micrOMEGAs: Version 1.3

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Abstract

We present the latest version of micrOMEGAs, a code that calculates the relic density of the lightest supersymmetric particle (LSP) in the minimal supersymmetric standard model (MSSM). All tree-level processes for the annihilation of the LSP are included as well as all possible coannihilation processes with neutralinos, charginos, sleptons, squarks and gluinos. The cross-sections extracted from CalcHEP are calculated exactly using loop-corrected masses and mixings as specified in the *SUSY Les Houches Accord*. Relativistic formulae for the thermal average are used and care is taken to handle poles and thresholds by adopting specific integration routines. The input parameters can be either the soft SUSY parameters in a general MSSM or the parameters of a SUGRA model specified at some high scale (GUT). In the latter case, a link with Suspect, SOFTSUSY, Spheno and Isajet allows to calculate the supersymmetric spectrum, Higgs masses, as well as mixing matrices. Higher-order corrections to Higgs couplings to quark pairs including QCD as well as some SUSY corrections (Δm_b) are implemented. Routines calculating $(g-2)_\mu$, $b \to s\gamma$ and $B_s \to \mu^+\mu^-$ are also included. In particular the $b \to s\gamma$ routine includes an improved NLO for the SM and the charged Higgs while the SUSY large $\tan\beta$ effects beyond leading-order are included. This new version also provides cross-sections for any $2 \to 2$ process as well as partial decay widths for two-body final states in the MSSM allowing for easy simulation at colliders.

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1 Introduction

We present `micrOMEGAs1.3`, a program which calculates the relic density of the lightest supersymmetric particle (LSP) in the minimal supersymmetric standard model (MSSM). The stable LSP, which occurs in supersymmetric models with R parity conservation, constitutes a good candidate for cold dark matter. Recent measurements from WMAP[1] have in fact constrained the value for the relic density within 10%,

$$.094 < \Omega h^2 < .128 \quad \text{at} \;\; 2\sigma.$$

Forthcoming experiments by the PLANCK satellite[2] will pin-down this important parameter to within 2%. One therefore needs to evaluate the relic density with high accuracy.

The relic density calculation is based on solving the equation characterizing the evolution of the number density of the LSP. For this, one needs to evaluate the thermally averaged cross-section for annihilation of the LSP, as well as, when necessary, coannihilation with other supersymmetric (SUSY) particles [3, 4, 5]. We use, as in `micrOMEGAs`1.1 [6], the method described in [7] for the relativistic treatment of the thermally averaged cross-section, and the generalization of [8] to the case of coannihilation. However we have improved our method for solving the density evolution equation, it is now solved numerically without using the freeze-out approximation. This improvement has not impaired the speed of the calculation.

The other main improvement to `micrOMEGAs1.3` is the use of loop corrected superparticle masses and mixing matrices. These masses and mixing matrices, as specified in the *SUSY Les Houches Accord* (SLHA)[9], are then used to compute exactly all annihilation/coannihilation cross-sections. This can be done whether the input parameters are specified at the weak scale or at the GUT scale in the context of SUGRA models or the like. In the last case, loop corrections are obtained from one of the public codes which calculate the supersymmetric spectrum using renormalization group equations (RGE) [10, 11, 12, 13]. These corrections to masses are critical for a precise computation of the relic density in two specific regions: the coannihilation region and the region where annihilation through a Higgs or Z exchange occurs near resonance. Note that these regions of the supersymmetric parameter space are among the ones where one predicts sufficiently high annihilation rates for the neutralino LSP to meet the WMAP upper bound on the relic density. In the first case, the critical parameter is the NLSP-LSP mass difference, in the latter the mass difference $2M_{LSP} - m_{H/Z}$ [14]. The Higgs masses are calculated either by one of the RGE codes or with `FeynHiggsFast`[15]. When annihilation occurs near a Higgs resonance, higher-order corrections to the width also need to be taken into account. As in `micrOMEGAs`1.1, QCD corrections to Higgs partial widths are included, furthermore we have added the important SUSY corrections, the Δm_b correction, that are relevant at large $\tan\beta$. These higher-order corrections also affect directly the Higgs-$q\bar{q}$ vertices and are taken into account in all the relevant annihilation cross-sections.

Besides the relic density measurement, other direct or indirect precision measurements constrain the supersymmetric models. In our package we calculate the supersymmetric contribution to $(g-2)_\mu$ and to $\Delta\rho$. We also include a new calculation of the supersymmetric contribution to $B_s \to \mu^+\mu^-$ and an improved calculation of the $b \to s\gamma$ decay rate. The latter includes an improved NLO for the SM and the charged Higgs contribution as well as the SUSY large $\tan\beta$ effects beyond leading-order, the Δm_b correction. The $b \to s\gamma$, $B_s \to \mu^+\mu^-$ or $(g-2)_\mu$ routines can be replaced or used as a stand-alone code.

