

Trilinear and Quartic Gauge Boson Couplings



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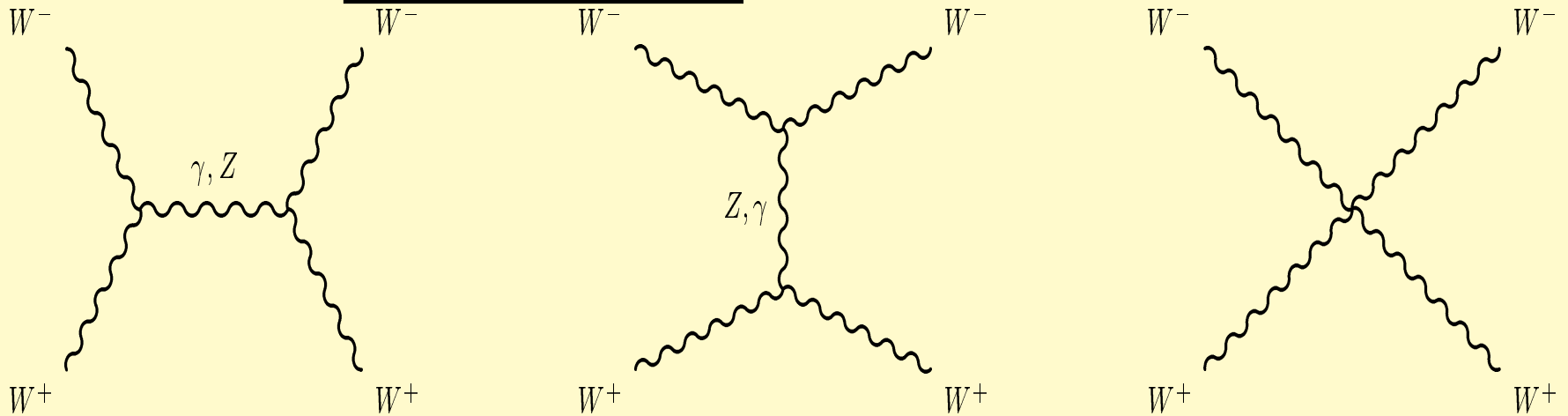
LAPTH-Annecy, France

OUTLINE

- Vector bosons self-couplings and the symmetry breaking connection
- Gauge Invariance and the Hierarchy of couplings: Higgs vs Higgsless description
- Present indirect limits as a yardstick for a meaningful future measurement/constraint
- Important channels. Outlook. Conclusions

Self-couplings: the Higgs and Symmetry Breaking Connection

$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ *Without Higgs*



If $g_{VVVV} \neq g_{VVV}^2 \implies \mathcal{M}_{LLLL} \propto E_W^4$

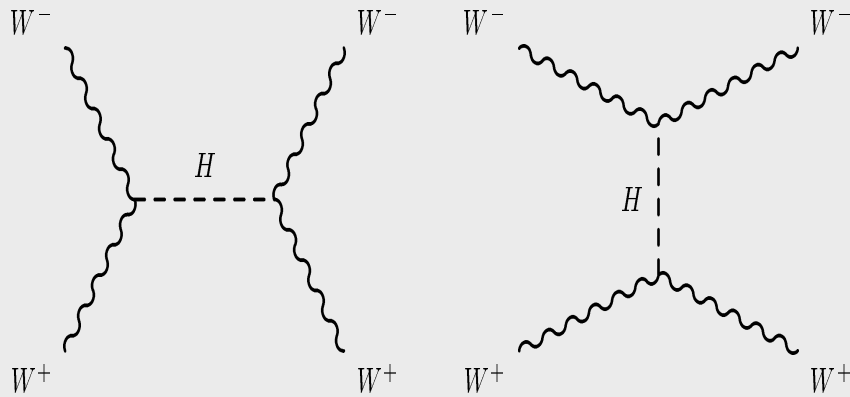
In the SM $\mathcal{M}_{LLLL} \sim \sqrt{2}G_F u \propto E_W^2$

Unitarity without Higgs requires $\sqrt{s_{WW}} \leq 1.2\text{TeV}$

Slight departure of the vector bosons self-couplings from SM values is enhanced at high energies

Higgs and Delayed Unitarity

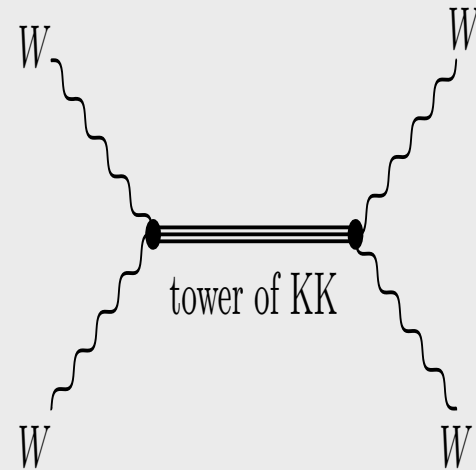
Higgs in SM



$$\mathcal{M}_{LLLL} \sim -\sqrt{2}G_F M_H^2 \left(\frac{s}{s-M_H^2} + \frac{t}{t-M_H^2} \right)$$

