One-loop electroweak corrections to double Higgs production in $e^+e^- \to \nu\ \nu\ HH$

-Current status of Higgs studies with GRACE-

KEK Minamitateya - LAPTH Collaboration

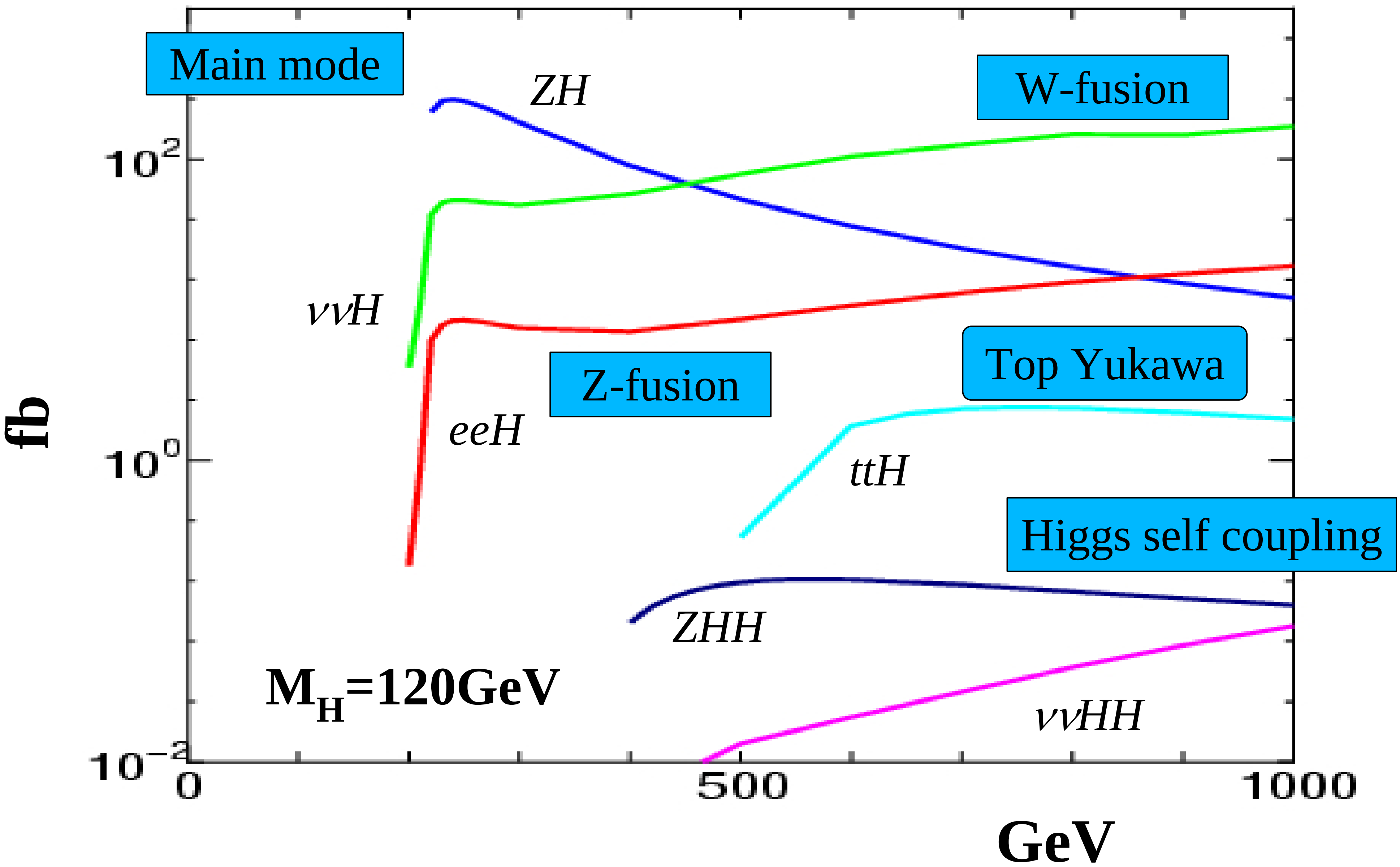
presented by Y.Yasui

ECFA 2004

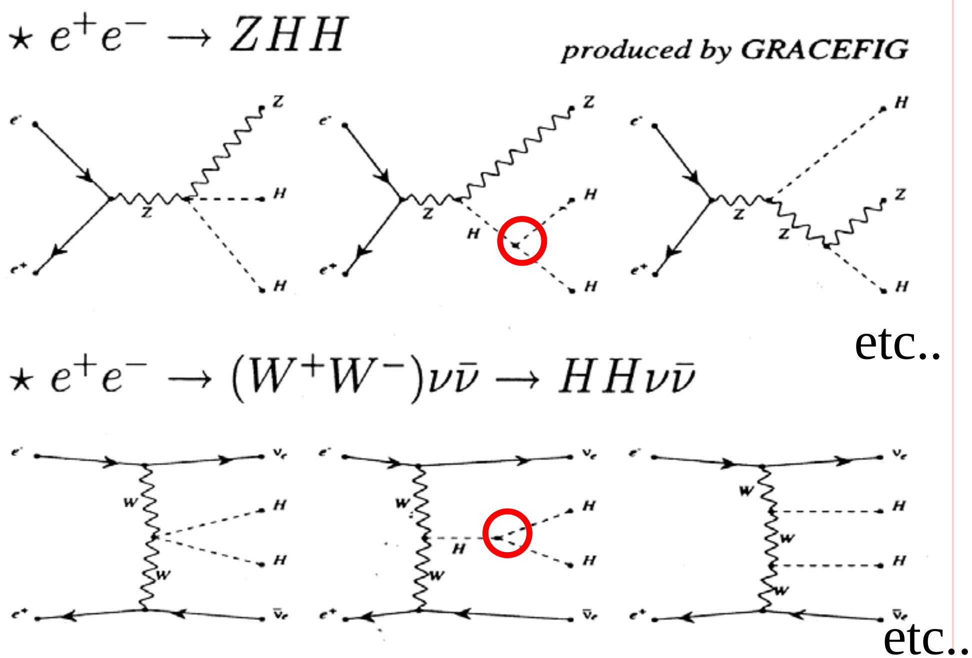
Durham

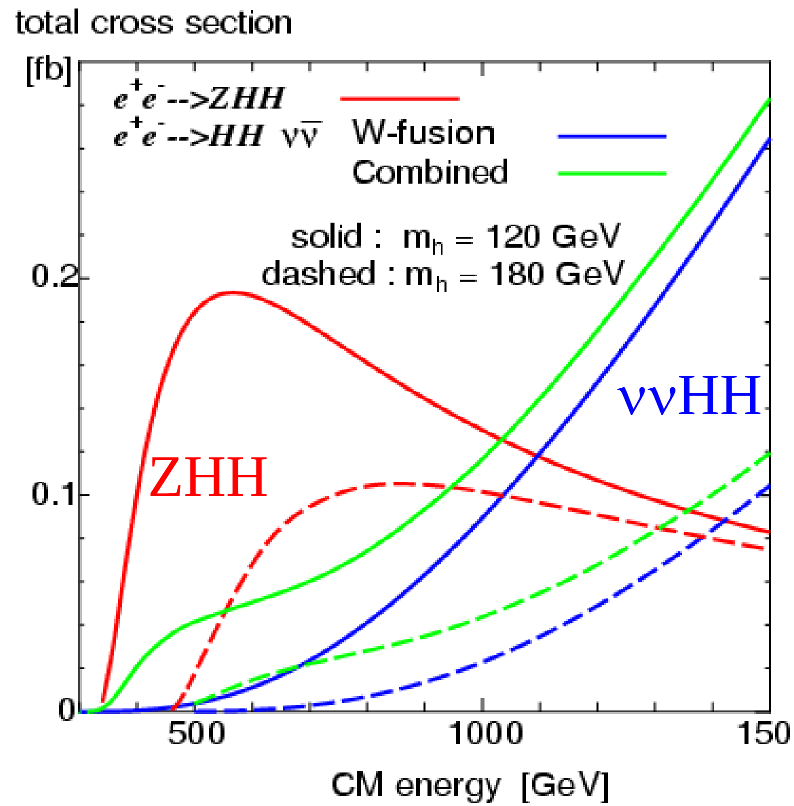
1 – 4 September 2004

Higgs Physics at TeV LC



Higgs self coupling





Simulation studies have been done

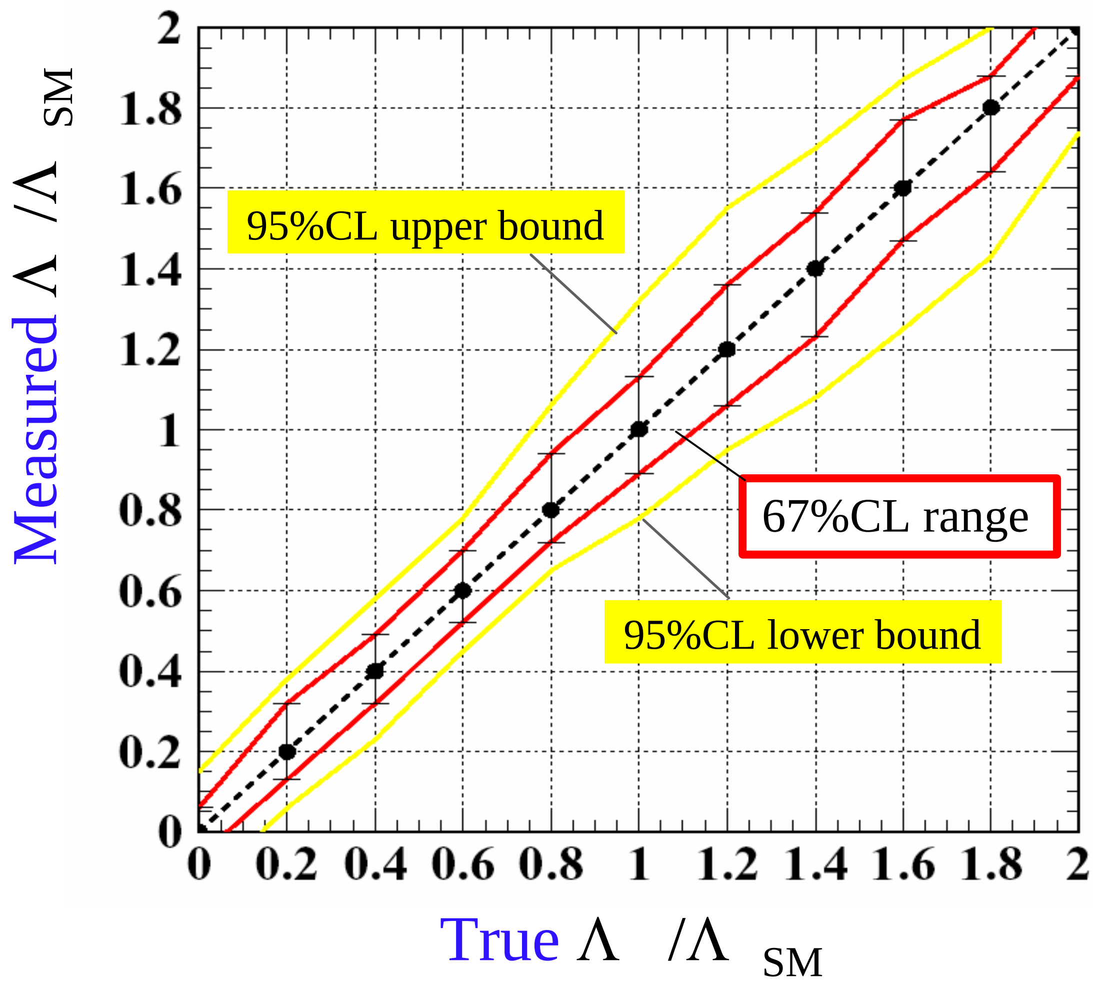
(by Japanese Higgs working group)

