

The Standard Model of Particle Physics: An Introduction to the Theory

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1 The Gauge symmetry principle

The central result of 20th century particle physics is the successful description of electromagnetic, weak and strong interactions as gauge field theories, resulting from an underlying symmetry called gauge invariance. Postulated some thirty years ago for the weak [1, 2, 3, 4] and strong interactions[5], the perturbative predictions of this “Standard Model” (SM) of particle physics have been tested and confirmed with amazing accuracy at LEP.

1.1 Electromagnetism as a prototype

One manifestation of the gauge invariance principle is the fact that in electrostatics the electric field, and hence the electrostatic force, depends only on the *difference* of potential. The gauge field idea is best known in the quantised version of Maxwell's equations. The quantum field creating and annihilating photons is the vector potential $A^\mu(x)$, which is related to the electromagnetic field strength by $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. The field strength, and thereby Maxwell's equations, are invariant under gauge transformations,

$$A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Lambda(x). \quad (1)$$

Gauge invariance is also familiar from the quantum mechanics description of a charged particle interacting with an electromagnetic field. Schrodinger's equation for a free particle of mass m , $(1/2m)(-i\vec{\nabla})^2\psi = i\partial\psi/\partial t$, is invariant under a *global* phase transformation $\psi \rightarrow \exp(i\lambda)\psi$. If one insists that the equation remains valid under *local* phase transformation, in other words that the transformation can be different at different points in space-time, $\lambda \rightarrow q\Lambda(x = (t, \vec{x}))$, one is then led to introduce a *compensating* vector field which transforms exactly like Eq. 1. This prescription gives the familiar Schrodinger's equation of a particle of charge q with the electromagnetic field $A^\mu = (V, \vec{A})$,

$$(1/2m) \left(-i\vec{\nabla} + q\vec{A}\right)^2 \psi = (i\partial/\partial t + qV) \psi. \quad (2)$$

This principle is carried over to the relativistic quantum case by requiring that all derivatives ∂_μ be replaced by *covariant derivatives* $D_\mu = \partial_\mu - iqA_\mu$, in analogy with Eq. 2 which clearly displays the combination of the space-time derivatives and the vector potential components. An important consequence of this, is that all charged particles couple exactly the same way to the electromagnetic field, the *coupling is universal*. The essential point is that charged fields and covariant derivatives of charged fields have identical local transformations,

$$\psi(x) \rightarrow U(x)\psi(x), \quad D_\mu\psi(x) \rightarrow U(x)D_\mu\psi(x), \quad U(x) = e^{iq\Lambda(x)} \quad (3)$$

The group of transformations, $U(x)$, in Eq. 3, is an Abelian $U(1)$ group since it does not matter in which order we apply successive transformations of the form $U(x)$.

Electromagnetism is a long range force where the messenger of the force, the photon is massless. As is known also, electromagnetic fields have only two transverse independent polarisations (one also speaks of the photon as having only 2 helicity states), despite the fact that the photon is a spin-1 described by a vector A_μ . In fact all these properties, the derivation of Maxwell's equations that *unify* the electric and magnetic fields are embodied in the simple Lagrangian describing (free) photons

$$\mathcal{L}_{\text{em}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \equiv \frac{1}{4} \left((\vec{E} + i\vec{B})^2 + (\vec{E} - i\vec{B})^2 \right). \quad (4)$$

$(\vec{E} \pm i\vec{B})$ displays the two helicity states of the photon, or polarisation of the field. A mass term for the photon would be represented by $m^2 A_\mu A^\mu$ which breaks gauge invariance as it is not invariant under Eq. 1. QED which is the gauge theory describing the interaction of electrons with the photons has been tremendously successful.

1.2 Weak interactions and non-Abelian gauge theories

Considering how elegant and powerful the gauge invariance principle is and how it dictates the form of the interaction, it was natural to extend it to the weak interactions. So much so, since processes as different as β -decay ($n \rightarrow p + e^- + \bar{\nu}_e$), muon decay ($\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$) or muon capture ($\mu^- + p \rightarrow n + \nu_\mu$) seemed to be of the same nature and have the same strength, pointing to universality. There are however some important differences with QED.

