Requirements on colliders to match the precicion of the SUSY relic density

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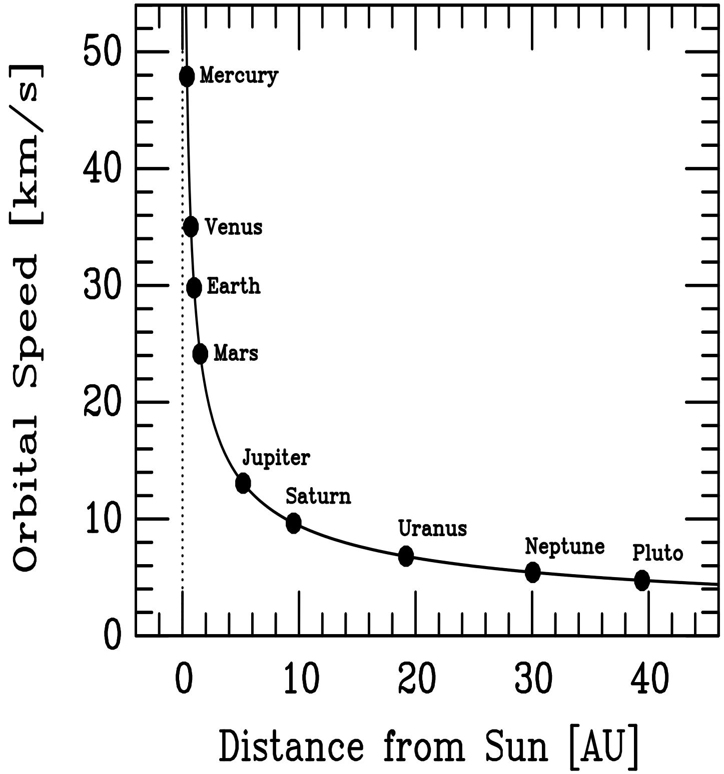
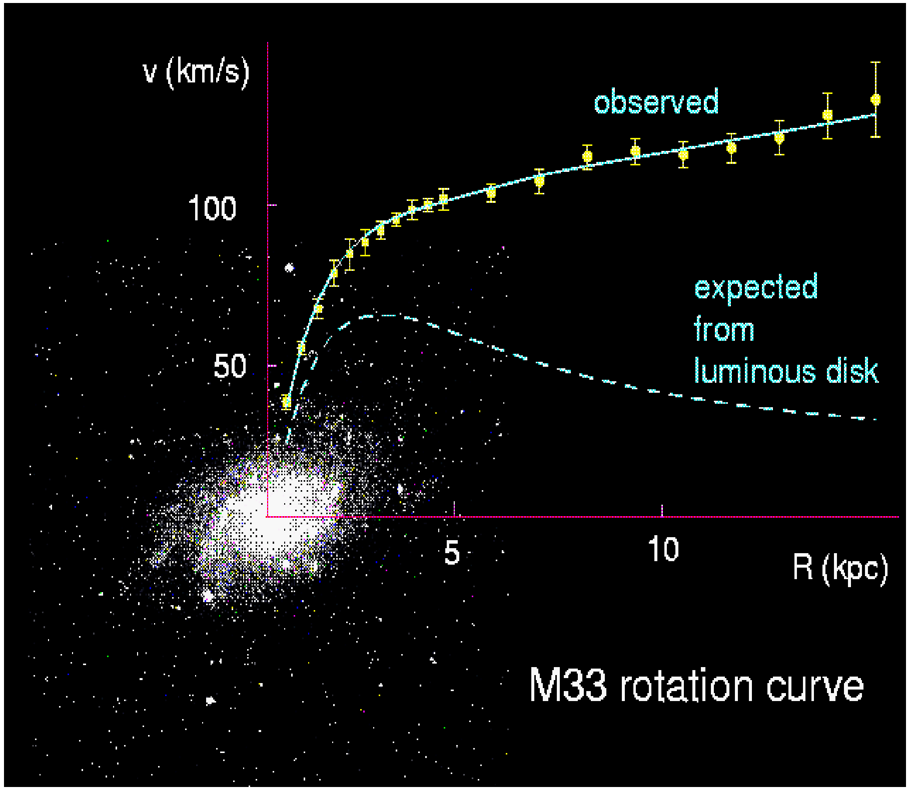
Propaganda: we are insignificant!





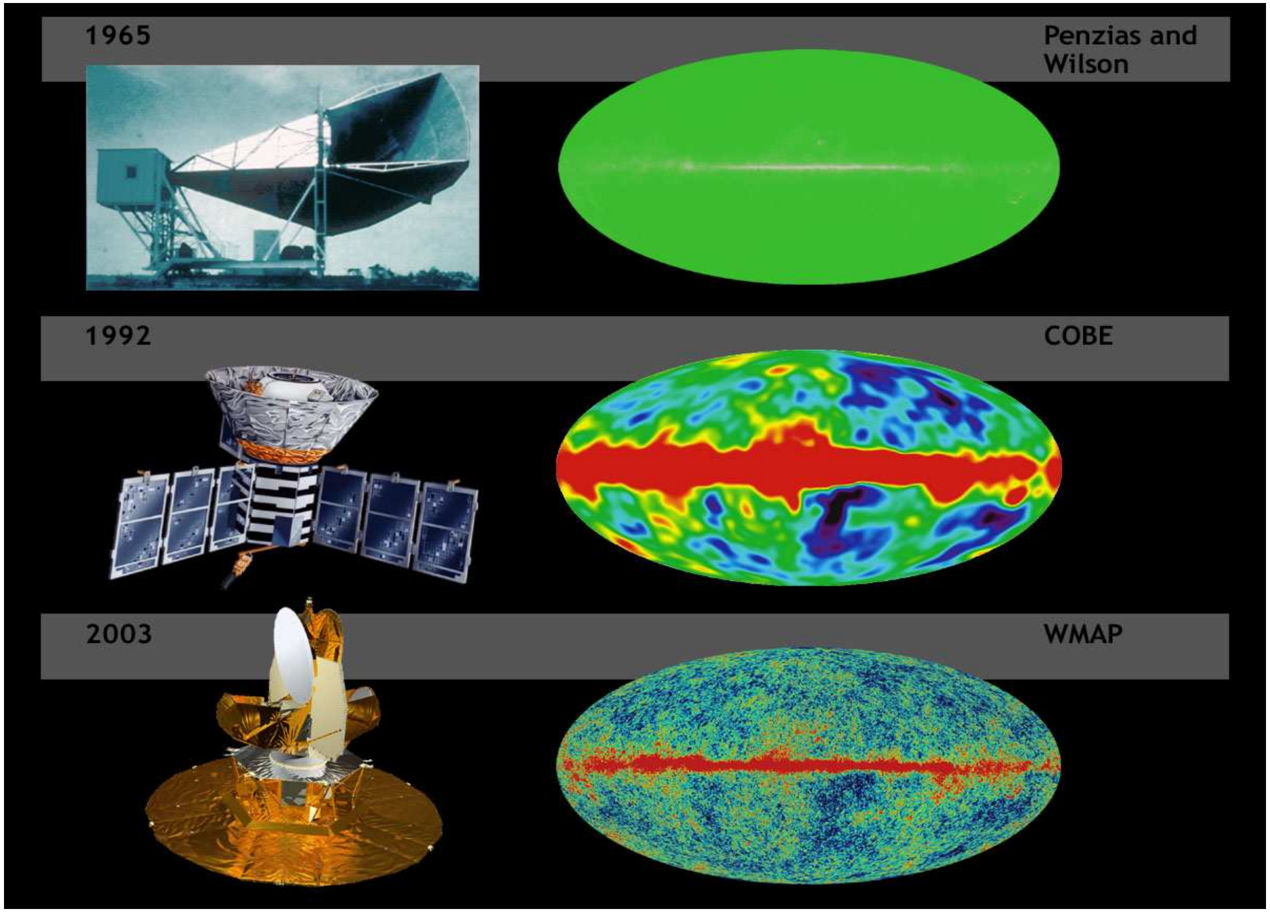
We are not in the centre of the universe

