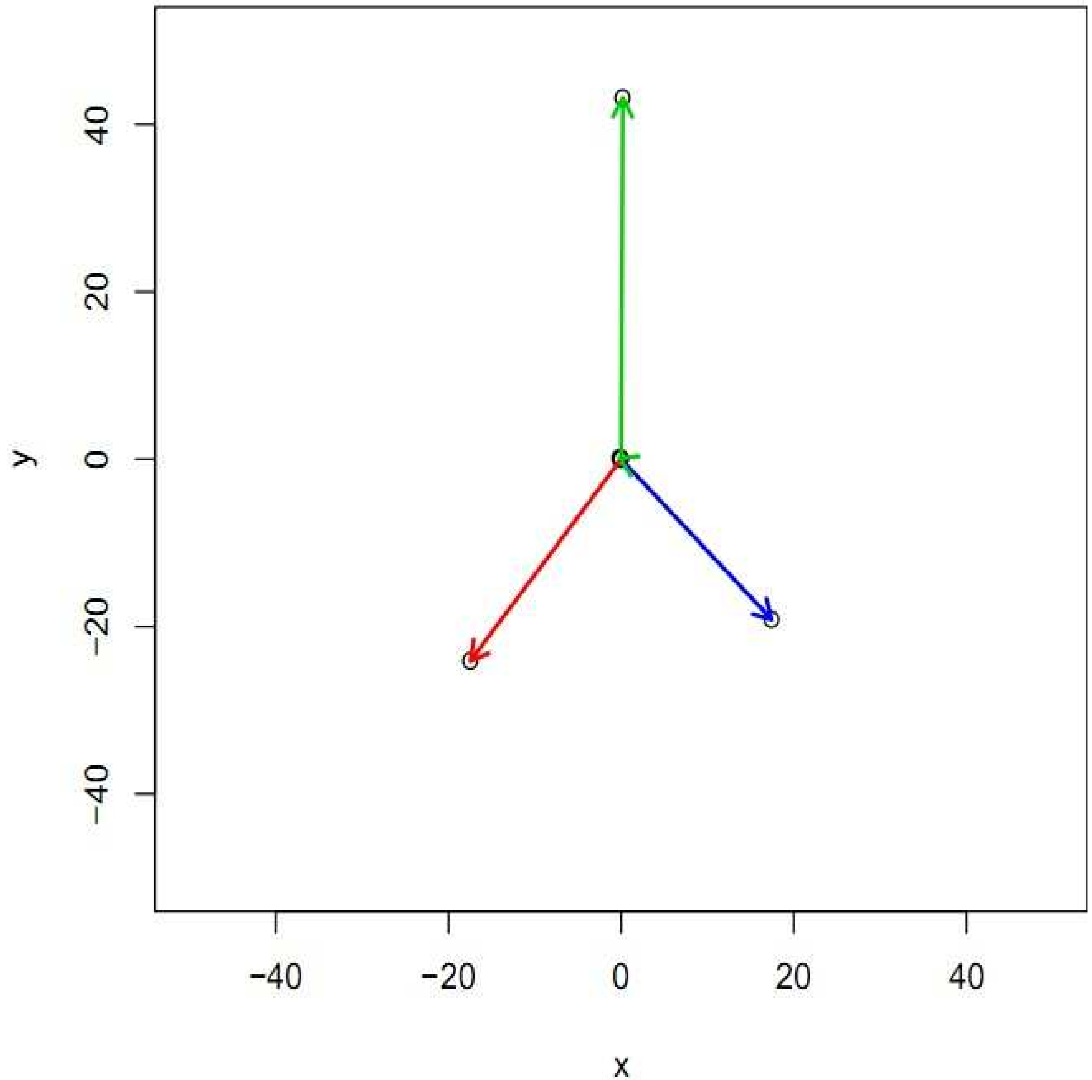

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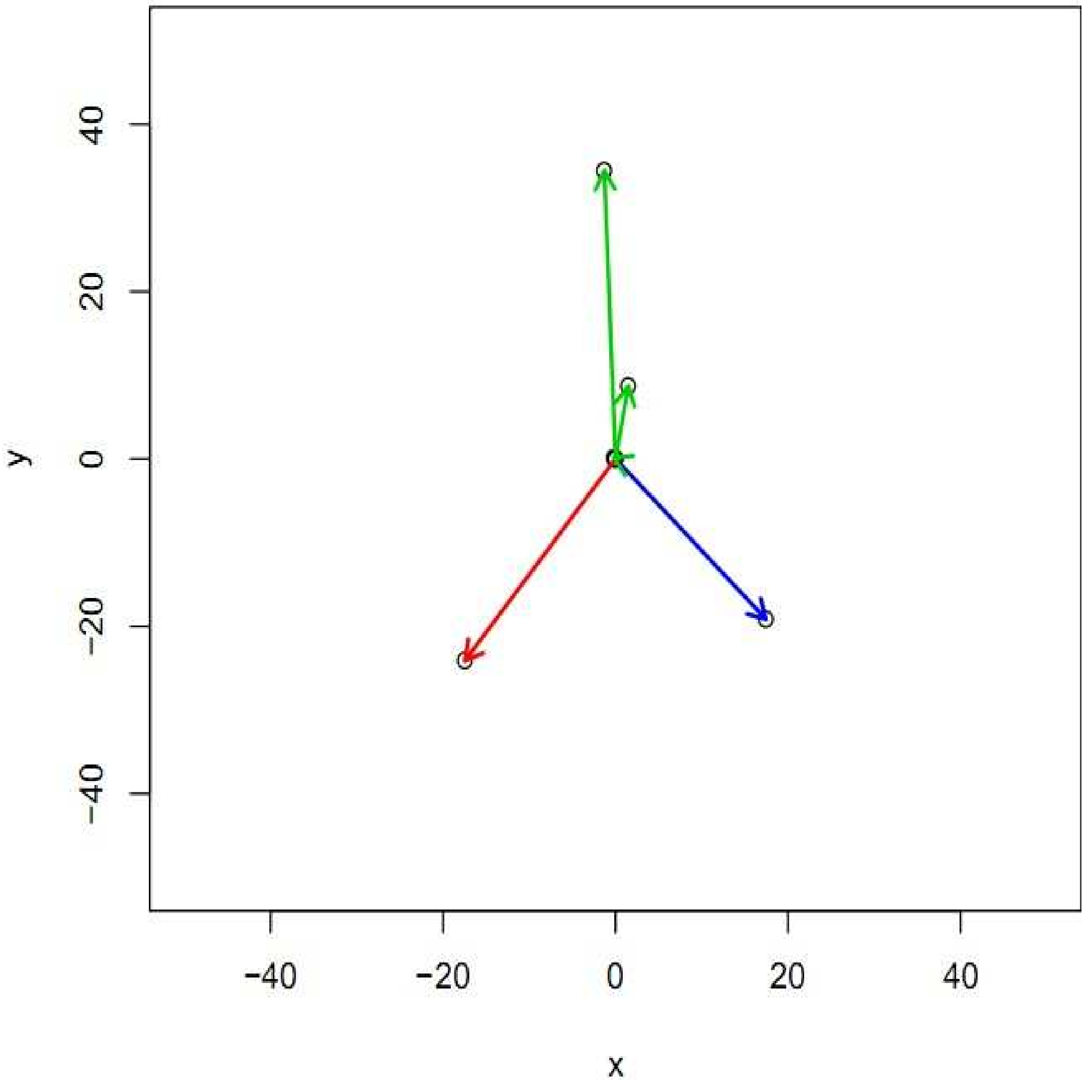

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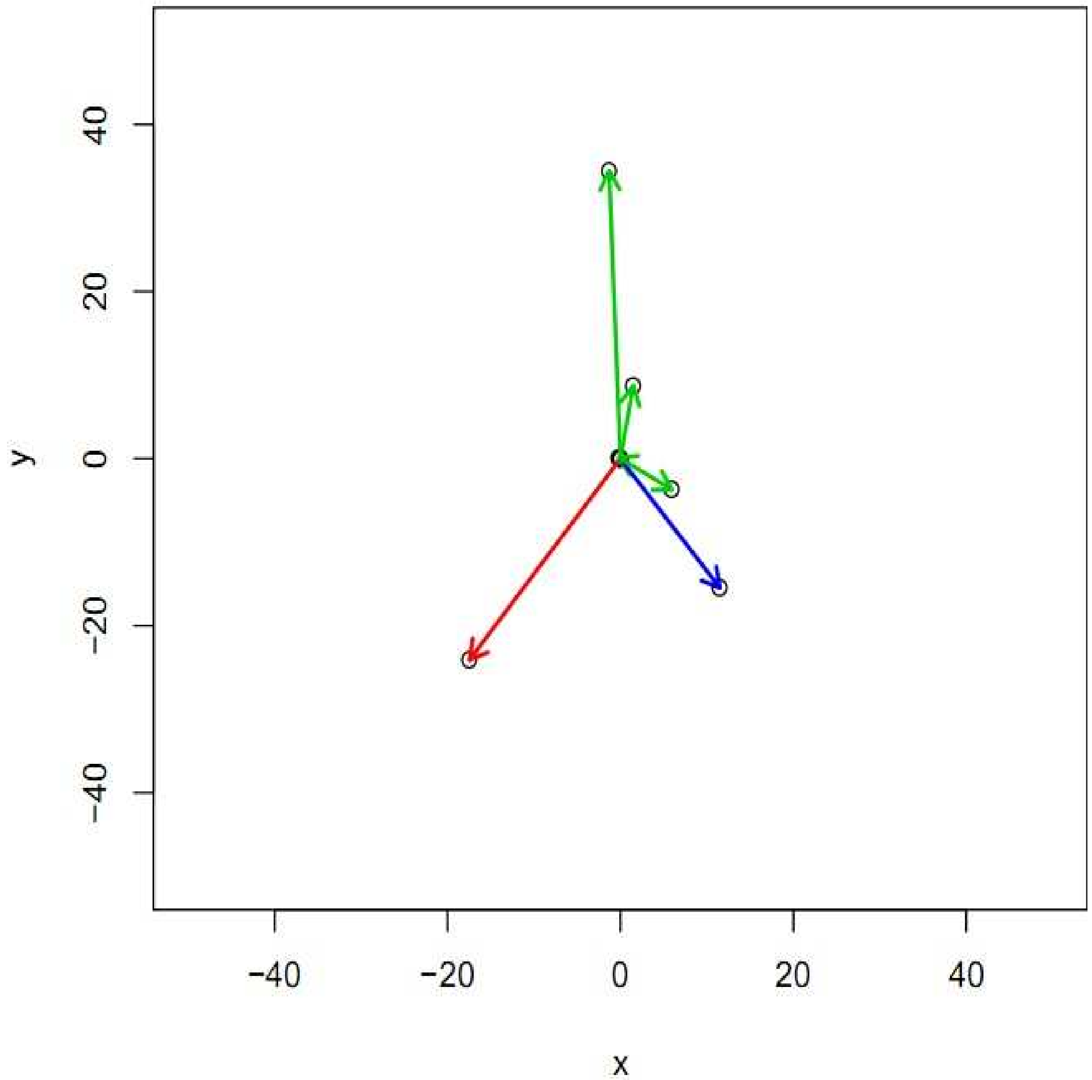