Unitarity implies $M_H \leq \frac{4\pi\sqrt{2}}{3G_F} \sim 700\text{GeV}$

Extra-dim with special boundary cnds



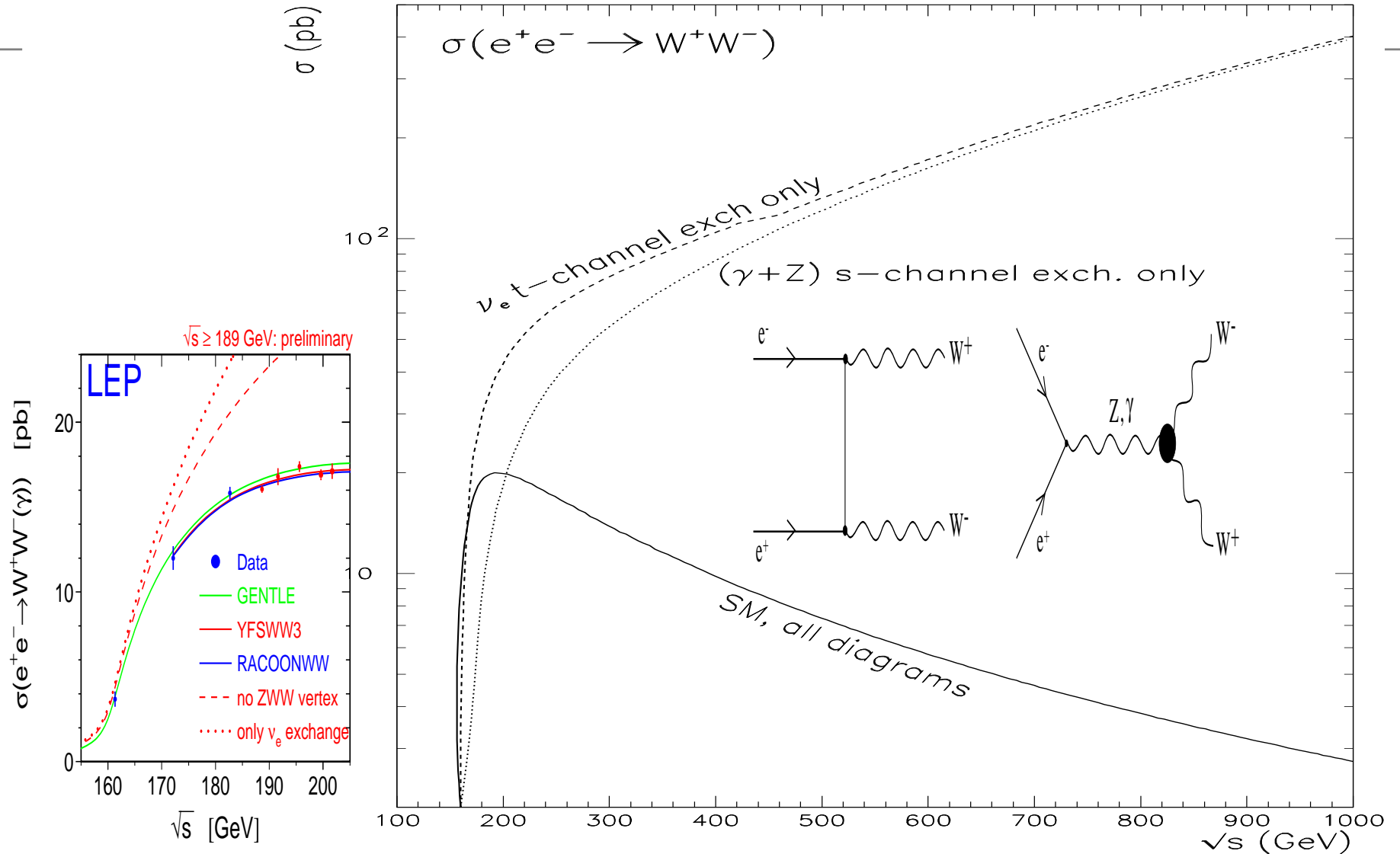
Unitarity is delayed up to a few TeV

cancellation from underlying

5-d gauge symmetry.