Within `micrOMEGAs1.3` all (co-)annihilation cross-sections are compiled by `CalcHEP` [16] which is included in the package. `CalcHEP` is an automatic program to calculate tree-level cross-sections for any process in a given model, here the MSSM. We provide a code that performs the calculation of cross-sections and decay widths that can be called independently of the relic density calculation. The input parameters are the parameters of the *SUSY Les Houches Accord*. Another new feature is the possibility to call `CalcHEP`, directly from a `micrOMEGAs1.3` session, and calculate interactively cross-sections for any process in the MSSM or in mSUGRA models. For this, all widths of supersymmetric particles are evaluated automatically at tree-level, including the available two-body decay modes. The relic density as well as other constraints are also calculated in the `CalcHEP` session.

In summary, the new program `micrOMEGAs1.3`

- Calculates complete tree-level matrix elements for all subprocesses.
- Includes all coannihilation channels, in particular channels with neutralinos, charginos, sleptons, squarks and gluinos.
- Calculates the relic density for any LSP, not necessarily the lightest neutralino.
- *Deals with two sets of input parameters: parameters of the MSSM understood to be specified at the EWSB scale or parameters of the SUGRA model specified at the GUT scale. Both mSUGRA or non-universal SUGRA models are included.
- *Provides an interface with the main codes to calculate the supersymmetric spectrum: `Suspect` [10], `Isajet` [13], `Spheno` [12] and `SOFTSUSY` [11].
- *Includes an interface with the *SUSY Les Houches Accord* [9] for supersymmetric model specifications and input parameters. This gives a lot of flexibility as any model for which the MSSM spectrum is calculated by an external code can be incorporated easily.

- *Includes loop corrected sparticle masses and mixing matrices.
- *Includes loop-corrected Higgs masses and widths. QCD corrections to the Higgs couplings to fermion pairs are included as well as, via an effective Lagrangian, the Δm_b correction relevant at large $\tan\beta$.
- *Provides exact numerical solution of the Boltzmann equation by Runge-Kutta.
- Outputs the relative contribution of each channel to $1/\Omega$
- *Computes cross-sections for any $2 \to 2$ process at the parton level.
- *Calculates decay widths for all particles at tree-level including all $1 \to 2$ decay modes.
- *Calculates NLO corrections to $b \to s\gamma$.
- *Calculates constraints on MSSM: $(g-2)_\mu$, $\Delta\rho$, $B_s \to \mu^+\mu^-$.
- *Supports both C and `Fortran`.
- Performs rapidly the relic density calculation, the limiting factor in the execution time of the program is the computation of the supersymmetric spectrum.

New features in the list above are denoted by a star. In this paper we emphasize mainly the new features of our package, full details can be found in the original reference [6]. In Section 2, we describe the main changes to our calculation of the relic density. We then give the parameters of the supersymmetric model used in our package. A description of the package follows in Section 4. Section 5 gives instructions for running the program as well as sample sessions. Finally in Section 6 we compare our results with those of `DarkSUSY`4.0 [17], the other public package that computes the relic density of supersymmetric dark matter.

2 Calculation of the relic density

The most complete formulae for the calculation of the abundance $Y(T)$ were presented in [7, 8] and we will follow their approach rather closely. The evolution equation for the abundance, defined as the number density divided by the entropy density, writes

$$\frac{dY}{dT} = \sqrt{\frac{\pi g_*(T)}{45}} M_p < \sigma v > (Y(T)^2 - Y_{eq}(T)^2) \tag{2.1}$$

where g_* is an effective number of degree of freedom [7], M_p is the Planck mass and $Y_{eq}(T)$ the thermal equilibrium abundance. $<\sigma v>$ is the relativistic thermally averaged annihilation cross-section of superparticles summed over all channels,

$$<\sigma v>=\frac{\sum\limits_{i,j} g_i g_j \int\limits_{(m_i+m_j)^2} ds\sqrt{s}K_1(\sqrt{s}/T)p_{ij}^2\sigma_{ij}(s)}{2T\big(\sum\limits_i g_i m_i^2 K_2(m_i/T)\big)^2}\,, \tag{2.2}$$

where g_i is the number of degree of freedom, σ_{ij} the total cross section for annihilation of a pair of supersymmetric particles with masses m_i, m_j into some Standard Model particles, and $p_{ij}(\sqrt{s})$ is the momentum (total energy) of the incoming particles in their center-of-mass frame.

Integrating Eq. 2.1 from $T=\infty$ to $T=T_0$ leads to the present day abundance $Y(T_0)$ needed in the estimation of the relic density,

$$\Omega_{LSP}h^2=\frac{8\pi}{3}\frac{s(T_0)}{M_p^2(100(\mathrm{km/s/Mpc}))^2}M_{LSP}Y(T_0)=2.742\times10^8\frac{M_{LSP}}{GeV}Y(T_0) \tag{2.3}$$

where $s(T_0)$ is the entropy density at present time and h the normalized Hubble constant. The present-day energy density is then simply expressed as $\rho_{LSP}=10.54\Omega h^2(GeV/m^3)$.

