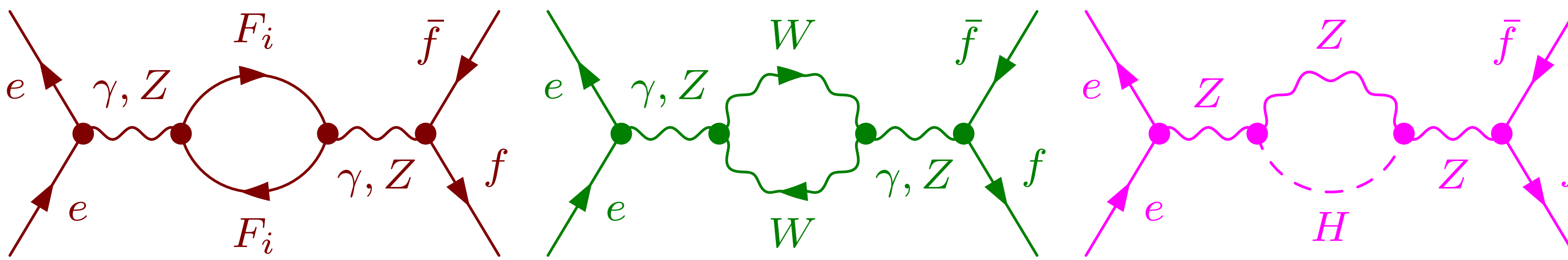
Elements of the Renormalisation of the Electroweak Theory
and
Precision Tests of the Standard Model



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Introduction

EW theory is the combination of two fundamental principles

- Gauge Symmetry Principle
- Hidden symmetry or *Spontaneous symmetry breaking*

This allows

✓ a correct quantum description

✓ high degree of precision (LEP,SLC,...)

Maxwell equations: Unify $\overrightarrow{E}$ and $\overrightarrow{B}$

Local conservation of the electric charge

$$\partial j = 0 \qquad j^\mu = (\rho, \overrightarrow{j})$$

$$\mathbf{div}\,\overrightarrow{E} = \rho \qquad \mathbf{div}\,\overrightarrow{B} = \mathbf{0}$$

$$\mathbf{Curl}\,\overrightarrow{E} + \frac{\partial \overrightarrow{B}}{\partial \mathbf{t}} = \mathbf{0} \qquad \mathbf{Curl}\,\overrightarrow{B} - \frac{\partial \overrightarrow{E}}{\partial \mathbf{t}} = \overrightarrow{j}$$

GI: electrostatic field (force) depends only on difference of potential

The quantum (for photons) is the vector potential $A^\mu(x) = (V, \overrightarrow{A})$,

$$\overrightarrow{B} = Curl\,\overrightarrow{A} \qquad \overrightarrow{E} = -GradV - \partial\overrightarrow{A}/\partial t.$$

Introduction: Electromagnetism as a prototype

Gauge invariance $A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x)$.

$$\mathbf{F}^{\mu\nu} = \partial^\mu \mathbf{A}^\nu - \partial^\nu \mathbf{A}^\mu$$

$$\partial_\mu \mathbf{F}^{\mu\nu} = \mathbf{j}^\nu \qquad \varepsilon_{\mu\nu\rho\sigma}\partial^\nu \mathbf{F}^{\rho\sigma} = \mathbf{0}.$$

All of this can be derived from the Lagrangian

$$\mathcal{L}_{\rm em} = -\frac{\mathbf{1}}{\mathbf{4}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} \equiv \frac{\mathbf{1}}{\mathbf{4}}\left((\overrightarrow{E} + \mathbf{i}\,\overrightarrow{B})^{\mathbf{2}} + (\overrightarrow{E} - \mathbf{i}\,\overrightarrow{B})^{\mathbf{2}}\right) .$$

$$(1/2m)(-i\overrightarrow{\nabla})^2\psi = i\partial\psi/\partial t$$

invariant under a *global* phase transformation

$$\psi \rightarrow \exp(i\lambda)\psi$$

what about invariance under *local* phase transformation?

$$\lambda \rightarrow q\Lambda(x = (t, \overrightarrow{x}))$$

Possible only if one introduces a *compensating* vector field which transforms exactly A_μ ◀.
This prescription gives

$$(1/2m)\left(-i\overrightarrow{\nabla} + q\overrightarrow{A}\right)^2\psi = (i\partial/\partial t + qV)\,\psi\ .$$