These processes are charged current processes representing $e^- \leftrightarrow \nu_e$ or $n \leftrightarrow p$ transitions and thus contrary to QED, they involve a change in the identity of the matter, spin-1/2 field. Moreover, the structure of the weak current suggested that only left-handed fields are involved. Chirality (left-handed and right-handed states) of a fermion corresponds, at high-energy, to the helicity state of the fermion. An electron field can be decomposed then as $e = e_L + e_R$. Electromagnetism treats these two components on an equal footing, preserving symmetry under parity. Indeed the gauge coupling of electrons to photons writes $\bar{e}\gamma^\mu A_\mu e = \bar{e}_L\gamma^\mu A_\mu e_L + \bar{e}_R\gamma^\mu A_\mu e_R$. γ^μ are a set of matrices which may be thought of as a relativistic generalisation of the Pauli spin matrices. Note in passing that the gauge interaction does not mix these two states. In contrast, the charged current to which one would like to associate a charged gauge boson, W^\pm , is of the form, $\bar{e}_L\gamma^\mu W_\mu^+ \nu_{e(L)}$. To make this resemble the electromagnetic current, one can write it as $\bar{E}_L\gamma^\mu W_\mu^+ \tau^+ E_L$. The entity $E_L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ should be considered as a doublet (one speaks of an isospin doublet) and τ^\pm are the raising and lowering Pauli matrices.

This two-level transition is also very familiar to us from quantum mechanics as is the use of the Pauli matrices τ . The smallest group of gauge transformation acting on the doublet E_L (and n, p) generalising Eq. 3, is the non-Abelian $SU(2)_L$, which besides τ^\pm has also ‘‘a neutral’’ generator τ^3 . There will therefore be 3 compensating *gauge fields*: W_μ^\pm, W_μ^3 . $SU(2)_L$ symmetry predicts the coupling of W^3 : $\bar{E}_L\gamma^\mu W_\mu^3 \tau^3 E_L = \bar{\nu}_e\gamma^\mu W_\mu^3 \nu_e - \bar{e}_L\gamma^\mu W_\mu^3 e_L$. Unfortunately this neutral current does not correspond to the electromagnetic current. For a start it involves neutral particles. On the other hand, it does include a part (left-handed) of the electromagnetic current. Therefore we do have a partial unification or at least a unified description of the weak and electromagnetic interaction. To fully reconstruct the electromagnetic current from the neutral isospin current, one must postulate the existence of at least another neutral current. In the standard model this is introduced via a $U(1)_Y$ neutral current, associated with the hypercharge Y and a gauge field B_μ . The latter couples to both the left-handed doublet and right-handed (e.g. e_R) isospin singlet. The photon and the Z will appear as a superposition of the fields B_μ and W_μ^3 .

It is also appropriate to say, at this stage, that the neutron and the proton are made up of quarks, u and d , that form a doublet under $SU(2)$ and which are bound by the strong force. Each quark carries a set of three colours. The messenger of the colour force, strong interaction, is the gluon. The gauge group here is $SU(3)$ which means in fact that we have eight types of gluons, corresponding to the eight generators of the fundamental representation of $SU(3)$ represented by the 8 Gell-Mann matrices T^a . e, ν_e, u, d form the first generation of matter particles of the SM model. One has discovered three such families. These fermions are listed in Table. 1 which gives their respective charge under the three gauge groups $SU(2)_L, U(1)_Y, SU(3)_c$.