- Background processes were calculated by GRACE (at tree level)

  $e^+e^- \to \nu\nu bbbb$ (~ $\nu\nu ZZ, \nu\nu Z\gamma^*$), $e^+e^- \to \nu\nu bbh$ (~ $\nu\nu Zh$), etc.
- Quick simulator

Measurement sensitivity

By Yamashita et.al. LCWS 2004



@1TeV

$\boldsymbol{I}_{lumi}$=1 ab^{-1}
Pol_{beam}= -80%

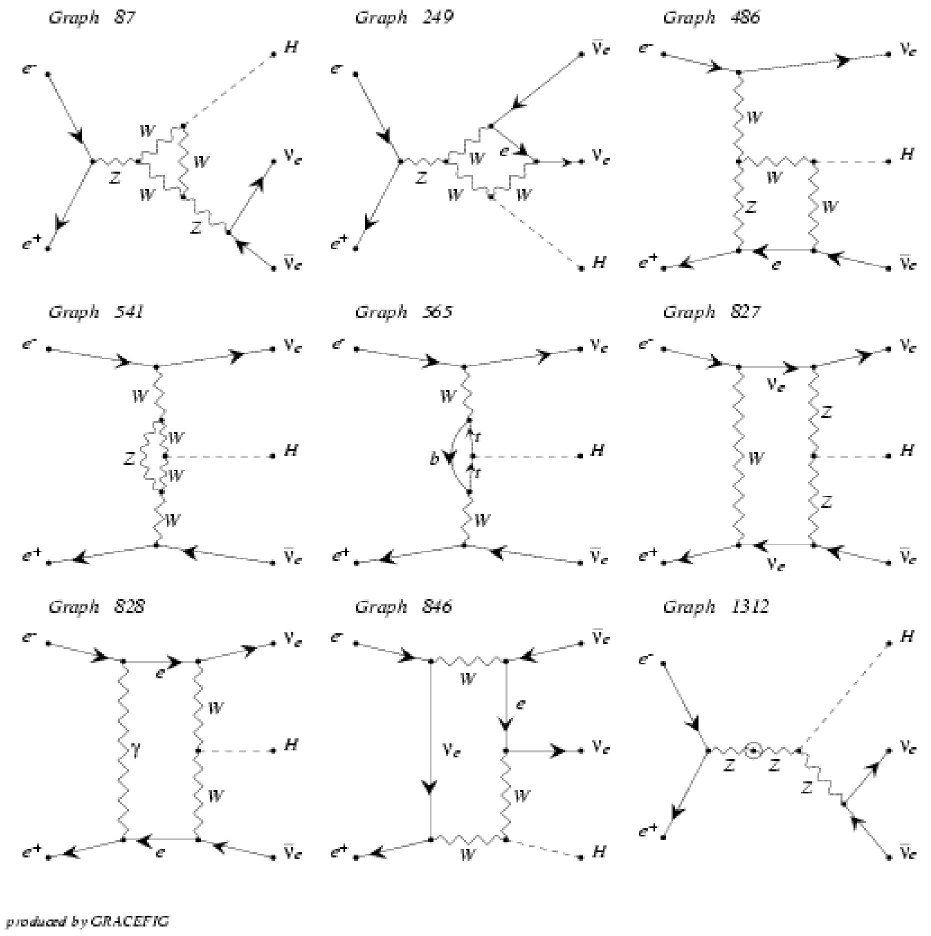
M_h=120 GeV
(SM Higgs Br)

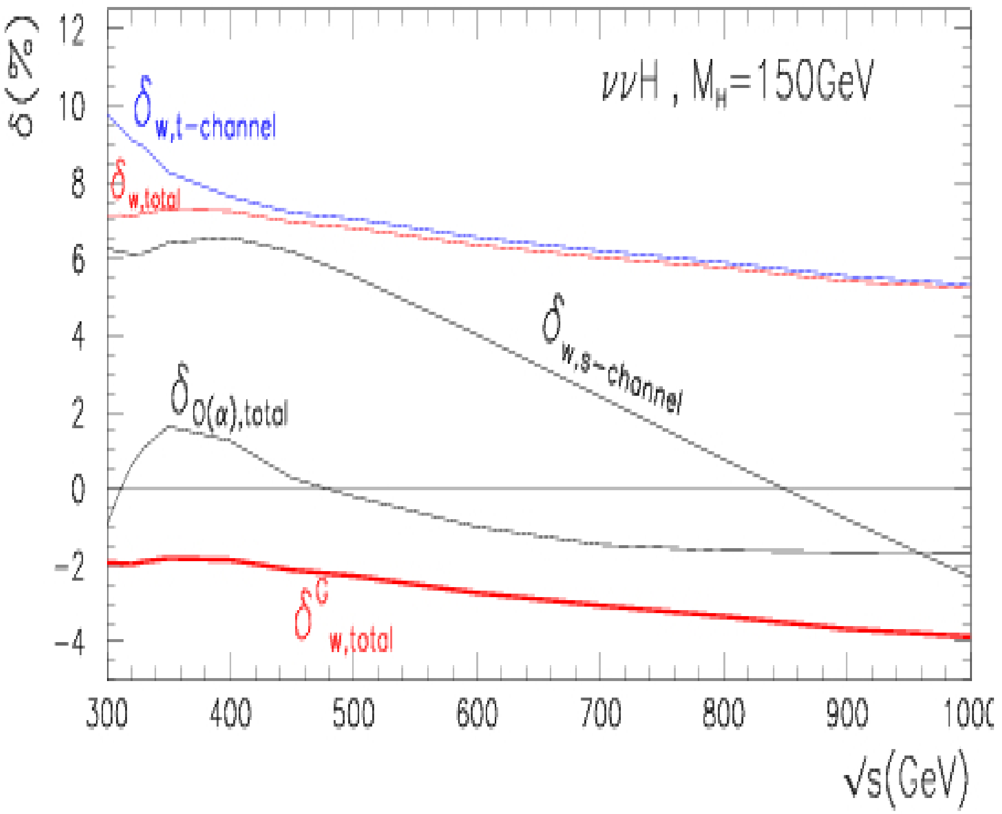
Use only $hh \rightarrow 4b$
($Br(hh \rightarrow 4b)$~47%)
Eff.(4b) 80%

Precise study ⇒ Radiative corrections are important!!