$$v = \sqrt{GM(r)/r}$$

Dark Matter= New Physics

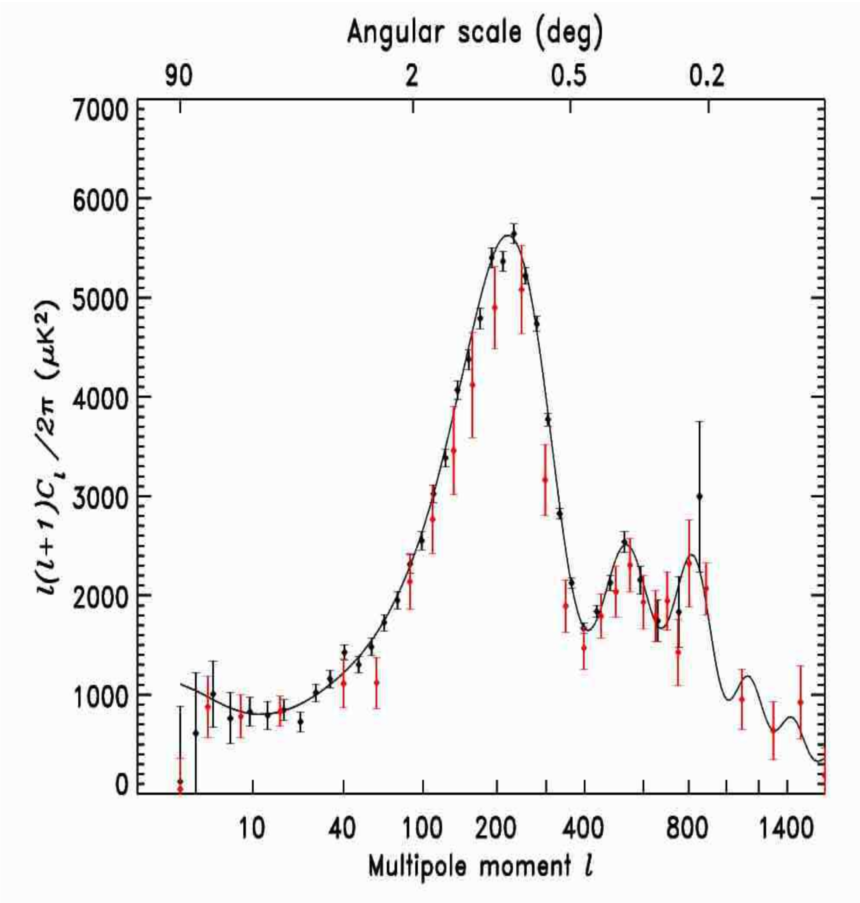
we are not made up of the same as most of universe

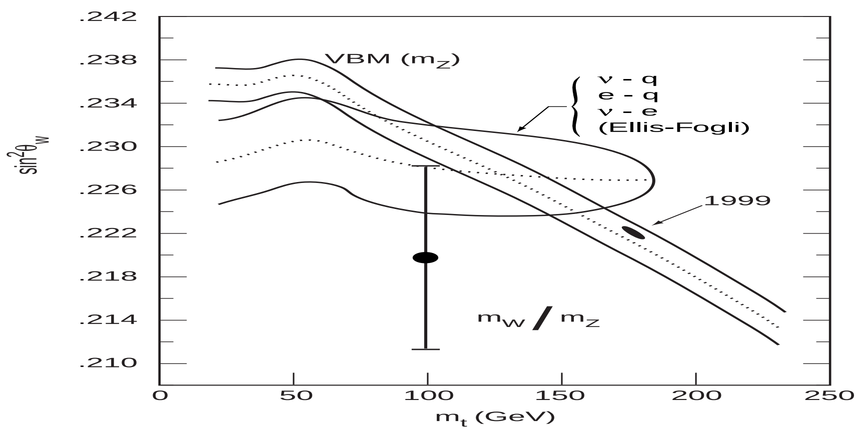
Cosmology in the era of precision measurement 1.

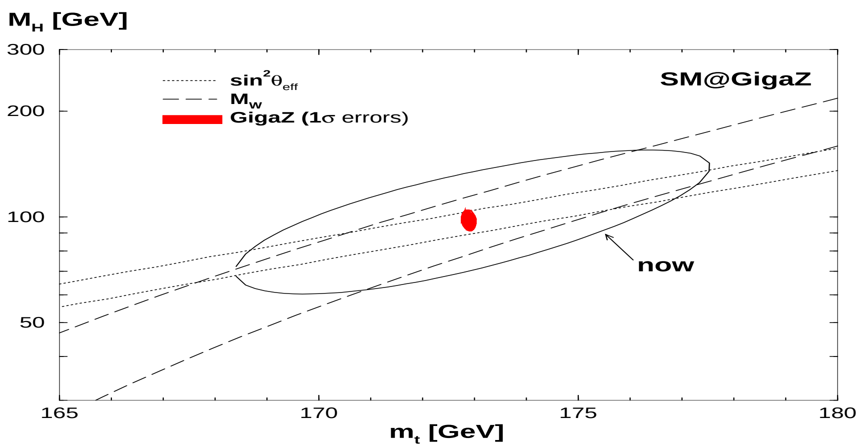


Pre-WMAP and WMAP vs Pre-LEP and LEP

Cosmology in the era of precision measurement 2.



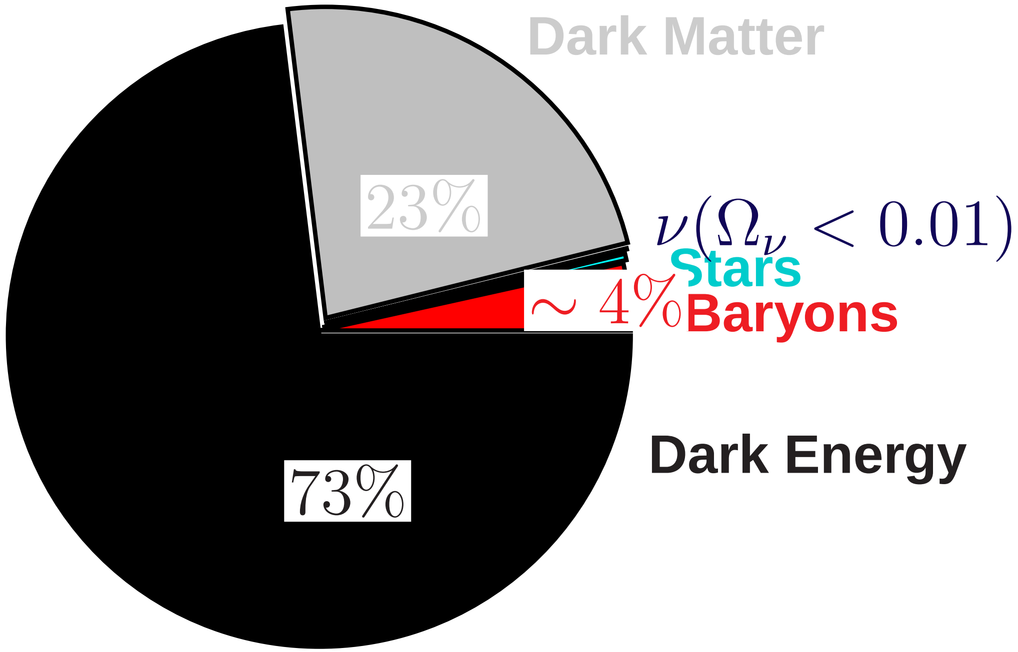




angular power spectrum of the CMB, pre-WMAP and WMAP

Planck+SNAP will do even better (per-cent precision) like from LEP to LHC+LC

matter budget and Precision 1.