Introduction: $U(1)$ Gauge Transformation

relativistic quantum: covariant derivative $\rightarrow$
Coupling is universal

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - iqA_\mu \rightarrow \mathcal{L}_{int} = q\bar{e}\gamma^\mu e A_\mu$$

$$\psi(x) \rightarrow U(x)\psi(x), \qquad D_\mu\psi(x) \rightarrow U(x)D_\mu\psi(x)$$

$$U(x) = e^{iq\Lambda(x)}$$

covariant derivatives and charged fields have identical local transformations.
QED: $U(1)$ Abelian group

Weak Interactions and non Abelian theories

It was found that

- β-decay: $n \to p + e^- + \bar{\nu}_e$,
- muon decay: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
- muon capture: $\mu^- + p \to n + \nu_\mu$

were of the same nature and have the same strength.

Universality, Gauge Interaction?

But important differences with QED: they involve

- a change in the identity of the fermion
- only left-handed field/component were found to interact:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu \; \bar{e}\gamma^\alpha(1-\gamma_5)\nu_e \qquad G_F \propto M^{-2}$$

Weak Interactions and non Abelian theories

- QED $\bar{e}\gamma^\mu A_\mu e = \bar{e}_L\gamma^\mu A_\mu e_L + \bar{e}_R\gamma^\mu A_\mu e_R$
- $A_\mu \rightarrow W_\mu^+$ but $\bar{e}_L\gamma^\mu W_\mu^+ \nu_{e(L)}$
- Construction along QED suggests $\bar{E}_L\gamma^\mu W_\mu^+ \tau^+ E_L$
-
$$E_L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$$
- the smallest group is $SU(2)$: $\tau^\pm$ needs τ_3.
- this implies 3 compensating fields: $W^\pm, W^3$
- $W_\mu^3 (\neq A_\mu)$:

$$\bar{E}_L\gamma_\mu W_\mu^3 \tau^3 E_L = \bar{\nu}_e\gamma_\mu W_\mu^3 \nu_e \; - \; \bar{e}_L\gamma_\mu W_\mu^3 e_L$$

Weak Interactions and non Abelian theories

- The neutral current W^3 contains part of the e-m current
- Postulate a new $U(1)$ field, B_μ corresponding to hypercharge Y
- photon will emerge as a combination of $W^3 - B$.

$$\psi(x) \rightarrow U(x)\psi(x) = e^{i\theta_3^a(x)T^a} e^{i\theta_2^j(x)\frac{\tau^j}{2}} e^{i\theta_1(x)Y} \psi(x) \ ,$$

$$D_\mu = \partial_\mu - ig_s T^a A_\mu^a - ig\frac{\tau^i}{2}W_\mu^i - ig'YB_\mu \ .$$

$$\mathcal{L}_{\text{matter}} = \sum_{j=Q,u_R,d_R,L,e_R,\nu_R} \overline{\psi}_j i\gamma^\mu D_\mu \psi_j \ .$$

Weak Interactions and non Abelian theories

	SU(3)	SU(2)$_L$	U(1)$_Y$	$Q = T_3 + Y$
$Q = (u_L, d_L)$	3	2	$\frac{1}{6}$	$(\frac{2}{3}, -\frac{1}{3})$
u_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L = (\nu_L, e_L)$	1	2	$-\frac{1}{2}$	$(0, -1)$
e_R	1	1	-1	-1
ν_R	1	1	0	0

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\mu W^i_\nu + g\epsilon^{ijk} W^j_\mu W^k_\nu \,.$$

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4} A^{a,\mu\nu} A^a_{\mu\nu} - \frac{1}{4} W^{i,\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \mathcal{L}_{\text{gauge fix.}} + \mathcal{L}$$