⇒ **Systematic study of the RC for Higgs physics at LC with GRACE**

$e^+e^- \to \nu\,\nu\,H$





GRACE Phys.Lett.B559(2003)252

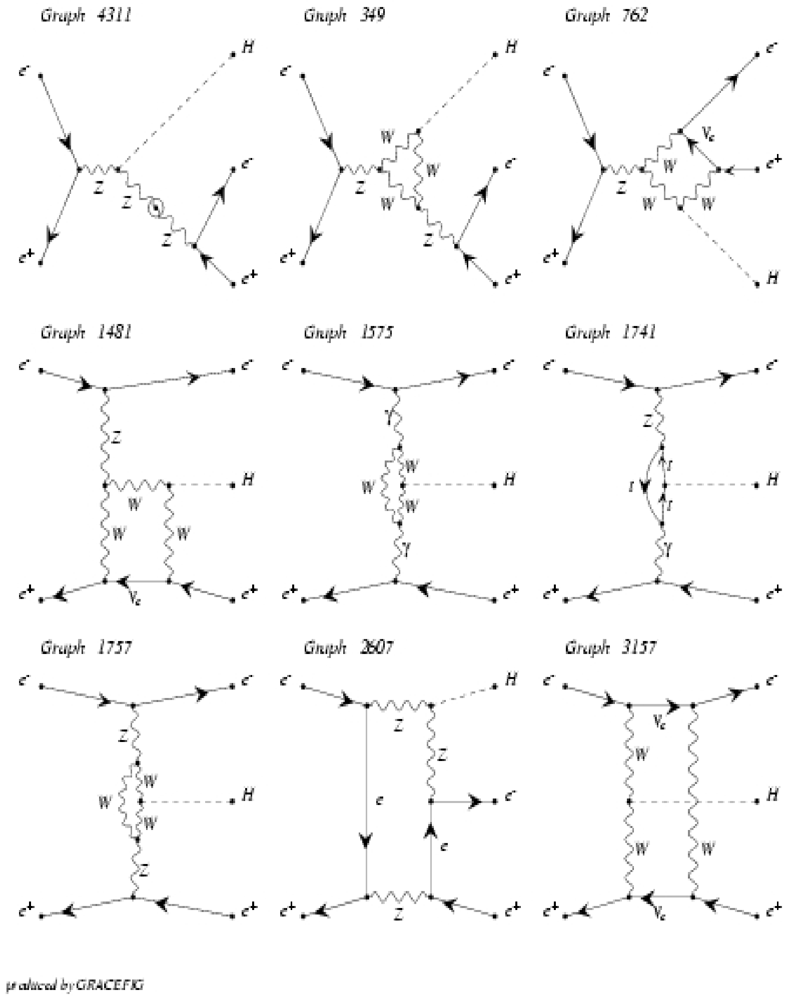
Denner et al. Phys.Lett.B575(2003)290

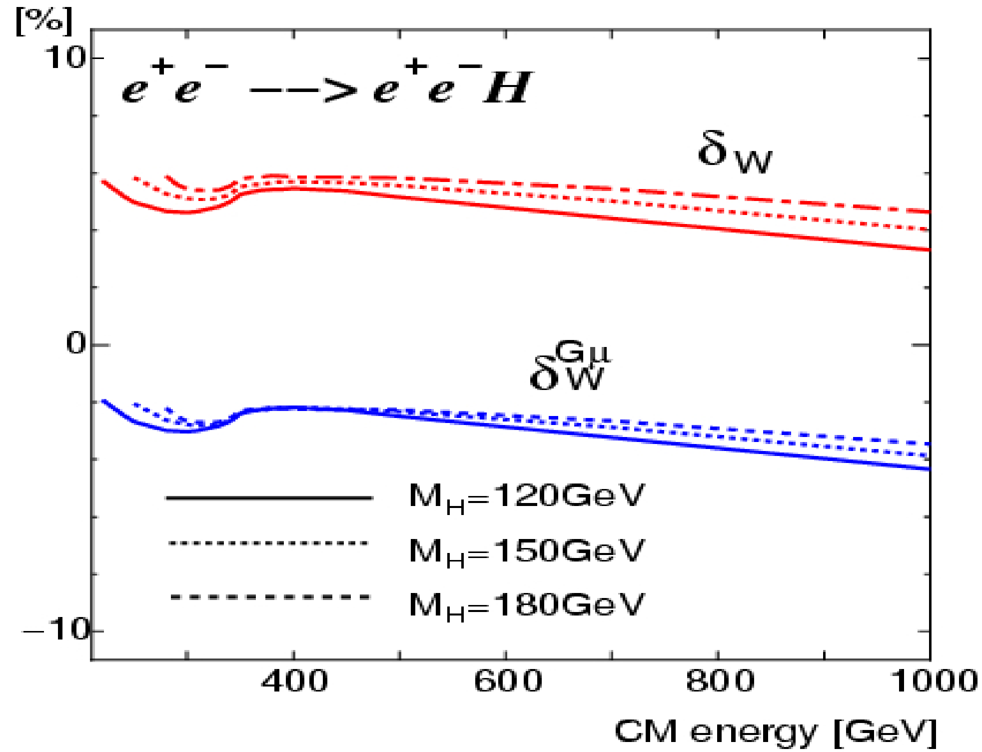
Nucl.Phys.B660(2003)289

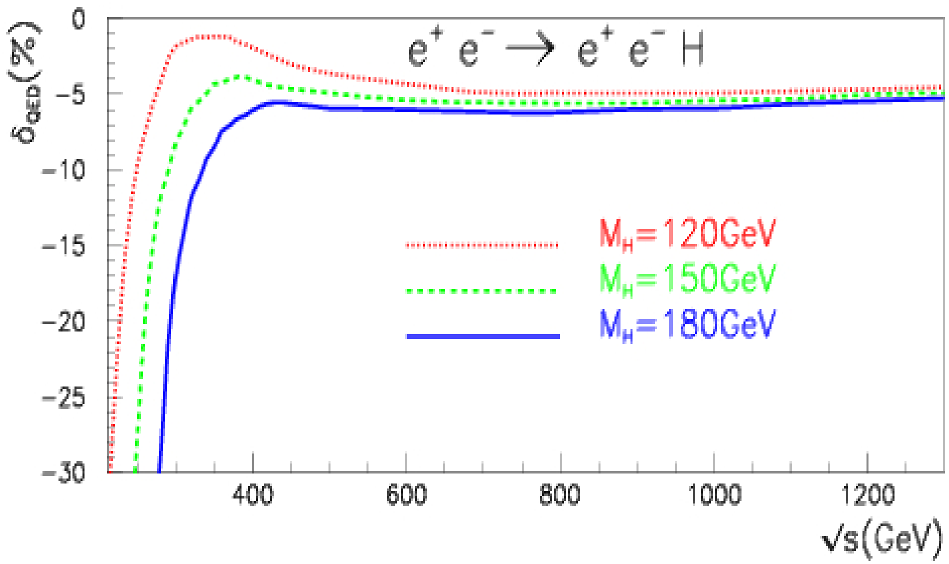
$$\delta_{O(\alpha)} = \sigma_{O(\alpha)} / \sigma_{tree} - 1$$