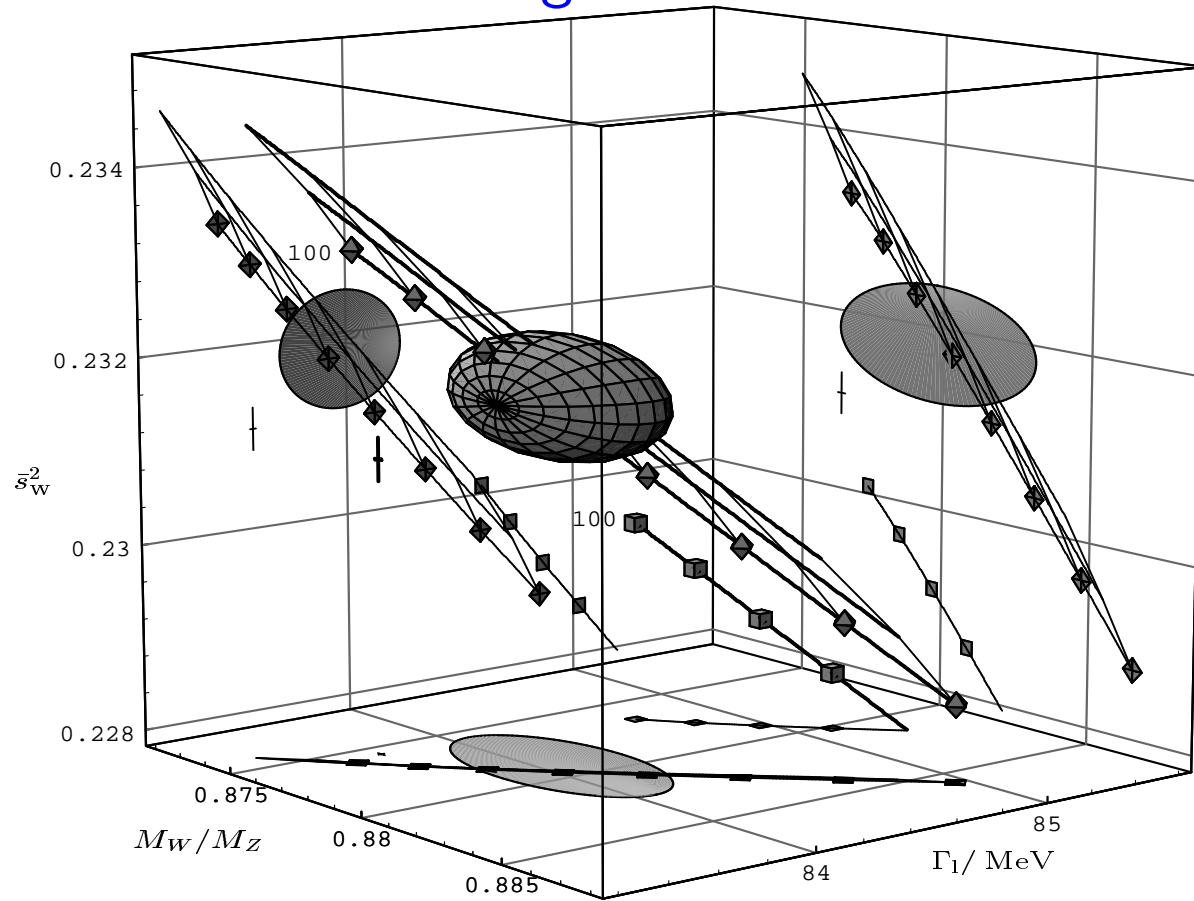
- expect some collective modes to effectively affect the self-interaction of the gauge bosons
- watch out for the longitudinal modes

Gauge Invariance: $g_{ffV} = g_{VVV}$



- LEP legacy: We know that WWV can not deviate too much (10%) from SM gauge value.
- But slightest deviations are revealed at higher energies (LHC?)

Gauge Invariance: Indirect Limits



Dittmaier, Schildknecht and Weiglein 1996.

Origin of VVVV and VVV: Gauge Invariance, Symmetry Breaking. I. SM

- $\mathcal{L}_{\text{Gauge}} = -\frac{1}{2} [\text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}) + \text{Tr}(\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu})]$ GI kinetic term

$$\mathbf{W}_{\mu\nu} = \frac{1}{2} \left(\partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + \frac{i}{2} g[\mathbf{W}_\mu, \mathbf{W}_\nu] \right) = \frac{\tau^i}{2} \left(\partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g \epsilon^{ijk} W_\mu^j W_\nu^k \right)$$

$$\mathbf{B}_{\mu\nu} = \frac{1}{2} (\partial_\mu B_\nu - \partial_\nu B_\mu) \tau_3 \quad \mathbf{B}_\mu = \tau_3 B_\mu$$

- $\mathcal{L}_{\text{Gauge}}$ describes **transverse states** (field strength)

- **longitudinal states**, $Z_\mu^L \propto \partial_\mu \phi_3$ $\epsilon_\mu^L = \frac{k_\mu}{M_Z} - M_Z \frac{s_\mu}{s.k}$ do not contribute much to $\mathcal{L}_{\text{Gauge}}$

- BUT to the mass and Symmetry Breaking Lagrangian

- $\mathcal{L}_M = M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \subset \mathcal{L}_{H,M=SB} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) - \lambda \left[\Phi^\dagger \Phi - \frac{\mu^2}{2\lambda} \right]^2$

- Self interacting Goldstone Bosons \equiv Self-interacting $V_L V_L \rightarrow V_L V_L$

- Symmetry Breaking has a custodial $SU(2)$ symmetry $\rightarrow \rho = 1$

Gauge invariance of Mass and SB: Covariant Derivatives

Origin of VVVV and VVV: Gauge Invariance, Symmetry Breaking. II. SM without Higgs

kinetic term still there but mass and longitudinals through a system of Goldstones without the Higgs (still gauge invariant): Non-Linear realisation of SB

$$\begin{aligned}\Sigma &= \exp\left(\frac{i\omega^i\tau^i}{v}\right) \quad (v = 246 \text{ GeV is the vev}) \quad \text{and} \quad \mathcal{D}_\mu\Sigma = \partial_\mu\Sigma + \frac{i}{2} (g\mathbf{W}_\mu\Sigma - g'B_\mu\Sigma\tau_3) \\ \mathcal{L}_M &= \frac{v^2}{4}\text{Tr}(\mathcal{D}^\mu\Sigma^\dagger\mathcal{D}_\mu\Sigma) \equiv -\frac{v^2}{4}\text{Tr}(\mathcal{V}_\mu\mathcal{V}^\mu) \quad \text{with} \quad \mathcal{V}_\mu = (\mathcal{D}_\mu\Sigma)\Sigma^\dagger\end{aligned}$$

Hierarchy of Operators: Most important effects

- POST-LEP 1&2: The SM is a gauge invariant (GI) theory with a custodial SU(2) symm.
- as in QED any deviation should be parameterized as a (higher order) GI operators
- This approach defines a hierarchy of couplings and order of magnitude for deviations