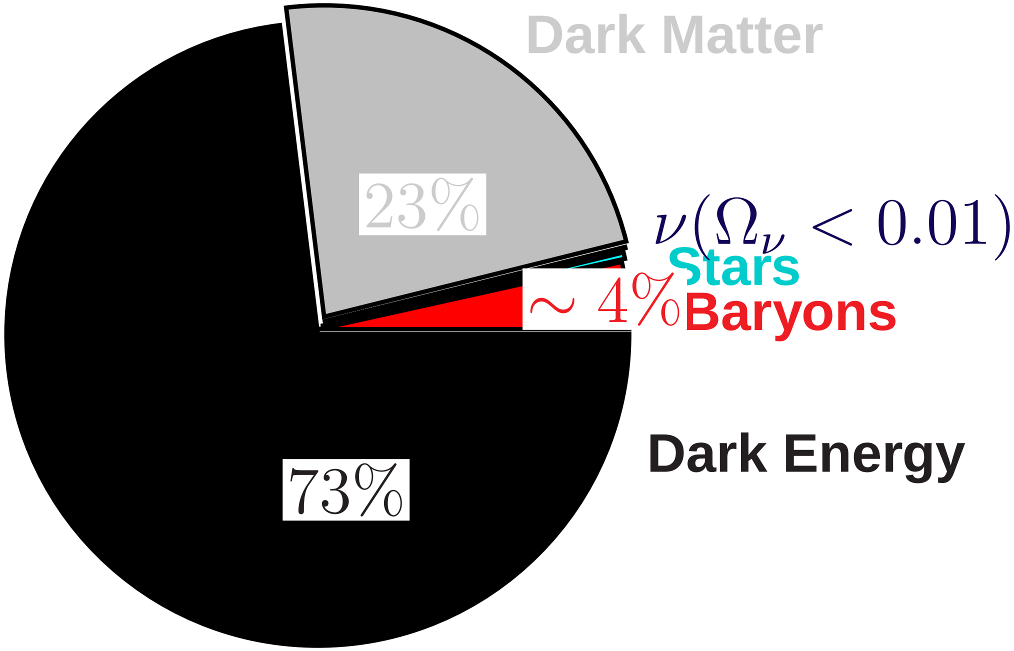


$$\mathbf{t_0 = 13.7 \pm 0.2}\ \textbf{Gyr}\ \mathbf{(1.5\%)}$$

$$\mathbf{\Omega_{tot} = 1.02 \pm 0.02\ (2\%)}$$

$$\mathbf{\Omega_{DM} = 0.23 \pm 0.04\ (17\%)}$$

matter budget and Precision 1.



$\mathbf{t_0 = 13.7 \pm 0.2}$ **Gyr** $\mathbf{(1.5\%)}$	$\alpha^{-1} = \mathbf{10 t_0 (10^{-7}\%)}$
$\mathbf{\Omega_{\rm tot} = 1.02 \pm 0.02 (2\%)}$	$\rho = \mathbf{\Omega_{\rm tot} (\sim 0.1\%)}$
$\mathbf{\Omega_{\rm DM} = 0.23 \pm 0.04 (17\%)}$	$\sin^2\theta_{\rm eff} = \mathbf{\Omega_{\rm DM} (0.08\%)}$

We should then be able to match the present WMAP precision!...

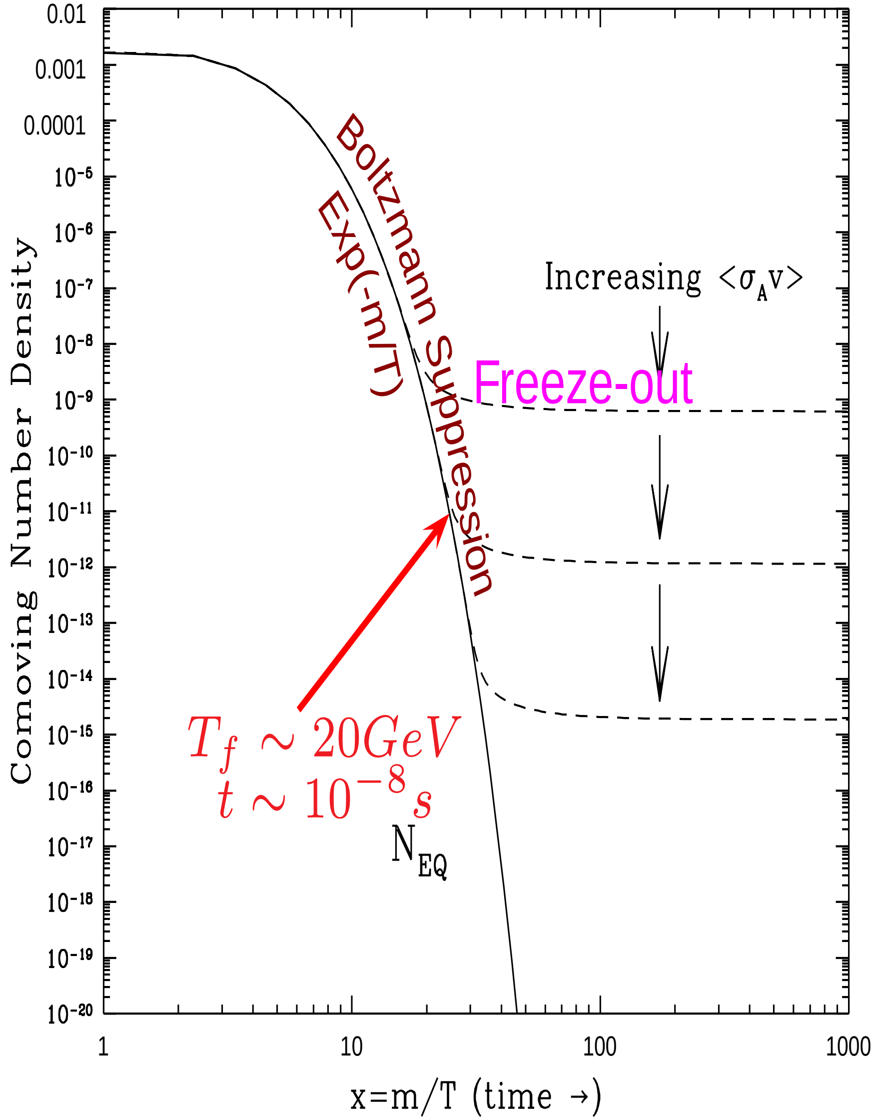
once we discover susy dark matter

This talk: Dark Matter not Dark Energy

Plan

- Dark Matter= neutralino LSP thermal relic
- Relic density and History of Universe
- mSUGRA scenarios
- Deriving accuracies in these scenarios
- Within mSUGRA on the GUT scale parameters
- in a model independent approach on the relevant physical parameters
- Outlook and conclusions

Relic Density: derivation



- At first all particles in thermal equilibrium
- universe cools and expands: interaction rate too small to maintain equilibrium
- (stable) particles can not find each other: freeze out and leave a relic density

dilution due to expansion

$$dN/dt = -3HN - <\sigma v> (N^2 - N_{eq}^2)$$

$$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to X \qquad X \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

$$\Omega_{\tilde{\chi}_1^0} = m_{\tilde{\chi}_1^0} N_{\tilde{\chi}_1^0} / \rho_{\rm cri}, \rho_{\rm cri} = 3H_0^2/8\pi G_N$$

$$\rho_{\rm cri} = h^2 1.910^{-29} gcm^{-3} \to$$

$$\Omega_{\tilde{\chi}_1^0} h^2 \propto 1/\sigma_{\tilde{\chi}_1^0}$$

$$\text{Einstein} \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{\Lambda}{8\pi G} \right)$$

Isotropic and Homogeneous

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$\text{conservation} \quad H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2}$$

$$\rightarrow \sum_M \Omega_M + \Omega_\Lambda + \Omega_k = 1 \quad \Omega_M = \frac{\rho_M}{\rho_c} \quad \rho_c = \frac{3H^2}{8\pi G}$$