The gauge coupling constants of these three fundamental interactions, which help define the generalisation of the gauge transformations (Eq. 3) and the covariant derivative are, respectively, g, g' and g_s . The gauge transformations

Table 1: Quantum numbers of the first generation of quarks and leptons, and of the Higgs doublet field, Φ , within the SM. The rows give the irreducible representations under colour $SU(3)$ and weak $SU(2)$, and the hyper-charges for the various multiplets. The electric charge (in units of e) is given in the last column. The assignments for the quarks and leptons of the second and third generations ((c, s, ν_μ, μ) and (t, b, ν_τ, τ)) are identical to the ones for the first generation. For the experimental determination of the number of neutrinos (and light generations) see Chap. 7.

	SU(3)	SU(2) _L	U(1) _Y	$Q = T_3 + Y$
$Q = (u_L, d_L)$	3	2	$\frac{1}{6}$	$(\frac{2}{3}, -\frac{1}{3})$
u_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L = (\nu_L, e_L)$	1	2	$-\frac{1}{2}$	$(0, -1)$
e_R	1	1	-1	-1
ν_R	1	1	0	0

and the covariant derivative are then

$$\begin{aligned} \psi(x) &\rightarrow U(x)\psi(x) = e^{i\theta_3^a(x)T^a} e^{i\theta_2^j(x)\frac{\tau^j}{2}} e^{i\theta_1(x)Y} \psi(x), \\ D_\mu &= \partial_\mu - ig_s T^a A_\mu^a - ig \frac{\tau^i}{2} W_\mu^i - ig' Y B_\mu. \end{aligned} \quad (5)$$

The covariant derivatives completely specify the interactions of all known fermions and gauge bosons and encode the universality of the gauge couplings (see Chap. 8 for experimental tests) via the matter Lagrangian

$$\mathcal{L}_{\text{matter}} = \sum_{j=Q, u_R, d_R, L, e_R, \nu_R} \bar{\psi}_j i \gamma^\mu D_\mu \psi_j. \quad (6)$$

An important observation concerning non-Abelian gauge theories is that the gauge fields are self-interacting. Not only the matter fields carry charge but also the gauge fields (isospin for $W^{1,2,3}$, colour for the gluons). This is imposed by the non-Abelian gauge transformations. For example the generalisation of the field strength, $F_{\mu\nu}$ for $SU(2)$ is

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk} W_\mu^j W_\nu^k. \quad (7)$$

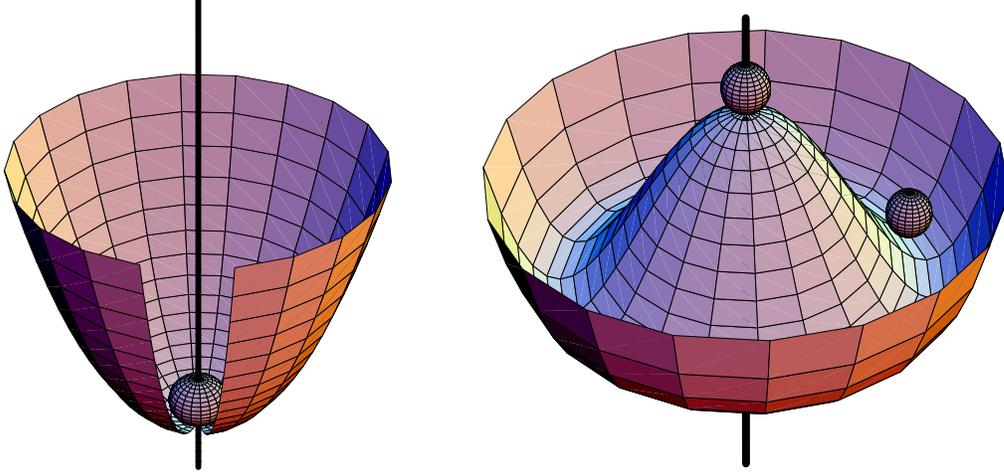
The first two terms enter the kinetic term as in the Abelian case, whereas the last term describes the self-interaction. The gauge Lagrangian is constructed as in Eq. 4 using the appropriate fields strengths for $SU(2)$, $U(1)$ and $SU(3)$. The gauge Lagrangian uniquely fixes the W^+W^-Z and $W^+W^-\gamma$ triple gauge vertices (TGV) which have been measured at LEP (see Chapter 9), as well as the quartic couplings such as $W^+W^-Z\gamma$ in terms of a single parameter, the gauge coupling g . One important property is that the non-Abelian gauge symmetry predicts the gyromagnetic moment of the W^\pm to be 2, like that of the charged elementary fermions.