QED example

$$\begin{aligned}\mathcal{L}_{eff.}^{QED} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\beta_1}{m^2} \frac{e^2}{16\pi^2} \left(F_{\mu\nu} \square F^{\mu\nu} + \frac{\epsilon_2}{m^2} F_{\mu\nu} \square^2 F^{\mu\nu} \right) \\ &+ \frac{1}{m^4} \frac{e^4}{16\pi^2} \left(\beta_2 (F_{\mu\nu} F^{\mu\nu})^2 + \beta_3 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right) + \dots + \mathcal{L}_{gauge\ fixing}\end{aligned}$$

SM need to describe new physics of longitudinal modes

decide about Linear (with Higgs) or Non-Linear (without Higgs) of Symm. Breaking

Operators for W Physics

Linear Realisation , Light Higgs	Non Linear-Realisation , No Higgs
$\mathcal{L}_B = ig' \frac{\epsilon_B}{\Lambda^2} (\mathcal{D}_\mu \Phi)^\dagger B^{\mu\nu} \mathcal{D}_\nu \Phi$	$\mathcal{L}_{9R} = -ig' \frac{L_{9R}}{16\pi^2} \text{Tr}(\mathbf{B}^{\mu\nu} \mathcal{D}_\mu \Sigma^\dagger \mathcal{D}_\nu \Sigma)$
$\mathcal{L}_W = ig \frac{\epsilon_w}{\Lambda^2} (\mathcal{D}_\mu \Phi)^\dagger (2 \times \mathbf{W}^{\mu\nu}) (\mathcal{D}_\nu \Phi)$	$\mathcal{L}_{9L} = -ig \frac{L_{9L}}{16\pi^2} \text{Tr}(\mathbf{W}^{\mu\nu} \mathcal{D}_\mu \Sigma \mathcal{D}_\nu \Sigma^\dagger)$
$\mathcal{L}_\lambda = \frac{2i}{3} \frac{L_\lambda}{\Lambda^2} g^3 \text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{W}^{\nu\rho} \mathbf{W}^\mu{}_\rho)$ <p>-----</p> <p>-----</p>	<p>-----</p> $\mathcal{L}_1 = \frac{L_1}{16\pi^2} (\text{Tr}(D^\mu \Sigma^\dagger D_\mu \Sigma))^2 \equiv \frac{L_1}{16\pi^2} \mathcal{O}_1$ $\mathcal{L}_2 = \frac{L_2}{16\pi^2} (\text{Tr}(D^\mu \Sigma^\dagger D_\nu \Sigma))^2 \equiv \frac{L_2}{16\pi^2} \mathcal{O}_2$

$W^+W^-Z, W^+W^-\gamma$ couplings

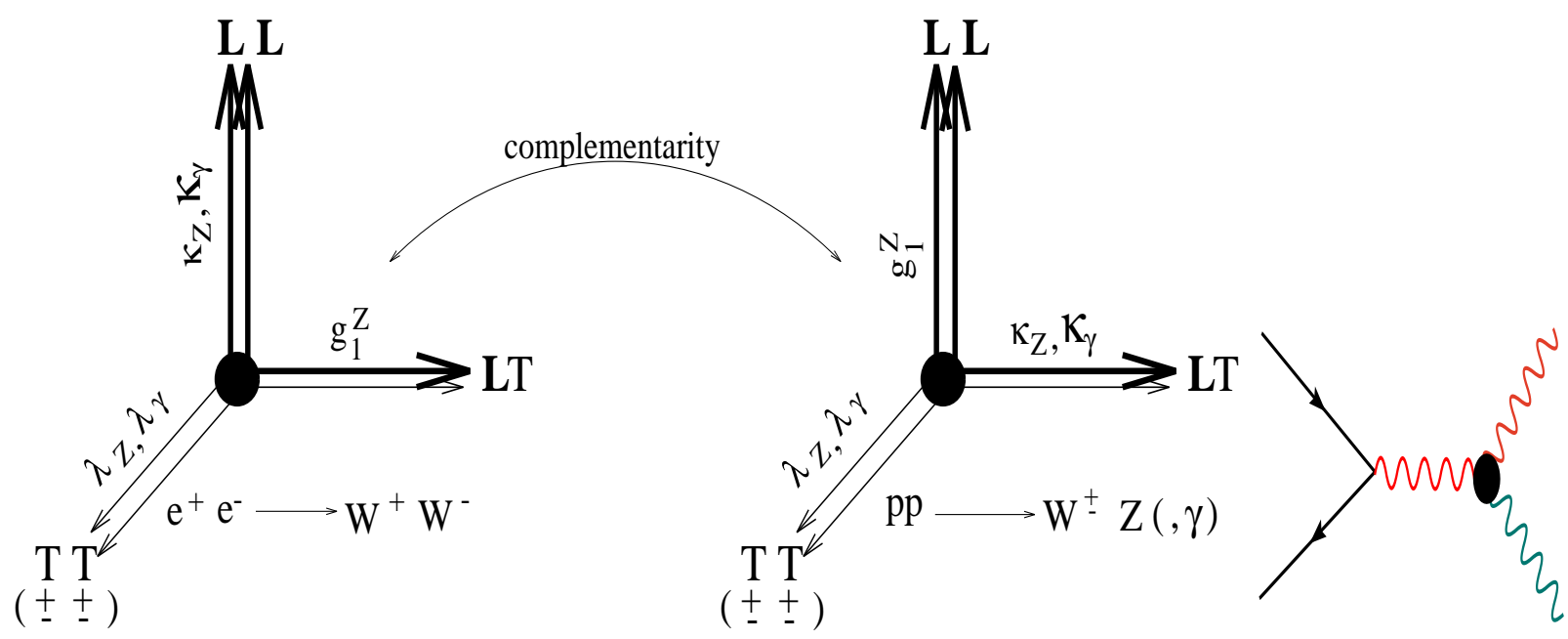
$$\begin{aligned}\Delta\kappa_\gamma &= \frac{e^2}{s_w^2} \frac{v^2}{4\Lambda^2} (\epsilon_W + \epsilon_B) = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} (L_{9L} + L_{9R}) \\ \Delta\kappa_Z &= \frac{e^2}{s_w^2} \frac{v^2}{4\Lambda^2} (\epsilon_W - \frac{s_w^2}{c_w^2} \epsilon_B) = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} \left(L_{9L} - \frac{s_w^2}{c_w^2} L_{9R} \right) \\ \Delta g_1^Z &= \frac{e^2}{s_w^2} \frac{v^2}{4\Lambda^2} \left(\frac{\epsilon_W}{c_w^2} \right) = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} \left(\frac{L_{9L}}{c_w^2} \right) \\ \lambda_\gamma &= \lambda_Z = \left(\frac{e^2}{s_w^2} \right) L_\lambda \frac{M_W^2}{\Lambda^2}\end{aligned}$$