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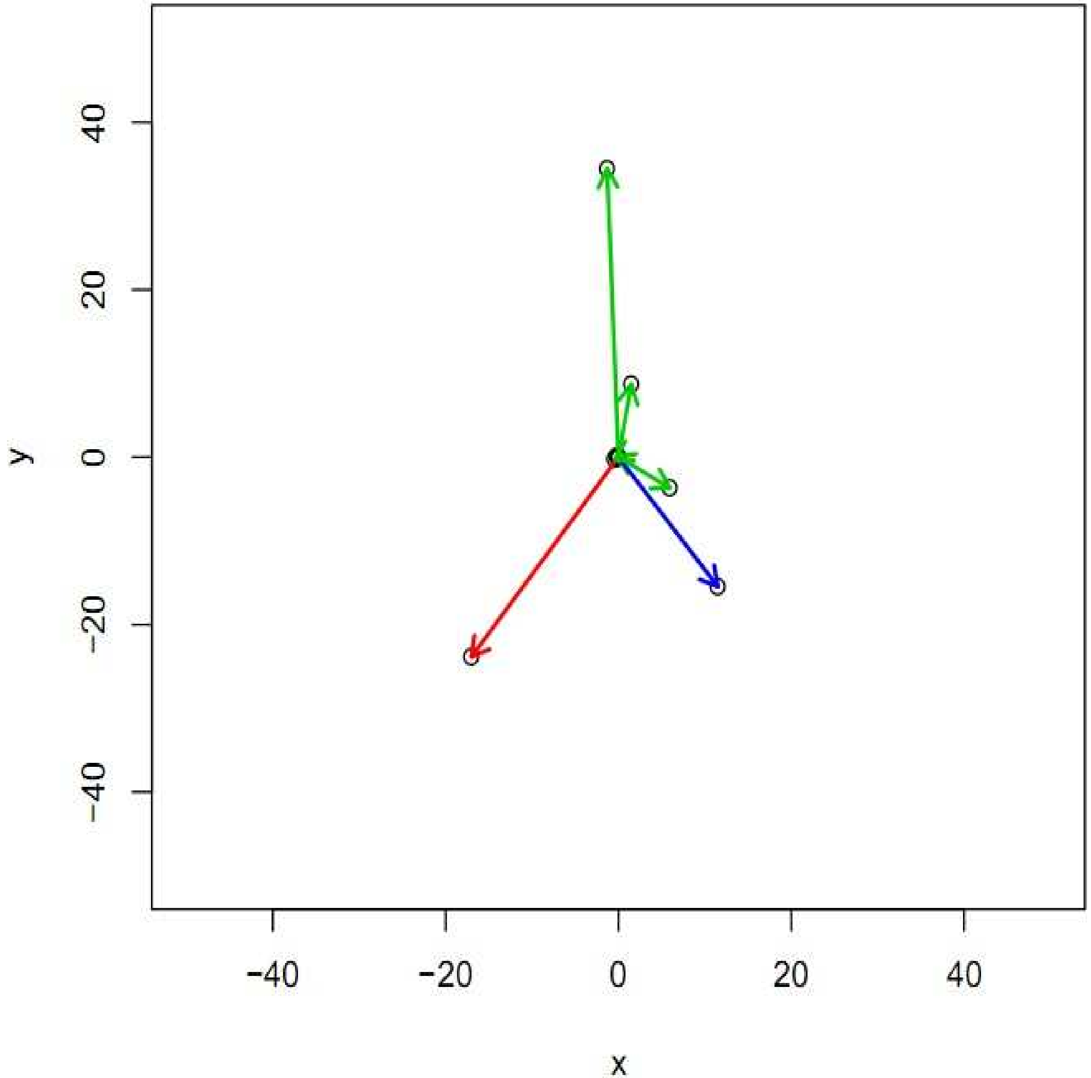
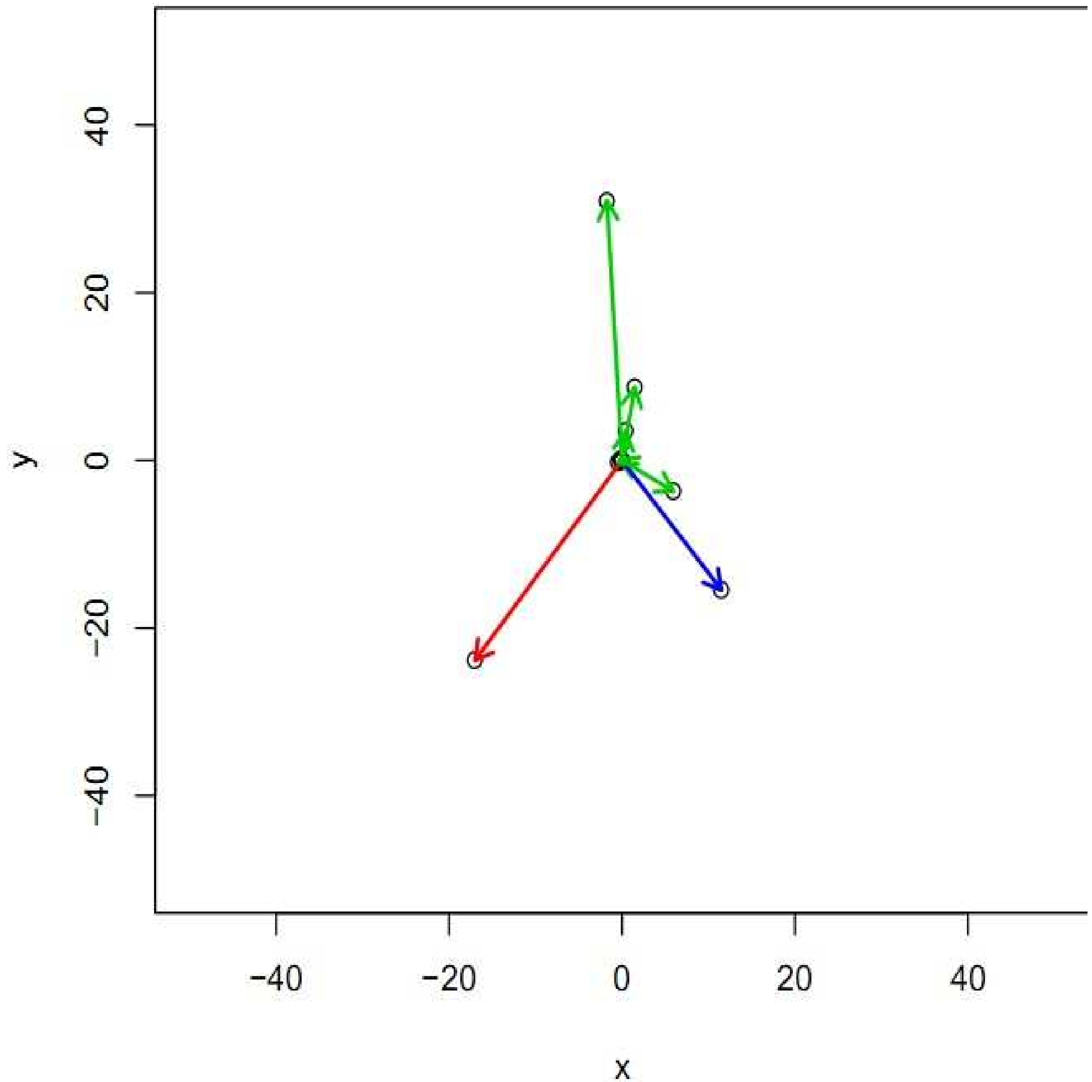
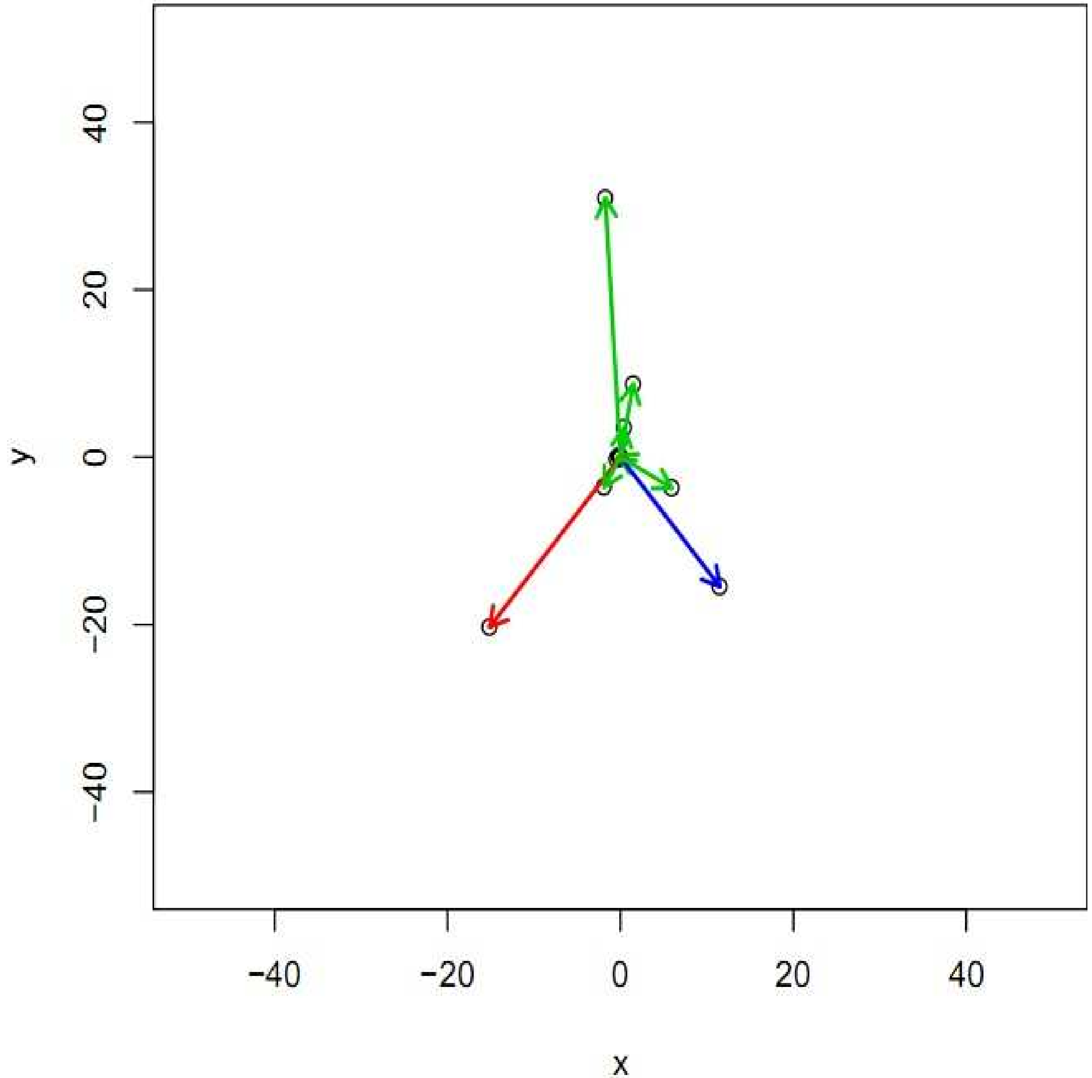