2 Spontaneous symmetry breaking and mass generation

The main blow to the construct so far is that the weak interactions describe a short range force, in other words the W^\pm and the Z are massive. As pointed out above for QED, a mass term introduced by hand destroys the gauge invariance of the theory. This major hurdle was solved by borrowing and adapting an idea that is encountered in many solid-state physics phenomena. In such systems, the Hamiltonian is invariant under a symmetry but the *ground state* of the system breaks this symmetry. Such is the case with a ferromagnet below the Curie temperature. In this case, rotational symmetry is broken by the ground state (all spins of the atoms aligned in the same direction) despite the fact that the dynamics (the Heisenberg spin-spin Hamiltonian) does not select any preferred direction. This spontaneous symmetry breaking is also at work in superconductivity where, with the Meissner effect, the fact that the magnetic field enters the solid over a very short range could be associated with a massive photon.

2.1 The Higgs mechanism and mass generation for the gauge bosons

Figure 1: *Scalar potentials.* The first panel shows a symmetric potential, under rotation around the vertical axis, with a stable minimum where the ball is resting. In this situation $\langle 0|\phi|0 \rangle = 0$. The second panel, resembling a mexican hat, illustrates spontaneous symmetry breaking. The symmetric configuration at the top of the hat is unstable. The system will pick up any stable configuration along the brim with $\langle 0|\phi|0 \rangle \neq 0$. The Goldstone mode therefore represents this azimuthal direction whereas the radial component is the Higgs field.



One usually thinks about (and most often defines) the vacuum as a state where all fields have zero expectation value. However it may happen, like what is depicted in Fig. 2, that the state with zero energy is not the most stable. The system will choose stability or minimum energy (bottom of the well) rather than the state with maximum symmetry (top of the mountain). Such potentials are provided by scalar fields, with a potential of the form $V = \lambda(|\phi|^2 - v^2/2)^2$ ($\lambda > 0$), where v is the vacuum expectation value (v.e.v) of the scalar field $\langle 0|\phi|0 \rangle = v/\sqrt{2}$. This scale is the origin of the mass of the gauge bosons and also the fermions. It is as if some of the charged fermions and gauge bosons moving in such vacua feel a resistance and behave like they have mass. To see how this works, it is most simple to take QED as an example[2] to illustrate how a mass for the photon can be introduced in a *gauge invariant* way.

One needs to take a charged scalar field ϕ . The latter can be represented by a complex field $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$. For our purposes it is best to rewrite this in polar coordinate as $\phi = (h + v)e^{i\theta/v}/\sqrt{2}$, where both h and θ have zero v.e.v. θ characterises the rotational symmetry of the potential. The interaction of this scalar field with the electromagnetic field is described by a fully gauge invariant Lagrangian constructed through the use of the covariant derivative, Eq. 3. Upon expansion of this Lagrangian we find

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}v^2 \left(eA_\mu + \frac{1}{v}\partial_\mu\theta \right)^2 + \frac{1}{2}\partial_\mu h\partial^\mu h - \frac{\lambda}{4}(h^2 + 2vh)^2 + \frac{1}{2} \left(eA_\mu + \frac{1}{v}\partial_\mu\theta \right) (h^2 + 2vh). \quad (8)$$