$$\delta^G_W = \delta_W - 3\Delta r$$

$e^+e^- \to \; e^+e^- H$

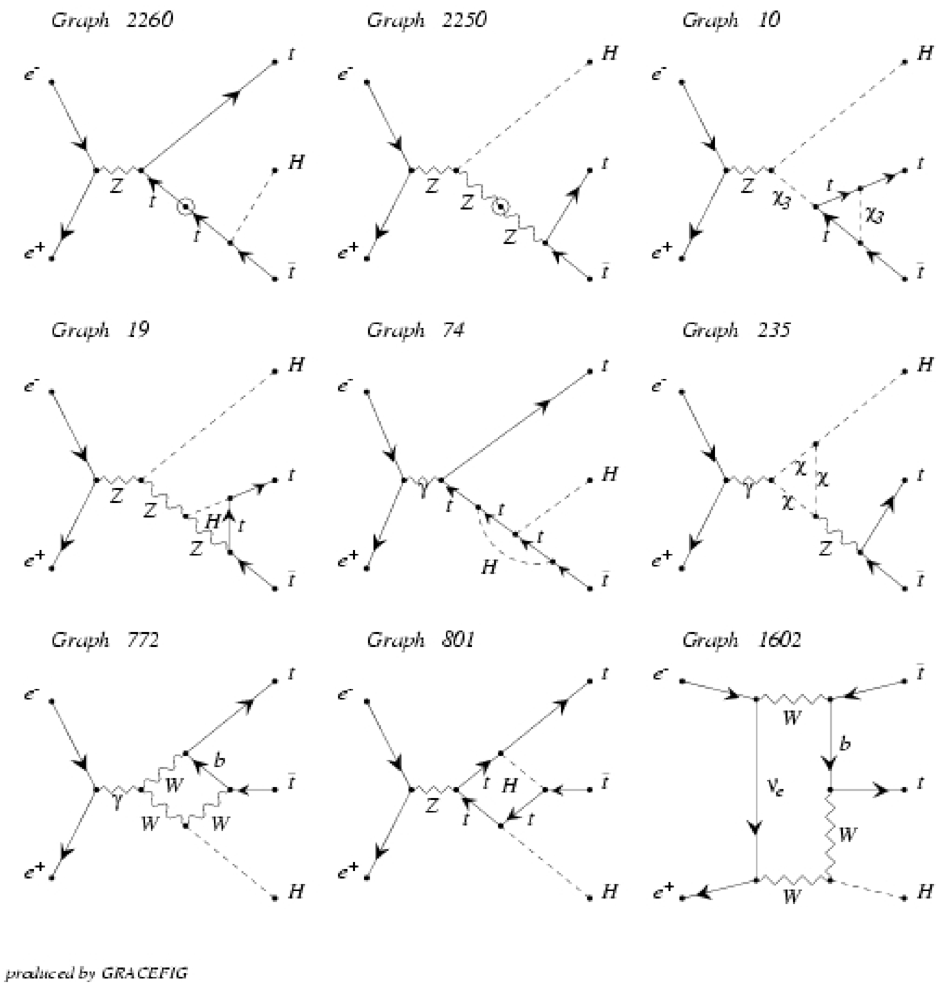


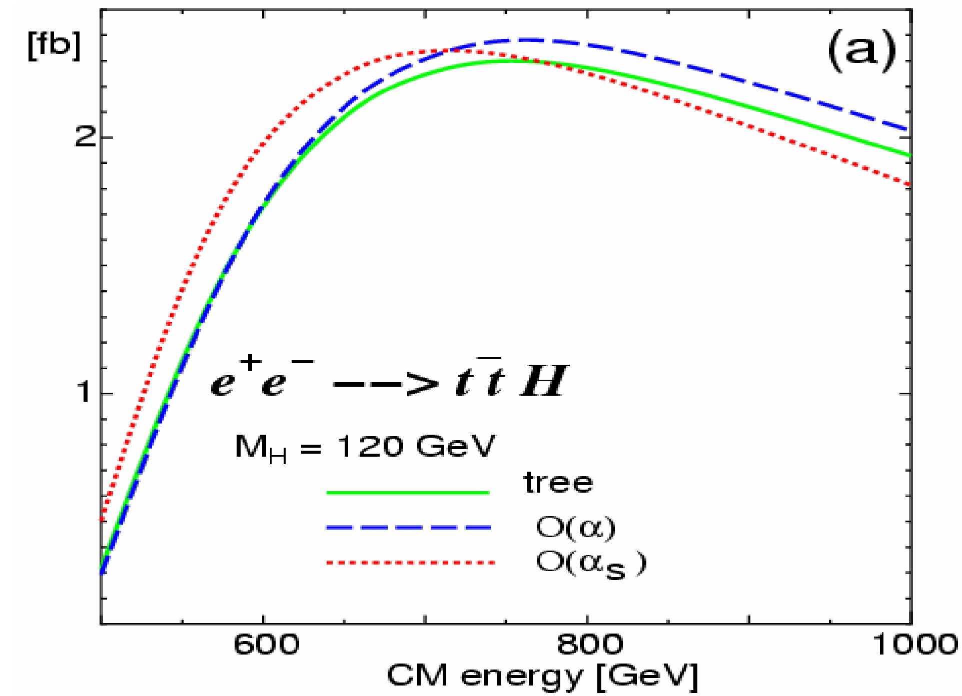


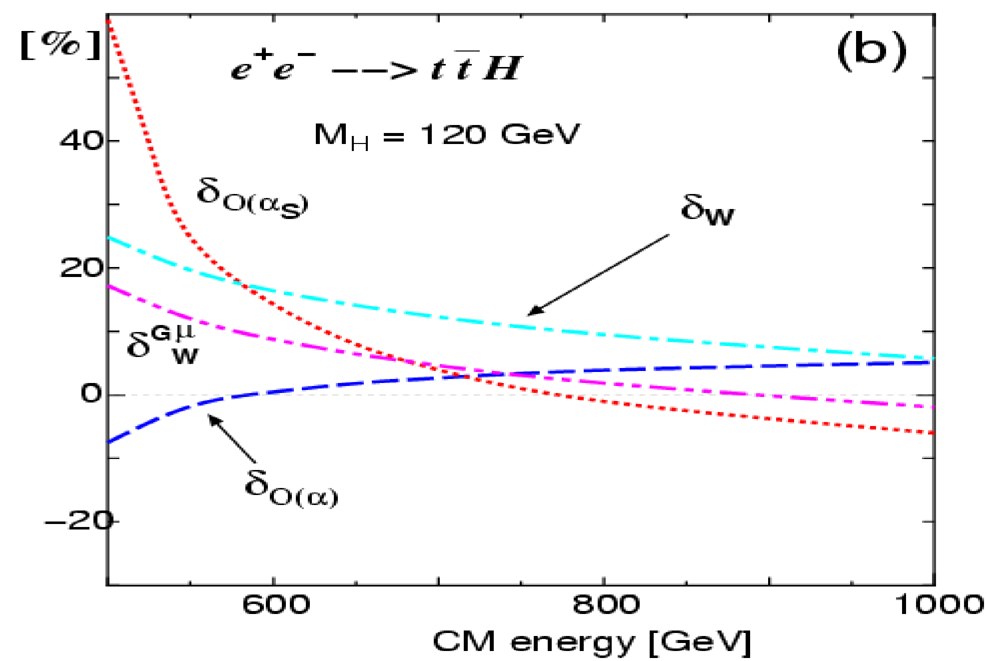


GRACE ***hep-ph/04070*** *65(PLB in press)*

$e^+e^- \to t\,t\,H$

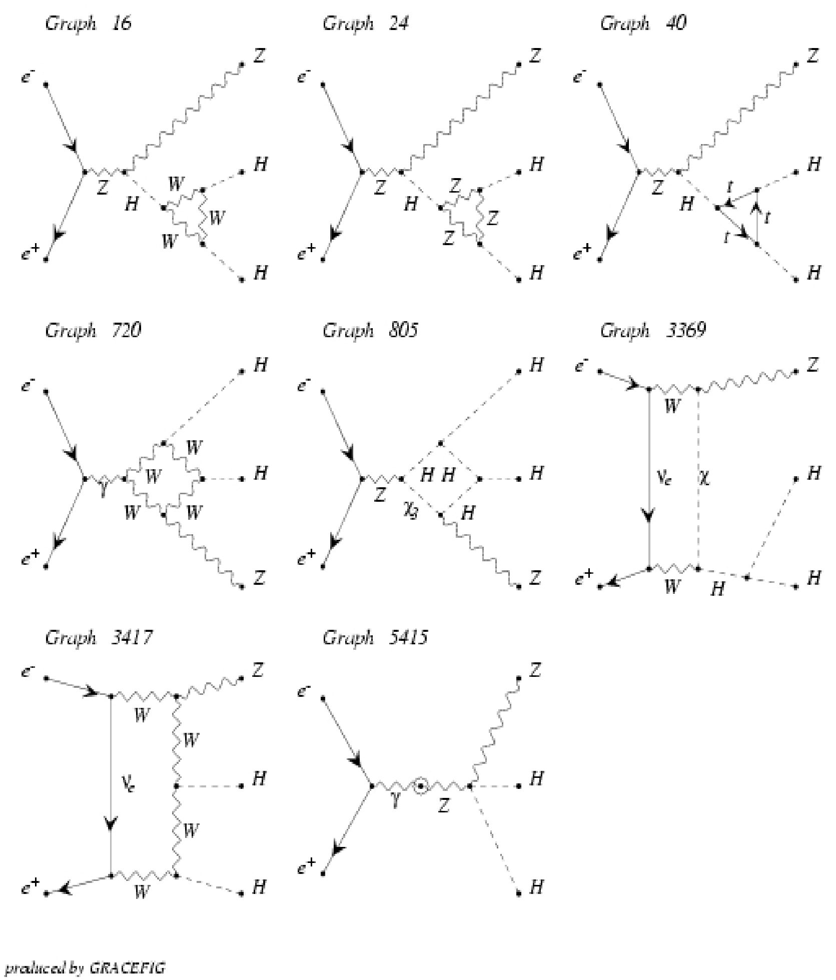


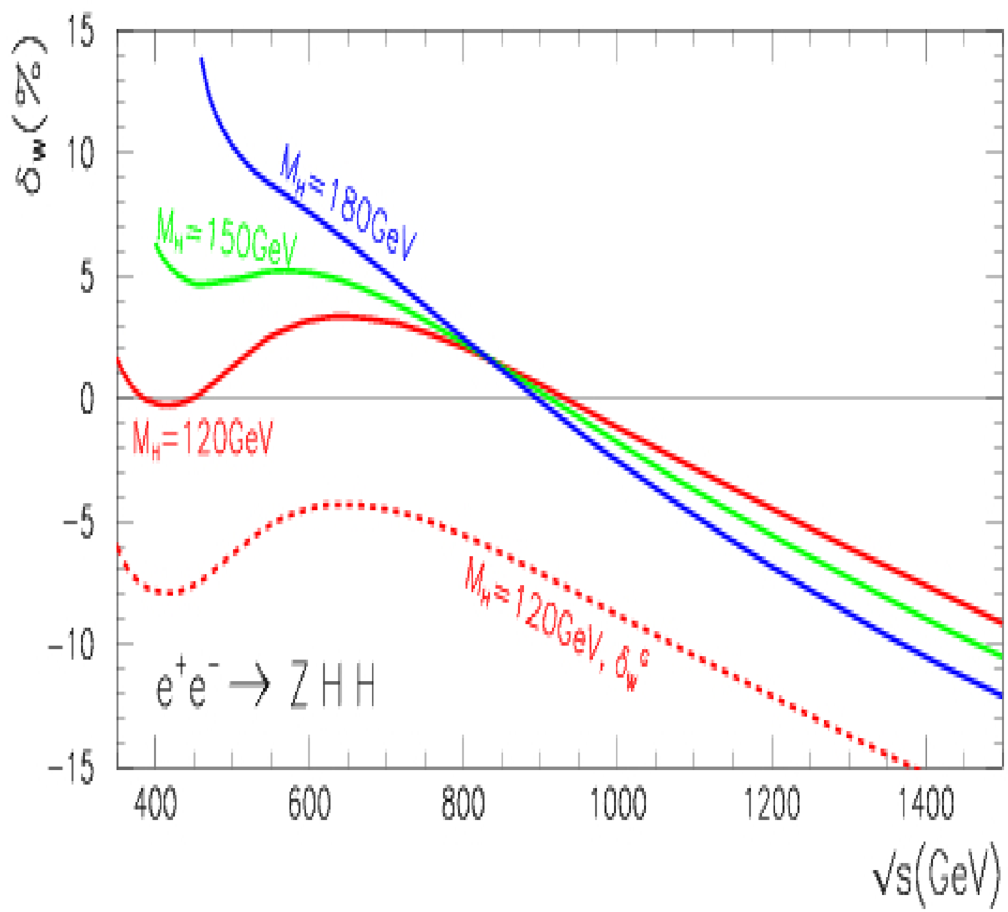




You et al. Phys.Lett.B571(2003)85 ??
GRACE Phys.Lett.B571(2003)163
Denner et al. Phys.Lett.B575(2003)290

$e^+e^- \rightarrow ZHH$





GRACE Phys.Lett.B576(2003)152

Zhang et al. Phys.Lett.B578(2004)349

Summary of Higgs studies by GRACE-loop

- ***Single Higgs production***
  - $e^+e^- \rightarrow$ ZH (full number of graphs = 341)
  - $e^+e^- \rightarrow$ $\nu\ \nu$ H (1350) Phys.Lett. B559 (2003) 252-262
    A.Denner et.al. PLB 560(2003)196, NPB 660(2003)289
  - $e^+e^- \rightarrow$ e^+e^- H (4470) hep-ph/0407065 (PLB in press)
- ***top Yukawa***