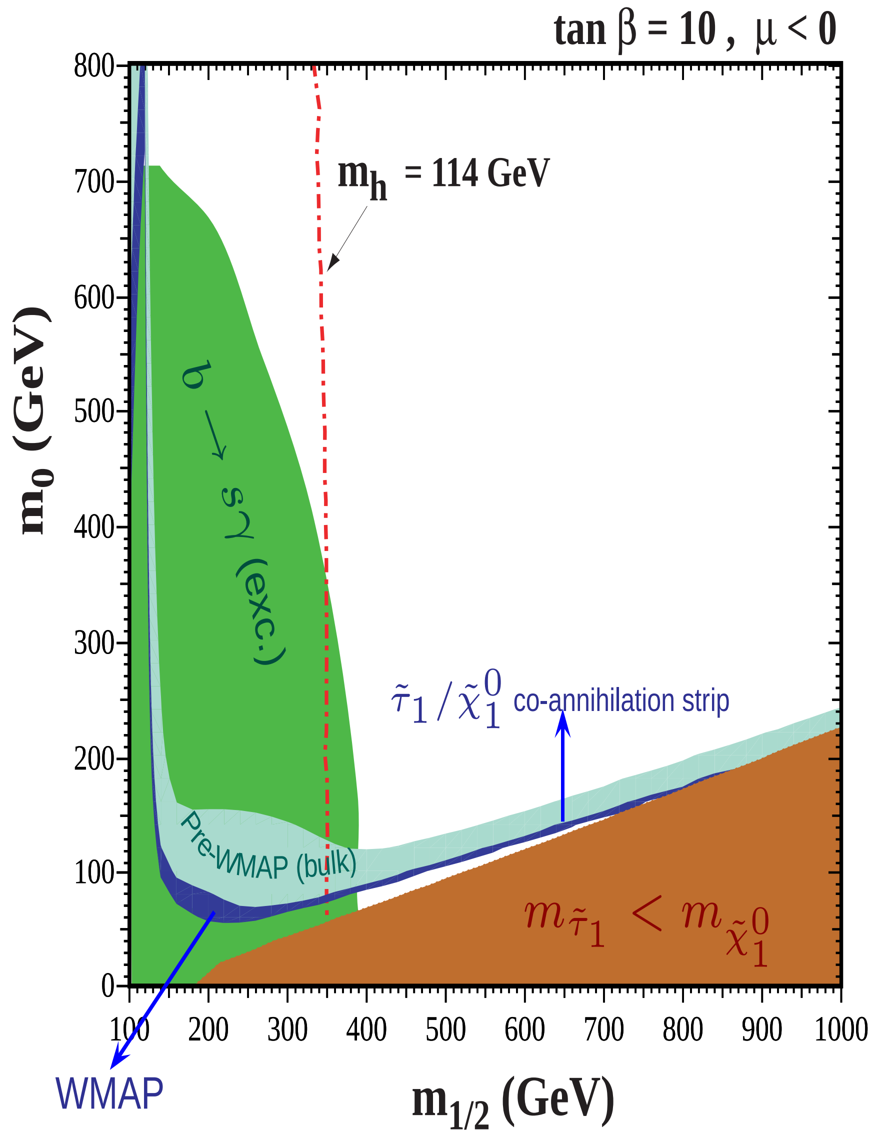
$$\text{Acceleration} \quad \left(\frac{\ddot{a}}{a} \right) = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i) \quad p = w\rho$$

$$\rho(a) \propto \frac{1}{a(t)^{3(1+w)}} \quad w_{rad} = 1/3 \;\; w_M = 0 \; w_\Lambda = -1$$

Relic Density: Loopholes and Assumptions

- At early times Universe is radiation dominated: $H(T) \propto T^2$ ◀
- Expansion rate can be enhanced by some scalar field (kination), extra dimension $H^2 = 8\pi G/3\ \rho(1 + \rho/\rho_5)$, anisotropic cosmology,...
- Entropy conservation (entropy increase will reduce the relic abundance

The mSUGRA inspired regions

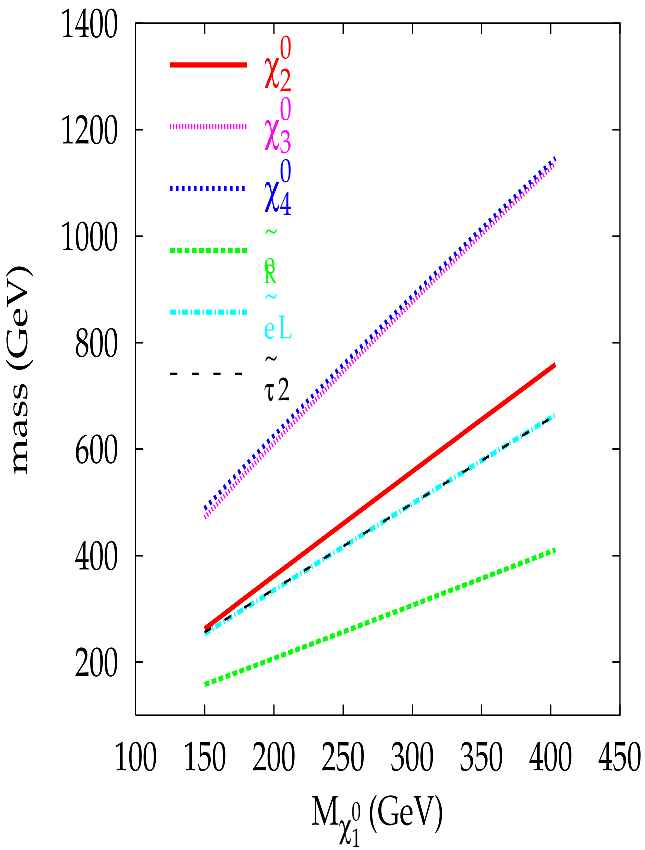


- Bulk region: bino LSP, $\tilde{l}_R$ exchange, (small $m_0, M_{1/2}$)
- $\tilde{\tau}_1$ co-annihilation: NLSP thermally accessible, ratio of the two populations $exp(-\Delta M/T_f)$ small m_0, $M_{1/2} : 350 - 900$GeV
- Higgs Funnel: Large $\tan\beta$, $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to A \to b\bar{b}, (\tau\bar{\tau})$, $M_{1/2} : 250 - 1100$GeV, $m_0 : 450 - 1000$GeV
- Focus region: small $\mu \sim M_1$, important higgsino component, requires very large TeV m_0

Use micrOMEGAs+SOFTSUSY $\rightarrow$...Softmicro..

- fix $A_0, \tan\beta, sgn(\mu)$ but scan on $M_{1/2}$
- WMAP strips imply $m_0 = f(M_{1/2})$: slopes
- RGE also needs SM input parameters!
- scale dependence of relic: default $M_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$: scale of EWSB conditions
- theoretical uncertainty: effect of different refinements in RGE and threshold corrections
- derive accuracy within mSUGRA, relying completely on mSUGRA. accuracies on high scale parameters and Sm inputs
- model independent approach: find out most relevant parameters and extract accuracy on these (weak scale parameters)
- accuracies derived in an iterative procedure and refer to the 10% WMAP precision