One sees clearly that the gauge field has acquired a mass, $m_\gamma = ev$ (second term in Eq. 8: $+e^2v^2A_\mu A^\mu/2$). Because of gauge invariance we can always make a (local) phase transformation on the field $\phi = (h + v)e^{i\theta/v}/\sqrt{2}$ such that the “phase” θ/v is set to zero. This gauge where $\theta \rightarrow 0$ in Eq. 8 is the *unitary gauge* where only the *physical states* h, A_μ are left. Counting the number of degrees of freedom before and after symmetry breaking, we find the same number of course. Before, one had two scalars and one massless spin-1 which has only two (transverse) states of polarisation. After symmetry breaking, one of the scalars, θ , transmutes into the longitudinal polarisation of the “heavy photon”. In fact it would be more appropriate to say that the gauge symmetry is hidden, once we choose a particular gauge, since the gauge symmetry is present in the dynamics. The θ field is a Goldstone boson, it corresponds, as seen in Eq. 8, to a massless pseudo-scalar. h is the Higgs scalar field whose mass is given by $\sqrt{2\lambda}v^2$. There is also a coupling hAA which proportional to the mass of the gauge boson.

A very similar approach is applied to the weak interaction. Since we need three massive gauge bosons, the three longitudinal states will be provided by three Goldstone bosons. This is most simply and economically provided

by a Higgs doublet, Φ , with quantum numbers such that the vacuum is left invariant under electromagnetic gauge transformations, so that the photon remains massless. Thus $Y_\Phi = -1/2$,

$$\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix} e^{i\omega^j \frac{\tau^j}{2v}} \quad \mathcal{L}_{\text{Higgs}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi^\dagger \Phi), \quad \text{with} \quad V(\Phi^\dagger \Phi) = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2. \quad (9)$$

where H is the physical Higgs field of the electroweak theory, $\omega^{1,2,3}$ the Goldstone bosons and v the vacuum expectation value.

2.2 Fermion masses

We already stressed the fact that QED treated both electron chiralities on the same footing, in particular both e_L and e_R have the same electric charge. Therefore the electron mass term $m_e(\bar{e}_R e_L + \bar{e}_L e_R)$ is gauge invariant in QED. According to Table 1 the SM has no left and right-handed multiplets with identical $SU(2)$ and $U(1)_Y$ charge, hence, a fermion mass term introduced by hand would break the symmetry of the SM. Once again, however, mass terms are possible via the Higgs mechanism. Let us consider the masses for the charged leptons. The left doublet L , right-handed singlet l_R and the Higgs field Φ combine so that they form a neutral symmetric object under $SU(2) \times U(1)$. The masses are introduced via Yukawa couplings y_l as

$$-\mathcal{L}_m^l = \sum_{i=e,\mu,\tau} y_i^l (\bar{L}_i \Phi l_{R,i} + \bar{l}_{R,i} \Phi^\dagger L_i) \longrightarrow \text{unitary gauge} \longrightarrow \sum_{i=e,\mu,\tau} \frac{y_i^l v}{\sqrt{2}} \left(1 + \frac{H}{v} \right) \bar{l}_i l_i, \quad \frac{y_i^l v}{\sqrt{2}} = m_i^l. \quad (10)$$

This exhibits a common important feature that applies also to quarks, namely that the couplings of the Higgs to fermions is proportional to the fermion mass. Because in the quark sector one is generating masses for both the up and down quark, one also induces mixing between the three-families in the charged current[6] (but not in the neutral currents[7]). This is also the source of CP-violation in the model. It is important to stress that although masses and mixing are introduced in a gauge invariant way, one nonetheless needs to introduce a large number of ad-hoc Yukawa couplings, contrary to the gauge boson masses that are expressed through the universal gauge coupling.

3 Salient features of the model

Within the SM the scale of all particle masses is set by the Higgs v.e.v., v . For the gauge bosons, the mass term is produced by the kinetic energy term of the Higgs doublet field, $\mathcal{L}_{\text{Higgs}}$ in Eq. 9. The latter reveals several key predictions of the SM which can be tested at LEP.