- At this order no \mathcal{C} violating tri-linear couplings in particular no $ZZZ, ZZ\gamma, Z\gamma\gamma$ couplings. These appear at higher orders. Should not receive high priority.
- For LHC fits and measurements better to switch to the L_i parameters

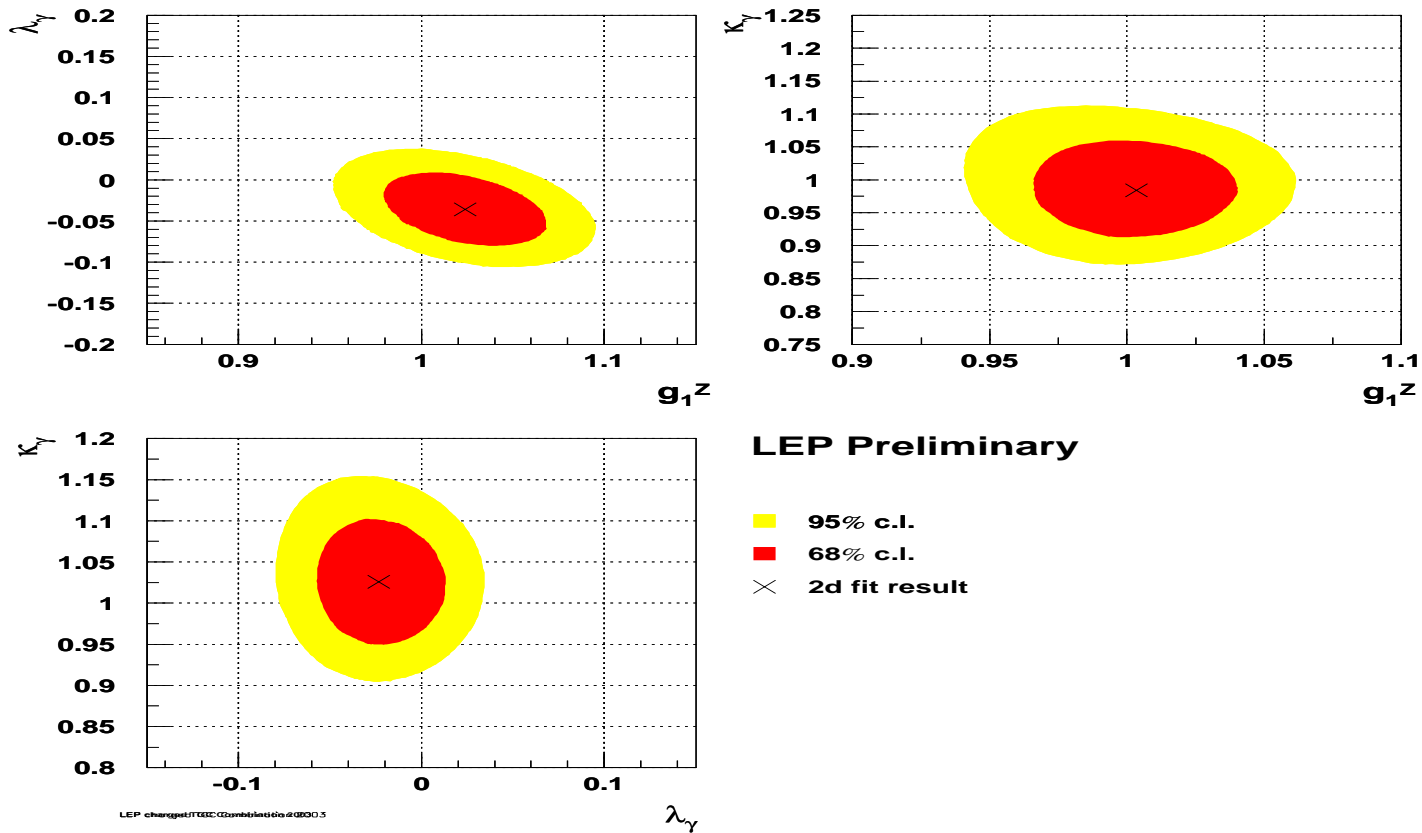
Pheno of tri-linear couplings

$$\begin{aligned}
 \mathcal{L}_{WWV} = & -ie \left\{ \left[A_\mu \left(W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^- \right) + \overbrace{(1 + \Delta\kappa_\gamma)}^{\kappa_\gamma} F_{\mu\nu} W^{+\mu} W^{-\nu} \right] \right. \\
 & + \frac{c_W}{s_W} \left[\overbrace{(1 + \Delta g_1^Z)}^{g_1^Z} Z_\mu \left(W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^- \right) + \overbrace{(1 + \Delta\kappa_Z)}^{\kappa_Z} Z_{\mu\nu} W^{+\mu} W^{-\nu} \right] \\
 & \left. + \frac{1}{M_W^2} \left(\lambda_\gamma F^{\nu\lambda} + \lambda_Z \frac{c_W}{s_W} Z^{\nu\lambda} \right) W_{\lambda\mu}^+ W^{-\mu}_\nu \right\}
 \end{aligned}$$

No ZZZ, ZZγ, Zγγ (higher order)



Current direct limits as a guide



5 – 10% accuracy $\Rightarrow |L_9| \sim 50$

$\mathcal{L}_{WWV_1V_2}$

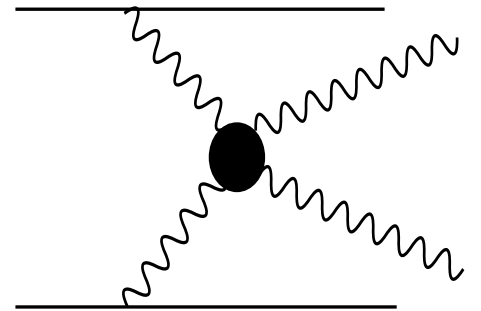
$$\begin{aligned}