  - $e^+e^- \rightarrow$ t t H (2327) Phys.Lett. B571 (2003) 163-172
    Y.You et.al.PLB 571(2003)85
    A.Denner et.al. PLB 575(2003)290, NPB 680 (2004)85
- ***Multi Higgs production***
  - $e^+e^- \rightarrow$ ZHH (5417) Phys.Lett. B576 (2003) 152-164
    R.Zhang et.al.PLB(2004)349
  - $e^+e^- \rightarrow$ $\nu_e \nu_e$ HH (19638) $\Rightarrow$ This talk New results!!

Current status of GRACE-loop

- **GRACE is a software system which performs automatic calculation of Feynman amplitudes**
- **Full EW 1-loop calculations in SM $2 \to 3$ processes are well established.**
- **We move to $2 \to 4$ processes!!**
  - $e^+e^- \to \nu_e \nu_e HH$ $(\nu_\mu \nu_\mu HH,\ \nu_\tau \nu_\tau HH)$

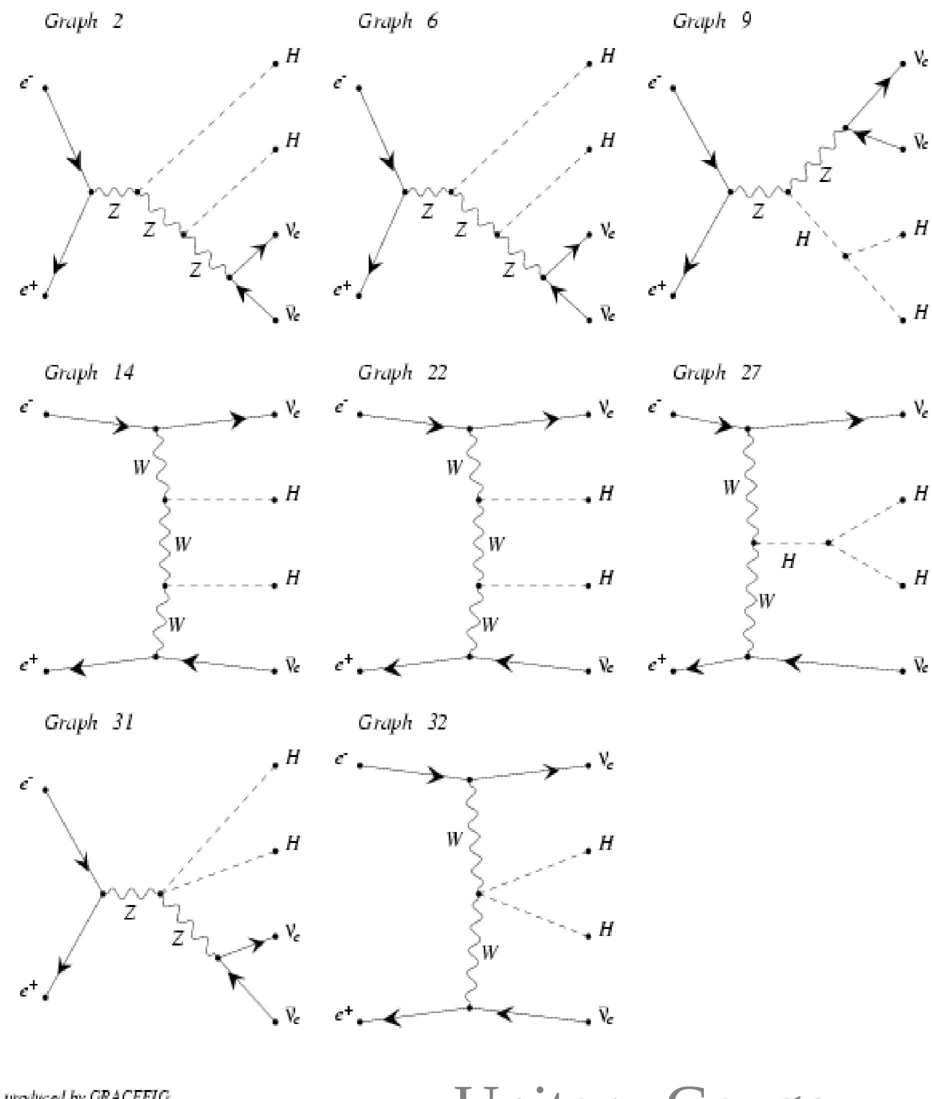
$$e^+e^- \to \nu_e \bar{\nu}_e HH$$

Number of diagrams

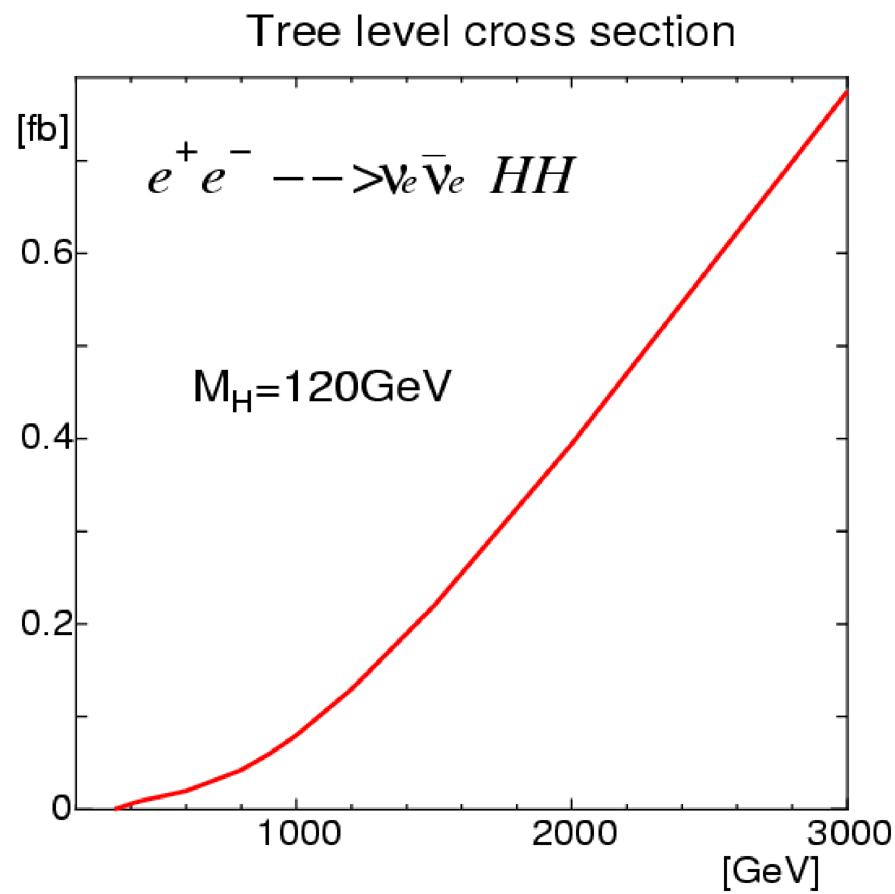
- **81 tree-level diagrams**
- **19638 one-loop diagrams**