Let us rewrite Eq. 2.1 in terms of $X=T/M_{LSP}$

$$\frac{dY}{dX}=A(X)(Y_{eq}(X)^2-Y(X)^2) \tag{2.4}$$

$$A(X)=\frac{M_{LSP}}{X^2}\sqrt{\frac{\pi g_*(M_{LSP}/X)}{45}}M_p<\sigma v> \tag{2.5}$$

First note that one will always have $Y(X)\approx Y_{eq}(X)$ when $A(X)Y_{eq}(X)\gg 1$. This is the case at $X\leq 1$ since the equilibrium abundance $Y_{eq}(X)\approx\mathcal{O}(1)$ [7, 8] and for a typical electroweak cross-section , $<\sigma v>\approx 10^{-10}GeV^{-2}$, and LSP mass, $M_{LSP}\approx 100GeV$, one has $A\approx 10^{10}$. Choosing a starting point for the solution of the numerical equation at small X will rapidly return the solution $Y=Y_{eq}$. On the other hand when $X>1$, $Y_{eq}(X)$ decreases exponentially as e^{-X}. Then neglecting the dependence on X in both $A(X)$ and $Y_{eq}(X)e^X$ we get

$$\Delta Y=Y(X)-Y_{eq}(X)=\frac{1}{2A} \tag{2.6}$$

where $\Delta Y\ll Y_{eq}$. In this approximation, ΔY does not depend on X, whereas $Y_{eq}(X)$ decreases exponentially. This can be used to find a starting point X_{f_1} for the numerical solution of the differential equation (2.4). In order to find X_{f_1} where $\Delta Y(X_{f_1})=\delta\ \ Y_{eq}(X_{f_1})$ one can solve

$$Y_{eq}(X_{f_1})'=A(X_{f_1})*Y_{eq}(X_{f_1})^2\delta(\delta+2) \tag{2.7}$$

In the `darkOmega` routine we use this equation to find X_{f_1}, $Y(X_{f_1})$ corresponding to $\delta = 0.1$ and solve the differential equation (2.4) by the Runge-Kutta method starting from this point [18]. We stop the Runge-Kutta run at point X_{f_2} where

$$Y_{eq}(X_{f_2}) < \frac{1}{10} Y(X_{f_2}) \quad . \tag{2.8}$$

Then we integrate Eq. 2.4 neglecting the term $Y_{eq}(X)$

$$\frac{1}{Y(X_0)} = \frac{1}{Y(X_{f_2})} + \int_{X_{f_2}}^{X_0} A(X) dX \quad . \tag{2.9}$$

Note that the temperature $T_0 = 2.725K$ corresponds to $X_0 \approx 10^{14}$. Thus without loss of precision we can set $X_0 = \infty$ for evaluating Y_0 since $A(X) \propto 1/X^2$.

Another routine `darkOmegaFO` performs the calculation in the *freeze-out* approximation[2]. Here we choose $\delta = 1.5$ as in Ref.[7] and omit the Runge-Kutta step ($X_f = X_{f_1} = X_{f_2}$). The precision of this approximation is about 2% although in some exotic cases the approximation works badly.

As in `micrOMEGAs`1.1, we include in the thermally averaged cross-section , Eq. 2.2, only the contribution of processes for which the Boltzmann suppression factor, B, is above some value B_ϵ

$$B = \frac{K_1((m_i + m_j)/T)}{K_1(2m_{LSP}/T)} \approx e^{-X\frac{(m_i+m_j-2m_{LSP})}{m_{LSP}}} > B_\epsilon \tag{2.10}$$

where m_i, m_j are the masses of the incoming superparticles. The recommended value is $B_\epsilon = 10^{-6}$[6].

In our program we provide two options to do the integrations, the *fast* one and the *accurate* one. The *fast* mode already gives a precision of about 1% which is good enough for all practical purposes. The *accurate* mode should be used only for some checks. In the *accurate* mode the program evaluates all integrals by means of an adaptative Simpson program. It automatically detects all singularities of the integrand and checks the precision. In the case of the *fast* mode the accuracy is not checked. We integrate the squared matrix elements over the scattering angle by means of a 5 points Gauss formula. For integration over s, Eq. 2.2, we use a restricted set of points which depends whether we are in the vicinity of a s-channel Higgs/Z/W resonance or not. We increase the number of points if the Boltzmann factor corresponding to m_{pole} is larger than $0.01 B_\epsilon$.