$\tilde{\tau}_1$ co-annihilation region: The Spectrum

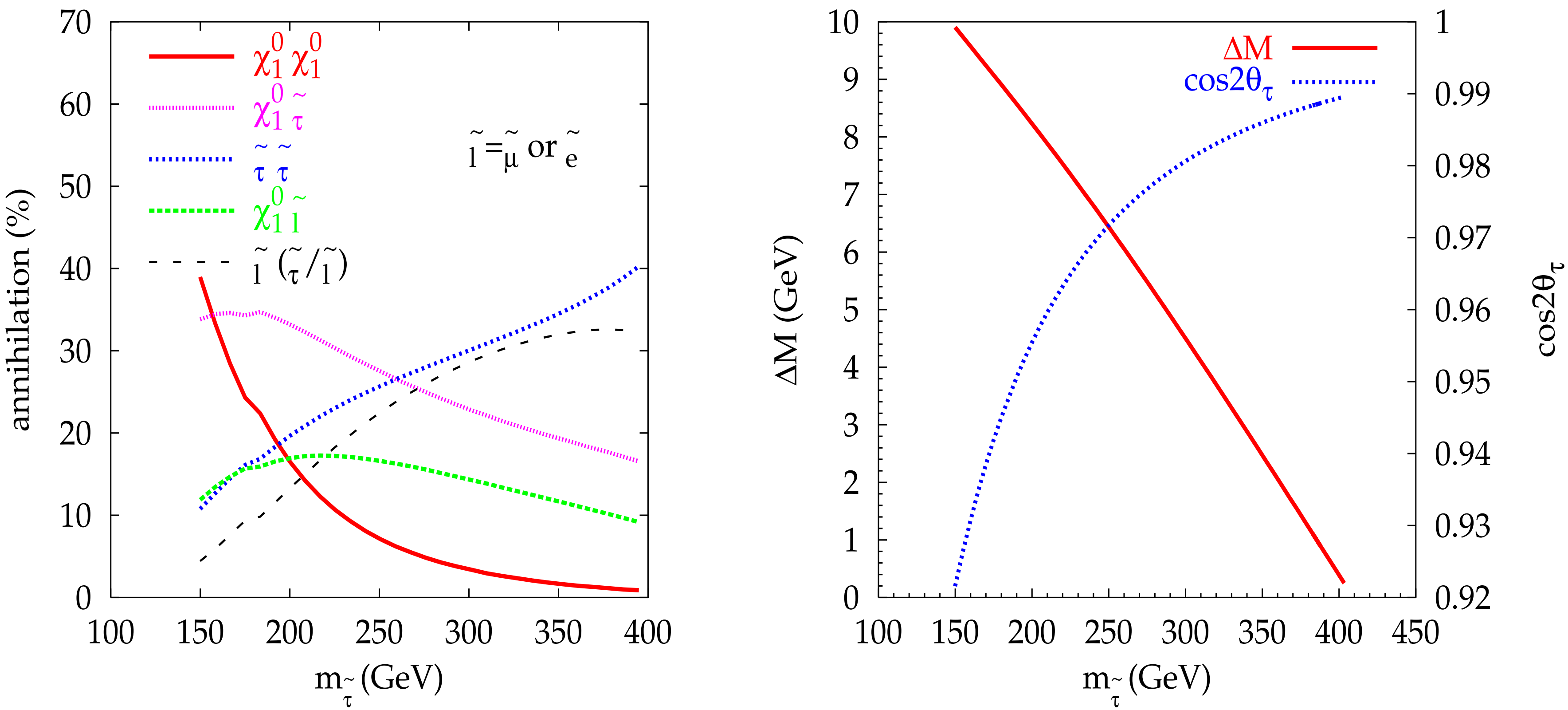


$$\mu > 0, \tan\beta = 10, A_0 = 10$$

$$m_0 = 5.84615 + 0.176374 M_{1/2} + 1.97802 \times 10^{-5} M_{1/2}^2$$

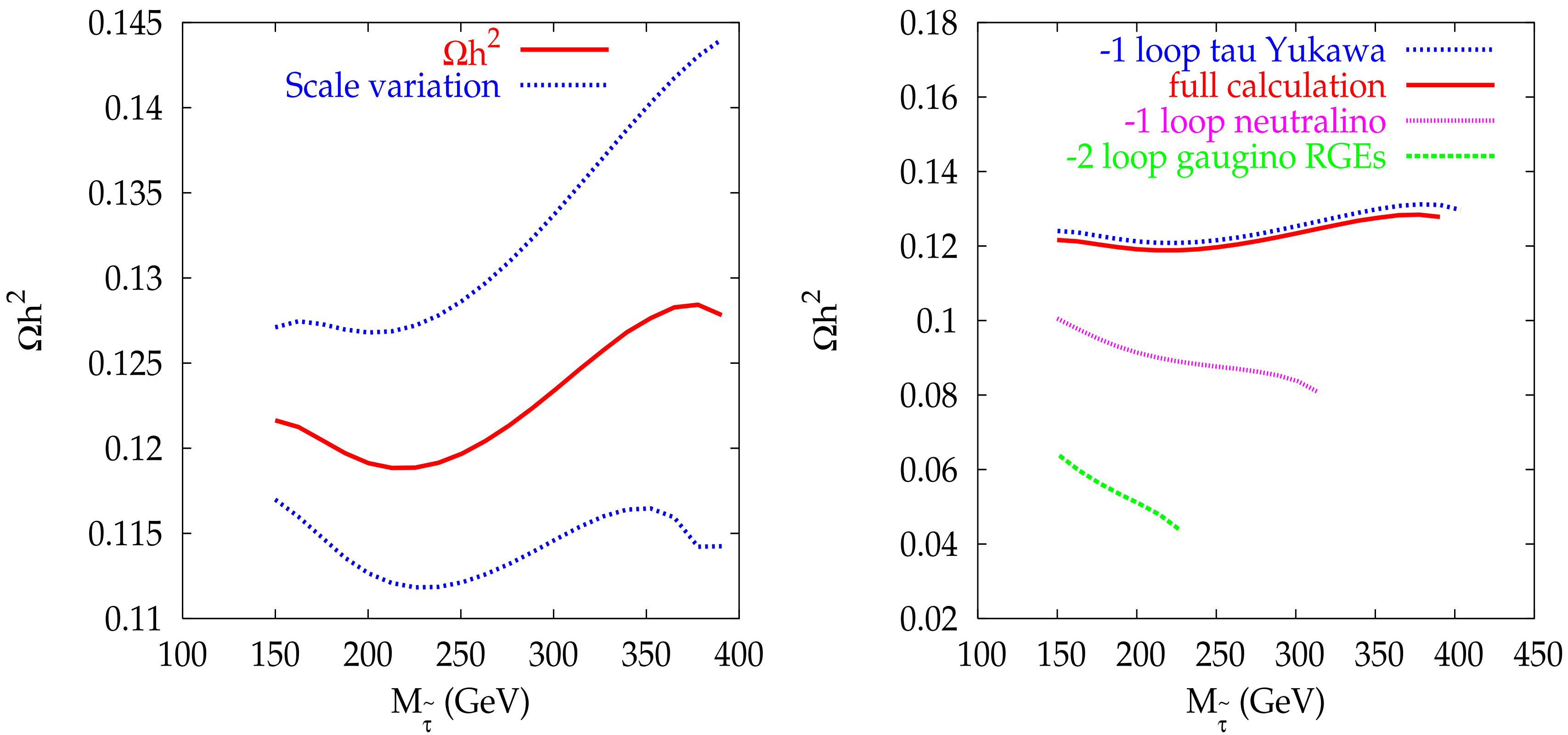
- At ILC will produce $(\tilde{\tau}_1, \tilde{\mu}_R, \tilde{e}_R)$,
- 500GeV, a window for $\tilde{e}_L$ and $\tilde{\chi}_1^0 \tilde{\chi}_2^0$.

$\tilde{\tau}_1$ co-annihilation region: The Landscape



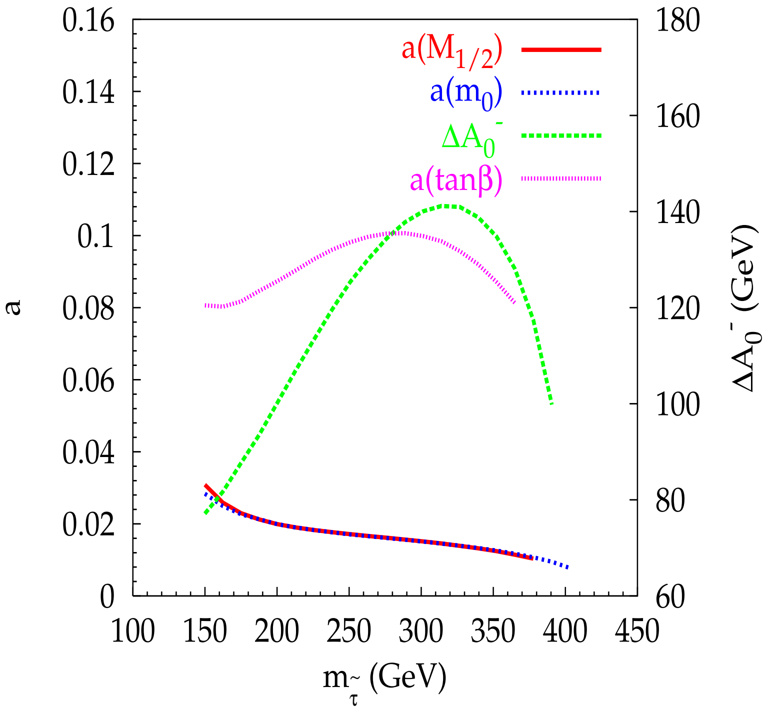
- Different co-annihilation channels are important
- $\Delta M = m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ from 10GeV to 1GeV. $\tilde{\tau}_1$ mixing angle small.
- Other relevant parameters $\mu \tan\beta$

$\tilde{\tau}_1$ co-annihilation region: Theory uncertainty



- Scale variation: From 5% (small $m_{\tilde{\tau}_1}$) to 20% large $m_{\tilde{\tau}_1}$)
- 2-loop gaugino RGE's ESSENTIAL as is 1-loop threshold correction to $\tilde{\chi}_1^0$