Neglecting electron-scalar coupling

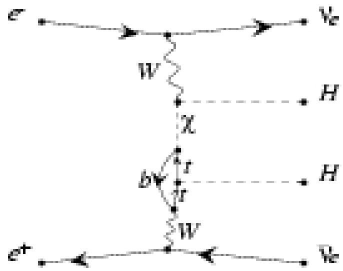
- **12 tree-level diagrams**
- **3416 one-loop diagrams**
  - **PEN 218, HEX 74**



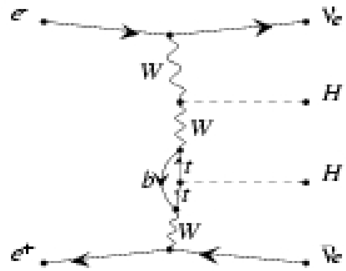
Unitary Gauge



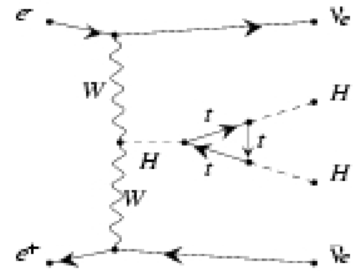
Graph 4403



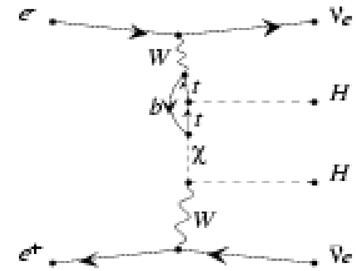
Graph 4471



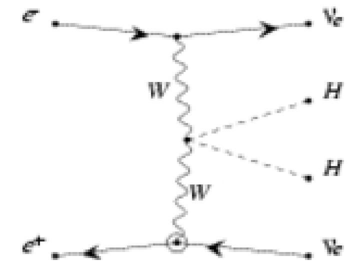
Graph 5384



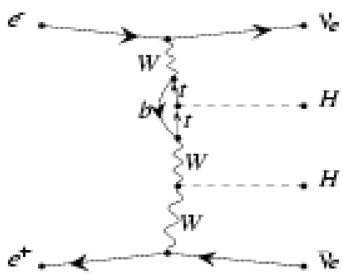
Graph 6652



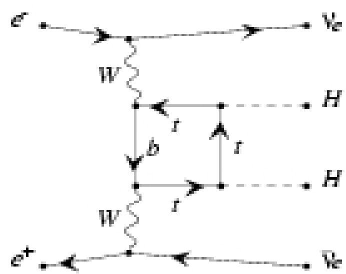
Graph 19205



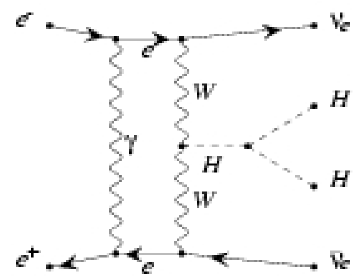
Graph 6686



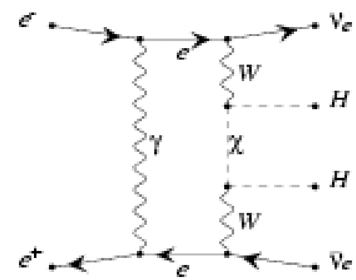
Graph 7205



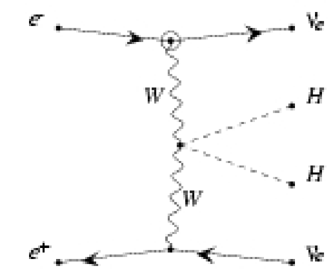
Graph 11477



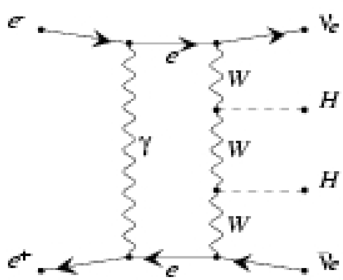
Graph 11496



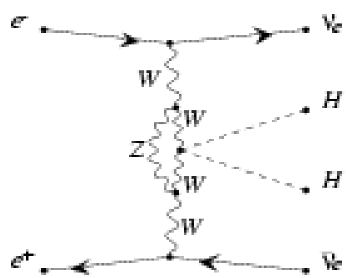
Graph 19208



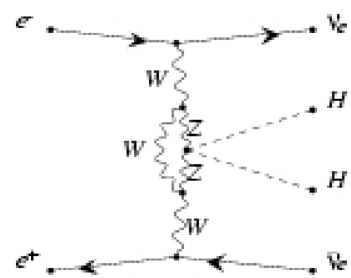
Graph 11497



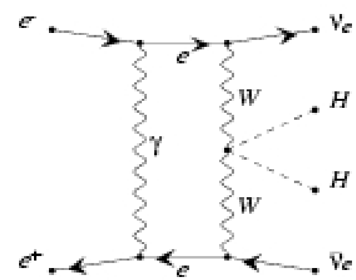
Graph 17308



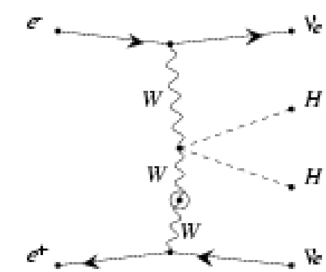
Graph 17310



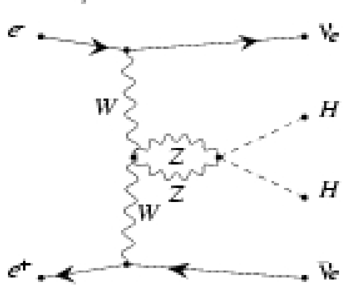
Graph 18283



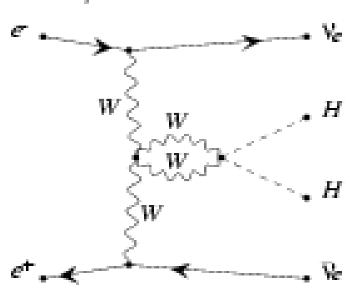
Graph 19631



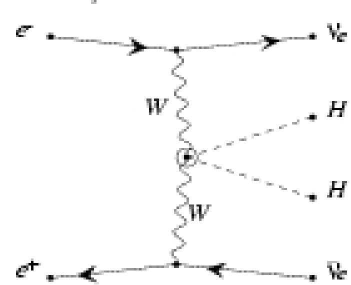
Graph 18857



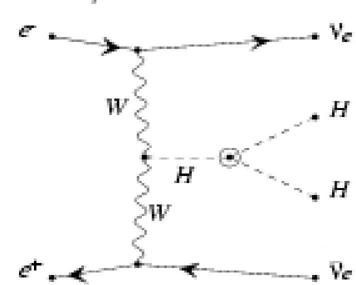
Graph 18858



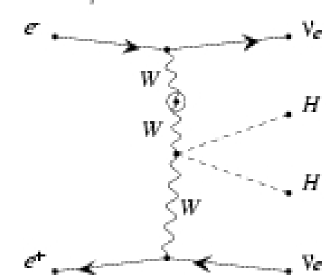
Graph 18873



Graph 18970



Graph 19635



GRACE-loop

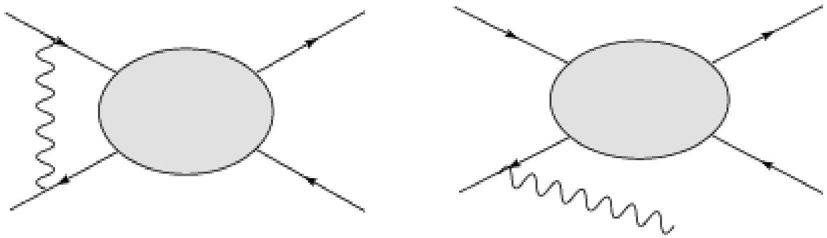
- **Full one-loop EW radiative corrections**
  - **On-mass shell renormalization scheme**
  - **Dimensional regularization n=4– 2ε**
    - ***FF* package for massive Box and Triangle integrals** **G. van Oldenborgh Comput. Phys.Commun 58(1991)1**
    - **N≥5 point integrals ⇒ Box integrals**
  - **Checks on calculation (in numerical way)**
    - **Ultraviolet Finiteness**
    - **Infrared Finiteness**
    - **Gauge parameter independence**