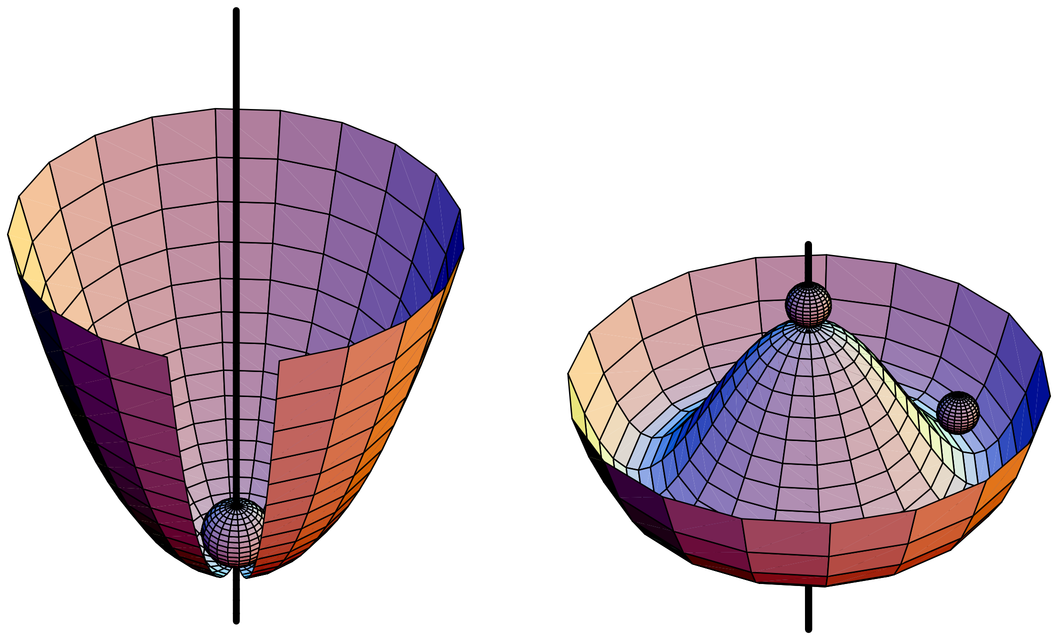
Gauge invariance and mass terms

1.
•$m^2 A_\mu A^\mu$ not invariant under $A_\mu \rightarrow A_\mu - \partial_\mu \Lambda$
•$m\bar{e}e = m(\bar{e}_L e_L + \bar{e}_R e_R)$
breaks GI and charge under $SU(2) \times U(1)$ (though ok under $U(1)_{em}$)

2. Idea of spontaneous symmetry breaking:
Hamiltonian is symmetric but not the background.
An example is a ferromagnet below the Curie Temperature. Rotational is broken in the ground state (all spins aligned in the same direction) but the dynamics described but the fully symmetric Heisenberg spin-spin Hamiltonian.

Higgs Potential

Such non symmetric backgrounds in QFT are introduced by a scalar potential that prefers stability rather than zero energy.



$V = \lambda(|\phi|^2 - v^2/2)^2$

$(\lambda > 0)$

$< 0|\phi|0 >= v/\sqrt{2}$

Massive QED as an example: take a charged scalar field, charge e

$\phi = (\phi_1 + i\phi_2)/\sqrt{2} = (h + v)e^{i\theta/v}/\sqrt{2}$

interaction $D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$

Invariance $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \chi \quad \phi \to e^{i\chi}\phi \; \equiv \frac{\theta}{v} \to \frac{\theta}{v} + \chi$

Higgs Potential and $U(1)$ mass

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\underbrace{(ev)^2}_{m_\gamma^2}\left(A_\mu + \frac{1}{ev}\partial_\mu\theta\right)^2$$

$$+ \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{\lambda}{4}(h^2 + 2vh)^2 + \frac{1}{2}\left(eA_\mu + \frac{1}{v}\partial_\mu\theta\right)(h^2 + 2vh) \,.$$

- This Lagragian is completely GI and yet $m_\gamma \neq 0$
- choose a gauge such that $\theta \to 0$: No Goldstone.
- there remains a Higgs, h with $m_h = \sqrt{2\lambda v^2}$
- Number of degrees of freedom is unchanged

Higgs mechansim and $SU(2) \times U(1)$

■ We now need to give mass to $W^\pm$ and the Z but not the photon.

◎ Need three Goldstones with a vacuum that remains invariant under $U(1)_{em}$.