Parton Shower movie



Parton Shower movie



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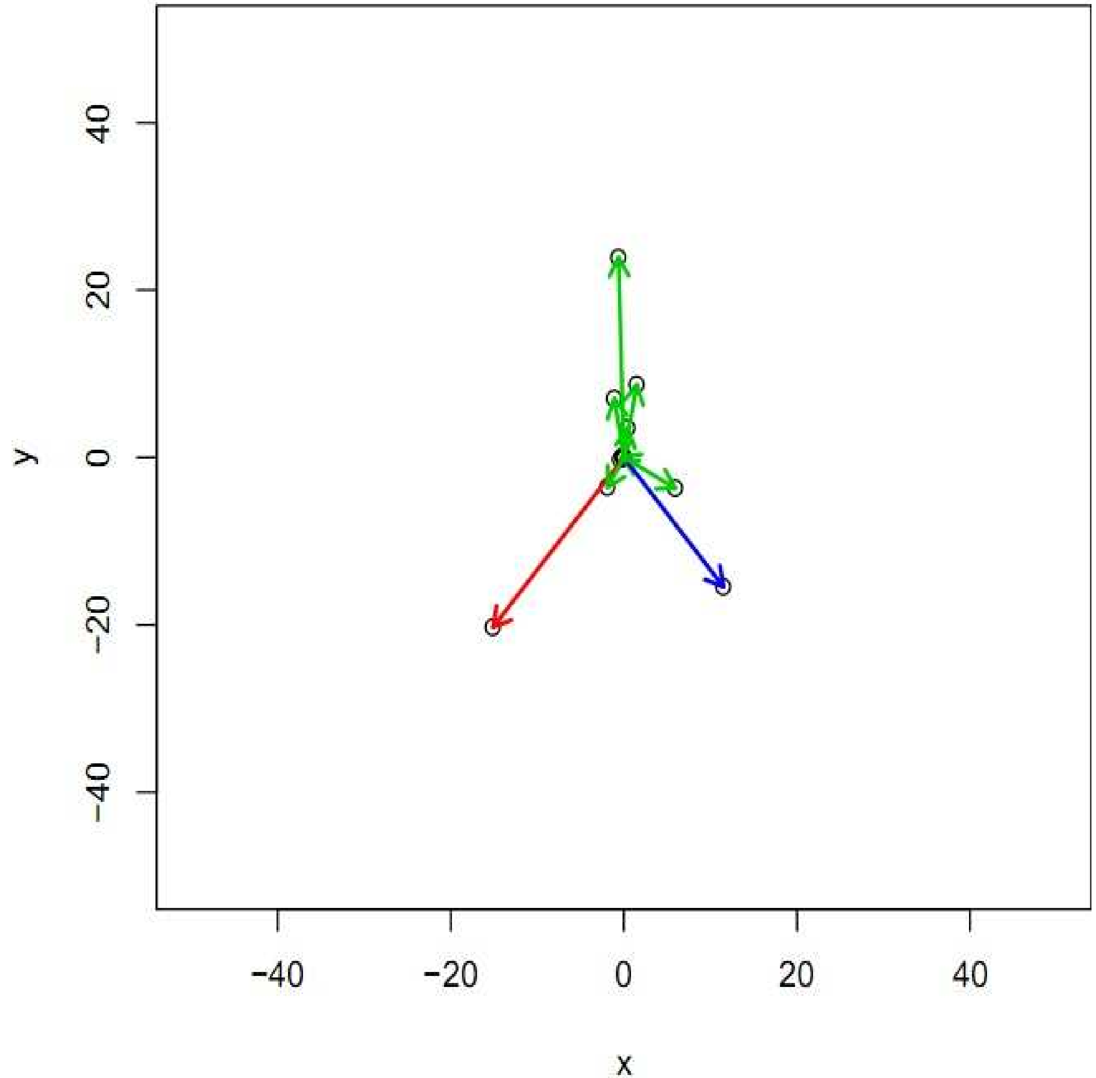
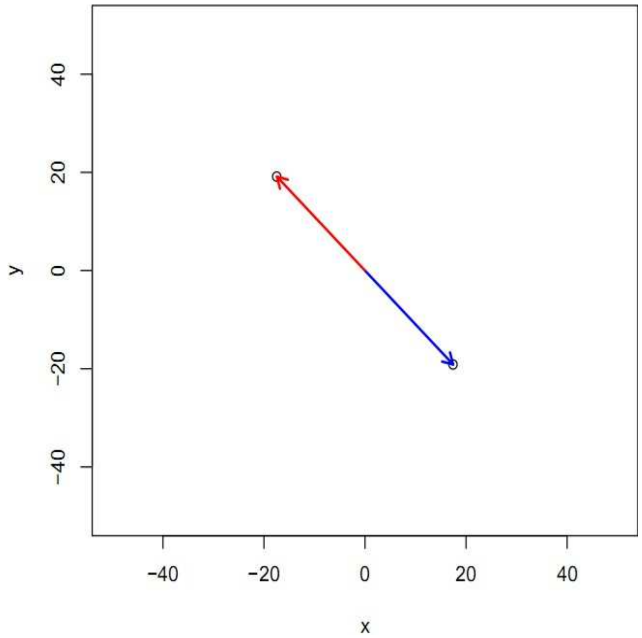
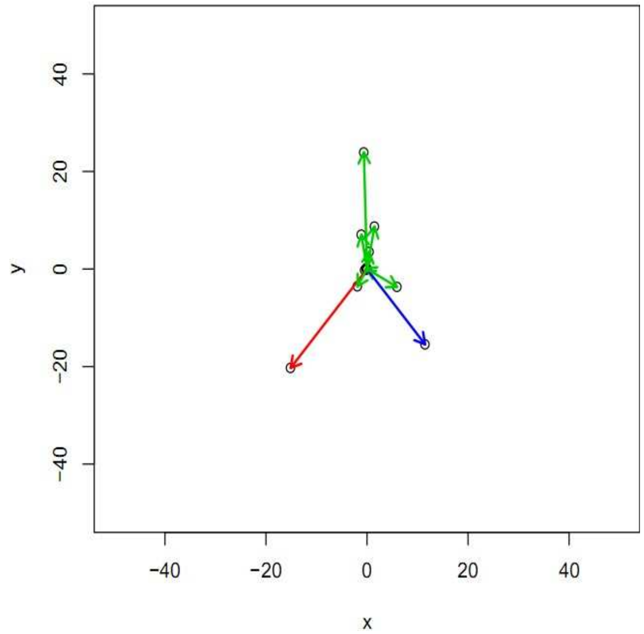