Ultraviolet finiteness

- **Dimensional regularization n=4-2ε**
  - **Regulator** $C_{UV} = 1/\varepsilon - \gamma_E + \log 4\pi$
  - **Different values of C_{UV}**
- **Checks performed with**
  - **One random phase space**
  - **Full set of diagrams**
  - **Quadruple precision**
  - **over 29 digits precision for** $e^+e^- \to \nu_e \nu_e HH$

Infrared finiteness

- Infrared div. is regulated by a photon mass λ
- $\delta_V + \delta_S \;\Rightarrow\; \lambda$ independent
  - Over 15 digits precision



- $k_\gamma < k_c$ factorisation for ISR can be used to extract weak correction in processes like $e^+e^- \to \nu_e \nu_e H$, $e^+e^- \to \nu_e \nu_e HH$ **(see later)**

  QED universal factor

$$\delta^{\rm QED}_{S+V}(k_c) \quad = \quad \frac{2\alpha}{\pi}\left((L_e - 1)\ln\frac{k_c}{E_b} + \frac{3}{4}L_e + \frac{\pi^2}{6} - 1\right),$$

$$L_e \equiv \ln(s/m_e^2)\,,\, s = 4E_b^2$$

  - *weak corrections* $\delta_W = \delta^{full}{}_{V+S}(k_c) - \delta^{QED}{}_{V+S}(k_c)$
- $\delta_H(k_c)$ by BASES MC calculation

Gauge invariance

- Independence on gauge parameters
- Linear-gauge
  - Gauge parameters appear in the denominator of the propagators

$$\frac{1}{k^2 - M_W^2}\left(g_{\mu\nu} - (1 - \xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right)$$

- too much complication for loop calculations
  - We use the Non-linear gauge

Non-linear gauge

$$\mathcal{L}_{\mathrm{GF}} \equiv -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z}(F^Z)^2 - \frac{1}{2\xi_A}(F^A)^2$$

$$\begin{aligned} F^\pm &\equiv (\partial_\mu \mp i e \tilde{\alpha} A_\mu \mp i g c_W \tilde{\beta} Z_\mu) W^{\mu +} \\ &\quad + \xi_W \frac{g}{2}(v + \tilde{\delta} H \mp i \tilde{\kappa} \chi_3)\chi^+ \\ F^Z &\equiv (\partial \cdot Z + \xi_Z \frac{g}{2 c_W}(v + \tilde{\varepsilon} H)\chi_3)^2 \\ F^A &\equiv \partial \cdot A \end{aligned}$$

Boudjema and Chopin Z.Phys. C73(1996) 85

Choose t'Hooft-Feynman gauge ($\xi_W = \xi_Z = \xi_A = 1$)

Vary parameters $\zeta = (\alpha, \beta, \delta, \kappa, \varepsilon)$

Numerical Check

- For each loop graph; g

$$d\sigma_g = d\sigma_g(\zeta) = 2\Re(\mathcal{T}_g^{loop} \cdot \mathcal{T}^{tree\dagger})$$

- Polynomial in $\zeta = (\alpha\ ,\beta\ ,\delta\ ,\kappa\ ,\varepsilon\ )$

$$d\sigma_g(\zeta) = d\sigma_g^{(0)} + \zeta\, d\sigma_g^{(1)} + \zeta^2 d\sigma_g^{(2)} + \zeta^3 d\sigma_g^{(3)}$$

- Independence of $\zeta \Rightarrow\ \Sigma_g\, d\sigma_g^{(1,2,3)} = 0$

$$d\sigma \equiv \sum_g d\sigma_g = \sum_g d\sigma_g^{(0)}$$

$$\mathrm{SUM}^{(i)} \equiv \frac{\sum_g d\sigma_g^{(i)}}{\mathrm{Max}_g(|d\sigma_g^{(i)}|)} = 0 \quad , \quad i = 1, 2, 3$$

Example of NLG check

$e^+e^- \to \quad \nu_e \, \nu_e \, HH$

$\alpha = \beta = \kappa = \varepsilon = 0, \;\; \delta \neq 0$ selected graphs 6202

graph#	δ^ 3	δ^ 2	δ^ 1	δ^ 0
3323		1.347497E-10	-7.175787E-09	4.196583E-09
3325		-2.732577E-11	-9.061324E-12	2.406820E-11
3326			-2.441878E-11	-5.573020E-11
...	...	...	...	...
19635		2.705654E-21	-2.331323E-23	-6.201892E-08
19636	7.212994E-33	1.610799E-32	8.070171E-33	1.243721E-32
19637		-5.411309E-21	-8.748376E-21	-1.348119E-20
19638	1.804316E-31	2.112921E-30	2.018737E-31	2.372393E-30
cnt	513	1988	3701	
max	1.2198E-09	1.1165E-08	2.3698E-08	5.3237E-07
sum	-6.5174E-37	-1.4245E-32	-2.1334E-29	9.2081E-07

δ= -2 : 0.92080890794613308296812723863187437E-06
δ= -1 : 0.92080890794613308296810594741121384E-06
δ= 0 : 0.92080890794613308296808462774505645E-06
δ= 1 : 0.92080890794613308296806327959540435E-06
δ= 2 : 0.92080890794613308296804190299250006E-06
.... :

$\alpha = \beta = \kappa = \delta = 0, \ \varepsilon \neq 0$ selected graphs 1610

	ε^ 3	ε^ 2	ε^ 1	ε^ 0
		3.2956E-11	5.0263E-09	-1.6596E-09
		3.6744E-33	1.1896E-31	-1.6733E-32
		4.9263E-10	4.8763E-09	2.6826E-09
	...	...	...	...
	4.0158E-20	3.4773E-20	3.4773E-20	3.0110E-20
	4.4546E-52	-4.0158E-20	-4.0158E-20	0.0000E+ 00
	1.0557E-30	9.1409E-31	9.1409E-31	7.9151E-31
	...	...	...	...
			-3.1743E-34	-4.5501E-32
			-4.9140E-22	-7.0438E-20
			-1.9855E-30	-4.2551E-30
cnt	76	288	1246	
max	1.0130E-09	2.0222E-08	2.5773E-08	5.3237E-07
sum	-9.2306E-36	-6.8760E-27	1.8680E-26	7.7555E-07

ε= -2 : 0.775554434942596894200137146934376 93E-06
ε= -1 : 0.77555443494259689423944548877472206E-06
ε= 0 : 0.77555443494259689426500186113027972E-06
ε= 1 : 0.77555443494259689427680626387551886E-06
ε= 2 : 0.77555443494259689427485869702519782E-06
.... :