2.1 Decays of the Higgs scalars

When the LSP is near a Higgs resonance, it annihilates very efficiently. The value of the neutralino annihilation cross-section depends on the total width if this width is larger

[2]This function was used in the original version of `micrOMEGAs`[6].

than $\approx T_f/X_f$, the freeze-out temperature. This is usually the case for large Higgs masses of 1TeV especialy at large $\tan\beta$ due to the enhancement in the $b\bar{b}$ channel. However the width of $h(H,A) \to b\bar{b}$ receives important QCD corrections. Typically for the heavy Higgses ($m_H > 1$TeV) the partial width into $q\bar{q}$ can vary easily by a factor of 2 from the tree-level prediction, due mostly to the running of the quark mass at high scale. To take these corrections into account we have redefined the vertices $hq\bar{q}, Hq\bar{q}$ and $Aq\bar{q}$ using an effective mass that reproduces the radiatively corrected Higgs decays [19]. The effective mass at the scale Q writes

$$M_{eff}^2(Q) = M(Q)^2\left[1 + 5.67a + (35.94 - 1.36n_f)a^2 + (164.14 - n_f(25.77 - 0.259n_f))a^3\right] \tag{2.11}$$

where $a = \alpha_s(Q)/\pi$, the scale of the reaction is set to $Q = 2m_{\tilde{\chi}_1^0}$, $M(Q)$ and $\alpha_s(Q)$ are the quark masses and running strong coupling in the $\overline{MS}$-scheme. We use NNLO expressions for the strong coupling constants [20] and for the running quark masses [19, 21]. The relation between the $\overline{MS}$ and the pole quark masses are implemented at three-loops [20, 21]. These are relevant for the top quark, since we use the pole mass as input following the *SUSY Les Houches Accord* [9]. For b-quark, although $m_b(m_b)^{\overline{MS}}$ is the input parameter, it is still necessary to compute the pole mass used as an input parameter to some of the RGE codes. We set $M_{eff}(Q) = M_{pole}$ at scales where the effective mass exceeds the value of the pole mass.

We also take into account the SUSY-QCD corrections [22] to $h, H, A \to b\bar{b}$ vertices that are important at large $\tan\beta$. Here we use the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{eff} = \sqrt{4\pi\alpha_{QED}}\frac{m_b}{1+\Delta m_b}\frac{1}{2M_W\sin\theta_W}&\left[-Hb\bar{b}\frac{\cos\alpha}{\cos\beta}\left(1+\frac{\Delta m_b\tan\alpha}{\tan\beta}\right)\right.\\ &\left.+iAb\bar{b}\tan\beta\left(1-\frac{\Delta m_b}{\tan\beta^2}\right)+hb\bar{b}\,\frac{1}{\cos\beta}\left(1-\frac{\Delta m_b}{\tan\alpha\tan\beta}\right)\right]\end{aligned} \tag{2.12}$$

where m_b is the effective b-quark mass described above, α_{QED} the electromagnetic coupling, $\tan\beta$ is the ratio of the vev's of the Higgs doublets and α is the Higgs mixing angle. Δm_b is a correction factor arising from loop contribution of SUSY particles. This factor is particularly important at large $\tan\beta$ and also contributes to $b \to s\gamma$ (all details are given in Appendix B).

In the large $\tan\beta$ case, when neutralino annihilation via s-channel Higgs exchange dominates, the inclusion of SUSY-QCD corrections can shift by about 15% the value for the relic density. There is an option to switch off this correction (see Section4.1).

The total width of the Higgs includes only the two-body final states that occur at tree-level. In the case of the light Higgs, this underestimates the width since the partial width to off-shell W or gg final states can reach 10%. However an accurate value for this very narrow width has in general not a strong impact on the relic density. On the other hand a precise value for the heavy Higgs width is necessary.

2.2 Neutralino "width"

We assume that the LSP is stable because of R-parity conservation, however it is necessary to introduce a width for this stable particle in order to avoid infinities in some processes. For example, in the coannihilation process like $\tilde{e}_L \chi_1^0 \to eX$ via t-channel exchange of χ_1^0 an infinity is caused by the pole in the propagator, this is due to the fact that one can have a real decay $\tilde{e}_L \to e\tilde{\chi}_1^0$. We assign a value of `sWidth` $\cdot M_{LSP}$ to the width of all supersymmetric particles . The default value for the variable `sWidth` is 0.01.

2.3 Loop corrections to the MSSM spectrum.

In the mSUGRA model, but also in the more general MSSM, annihilation of the LSPs near a Higgs or Z resonance and/or coannihilation processes are often the dominant reactions in models where $\Omega h^2 \approx 0.1$ [23]. For an accurate calculation of the relic density it is then very important to have the exact relations between particle masses. In particular, the direct annihilation of a pair of neutralinos ($\tilde{\chi}_1^0$) depends sensitively on the mass difference with the Higgs or Z, $2m_{\tilde{\chi}_1^0} - M_{H/Z}$, when the annihilation occurs near the resonance. Furthermore coannihilation processes depend strongly on the NLSP-LSP mass difference.