 & - e^2 \left\{ (A_\mu A^\mu W_\nu^+ W^{-\nu} - A^\mu A^\nu W_\mu^+ W_\nu^-) \right. \\
 & + 2 \frac{c_w}{s_w} \left(1 + \frac{l_{9l}}{c_w^2}\right) \left(A_\mu Z^\mu W_\nu^+ W^{-\nu} - \frac{1}{2} A^\mu Z^\nu (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) \right) \\
 & + \frac{c_w^2}{s_w^2} \left(1 + \frac{2l_{9l}}{c_w^2} - \frac{l_-}{c_w^4}\right) (Z_\mu Z^\mu W_\nu^+ W^{-\nu} - Z^\mu Z^\nu W_\mu^+ W_\nu^-) \\
 & + \frac{1}{2s_w^2} (1 + 2l_{9l} - l_-) (W^{+\mu} W_\mu^- W^{+\nu} W_\nu^- - W^{+\mu} W_\mu^+ W^{-\nu} W_\nu^-) \\
 & - \frac{l_+}{2s_w^2} ((3W^{+\mu} W_\mu^- W^{+\nu} W_\nu^- + W^{+\mu} W_\mu^+ W^{-\nu} W_\nu^-) \\
 & + \left. \frac{2}{c_w^2} (Z_\mu Z^\mu W_\nu^+ W^{-\nu} + Z^\mu Z^\nu W_\mu^+ W_\nu^-) + \frac{1}{c_w^4} Z_\mu Z^\mu Z_\nu Z^\nu \right) \left. \right\} \\
 & \text{with } l_{9l} = \frac{e^2}{32\pi^2 s_w^2} L_{9L} \quad ; \quad l_{\pm} = \frac{e^2}{32\pi^2 s_w^2} (L_1 \pm L_2)
 \end{aligned}$$

No Anomalous $WW\gamma\gamma$, No $V_1V_2\gamma\gamma$

New $ZZZZ$, New $WWWW$ and $WWZZ$ structures.

Best probed in WW scattering ($L_{1,2}$) $\implies \implies$

(higher order structures see Bélanger et al.,)