Input parameters

M_Z=91.1876GeV Γ_Z=2.4952GeV

M_W=80.4163GeV

M_H=120GeV

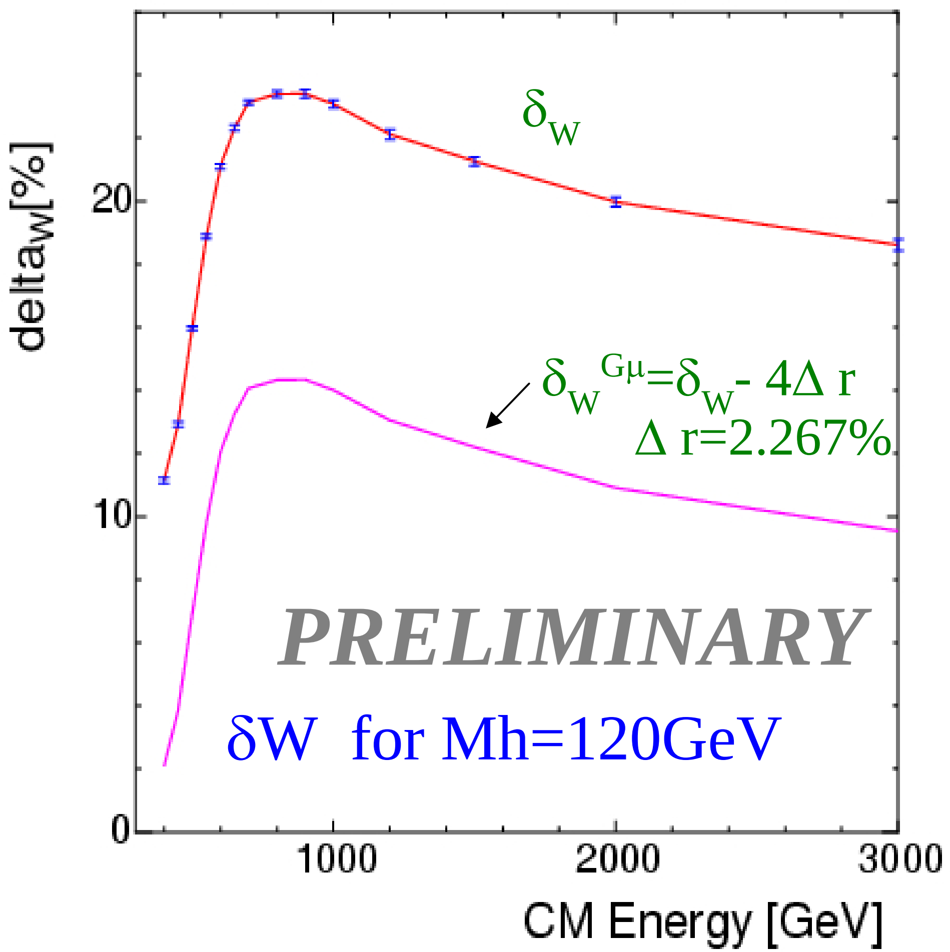
M_t=180GeV M_b=4.7GeV

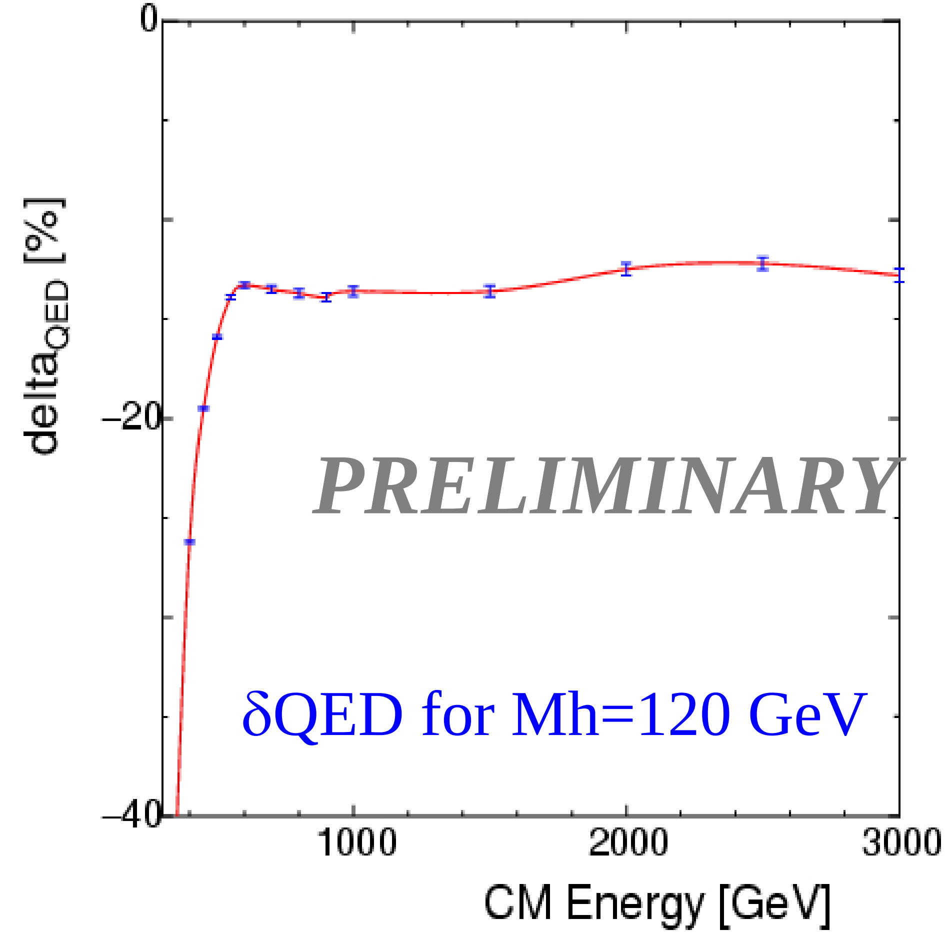
M_s=92MeV M_c=1.5GeV

M_u=M_d=63MeV for the denominators and vacuum polarization

α =1/137.0359895

$$\delta=\sigma(O(\alpha))/\sigma(\text{tree})-1$$

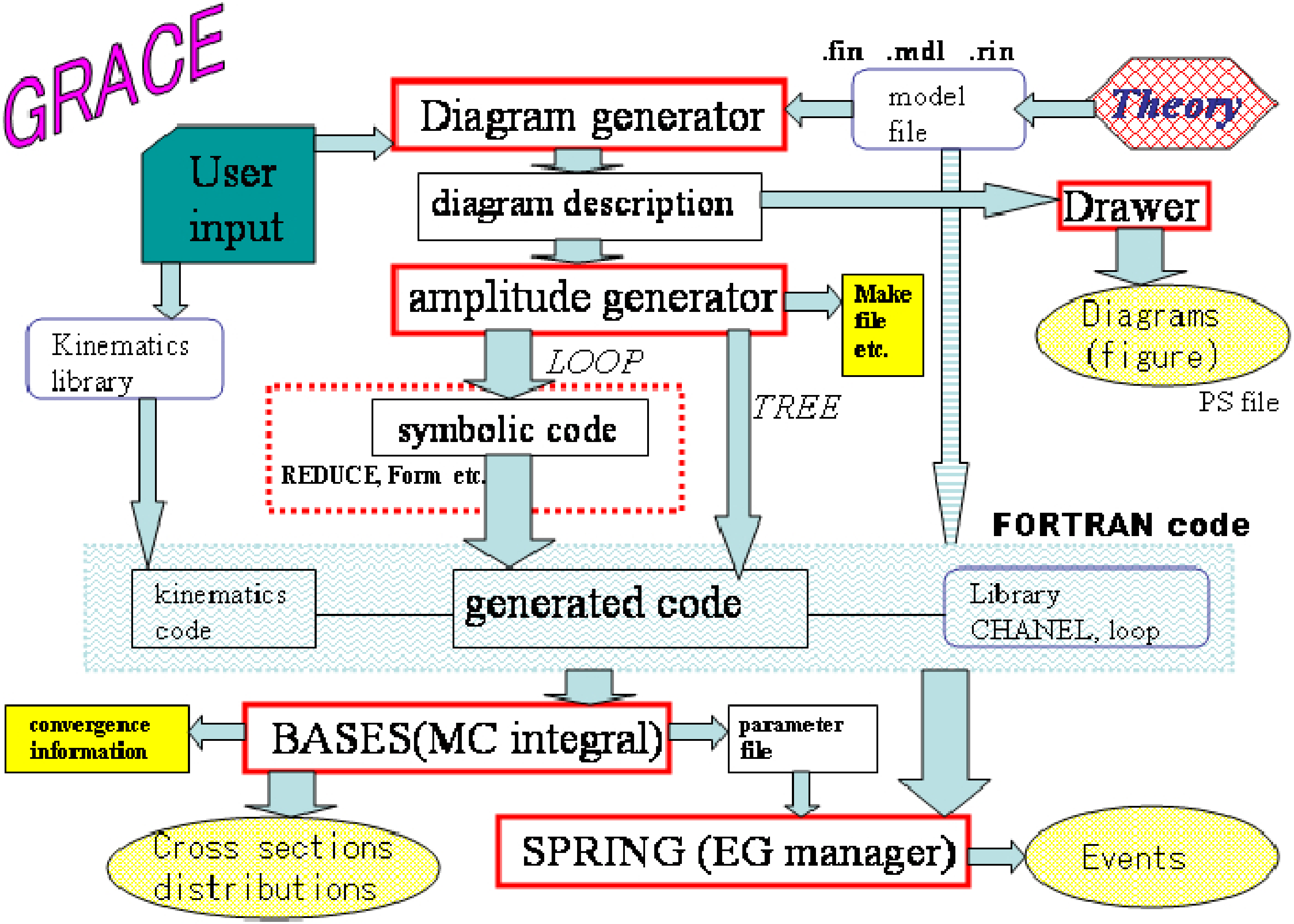




KEK IBM RS600 (Power 3 :375MHz)
IDRIS IBM (Power 4: 1.3/1.7GHz)

Summary

- **GRACE-loop**
  - **Full EW one-loop calculations are already established for $2 \to 2$ and $2 \to 3$ processes in SM**
  - **We started $2 \to 4$!!**
- **Higgs study**
  - **Systematic study of the RC for Higgs physics at LC**
    - $e^+e^- \to \nu \nu H$, $e^+e^- \to e^+e^- H$, $e^+e^- \to t t H$, $e^+e^- \to ZHH$
  - **New Results** $e^+e^- \to \nu_e \nu_e HH$
    - **EW corrections are sizable *O(10)%***


GRACE
.fin .mdl .rin
Diagram generator
model file
Theory
User input
diagram description
Drawer
amplitude generator
Make file etc.
Diagrams (figure)
PS file
Kinematics library
LOOP
TREE
symbolic code
REDUCE, Form etc.
FORTRAN code
kinematics code
generated code
Library CHANEL, loop
convergence information
BASES(MC integral)
parameter file
Cross sections distributions
SPRING (EG manager)
Events