In this new version of `micrOMEGAs1.3`, we provide an option to calculate loop corrections to all sparticle masses [3]. Within the MSSM defined at the EWSB scale, loop corrections are implemented by a call to `Suspect`[10], within the SUGRA or other model defined at the GUT scale, the loop corrections are done by any of the four public codes (`Suspect` ,`SOFTSUSY` , `Spheno` ,`Isajet`) for calculating the supersymmetric spectrum based on renormalization group equations. Because it is a mass difference rather than the absolute mass that has a large impact on the prediction of the relic density, even radiative corrections at the percent level, such as is often the case for neutralinos, need to be taken into account. Indeed large shifts in the prediction of the relic density between tree-level and loop-corrected masses can be found. Typically the prediction for the relic density can change by 20%, but in some scenarios corrections can reach 100% or even more. We not only use the loop-corrected sparticle masses but also the corresponding mixing matrix elements. In this way we take into account some of the loop corrections in the evaluation of the matrix elements for different processes. This however means, since

[3] Pole masses in the calculation of the relic density were first used in Ref. [24]

Table 1: Standard Model parameters

name	default	definition
AlfEMZ	0.00781653	electromagnetic coupling $\alpha_{em}(M_Z)$
AlfSMZ	0.1172	strong coupling, $\alpha_s^{\overline{MS}}(M_Z)$ for $n_f = 5$
SW	0.481	Weinberg angle, $\sin\theta_W$
MZ	91.1884	Z mass
Ml	1.777	tau-lepton pole mass
Mtp	175.0	t-quark pole mass
MbMb	4.23	$\overline{MS}$ scale independent b-quark mass Mb(Mb)

it is only a partial implementation of loop corrections, that theoretical inconsistencies in the model could occur, in particular problems with unitarity violation in some processes. This would mainly show up in processes with production of gauge particles, however at much higher energies that are typically involved in the LSP annihilation processes.

3 The MSSM parameters.

In our package, we compute various matrix elements and cross-sections for $2 \to 2$ processes within the framework of the MSSM. The model file corresponding to the specific implementation of the MSSM was derived with LanHEP[25], a program that generates the complete set of particles and vertices once given a Lagrangian [26, 27]. Names are attached to the parameters of the MSSM, including those of the SM, and their values can be set with an instruction. For example, the command `assignVal("Mtp",180.)` assigns the value $m_t = 180$GeV to the pole mass of the t-quark.

The list of parameters of the Standard Model and their default values is presented in Table 1. All quarks and leptons of the first two generations are assumed massless. The default values for the electromagnetic coupling and the Weinberg angle correspond to the values in the $\overline{MS}$ scheme at the M_Z scale.

The parameters of the MSSM are described in Table 2. We follow the conventions of the *SUSY Les Houches Accord* [9]. The masses of the third generation fermions are ordered, for example $m_{\tilde{t}_1}$ corresponds to the lightest top-squark. In this list, the number of parameters exceeds the number of MSSM independent parameters. They correspond to physical parameters, masses and mixings. This extended set of parameters is however necessary when one wants to use effective masses and vertices that include loop corrections. Our computation of matrix elements for cross-sections is based on this set of parameters. Note that the trilinear muon coupling, A_μ, is added to the parameter list even though it does not contribute to matrix elements or to the spectrum since the muon is assumed to be massless. This parameter is however important for evaluating the muon anomalous magnetic moment.

Table 2: MSSM parameters of the SUSY Les Houches Accord

name	comment	name	comment
tb	$\tan\beta$	MSnl	τ-sneutrino mass
alpha	Higgs α angle	MSe$\frac{L}{R}$	masses of left/right selectrons
mu	Higgs μ parameter	MSm$\frac{L}{R}$	left/right smuon masses
Mh	Mass of light Higgs	MSli	i=1,2 masses of light/heavy $\tilde{\tau}$
MH3	Mass of CP-odd Higgs	MSu$\frac{L}{R}$	masses of left/right u-squarks
MHH	Mass of Heavy Higgs	MSs$\frac{L}{R}$	masses of left/right s-squarks
MHc	Mass of charged Higgs	MSti	i=1,2 masses of light/heavy t-squarks
Al	$\tilde{\tau}$ trilinear coupling	MSd$\frac{L}{R}$	masses of left/right d-squarks
Am	$\tilde{\mu}$ trilinear coupling	MSc$\frac{L}{R}$	masses of left/right c-squarks
Ab	$\tilde{b}$ trilinear coupling	MSbi	i=1,2 masses of light/heavy b-squarks
At	$\tilde{t}$ trilinear coupling	Zn$_{ij}$	i,j=1,..,4; neutralino mixing matrix
MNEi	i=1,2,3,4; neutralino masses	Zu$_{ij}$	i=1,2;j=1,2; chargino U mixing matrix
MCi	i=1,2 chargino masses	Zv$_{ij}$	i=1,2;j=1,2; chargino V mixing matrix
MSG	mass of gluino	Zl$_{ij}$	i=1,2;j=1,2; $\tilde{\tau}$ mixing matrix
MSne	e-sneutrino mass	Zt$_{ij}$	i=1,2;j=1,2; $\tilde{t}$ mixing matrix
MSnm	μ-sneutrino mass	Zb$_{ij}$	i=1,2;j=1,2; $\tilde{b}$ mixing matrix

The values of the SLHA parameters can either be set by an external program, here a call to one of the RGE codes that calculate the supersymmetric spectrum, or by specifying the MSSM parameters at the weak scale. In either case one needs to specify a set of *independent* parameters as described below.