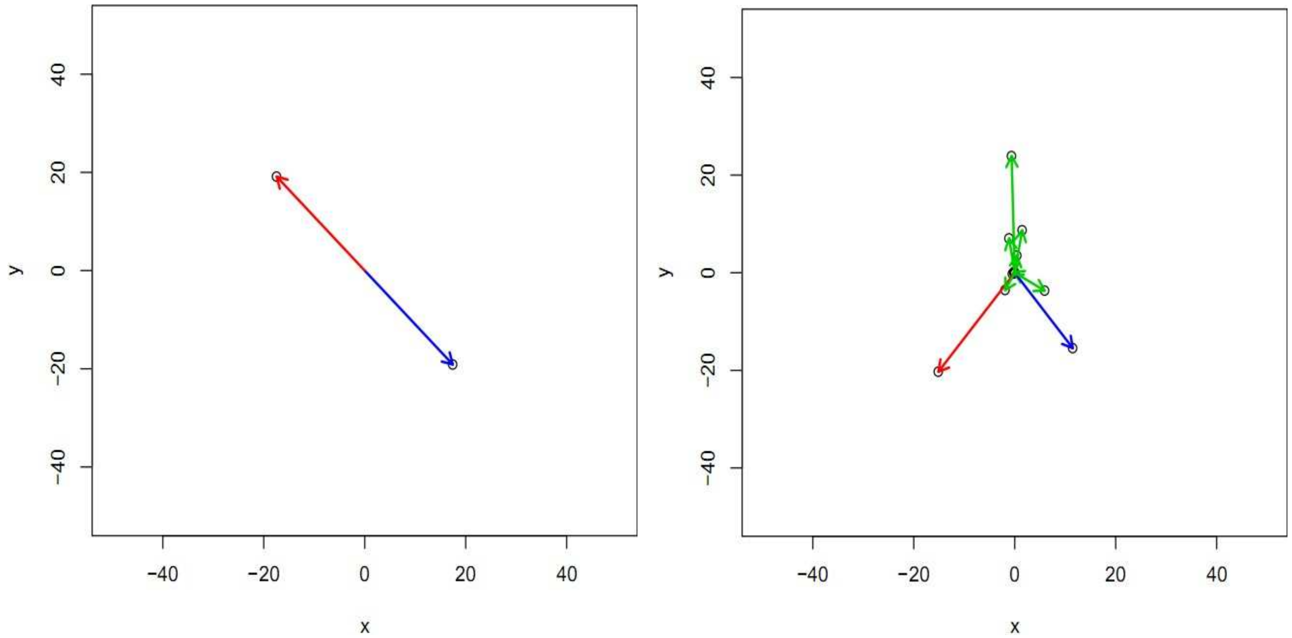
Parton Shower movie





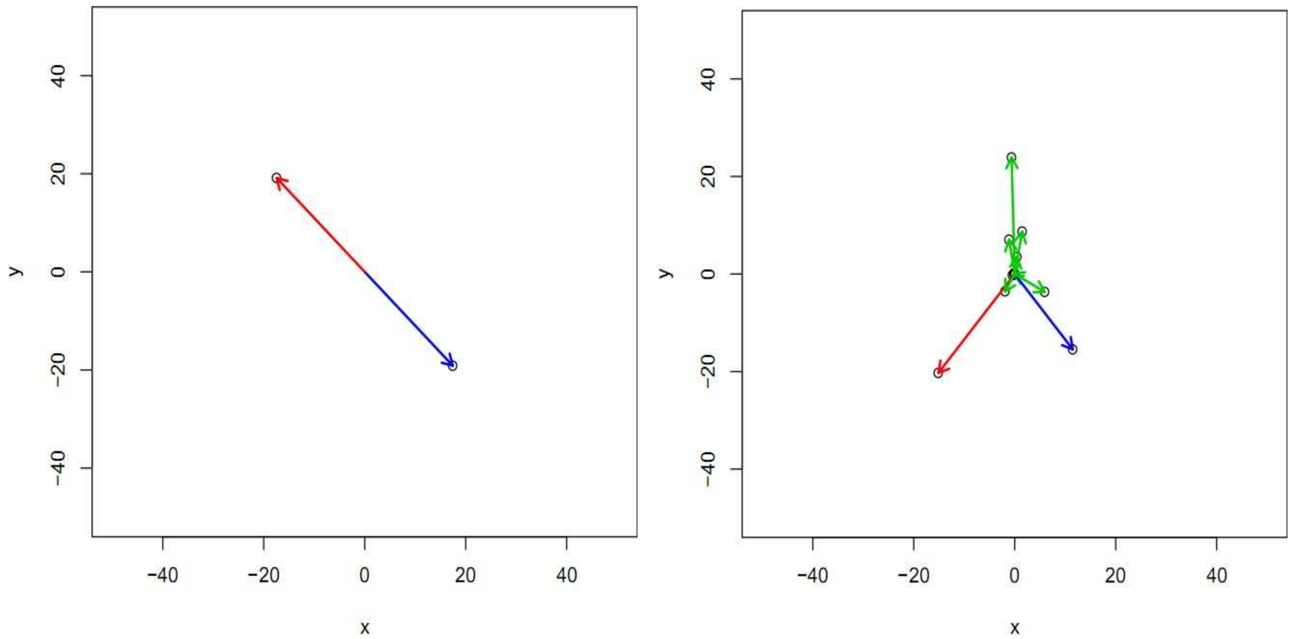
- The topology generated by the PS can be quite complicated

Parton Shower movie



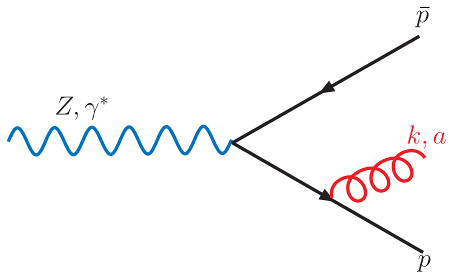
- The topology generated by the PS can be quite complicated
- These are events shape that can not be described by fixed order pert. calculations

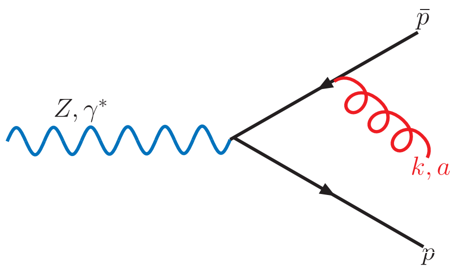
Parton Shower movie

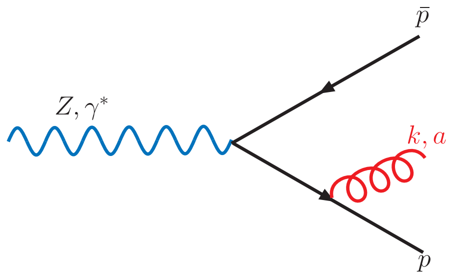


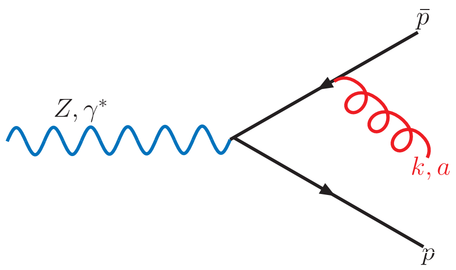
- The topology generated by the PS can be quite complicated
- These are events shape that can not be described by fixed order pert. calculations
- Total cross section still given by hard scattering (usually LO), experiments usually normalise to data

Origin and justification of PS: soft and collinear divergencies







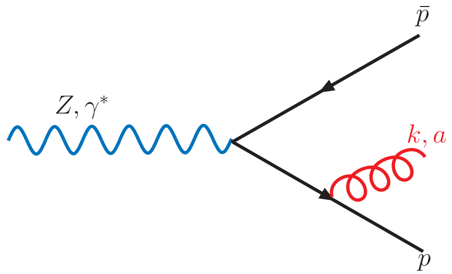


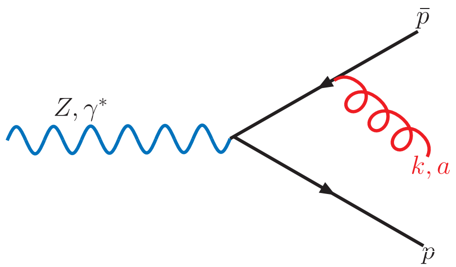
$$\mathcal{A}_\mu = \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \quad m_q = 0$$

$$+ \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon} v(\bar{p})$$

$$= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k}\right)t_a$$

$$2p.k = 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \quad \rightarrow 0 \text{ for } E_g \rightarrow 0, \quad \theta_{pk} \rightarrow 0$$