1. The Higgs Lagrangian generates mass terms for the charged W^\pm and for Z fields. W_μ^3 and B_μ combine to give the massless photon and the Z ,

$$\begin{aligned} Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g W_\mu^3 - g' B_\mu) \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu. \end{aligned} \quad (11)$$

This relation defines the weak mixing angle, or, more precisely, $\sin \theta_W$ and $\cos \theta_W$.

2. The W and Z masses are given by

$$M_W = \frac{gv}{2} \quad \text{and} \quad M_Z = \sqrt{g^2 + g'^2} \frac{v}{2} = \frac{M_W}{\cos \theta_W}. \quad (12)$$

The mass relation $M_W^2 = \rho M_Z^2 \cos^2 \theta_W$ with $\rho = 1$ is a consequence of the breaking the electroweak symmetry by a scalar doublet Higgs field.

3. The mass terms for the W and Z are directly related to HWW and HZZ couplings, of strength $2M_W^2/v$ and $2M_Z^2/v$, respectively. This coupling of a single scalar to two gauge bosons requires the existence of a v.e.v. for the Higgs doublet field. Normally, gauge bosons couple to pairs of scalars only.
4. The Higgs boson mass is given by $M_H^2 = 2\lambda v^2$. Since the quartic coupling λ is a free parameter, there is no intrinsic prediction for M_H within the SM. Searches for this missing boson are reviewed in Chap. 11.
5. The same Higgs mechanism is responsible for the mass of the fermions. The coupling of the Higgs being proportional to the mass of the fermion, an intermediate mass Higgs, $M_H < 2M_W$, will decay predominantly to b -quarks.

It is important to stress that the model gives a very nice unified description of the weak and electromagnetic interactions. However the model does not fully unify these interactions, since we still have *two* (independent) gauge couplings g, g' . Viewed another way the model does not *predict* $\sin\theta_W$. Nonetheless the gauge principle automatically leads to the universality of the gauge coupling. Most importantly for LEP1, the neutral interaction Lagrangian derived from Eq. 6, summarises the main properties and parameters of Z physics:

$$\mathcal{L}_{\text{matter}}^{NC} = e \sum_i Q_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu + \frac{e}{2 \sin\theta_W \cos\theta_W} \sum_i \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma_5) \psi_i Z_\mu, \quad (13)$$

with the vector and axial-vector couplings given by ($s_W^2 = \sin^2\theta_W$)

$$g_V^i = T_3(i) - 2Q_i s_W^2; \quad g_A^i = T_3(i) \quad \text{and} \quad g = \frac{e}{\sin\theta_W}, \quad g' = \frac{e}{\cos\theta_W}. \quad (14)$$

4 The need for radiative corrections

Leaving aside fermion masses, with the assignment of the quantum numbers given in Table 1, the fermion and gauge field Lagrangians considered above are completely determined in terms of just five free parameters. These can be taken as the gauge couplings g_s, g, g' , the Higgs v.e.v, v , and the quartic Higgs coupling, λ . In particular the three weak parameters g, g', v alone allow to determine all the properties of the gauge weak bosons (masses and self-couplings) as well as their interaction to matter. This is the power and beauty of gauge invariance in that with an extremely limited number of parameters one can describe a very large number of processes and observables. For example at the Z peak, at LEP1 and SLC, the mixing angle $s_W = \sin\theta_W$ can be extracted from the ratio of a number of cross sections that measure the coupling g_V^f/g_A^f . From the observables with leptons in the final states, the latest LEP measurement of s_W^2 from leptonic observables, which we will identify as $s_{\text{eff},1}^2$, gives an average value

$$s_W^2 = s_{\text{eff},1}^2 = 0.23159 \pm 0.00018 \quad (\text{i.e. } 0.08\% \text{ precision}). \quad (15)$$