3.1 Input parameters at the GUT scale

Within the context of the SUGRA scenario for supersymmetry breaking the MSSM parameters can be evaluated at the weak scale starting from a set of scalar masses, gaugino masses, trilinear couplings defined at the GUT scale. The GUT scale input parameters are listed in Table 3. Only one parameter, $\tan\beta$, is defined at M_Z. We implicitly assume that the first two generations are identical. The parameters for the mass of the Higgs doublet can be entered with a negative sign, in this case they will be understood as $M^2_{H_U} = -|M_{H_U}|^2$

We treat the mSUGRA model as a special case of the general SUGRA. Since simplifying relations are imposed on masses and couplings, in the mSUGRA model one has to specify only a small number of input parameters at the GUT scale: $M_0, M_{1/2}, A_0, \tan\beta, sgn(mu)$. These correspond to `m0` $= Mli = Mri = Mqi = Mui = Mdi = MHu = MHd$

- common scalar mass at GUT scale;

`mhf`$= MG1 = MG2 = MG3$ - common gaugino mass at GUT scale;

`a0`$=At = Ab = Al$ - trilinear soft breaking parameter at GUT scale;

`tb`- $\tan\beta$ or the ratio of vacuum expectation values at MZ;

`sgn`- $+/-1$, sign of μ, the Higgsino mass term.

Four different routines read the parameters of Table 2 and pass them to the corresponding packages that solves the RGE equations and calculate the MSSM masses and mixing matrices. The routines `suspectSUGRA` [10],`softsusySUGRA` [11], `sphenoSUGRA` [12], `isajetSUGRA` [13] are described in section4.1. Note that some of the standard parameters of Table 1 also play a role in the low energy boundary conditions implemented in the RGE codes. They are passed to RGE routines implicitly. We assume that the second generation is identical to the first one and only parameters of the first generation are used.

Table 3: Independent GUT-scale parameters.

name	comment	name	comment
tb	$\tan\beta$ (at M_Z)	Ml1	Left-handed slepton mass for $1^{st}/2^{nd}$ gen.
At	$\tilde{t}$ trilinear coupling	Ml3	Left-handed slepton mass for 3^{rd} gen.
Ab	$\tilde{b}$ trilinear coupling	Mr1	Right-handed slepton mass for $1^{st}/2^{nd}$ gen.
Al	$\tilde{\tau}$ trilinear coupling	Mr3	Left-handed slepton mass for 3^{rd} gen.
MG1	U(1) Gaugino mass	Mq1	Left-handed squark mass for $1^{st}/2^{nd}$ gen.
MG2	SU(2) Gaugino mass	Mq3	Left-handed squark mass for 3^{rd} gen.
MG3	SU(3) Gaugino mass	Mu1	Right-handed u-squark mass for $1^{st}/2^{nd}$ gen.
sgn	sign of μ at the EWSB scale	Mu3	Right-handed u-squark mass for 3^{rd} gen.
MHu	Mass of first Higgs doublet	Md1	Right-handed d-squark mass for $1^{st}/2^{nd}$ gen.
MHd	Mass of second Higgs doublet	Md3	Right-handed d-squark mass for 3^{rd} gen.

3.2 Input parameters at the weak scale

The parameters of the *SUSY Les Houches Accord* can also be calculated starting from the set of *independent* MSSM parameters at the EWSB scale[4] listed in Table 3[26]. This can be done either at tree-level or with loop corrections (see Section 4.1). The names of the independent parameters of the MSSM are identical to the GUT scale parameters safe for `MHu,MHd` which are conveniently replaced by μ and M_A(MH3). Furthermore at the EWSB scale one must define the sfermion masses for all three generations. Here MH3 and MG3 are the pole masses of the CP-odd Higgs and of the gluino. All other parameters

[4]This set of parameters was used in the previous version of `micrOMEGAs`[6].

are treated as running ones. When evaluating loop corrections to pole masses starting from the independent set of parameters, it is assumed that the parameters are specified in the $\overline{DR}$ scheme at the EWSB scale, $Q = \sqrt{m_{\tilde{t}_1} \cdot m_{\tilde{t}_2}}$.