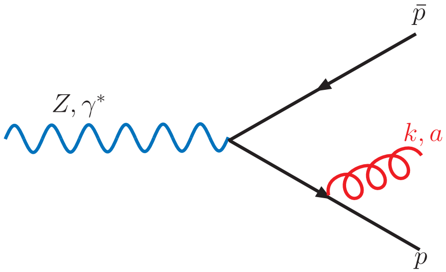
$$\mathcal{A}_\mu = \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \quad m_q = 0$$

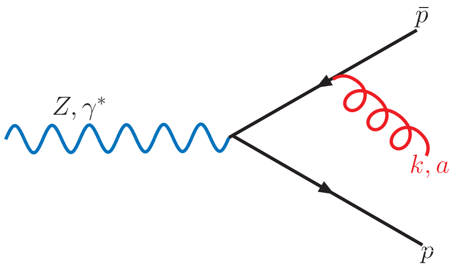
$$+ \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon} v(\bar{p})$$

$$= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k}\right)t_a$$

$$2p.k = 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \longrightarrow 0 \text{ for } E_g \to 0, \quad \theta_{pk} \to 0$$

$$\mathcal{A}_{\text{soft}}(k \to 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_0 \quad \text{diverges } k \to 0$$





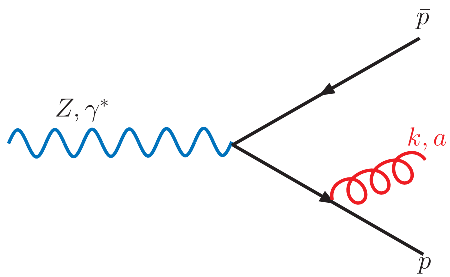
$$\mathcal{A}_\mu = \bar{u}(p)\not\epsilon(-ig_s t_a)\frac{-i}{\not p + \not k}\Gamma_\mu v(\bar{p}) \quad m_q = 0$$

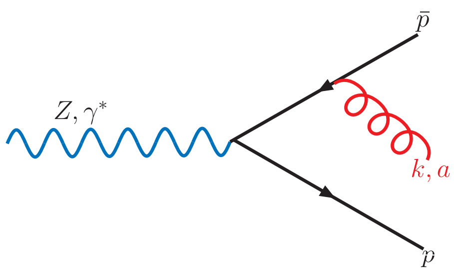
$$+ \bar{u}(p)\Gamma_\mu \frac{i}{\not\bar{p} + \not k}(-ig_s t_a)\not\epsilon v(\bar{p})$$

$$= -g_s\left(\frac{\bar{u}(p)\not\epsilon(\not p + \not k)\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not p + \not k)\not\epsilon v(\bar{p})}{2\bar{p}.k}\right)t_a$$

$$2p.k = 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \to 0 \text{ for } E_g \to 0, \quad \theta_{pk} \to 0$$

$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a\left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_{0g}$$





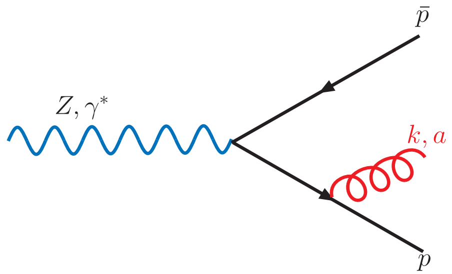
$$\begin{aligned}
\mathcal{A}_\mu &= \bar{u}(p)\not{\epsilon}(-ig_s t_a)\frac{-i}{\not{p}+\not{k}}\Gamma_\mu v(\bar{p}) \qquad m_q = 0 \\
&+ \bar{u}(p)\Gamma_\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s t_a)\not{\epsilon} v(\bar{p}) \\
&= -g_s\left(\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p)\Gamma_\mu(\not{p}+\not{k})\not{\epsilon} v(\bar{p})}{2\bar{p}.k}\right)t_a \\
2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \;\to\; 0 \text{ for } E_g \to 0, \;\; \theta_{pk} \to 0
\end{aligned}$$

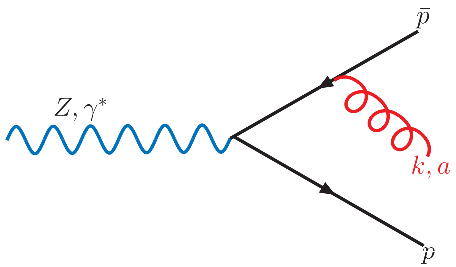
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right)\mathcal{A}_{0g}$$

Universal Radiator Factor

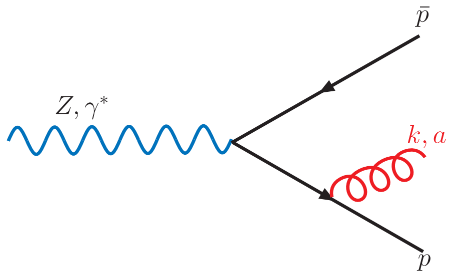
We have **factorisation** of the soft emission (long distance) from the short distance *i.e.* the hard process

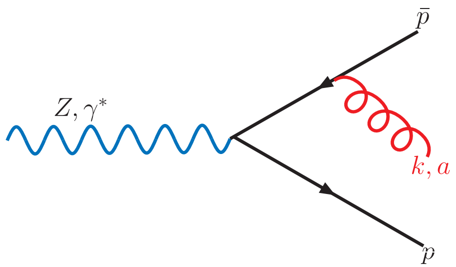
Squaring soft/collinear





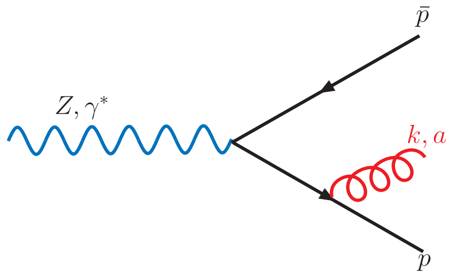
Squaring soft/collinear

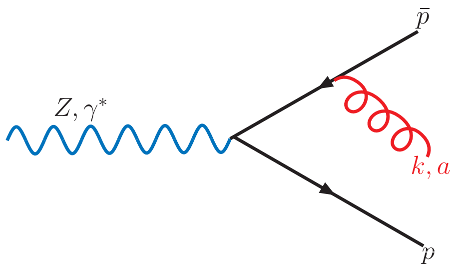




$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

Squaring soft/collinear

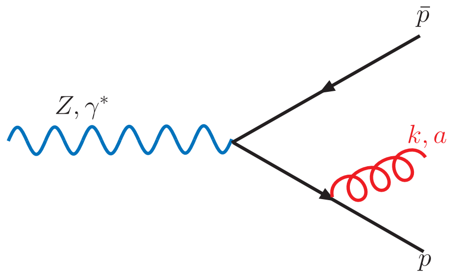




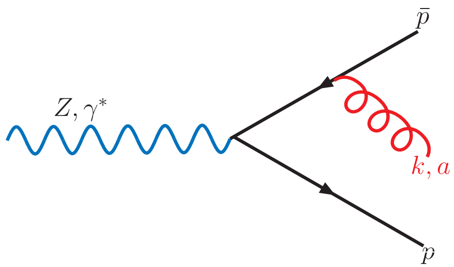
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

$$|\mathcal{M}_{1g}|^2 = \sum_{a,pol.(\epsilon)} |\mathcal{A}_{1g}(k \to 0)|^2 = C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k} |\mathcal{M}_{0g}|^2$$

Squaring soft/collinear

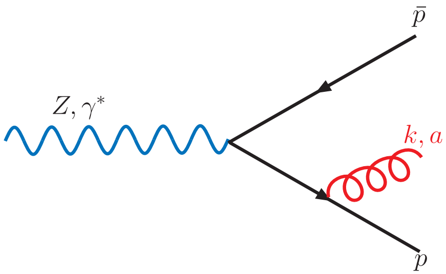


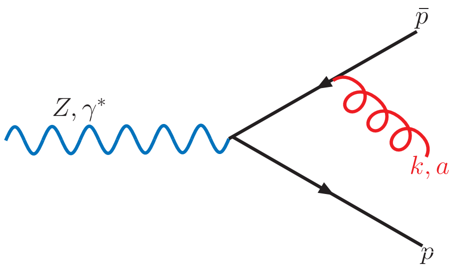
$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k}\right) \mathcal{A}_{0g}$$



Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}}\right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$





$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$

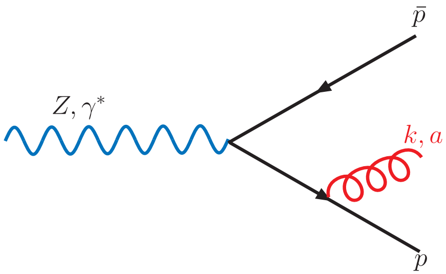
Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}} \right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

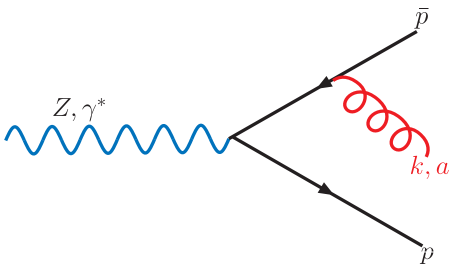
$$\theta = \theta_{\angle pk}\ ,\ \phi = \text{azimuth}$$

$$\mathrm{d}\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$

Squaring soft/collinear



$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$



Phase Space

$$|\mathcal{M}_{1g}|^2 \mathrm{d}\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 \mathrm{d}\Phi_{q\bar{q}} \right) \mathrm{d}\mathcal{S}; \quad \mathrm{d}\mathcal{S} \simeq \frac{\mathrm{d}^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k \; \bar{p}.k}$$