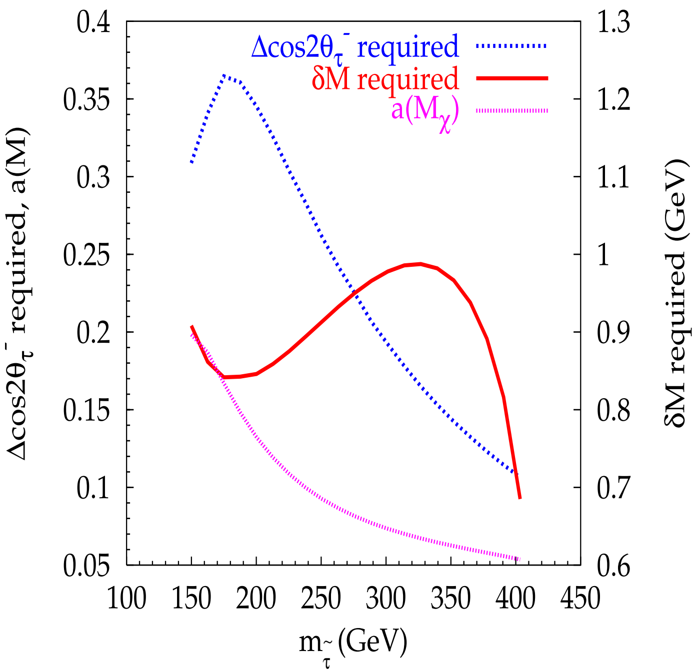
$\tilde{\tau}_1$ co-annihilation region: accuracy within mSUGRA



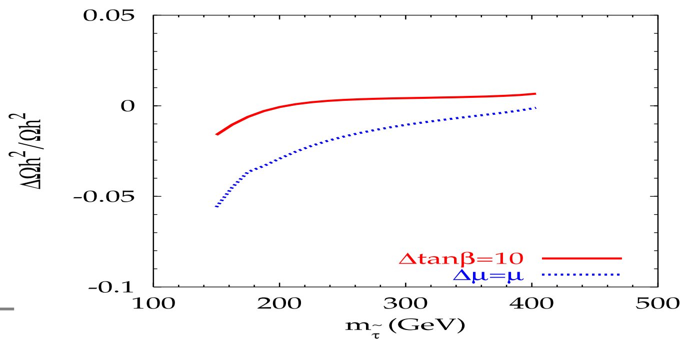
- accuracy on $m_0, M_{1/2}$ demanding: 3% : 1% may be achievable at LHC
- for tgb require 10percent.

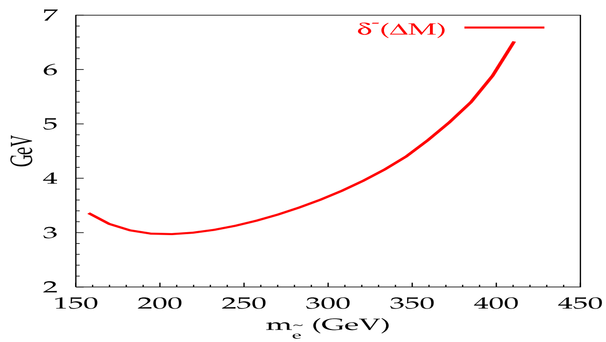
- since spectrum has decay chain similar to bulk (unless mass difference too small), LHC might match WMAP
- But more study is needed. LHC/LC obviously helps

$\tilde{\tau}_1$ co-annihilation region: Model Independent



- ΔM must be measured to less than 1GeV
- mixing angle accuracy should be feasible at ILC
- accuracy on LSP mass not demanding but this is because we have constrained ΔM.
- other slepton masses need also be measured
- in terms of physical parameters residual $\mu \tan\beta$ accuracies not demanding
- Preliminary studies indicate these accuracies will be met for the lowest $m_{\tilde{\chi}_1^0}$





Higgs Funnel: Relevant parameters

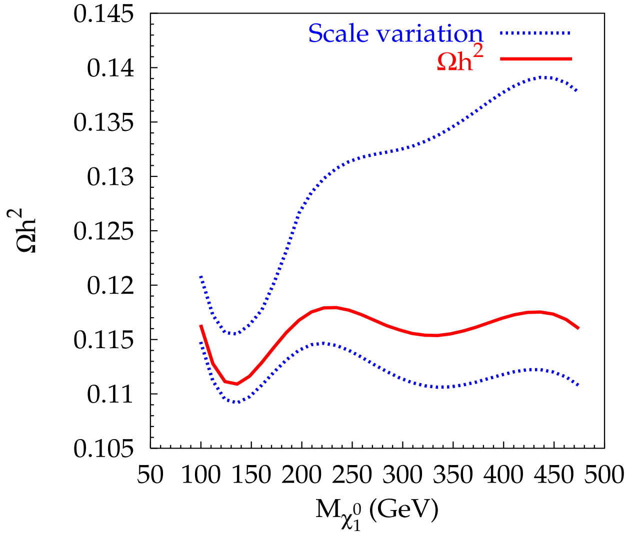
$$< \sigma v >^{-1}_{v=0} \propto \frac{\left((2m_{\tilde{\chi}_1^0})^2 - M_A^2\right)^2 + \Gamma_A^2 M_A^2}{m_{\tilde{\chi}_1^0}\ \hat{\Gamma}_A\ g^2_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 A}} \sim \frac{4 m_{\tilde{\chi}_1^0} \Gamma_A}{g^2_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 A}} \left(4 \left(\frac{M_A - 2m_{\tilde{\chi}_1^0}}{\Gamma_A} \right)^2 + 1 \right).$$

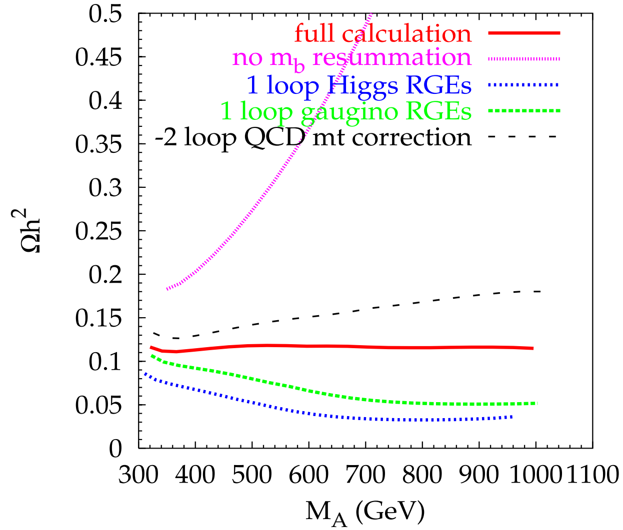
$$\tilde{g}_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 A} \propto \frac{M_Z}{M_1^2 - \mu^2} (M_1 s_{2\beta} + \mu) \sim -\frac{M_Z}{\mu} \sim -\frac{M_Z}{m_{\tilde{\chi}_3^0}},$$

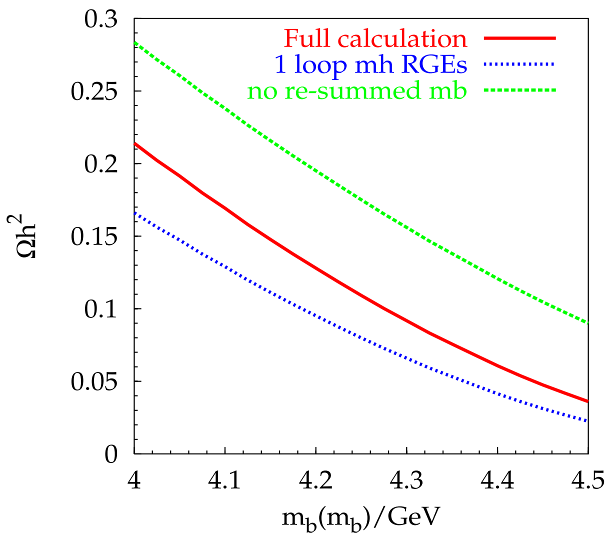
Higgs width in $b\bar{b}$ LHC+LC (?):