♦ most simple choice is a doublet $\Phi \to Y_\Phi = -1/2$

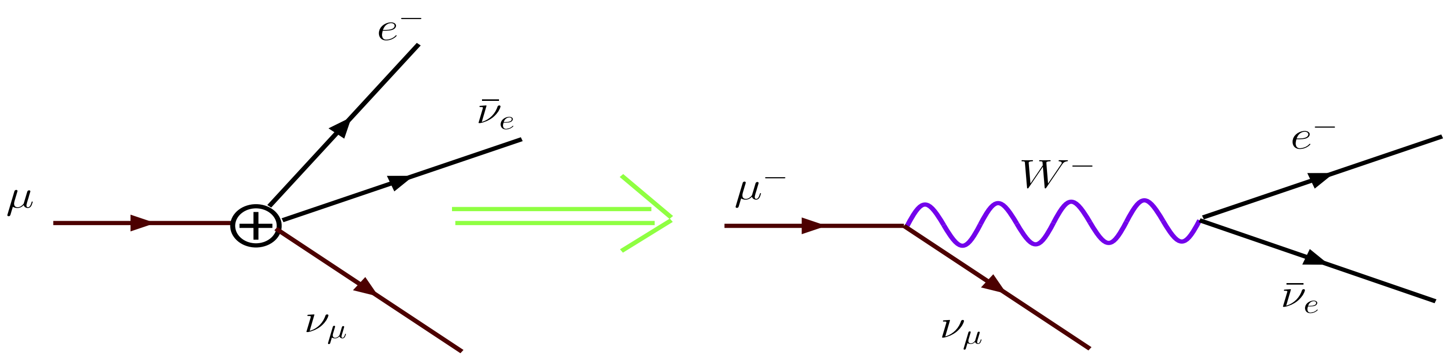
$$\Phi = \left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{array} \right) e^{i\omega^j \frac{\tau^j}{2v}}$$

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi^\dagger \Phi), V(\Phi^\dagger \Phi) = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 .$$

★ same Higgs gives mass to all fermions

Salient Features: 1

- $Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu = \frac{1}{\sqrt{g^2+g'^2}} \left(g W_\mu^3 - g' B_\mu\right)$
  $A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$
- $M_W = \frac{gv}{2}$ and $M_Z = \sqrt{g^2+g'^2}\,\frac{v}{2} = \frac{M_W}{\cos\theta_W}$
- $\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i \gamma^\mu (1-\gamma^5) \left(T^+ W_\mu^+ + T^- W_\mu^-\right) \psi_i$



Fermi Theory *Standard Model*

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$

Neutral currents: 1

$$\mathcal{L}_0 = \sum_i Q^i \bar{\psi}_i \gamma^\mu \psi_i A_\mu \quad (QED)$$

$$+ \frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma^5) \psi_i Z_\mu \quad (NC)$$

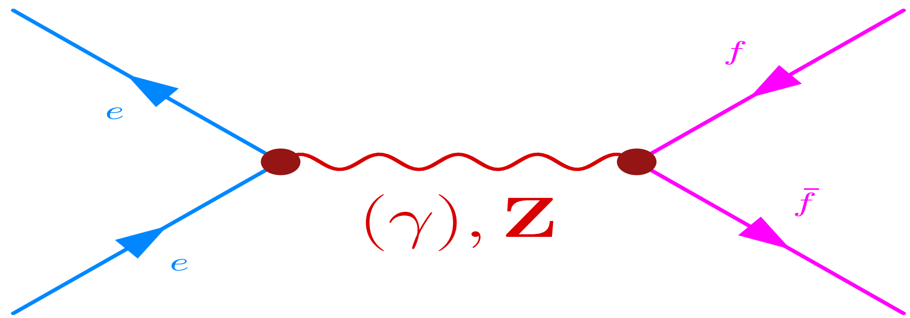
$$g_V^i = T_3(i) - 2Q_i s_W^2; , \qquad g_A^i = T_3(i)$$

◀

$$\mathcal{L}_{NC} = \frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i \gamma^\mu \left(g_L^i (1-\gamma^5) + g_R^i (1+\gamma^5) \right) \psi_i Z_\mu$$

$$g_V = g_L + g_R \qquad g_A = g_L - g_R$$

Phenomenology at the Z pole (1)



$$\frac{d\sigma_Z^f}{d\Omega} = \frac{9}{4}\frac{s/M_Z^2\Gamma_{ee}\Gamma_{f\bar{f}}}{(s-M_Z^2)^2+s^2\Gamma_Z^2/M_Z^2}\Big[\ (1+\cos^2\theta)(1-P_eA_e) + 2\cos\theta A_f(-P_e+A_e)\Big]$$

$$\Gamma(Z\to f\bar{f}) = \frac{\alpha M_Z}{3}\frac{1}{s_W^2c_W^2}(g_V^{f\,2}+g_A^{f\,2})\ N_c^f$$

$$A_f = \frac{2g_V^fg_A^f}{(g_V^f)^2+(g_A^f)^2} = \frac{(g_L^f)^2-(g_R^f)^2}{(g_L^f)^2+(g_R^f)^2}\,,\quad A_\ell = \frac{2(1-4s_W^2)}{1+(1-4s_W^2)^2}\sim 0.15$$