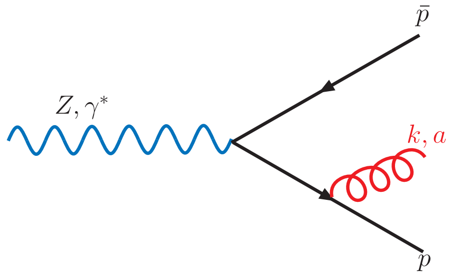
$$\theta = \theta_{\angle pk} \;, \; \phi = \text{azimuth}$$

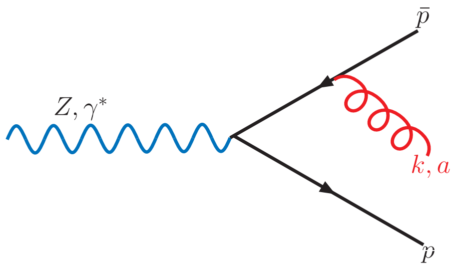
$$\mathrm{d}\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$

- $\mathrm{d}\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $\mathrm{d}\mathcal{S}$ diverges for $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$, collinear divergence

Squaring soft/collinear

$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$





Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k \; \bar{p}.k}$$

$$\theta = \theta_{\angle pk} \;,\; \phi = \text{azimuth}$$

$$d\mathcal{S} \quad \simeq \quad \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$$x_i = 2E_i/E_{\text{tot}} \quad p \to 1, \quad k \to 3$$

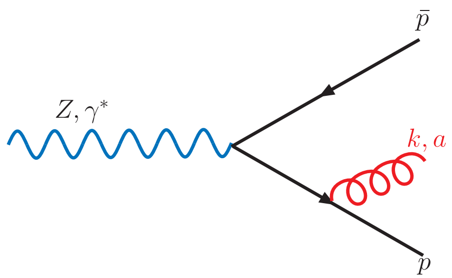
$$d\mathcal{S}_\phi \quad = \quad \frac{\alpha_s C_F}{2\pi} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

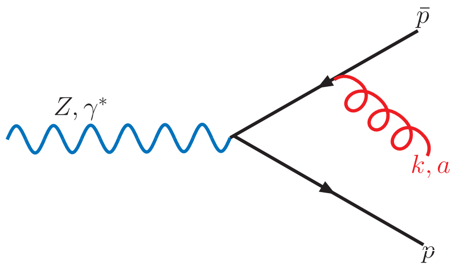
$$= \quad \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right) d\cos\theta dx_3$$

- $d\mathcal{S}$ diverges for $\omega \longrightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $d\mathcal{S}$ diverges for $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$, collinear divergence

Squaring soft/collinear

$$\mathcal{A}_{1g}(k \to 0) = -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_{0g}$$





Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p.\bar{p}}{p.k\ \bar{p}.k}$$

$$\theta = \theta_{\angle pk}\ ,\ \phi = \text{azimuth}$$

$$d\mathcal{S} \simeq \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$$x_i = 2E_i/E_{\text{tot}} \quad p \to 1, \quad k \to 3$$

$$d\mathcal{S}_\phi = \frac{\alpha_s C_F}{2\pi} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$= \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2\theta} \frac{1 + (1-x_3)^2}{x_3} - x_3 \right) d\cos\theta dx_3$$

- $d\mathcal{S}$ diverges for $\omega \to 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $d\mathcal{S}$ diverges for $\theta \to 0$ and $\theta \to \pi$, collinear divergence
- collinear divergence for $x_1 \to 1$ or $x_2 \to 1$ and Infrared divergence for $x_3 \to 0$

Splitting

$$\mathrm{d}\mathcal{S}_\phi \simeq \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\sin^2\theta}\frac{1+(1-x_3)^2}{x_3}\right)\mathrm{d}\cos\theta\,\mathrm{d}x_3$$

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta}=\frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar\theta}{1-\cos\bar\theta}\sim\frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar\theta^2}{\bar\theta^2} \quad \text{for} \quad \theta,\bar\theta\sim 0\ll 1$$

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q and $\bar q$ as independent emitters, notion of splitting as a probability

$$\mathrm{d}\sigma_{1g} \sim \mathrm{d}\sigma_{0g} \sum_{q\to qg}^{\bar q\to \bar q g} C_F \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\frac{1+(1-z)^2}{z}\mathrm{d}z \qquad (z\equiv x_3)$$

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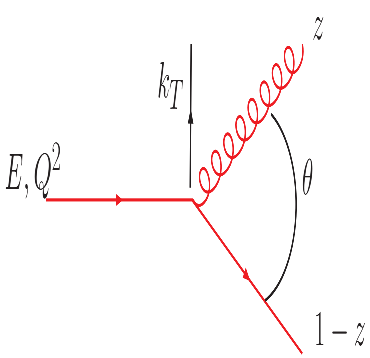

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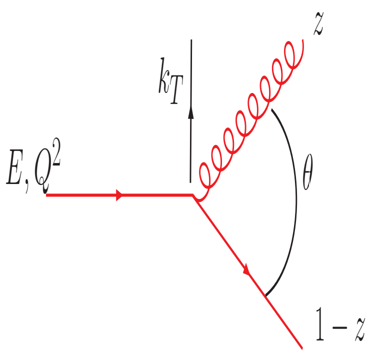


different choices of the evolution variables, equivalent in the collinear limit (diff. in practice/different codes)

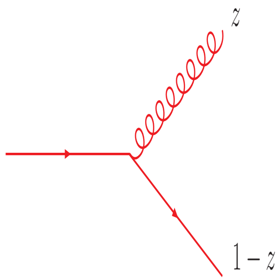
$$Q^2 = E^2 z(1-z)\theta^2 \qquad k_T^2 = E^2 z^2(1-z)^2\theta^2$$

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}Q^2}{Q^2} = \frac{\mathrm{d}k_T^2}{k_T^2}$$

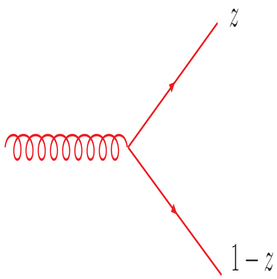
DGLAP

This generalises to different parton branching (gluon, quarks)

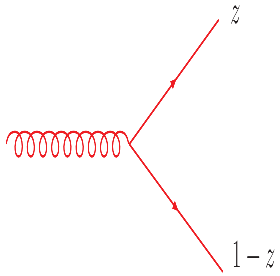
$$\mathrm{d}\sigma_{bc} \sim \mathrm{d}\sigma_a \frac{\alpha_s}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2} P_{a\to bc}(z)\mathrm{d}z$$



$$P_{gq}(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

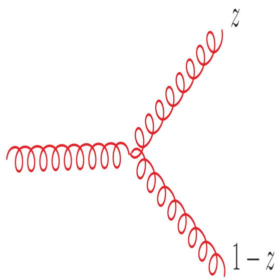


$$P_{qg}(z) = C_F \left(\frac{1+(1-z)^2}{z} \right)$$



$$P_{qg}(z) = T_R \left(z^2 + (1-z)^2 \right) \qquad T_R = \frac{n_f}{2}$$

(divergences at $z = 0, 1$ dealt with soft/virtual corr.)

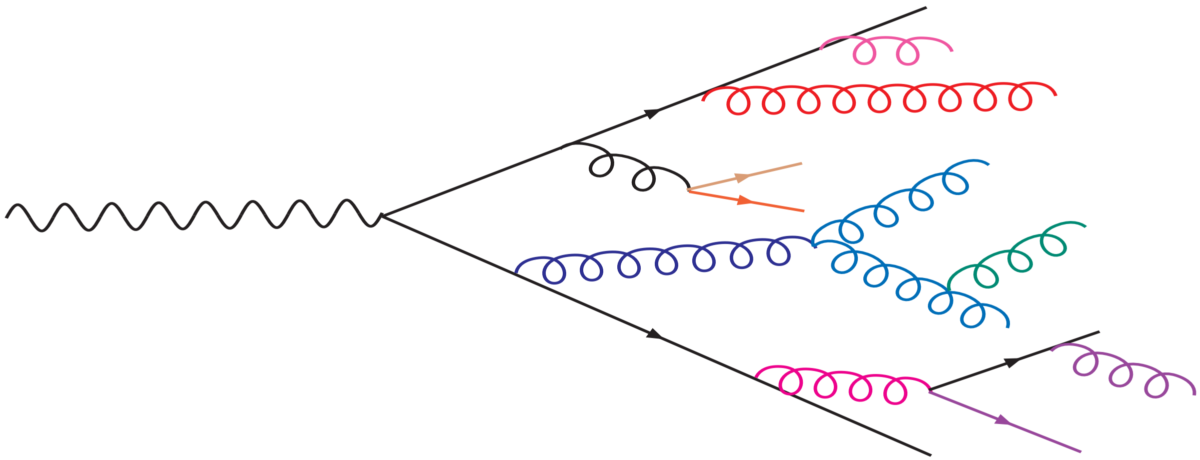


$$P_{gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right) \qquad C_A = 3 \qquad (C_F = 4/3)$$