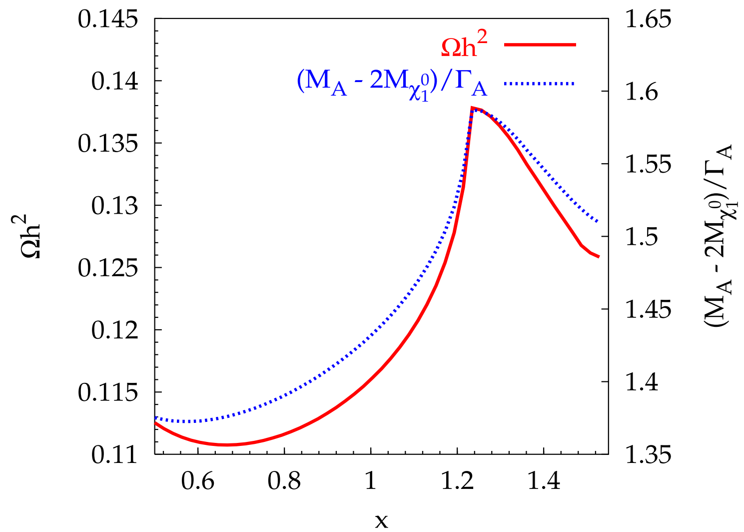
$$\begin{aligned} h_{bb} &\simeq \sin(\beta - \alpha) - \cos(\alpha - \beta) \tilde{A}_{bb}, \\ A_{bb} &= \tilde{A}_{bb} \left(1 - \frac{\Delta m_b}{\tan \beta^2} \right) \simeq \tilde{A}_{bb}; \quad \tilde{A}_{bb} = \frac{\tan \beta}{1 + \Delta m_b}. \\ m_b(M_Z)^{\overline{DR}}_{MSSM} &= m_b(M_Z)^{\overline{DR}}_{SM} / (1 + \Delta m_b) \end{aligned}$$

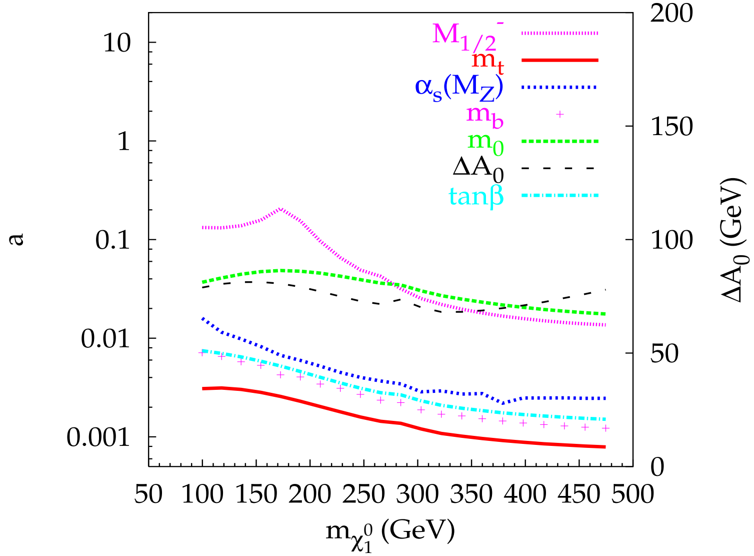
Higgs Funnel: Theoretical and Input Uncertainties

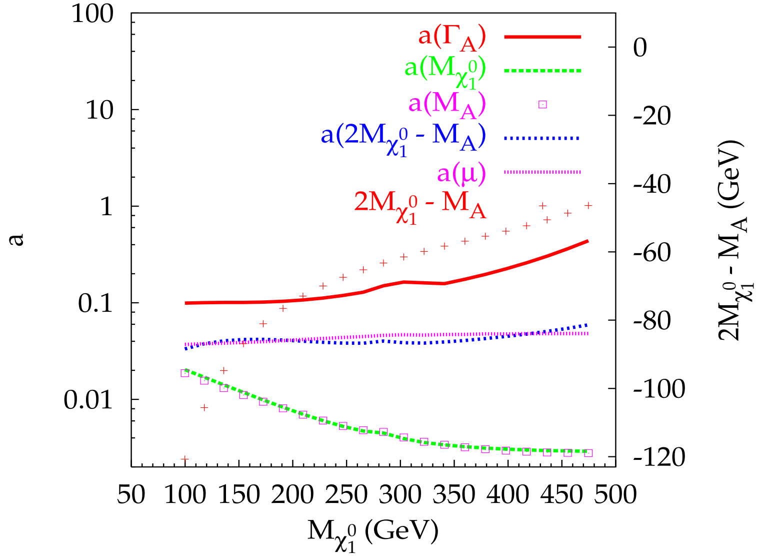












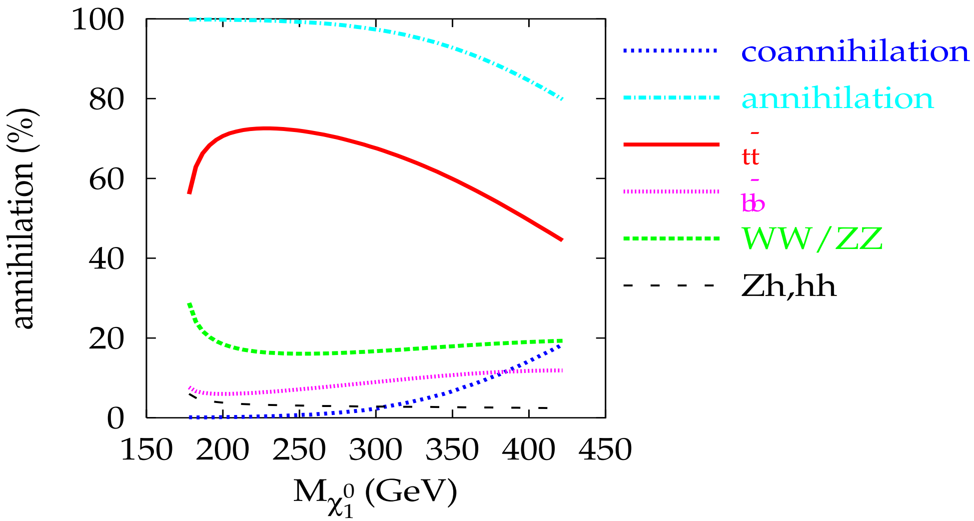
mSUGRA

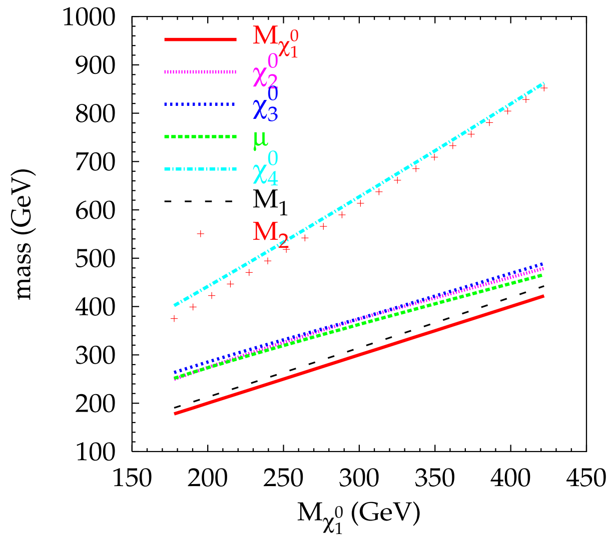
- $\Delta m_t = 0.6 : 0.2$GeV
- $\alpha_s, m_b, \tan\beta$ at permil!
- $m_0, M_{1/2} = 2 : 5\%$

Model Independent

- μ is large and need to measured at 5%
- $a(\Gamma_A) \sim 10\%$
- $a(M_A) \simeq m_{\tilde{\chi}_1^0} \sim 2 : 0.2\%$
- residual (not in width) $\tan\beta$ is small: $10\% \tan\beta \to 1\%$ relic