Phenomenology at the Z pole (2)

$$
\begin{aligned}
A_{FB}^f &\equiv \frac{\sigma_F^f - \sigma_B^f}{\sigma_F^f + \sigma_B^f} = \frac{3}{4} A_e A_f, \\
A_{LR}^f &\equiv \frac{1}{P_e} \frac{\sigma^f(-|P_e|) - \sigma^f(+|P_e|)}{\sigma^f(-|P_e|) + \sigma^f(+|P_e|)} = A_e \\
\bar{A}_{FB}^f &\equiv \frac{\left(\sigma_F^f(-|P_e|) - \sigma_B^f(-|P_e|)\right) - \left(\sigma_F^f(+|P_e|) - \sigma_B^f(+|P_e|)\right)}{\sigma_F^f(-|P_e|) + \sigma_B^f(-|P_e|) + \sigma_F^f(+|P_e|) + \sigma_B^f(+|P_e|)} \\
&= \frac{3}{4} P_e A_f.
\end{aligned}
$$

Z Lineshape: scan at the pole

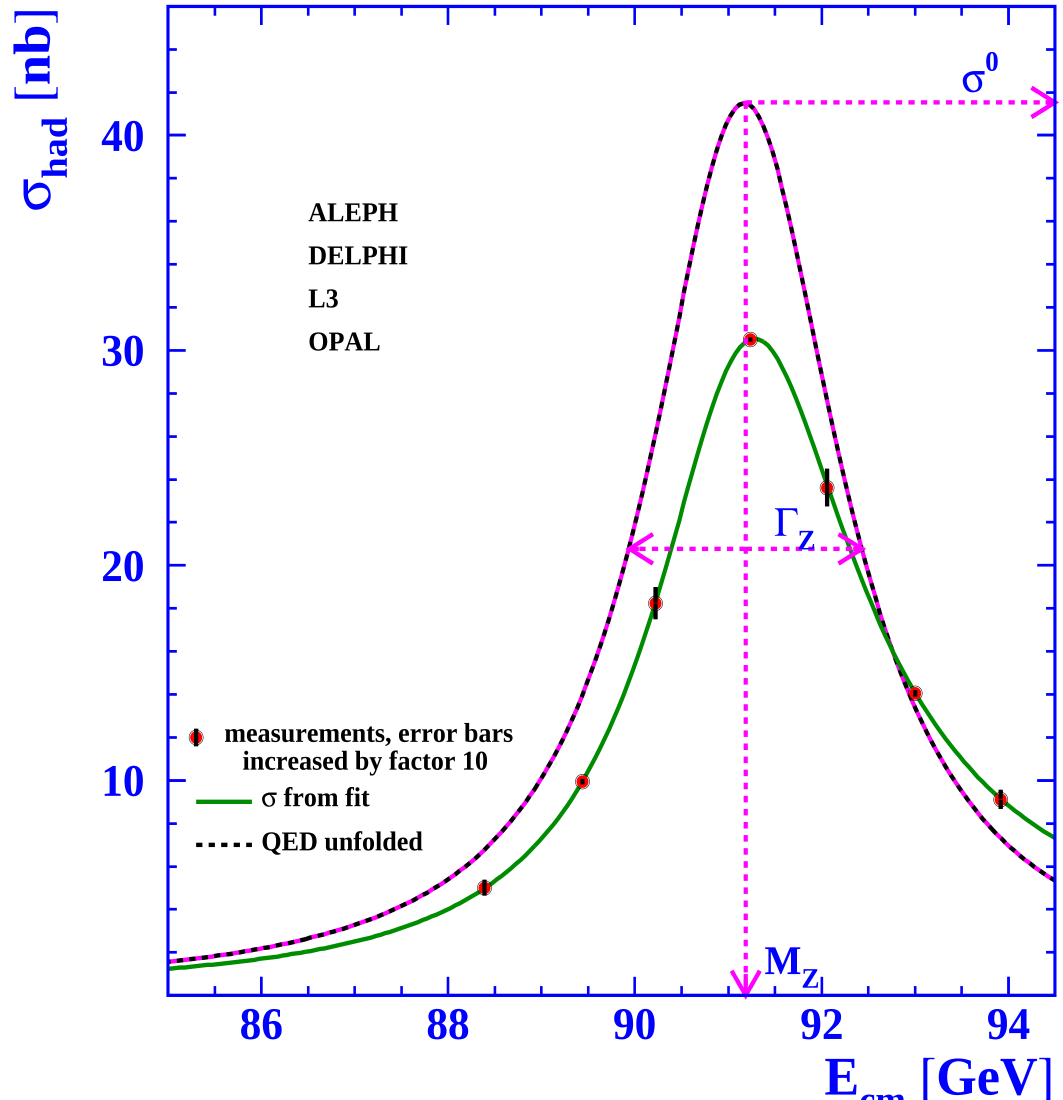
$$\sigma_{had} = 12\pi \frac{\Gamma_{ee}\Gamma_{had}}{M_Z^2\Gamma_Z^2}$$