We have also seen that this angle also describes the ratio M_W^2/M_Z^2 and therefore could be extracted from a combination of entirely different experiments (LEP1 and LEP2). The latest data [8] give $M_Z = 91.1875 \pm 0.0021 \text{ GeV}$ ($\Delta M_Z/M_Z \sim 2 \cdot 10^{-5}$) and $M_W = 80.451 \pm 0.033 \text{ GeV}$ ($\Delta M_W/M_W \sim 5 \cdot 10^{-4}$), leading to

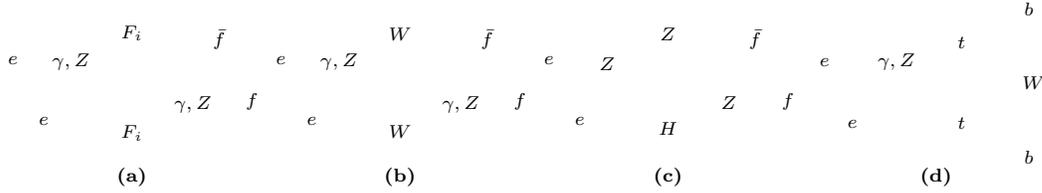
$$s_M^2 = 0.22162 \pm 0.00067 \quad (\text{i.e. } 0.3\% \text{ precision}). \quad (16)$$

Not only do we get a much better precision on the effective mixing angle extracted from the Z observables, but more telling is that this value is about 14σ away from that extracted from the mass ratio, M_W^2/M_Z^2 . The Born approximation which has been discussed so far, is insufficient to explain the LEP measurements which are described in Chap. 10.

Improving on the Born approximation necessitates the inclusion of radiative corrections which are contributions from the quantum fluctuations of the vacuum. They are represented by loops. A selection of radiative corrections to fermion pair production in e^+e^- is shown below.

Computing some of the closed loops naively can lead to infinite and inconsistent results, whereas they are supposed (considering the small electroweak coupling expansion parameter) to be small corrections! The infinities appear because one is summing over all possible states with all possible energies. Mathematically within the loops

Figure 2: Some the diagrams representing the quantum loop corrections to $f\bar{f}$ production at e^+e^- . One can indirectly access virtual particles. The first tree diagrams are self-energy diagrams. Diagram (d) is a vertex correction involving virtual top particles which contribute to $b\bar{b}$ production.



one is integrating over all possible momenta, k , with integrals of the form $\int d^4k/(k^2 + m^2)^n$ with $n = 1, 2, 3..$ that diverge for small n at high momenta. This problem had occurred in QED but was side-stepped with the realisation that as long as these infinities do not show up in measurable quantities expressed in terms of physical parameters, these infinities were not disastrous. Take for instance the vacuum polarisation of the photon, first figure in the self-energy diagrams. This is calculated following some simple set of rules derived from the QED Lagrangian. The divergence can be eliminated mathematically by invoking that the parameter e appearing in the Lagrangian is a mathematical quantity which is also infinite and can absorb the corresponding divergence of the loop. Differences in the value of e at different energies are on the other hand finite. Thus by choosing a reference value, and therefore defining the parameter e as a physical parameter extracted from some observable, all subsequent observables will be finite. In a renormalisable theory, like QED, once the fundamental parameters of the theory have been properly defined in terms of a few physical observables, all other observables are precisely determined.

Before the advent of spontaneous symmetry breaking this program could not be achieved for the weak interactions. Infinities appeared all over the place and required more than just a rescaling of the fundamental parameters of the model. Lacking gauge invariance in the dynamics of the system meant that many infinities seemed to be uncorrelated. Gauge invariance, through the Higgs mechanism, was essential for proving the renormalisability of the standard model. In fact it was only when this proof was achieved that the model received all its recognition and popularity. Technically, for the standard model, one also had to devise a new regularisation method (dimensional regularisation)[4] to mathematically handle the divergencies and to keep the symmetries of the model at the quantum level. The model is then highly predictive and is amenable to high precision measurements. One can say that with the advent of LEP/SLC and some other low-energy precision measurements the model has been crowned and elevated to the status of a theory since the model has been verified at better than .1%.