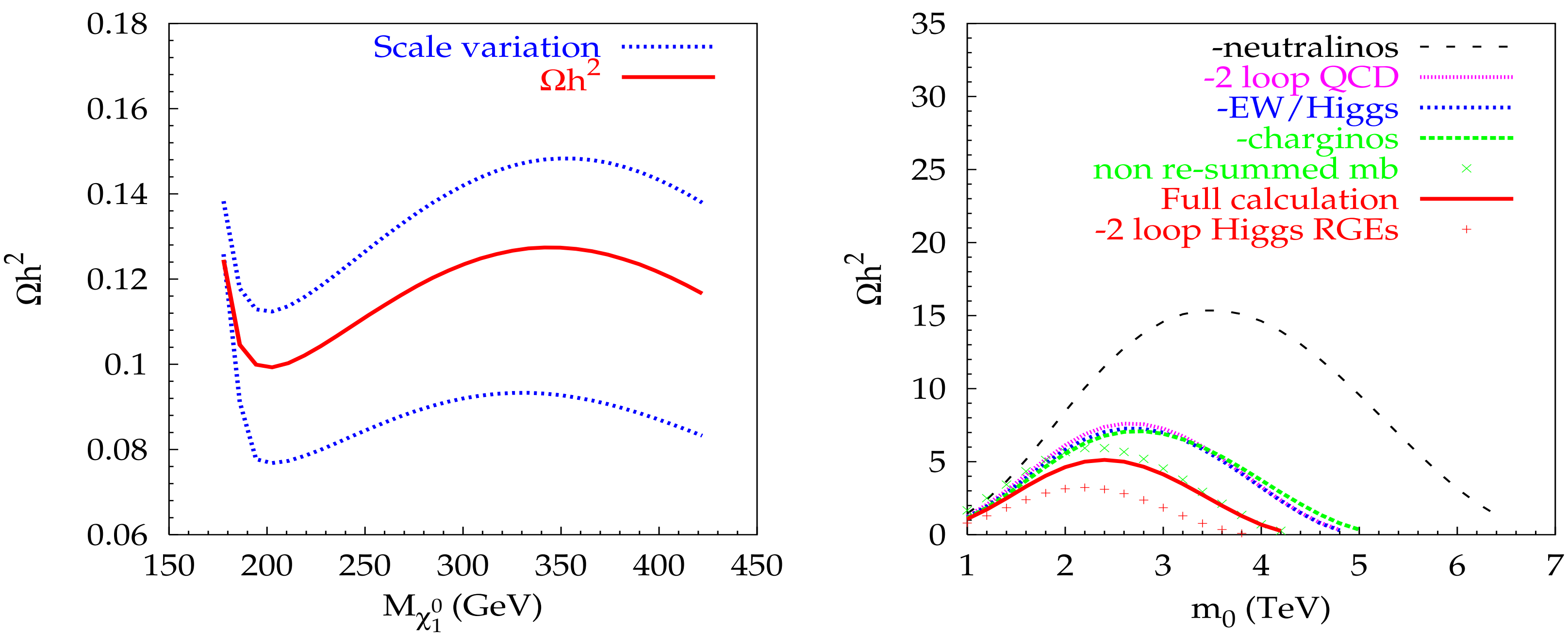
Focus: The Landscape, $m_0, m_{\tilde{f}} > 1\textbf{TeV}$, $\tan\beta = 50$





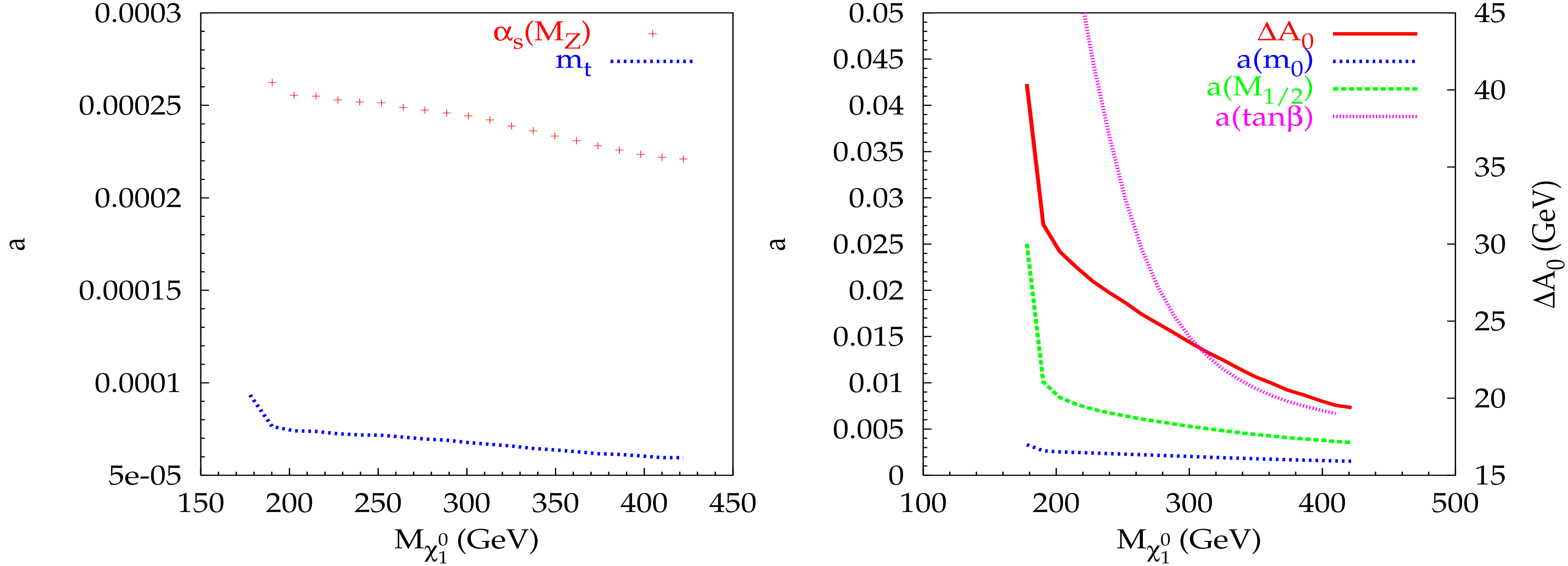
- $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to Z \to t\bar{t}$ important
- A exchange small ($M_A > 1$TeV), here it's Goldstone dominance
- Higgsino component implies WW, ZZ channel
- co-annihilation not the most important within WMAP.
- Higgsino component implies μ measurement

Focus: Uncertainties



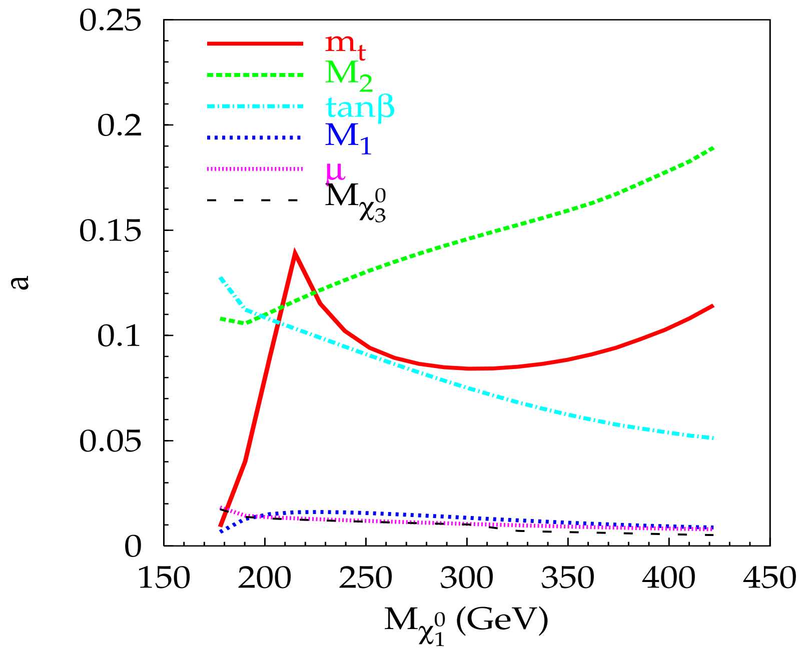
- $\tan\beta = 50$ required for SOFTSUSY, lower $\tan\beta$ imply lower top mass
- Position of the focus critically dependent on $m_t{}^{\overline{DR}}_{MSSM}(m_t)$
- in comparison scale dependence small: $20 : 30\%$

Focus: Accuracies within mSUGRA



- Most control top mass, require $\Delta m_t \sim 20$MeV
- $a(m_0) \sim 0.2\%$ IMPOSSIBLE $a(M_{1/2}) \sim 0.5\%$
- $a(\tan\beta) \sim 1 - 5\%$

Focus: Accuracies in relevant parameters M_1, μ



- Here $a(m_t) \sim 10\%$
- accuracies on $M_1(m_{\tilde{\chi}_1^0}), \mu(m_{\tilde{\chi}_2^0}) \sim 1\%$

Summary

- Within mSUGRA accuracies on $(m_0, M_{1/2}) \sim 1\%$ sometimes less, given enough energy LHC+LC should do the job
- Within mSUGRA need to control RGE codes
- some focus scenarios are too tough may not be able to match WMAP and PLANCK
- model independent analysis in mSUGRA inspired though gives the edge to LC plus some help from LHC
- Need to extend studies to less *accidental* scenarios (non gaugino unification)
- what about a study with non standard cosmology?