$$R_\ell \equiv \frac{\Gamma_{had}}{\Gamma_\ell} \sim 21$$

$$R_{b,c} \equiv \frac{\Gamma_{bb,cc}}{\Gamma_{had}}$$

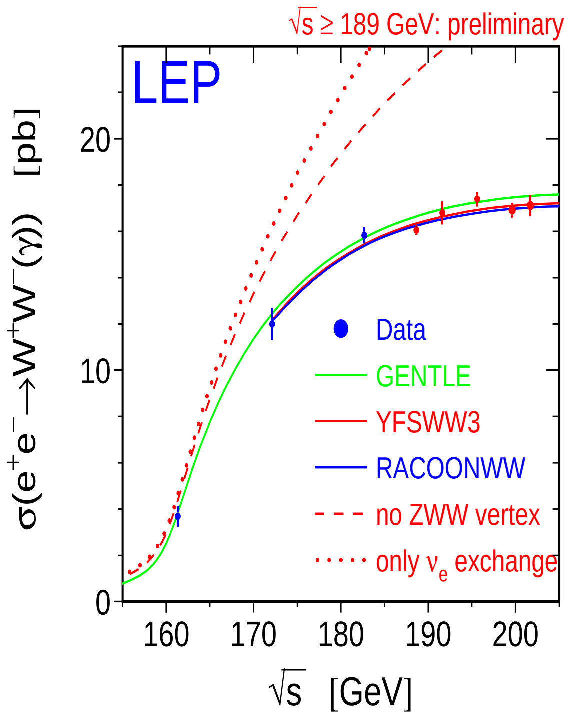
$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had,e,\mu,\tau}$$