Table 4: Set of independent MSSM parameters at the weak scale.

name	comment	name	comment
tb	$\tan\beta$	MG3	SU(3) Gaugino mass (gluino mass)
mu	Higgs μ parameter	Mli	Left-handed slepton mass for i^{th} generation
At	$\tilde{t}$ trilinear coupling	Mri	Right-handed selectron mass for i^{th} generation
Ab	$\tilde{b}$ trilinear coupling	Mqi	Left-handed squark mass for i^{th} generation
Al	$\tilde{\tau}$ trilinear coupling	Mui	Right-handed u-squark mass for i^{th} generation
Am	$\tilde{\mu}$ trilinear coupling	Mdi	Right-handed d-squark mass for i^{th} generation
MG1	U(1) Gaugino mass	MH3	Mass of Pseudoscalar Higgs
MG2	SU(2) Gaugino mass		

Two options are available to specify the weak scale MSSM parameters, either from a file using the function `ewsbInitFile` or directly as argument of the function `ewsbMSSM`. Either option will evaluate the supersymmetric spectrum at tree-level or to one-loop according to the value of the parameter `LCOn`, see section 4.1.

After evaluation of the spectrum in the context of the SUGRA or MSSM models, the function `calcDep` chooses the lightest supersymmetric particles and calculates the running masses of quarks at the LSP scale as well as various widths.

4 Functions of `micrOMEGAs`

The routines presented below belong to the `micromegas.a` library. They are available both in the C and Fortran versions. If for some reason a Fortran call differs from the C one, we present the Fortran version in brackets "[ ]". The types of the functions and their arguments are specified in Appendix A. Examples of implementation are presented in Section 5.4. Note that after assignments of the MSSM parameters the user has to call the initialization procedure `calcDep` (Sec. 4.1). Other routines of the package can only be used after making this call.

4.1 Variable assignment and spectrum calculation

•`assignVal(name,val)`
changes values of the parameters. `name` is one of the names presented in Tables 1,2, `val` is the value to be assigned. The function returns 0 when it successfully recognizes the

parameter name and 1 otherwise.

•`assignValW(name,val)`
the same routine as `assignVal`, instead of returning an error code it writes a warning on the screen.

•`suspectSUGRA(tb,MG1,MG2,MG3,Al,At,Ab,sgn,MHu,MHd,Ml1,Ml3,Mr1,Mr3,Mq1,Mq3, Mu1,Mu3,Md1,Md3)`
calculates the values of the MSSM parameters in the SUGRA scenario using the `Suspect` package. Returns 0 when the spectrum is computed succesfully, 1 in case of non-fatal problems (see the manual for the meaning of non-fatal errors [10]), and (-1) if no solution to RGE can be found for a given set of boundary conditions. This routine assigns values for the parameters in Table 2. The result depends on the input values of the SM parameters, in particular on the quark masses, $m_t^{pole}, m_b(m_b)$ (`Mtp`, `MbMb`) and on the strong coupling constant $\alpha_s(M_Z)$ (`AlfSMZ`). These parameters play a role in the low energy boundary conditions and are passed implicitly.

•`softSusySUGRA(tb,MG1,MG2,MG3,Al,At,Ab,sgn,MHu,MHd,Ml1,Ml3,Mr1,Mr3,Mq1,Mq3, Mu1,Mu3,Md1,Md3)`
same as above for `SOFTSUSY`.

•`sphenoSUGRA(tb,MG1,MG2,MG3,Al,At,Ab,sgn,MHu,MHd,Ml1,Ml3,Mr1,Mr3,Mq1,Mq3, Mu1,Mu3,Md1,Md3)`
same as above for `Spheno`.

•`isajetSUGRA(tb,MG1,MG2,MG3,Al,At,Ab,sgn,MHu,MHd,Ml1,Ml3,Mr1,Mr3,Mq1,Mq3, Mu1,Mu3,Md1,Md3)`
same as above for `Isajet`. This function depends only on m_t^{pole}, other SM parameters, and in particular $m_b(m_b)$ and α_s, are fixed internally. `Isajet` does not calculate the trilinear muon coupling, we use the approximate relation for mSUGRA models, $A_\mu = A_0 - 0.7M_{1/2}$.

Note that only the `Suspect` code is included in our package. Other codes should be installed independently by the user and linked to `micrOMEGAs` as explained in Section 5.1.

• `ewsbMSSM(tb,MG1,MG2,MG3,Am,Al,At,Ab,MH3,mu,Ml1,Ml2,Ml3,Mr1,Mr2,Mr3, Mq1,Mq2,Mq3,Mu1,Mu2,Mu3,Md1,Md2,Md3,LCOn)`
calculates the supersymmetric spectrum at tree-level or one-loop from the set of independent MSSM parameters at the EWSB scale as specified by the parameter `LCOn`. The Higgs sector parameters, masses and mixing angle α, are calculated with `FeynHiggsFast` [15].

`LCOn=0` - tree level formulae for super particles masses;

`LCOn=1` - `Suspect` is used to evaluate loop corrections to masses of super particles.

•`ewsbInitFile(filename,LCOn)`

reads the input file `filename` which specifies the set of independent MSSM parameters at the EWSB scale and calculates the supersymmetric spectrum at tree-level or one-loop as set by the parameter `LCOn` (same as above).

The function returns:

`0` - when the input has been read correctly;

`-1` - if the file does not exist or can not be opened for reading;

`-2` - if some parameter from Table 4 is missing as displayed on the sceen;

`-3` - if the spectrum cannot be calculated;

`n` - when the line number `n` has been written in the wrong format.