Gluons radiate the most

$P(z, \phi)$ can be defined for polarisation effects

Ex. Final State PS



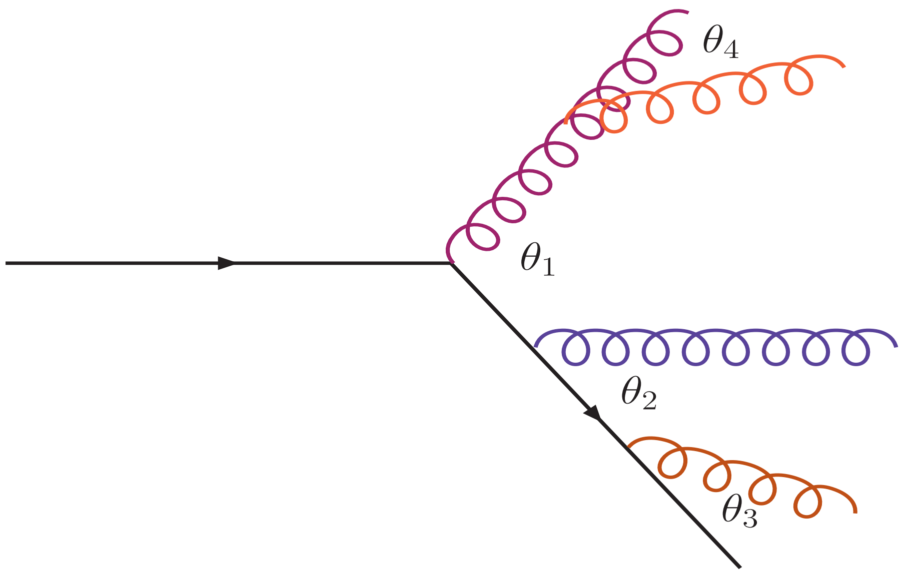
Need soft/collinear cut-offs to stay away from non perturbative physics. Details are model/code dependent

$Q > m_0 = min(m_{ij}) \sim 1$GeV

$z_{min}(E,Q) < z < z_{max}(E,Q)$

$k_T > k_{T,min} \sim 0.5$GeV

Radiation is angle ordered



$\theta_1 > \theta_2 > \theta_3$ and $\theta_1 > \theta_4$

On average, emissions have decreasing angles with respect to emitters

the jet is squeezed

The Probability of real emission exponentiates

- Conservation of total probability $\mathcal{P}_{\rm something} + \mathcal{P}_{\rm nothing} = 1$!
- Product of probabilities as time evolves $T \sim 1/Q$ evolves
  $\mathcal{P}_{\rm nothing}(0 < t < T) = \mathcal{P}_{\rm nothing}(0 < t \leq T_1)\mathcal{P}_{\rm nothing}(T_1 < t \leq T)$
- subdivide further $T_i = (i/n)T, 0 \leq i \leq n$

$$\begin{aligned}
\mathcal{P}_{\rm nothing}(0 < t < T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\rm nothing}(T_i < t < T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\rm something}(T_i < t < T_{i+1})) \\
&= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\rm something}(T_i < t < T_{i+1})\right) \\
&= \exp\left(-\int_0^T \frac{{\rm d}\mathcal{P}_{\rm something}(t)}{{\rm d}t}{\rm d}t\right)
\end{aligned}$$

The Probability of real emission exponentiates

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$$
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&= \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
\end{aligned}
$$

$$
\mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T)\exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
$$

The Probability of real emission exponentiates

$$\mathrm{d}\mathcal{P}_{\mathrm{first}}(T) = \mathrm{d}\mathcal{P}_{\mathrm{something}}(T)\exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\mathrm{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

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$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\mathrm{d}z \exp\left(-\sum_{bc}\int_{Q^2}^{Q^2_{\max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int_{Q_0^2/Q^2}^{1-Q_0^2/Q^2}\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\mathrm{d}z'\right)$$

Q_0 =low cut-off scale

The Probability of real emission exponentiates

$$\mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T)\exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\mathrm{d}z\,\exp\left(-\sum_{bc}\int_{Q^2}^{Q^2_{\text{max}}}\frac{\mathrm{d}Q'^2}{Q'^2}\int_{Q_0^2/Q^2}^{1-Q_0^2/Q^2}\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\mathrm{d}z'\right)$$

Q_0 =low cut-off scale

$\Delta(Q^2, Q^2_{\text{max}})$, Sudakov form factor

(probability of emitting no radiation between these 2 scales)

$\mathcal{P}_{\text{nothing}}$

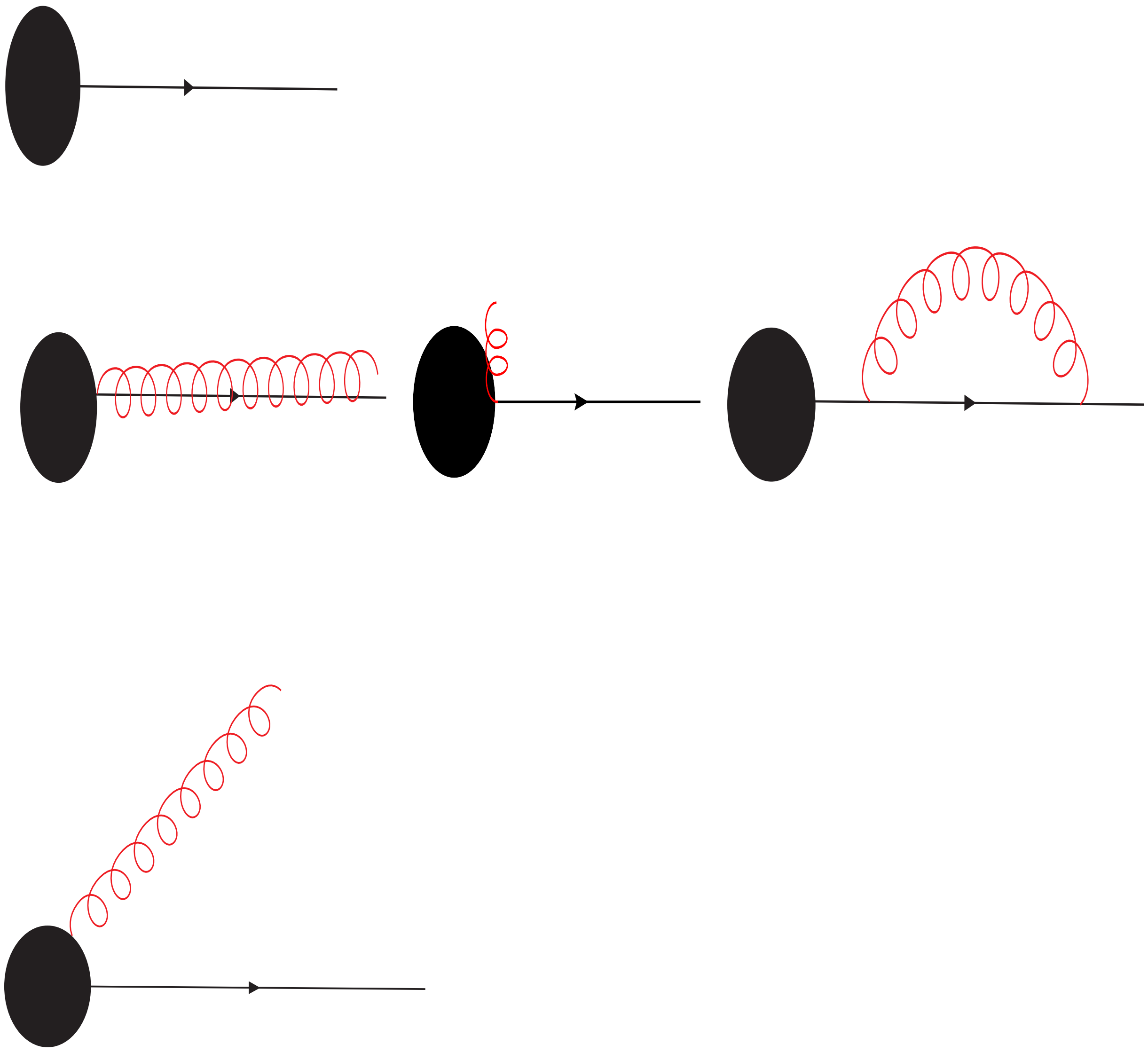
(a given parton only branches once)

- Start with a parton at high Q^2_{max} (typical of hard process)
- Work out the scale of the next branching, Q^2 by generating a random number $R \in [0,1]$ and solving $R = \Delta(Q^2_{max}, Q^2)$
- if no solution $Q^2 > Q^2_0$ `stop`
- otherwise work out the type of the branching
- generate the momenta of the decay products using the splitting functions
- repeat the procedure for the newly produced partons

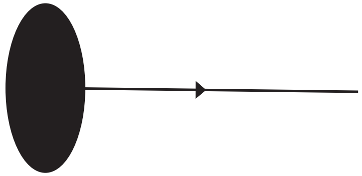
(some) differences between the MC for PS

- key difference is the evolution/scale variable
  - Angle θ (ordering `HERWIG`
  - Virtuality Q^2
  - Transverse momentum k_T
- $\int dQ^2/Q^2 = \log Q^2$, LL the same but important sub-leading differences
- soft emission (coherence), recall this factorises at the amplitude level...