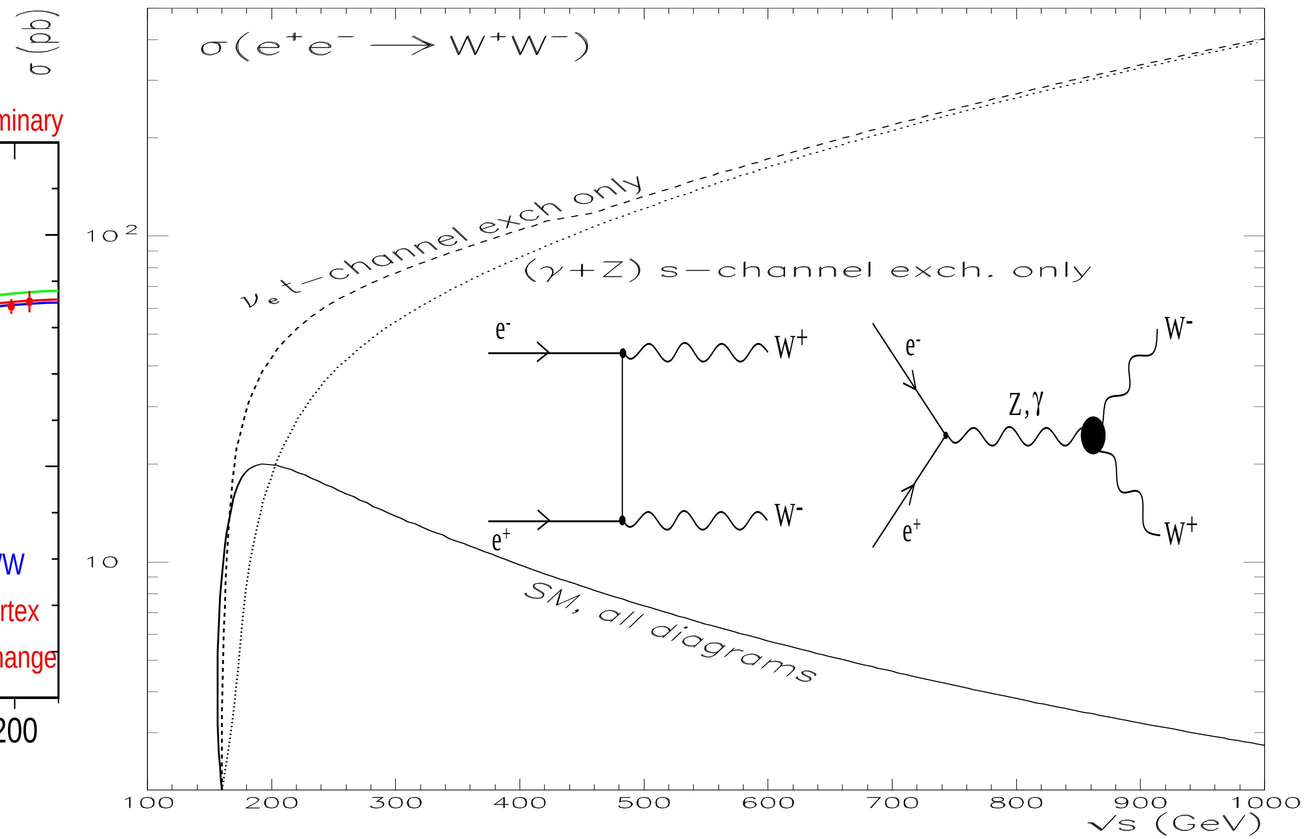
$$N_\nu = \frac{\Gamma_{inv}/\Gamma_\ell}{(\Gamma_\nu/\Gamma_\ell)_{SM}}$$



Measurements of key quantities at the Z peak

Quantity	Group(s)	Value
$\mathbf{M_Z}$ [GeV]	LEP	91.1876 ± 0.0021
$\mathbf{\Gamma_Z}$ [GeV]	LEP	2.4952 ± 0.0023
$\mathbf{\Gamma}(\mathrm{had})$ [GeV]	LEP	1.7444 ± 0.0020
$\mathbf{\Gamma}(\mathrm{inv})$[MeV]	LEP	499.0 ± 1.5
$\mathbf{\Gamma}(\ell^+\ell^-)$[MeV]	LEP	83.984 ± 0.086
σ_{had} [nb]	LEP	41.541 ± 0.037
$\mathbf{R_e}$	LEP	20.804 ± 0.050
$\mathbf{R}_\mu$	LEP	20.785 ± 0.033
$\mathbf{R}_\tau$	LEP	20.764 ± 0.045
$\mathbf{A_{FB}(e)}$	LEP	0.0145 ± 0.0025
$\mathbf{A_{FB}}(\mu)$	LEP	0.0169 ± 0.0013
$\mathbf{A_{FB}}(\tau)$	LEP	0.0188 ± 0.0017
$\mathbf{R_b}$	LEP + SLD	0.21664 ± 0.00065
$\mathbf{R_c}$	LEP + SLD	0.1718 ± 0.0031
$\mathbf{R_{s,d}/R_{(d+u+s)}}$	OPAL	0.371 ± 0.023
$\mathbf{A_{FB}(b)}$	LEP	0.0995 ± 0.0017
$\mathbf{A_{FB}(c)}$	LEP	0.0713 ± 0.0036
$\mathbf{A_{FB}(s)}$	DELPHI,OPAL	0.0976 ± 0.0114
$\mathbf{A_b}$	SLD	0.922 ± 0.020
$\mathbf{A_c}$	SLD	0.670 ± 0.026
$\mathbf{A_s}$	SLD	0.895 ± 0.091
$\mathbf{A_{LR}(hadrons)}$	SLD	0.15138 ± 0.00216
$\mathbf{A_{LR}(leptons)}$	SLD	0.1544 ± 0.0060
$\mathbf{A}_\mu$	SLD	0.142 ± 0.015
$\mathbf{A}_\tau$	SLD	0.136 ± 0.015





M_W is also measured at Tevatron via $p\bar{p} \to W^*$

Production: $q\bar{q} \rightarrow g \rightarrow t\bar{t}$

Decay $t \rightarrow Wb$ almost 100% of the time