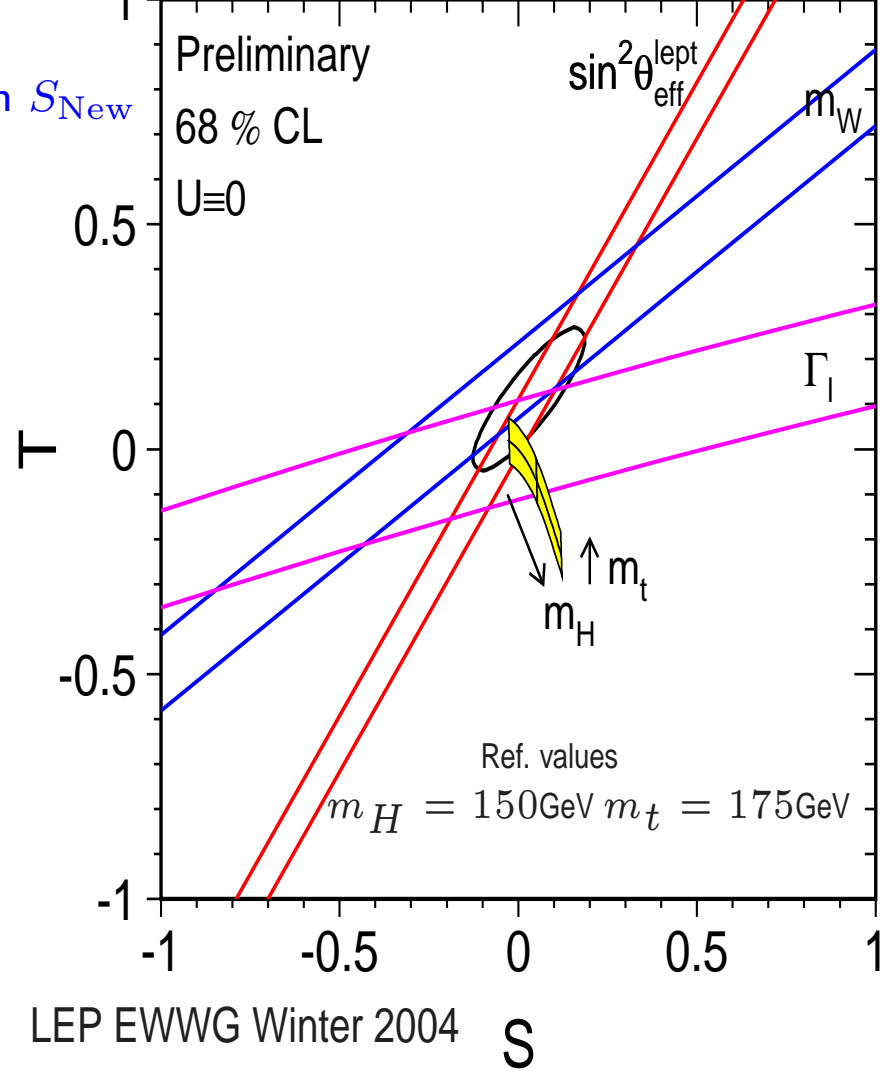
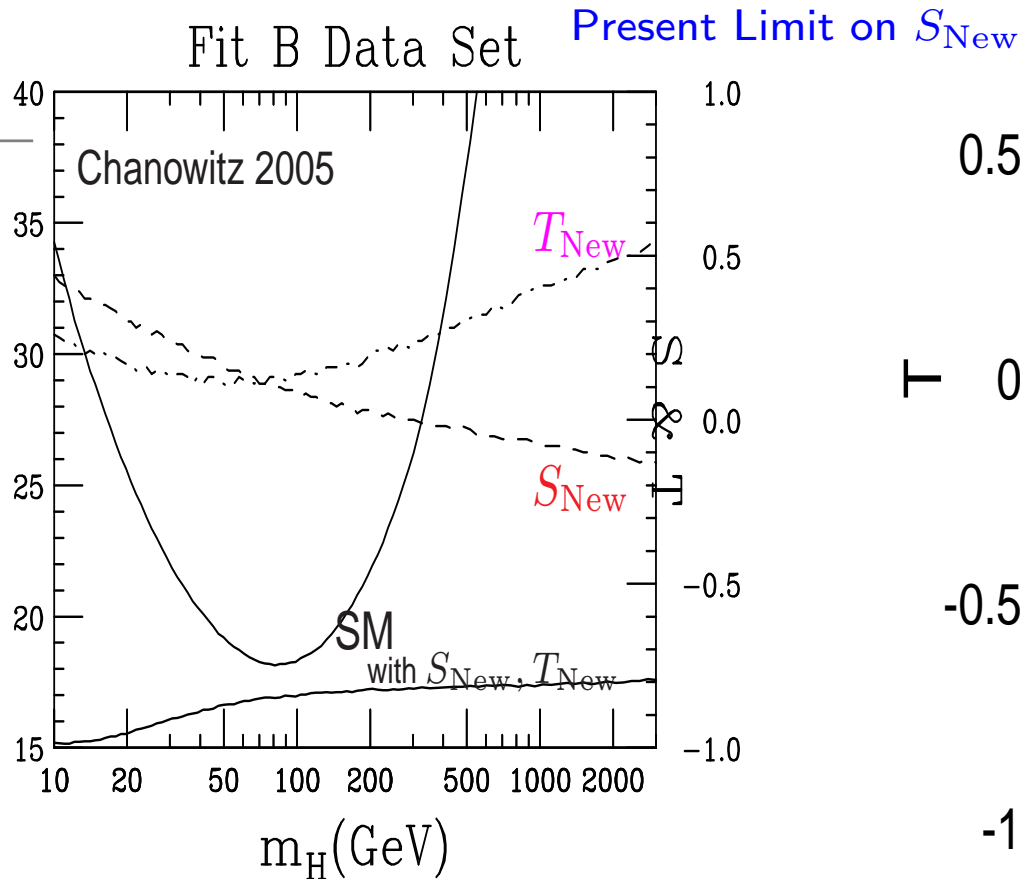
Catch 22: Contribution to 2-point functions?.....!