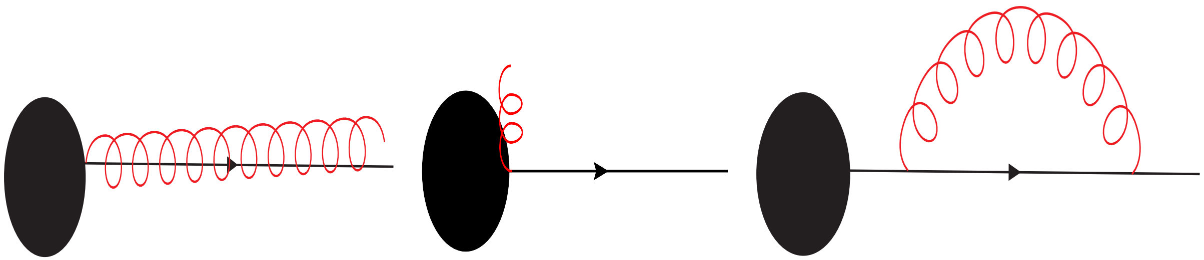
what difference?

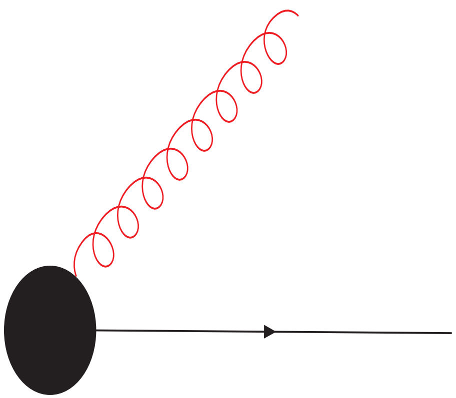


what difference?



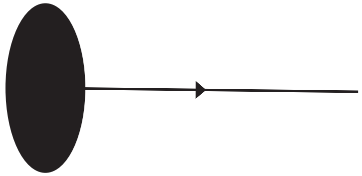
no radiation

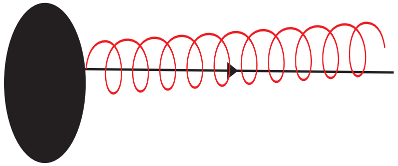




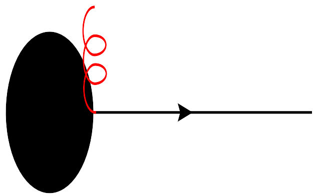
what difference?

no radiation

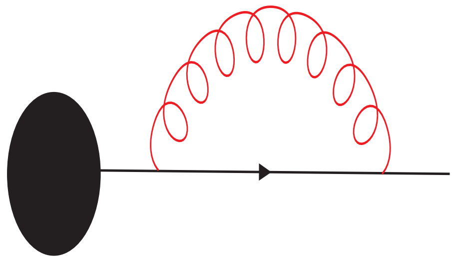




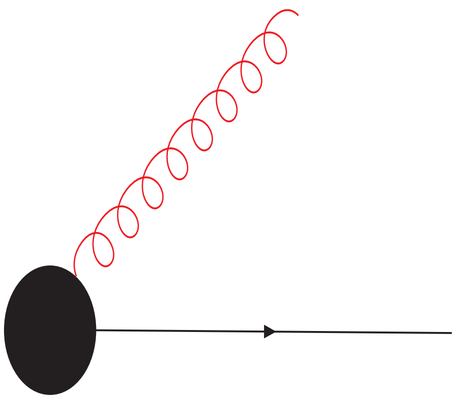
collinear



soft



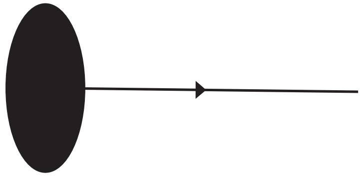
virtual/loop



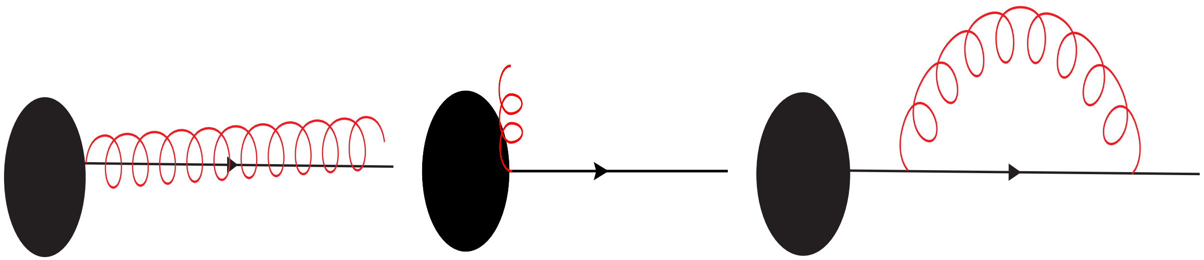
Resolved $k_T > Q_0$

Resolved and

what difference?

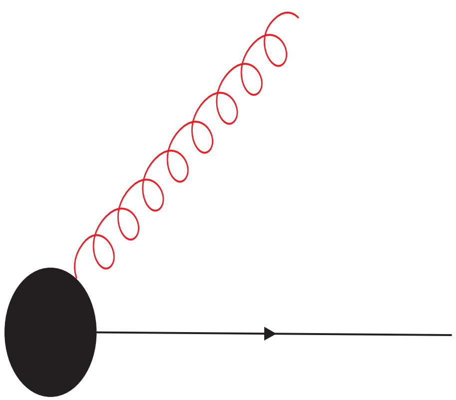


no radiation



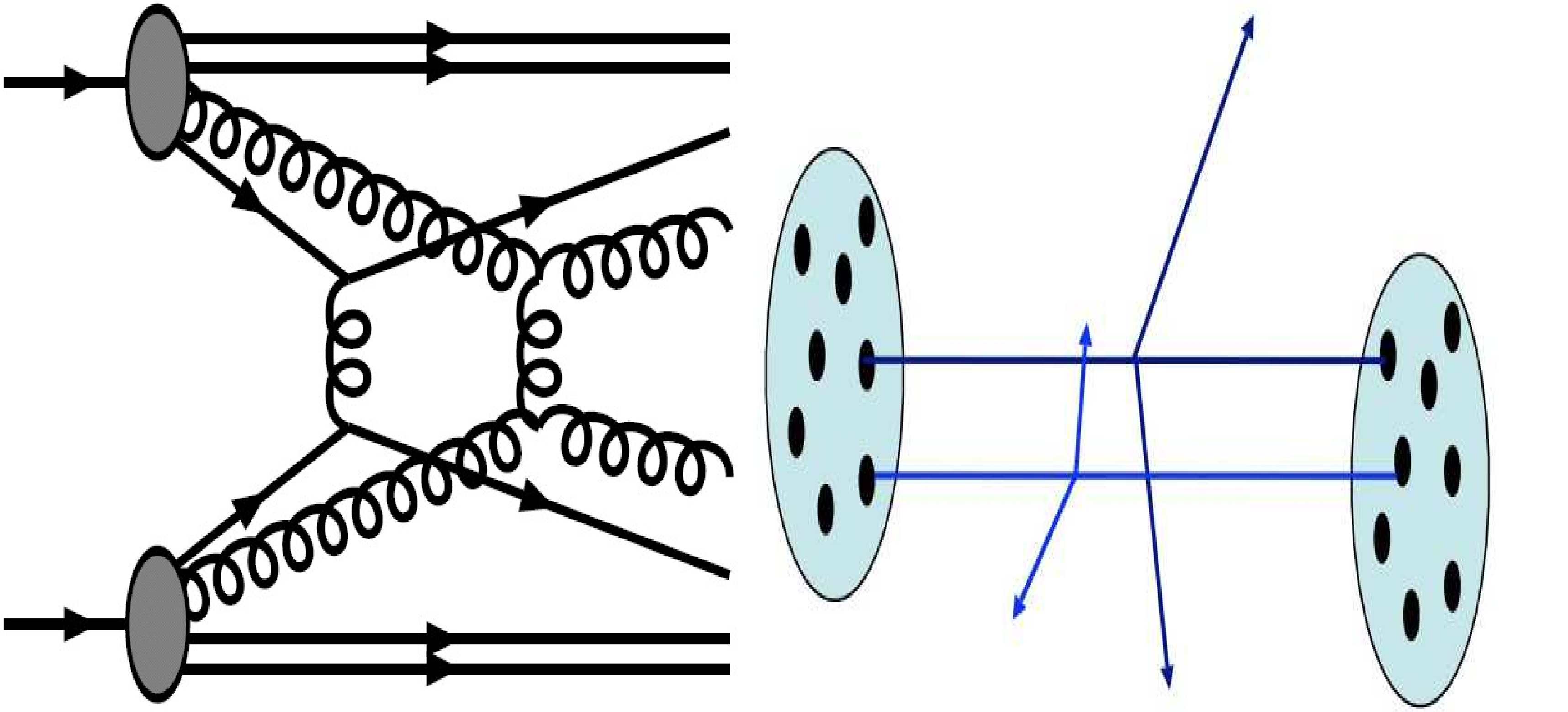
collinear soft virtual/loop

Unresolved from no radiation, $k_T < Q_0$. With addition of virtual, divergence tamed



Resolved $k_T > Q_0$

Multiple Parton Interaction



- For small $p_{T,\min}$ and high energy inclusive parton-parton cross section is larger than proton-proton cross section
- More than one parton (per proton) scatter
- calls for a model of spatial distribution within the proton (perturbation theory gives n-scatter distribution
- UE is not understood. It is modelled through some data that do not give strong constraints and hence extrapolation can be doubted.(minimum bias)