Renormalisation helps not only get finite results and highly improves the tree-level approximation, but the radiative corrections can give access, as we see in Figure 2, to new particles (like the Higgs, the top...) that are too heavy to be produced directly at LEP. Combining precision measurements, one can weigh the effects of these new states and literally get a bound or a measure on the masses of these particles. In this light it is not difficult also to understand that some equalities or natural relations (like the different definitions of $\sin \theta_W$) at the tree-level will be corrected. This is the case for the different manifestations of s_W^2 in section 3.

The most precisely measured quantities of the standard model are the QED fine structure constant, $\alpha = e^2/4\pi$, the mass of the Z , M_Z and G_μ the Fermi constant derived from muon decay (which at tree-level is $G_\mu/\sqrt{2} = g^2/8M_W^2$). Using these parameters, in lieu of the three fundamental parameters g , g' and v of the original Lagrangian (this is just a trade-off), one finds that

$$\begin{aligned}
 s_M^2 &\simeq s_Z^2 \left(1 + \frac{c_W^2}{c_W^2 - s_W^2} \Delta\hat{r} \right) ; & \Delta\hat{r} &\simeq -\frac{c_W^2}{s_W^2} \varepsilon_1 + \frac{c_W^2 - s_W^2}{s_W^2} \varepsilon_2 + 2\varepsilon_3 \\
 s_{\text{eff},1}^2 &\simeq s_Z^2 \left(1 + \frac{c_W^2}{c_W^2 - s_W^2} \Delta\kappa \right) ; & \Delta\kappa &\simeq \left(-\varepsilon_1 + \frac{\varepsilon_3}{c_W^2} \right) ,
 \end{aligned}$$

$$s_Z^2 = \frac{1}{2} \left(1 - \sqrt{1 - \frac{4\pi\alpha(M_Z^2)}{\sqrt{2}G_\mu M_Z^2}} \right) ; \quad \alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha(M_Z^2)} . \quad (17)$$

The most important corrections for LEP physics are encoded in the new quantum functions $\Delta\alpha(M_Z^2), \varepsilon_{1,2,3}$ [9]. $\Delta\alpha(M_Z^2)$ is a purely QED corrections representing the screening of the electric charge. It contains no New Physics parameters. It is given solely in terms of known parameters below the Z scale and correspond to the effective electric charge at the Z mass. s_Z^2 constitutes a QED-only improved mixing angle which nonetheless takes into account a rather large correction of about 6%. ε are genuine electroweak corrections that are sensitive to the top mass and the Higgs mass of the SM, they are also the most likely probe of a contribution beyond the SM. For example,

$$\varepsilon_1 = \frac{3G_\mu M_Z^2}{8\pi^2\sqrt{2}} \left\{ \left(\frac{m_t^2}{M_Z^2} - 1 \right) - 2s_M^2 \ln(M_H/M_Z) \right\} + \dots + \varepsilon_1^{\text{NP}} . \quad (18)$$

5 Conclusions

The confrontation of the precision tests with theory beyond the tree-level has not only allowed to extract the mass of the top-quark soon after LEP1 started operation, but has also put strong constraints on the Higgs mass. Most models without a light Higgs are now strongly disfavoured. This has added impetus to supersymmetric models which predicts a light Higgs, see Chapter 12. Another argument for supersymmetric grand unification derives from the very precise measurement of $s_{\text{eff},1}^2$ together with $\alpha_s(M_Z^2)$ (see Chap. 14,15): they suggest that the three gauge couplings g, g', g_s unify at some high scale with supersymmetric particles at or below the TeV scale. Apart from the still missing Higgs particle, the Standard Model has passed all stringent tests with flying colours. It is essential that one discovers the Higgs to confirm that the pattern of symmetry breaking and mass generation is indeed as described by the model or else that the model is in need of more symmetry (like supersymmetry) or new underlying dynamics.

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