- ◇ I only listed operators giving contribution to VVV and $VVVV$ but...
- ◇ A general approach would also include contributions to 2-point function (see QED example)

$$\mathcal{L}_{WB} = gg' \frac{\epsilon_{WB}}{\Lambda^2} (\Phi^\dagger \times \mathbf{W}^{\mu\nu} \Phi) B_{\mu\nu}$$

$$\mathcal{L}_{10} = gg' \frac{L_{10}}{16\pi^2} \text{Tr}(\mathbf{B}^{\mu\nu} \Sigma^\dagger \mathbf{W}^{\mu\nu} \Sigma) \longrightarrow L_{10} = -\pi S_{\text{New}} \simeq \frac{4\pi s_W}{\alpha} \epsilon_3$$

- ◇ LEP-Tevatron precision EW physics implies strong constraint on $S_{\text{New}}(L_{10})$ and consequently on the other operators
- ◇ This constraint could be considered as a yardstick for future measurements



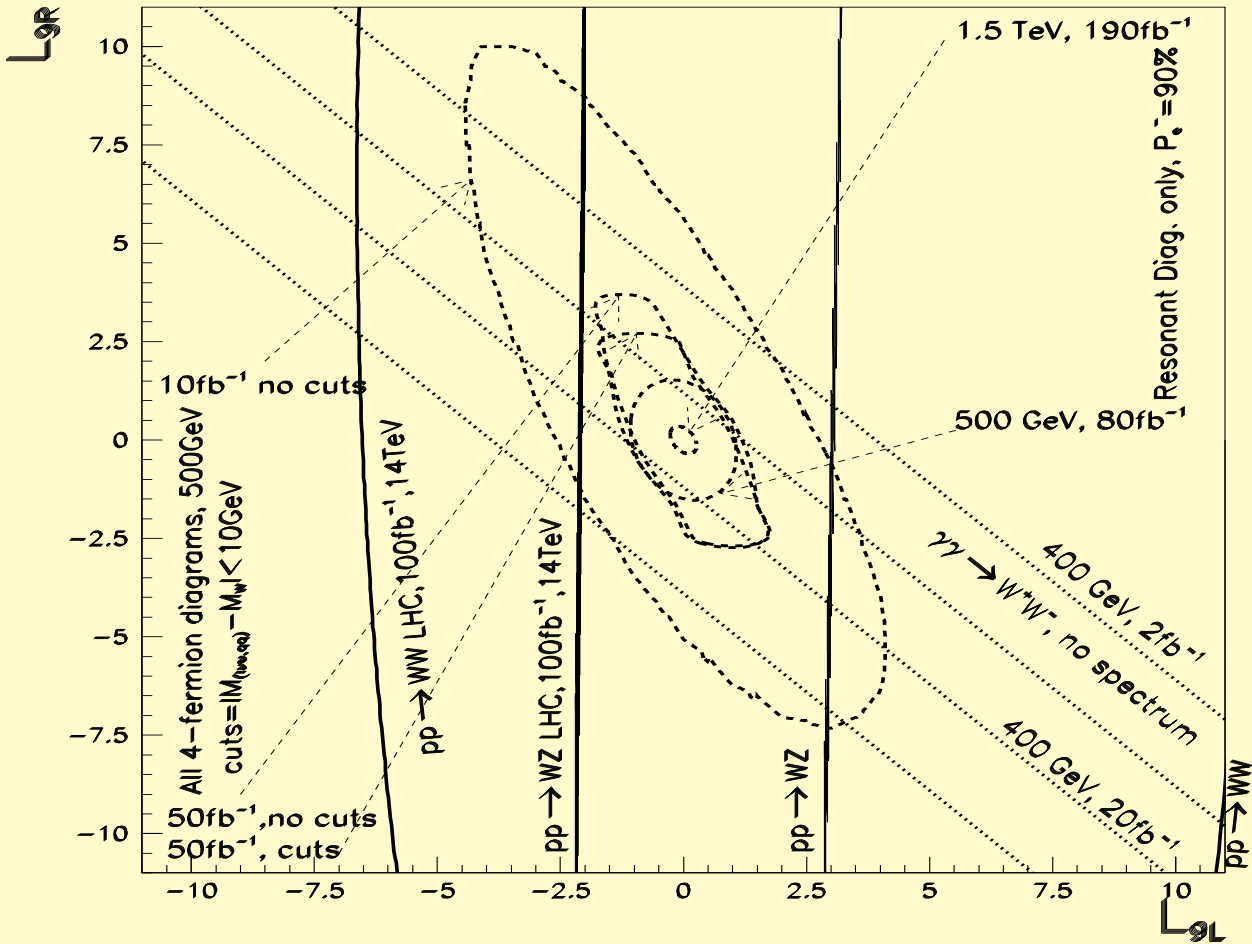
For $M_H = 1\text{TeV}$ need $S_{New} = -0.07$ and $T_{New} = +0.3 \implies$ **Constraint $L_{10} < 1$**

future L_i measurements of this order! $L_i \sim 1$

unless some global symmetry makes $L_{10} = 0$ ($SU(2)_A$?)

Future measurements I.

The S, T, U Post-LEP



From WW scattering $L_{1,2} \sim 1$ @ LHC

Future measurements and Conclusions.

- Especially if Higgs not found precision measurements of (longitudinal) vector bosons self-couplings is crucial
- Update simulations in the channels $pp \rightarrow WZ, W\gamma(WW?)$ including NLO (MCFM)
- $pp \rightarrow (W, Z)X$ (VV fusion to single V): worth it?
- Can more be done to optimize $WW \rightarrow WW$?
- Quartic through $pp \rightarrow WWZ, ZZZ, .$ though falls like $1/\hat{s}$ could it help? combined with WW fusion?
- work out in detail consequences of some extra-dim models, (Little Higgs?)...

...could constitute a nice subgroup at Les Houches 2005....

Official deadline for registration **TOMORROW**

<http://lappweb.in2p3.fr/conferences/LesHouches/Houches2005/application.html>