For example, the correct format of a line is

```
MG3 1500.
```

•`readLesH(filename,LE)`
reads the input file in the *SUSY Les Houches Accord* format [9]. If `LE=1` the SM parameters of Table 1 as well as $\tan\beta$ are also read from a SLHA output file.

•`calcDep(dMbOn)`
initializes internal parameters for subsequent calculations. In particular, the running masses of quarks, the strong coupling constant as well as the widths of gauge bosons, Higgses and superparticles. Running parameters are evaluated at the LSP scale. This routine also sorts the superparticles and selects the LSP. The parameter `dMbOn`$= 0$ switches off SUSY-QCD corrections, Δm_b, see Section 2.1.

4.2 Display of parameters.

•`findVal(name,&val) [findVal(name,val) ]`
assigns to the variable `val` the value of the parameter `name`. It returns `zero` if such variable indeed exists and `1` otherwise. This function can be applied to any of the parameters in Table 1,2 as well as to particle masses and widths specified in Tables 5,6,7.

•`findValW(name)`
returns the value corresponding to the variable `name`. If `name` is not defined `findValW` writes a warning on the screen.

•`printVar(file,N) [ printVar(N) ]`
prints the first N records of the full list of model parameters. The first 7 parameters correspond to Table 1, the following 75 parameters correspond to the list in Table 2. To see the parameters on the screen, substitute `file=stdout`. In the Fortran version, only display on the sceen is possible.

•`printMasses(file,sort) [ printMasses(sort) ]` prints into the file the masses of the supersymmetric particles as well as all Higgs masses and widths. The Fortran version writes down on the screen. If sort$\neq 0$, the masses are sorted in increasing order.

•`lsp()` [`lsp(name)`]
returns the name of the LSP. The relic density can be calculated with any particle being the LSP even though only the neutralino and the sneutrino can be dark matter candidates. If the user wants to impose a specific LSP, the nature of the LSP must be checked after calling `calcDep`.

•`lspmass_()` [`lspmass()`]
returns the mass of the lightest supersymmetric particle in GeV.

4.3 Calculation of relic density.

•`darkOmega(&Xf,fast,Beps)` [`darkOmega(Xf,Fast,Beps)`]
This is the basic function of the package which returns the relic density Ωh^2 (Eq. 2.3). The procedure for solving the evolution equation using Runge-Kutta was described in Section 2. The value of the freeze-out parameter `Xf` is returned by the function and equals $(X_{f_1} + X_{f_2})/2$, (see the definition in Eq. 2.7, 2.8). The parameter `Beps` defines the criteria for including a given channel into the sum for the calculation of the thermally averaged cross-section, Eq. 2.10; 10^{-6} is the recommended value.

If `fast=0`, we use an integration routine that increases the number of points until an accuracy of 10^{-3} is reached. If `fast=1` the accuracy is not checked, but a set of points is chosen according to the behaviour of the integrand: poles, thresholds, Boltzman suppression at large energy. The accuracy of this mode is about 1%. Finally, `fast=2` corresponds to the calculation of relic density using the widely-used approximation [5] based on the expansion in terms of velocity

$$p \cdot \sigma(p) = A + B \cdot p^2.$$

The recommended mode is `fast=1`.

If some problem is encountered, `darkOmega` returns (-1).

•`darkOmegaFO(&Xf,fast,Beps)` [`darkOmegaFO(Xf,fast,Beps)`]
calculates the relic density as the function `darkOmega` described above, but using the freeze-out approximation.

•`printChannels(Xf,cut,Beps,prcnt,f)` [`printChannels(Xf,cut,Beps,prcnt)`]
prints the relative contribution to Ω^{-1} for all subprocesses for which this contribution exceeds the value chosen for `cut`. If `prcnt=1` the contribution is given in percent, otherwise

the absolute value is displayed. It is assumed that the `Xf` parameter was first evaluated by `darkOmega`. In the C version, the output is directed to the file `f`, the Fortran version writes on the screen. Actually this routine evaluates the partial contributions to the integral of Eq. 2.9 without the $1/Y_f$ term and returns the corresponding value for Ωh^2 .

4.4 Routines for constraints.

•`deltarho_() [ delrho()]`
calculates, by a call to a `Suspect` routine, the $\Delta\rho$ parameter which describes the MSSM corrections to electroweak observables. It contains stop/sbottom contributions, as well as the two-loop QCD corrections due to gluon exchange and the correction due to gluino exchange in the heavy gluino limit [28]. Precise measurements of SM electroweak observables allow to set the limit $\Delta\rho < 2 \cdot 10^{-3}$.

•`bsgnlo_() [ bsgnlo() ]`
returns the value of the branching ratio for $b \to s\gamma$. For $b \to s\gamma$ we have improved on the results of [29] by including some very recent new contributions beyond the leading order that are especially important for high $\tan\beta$. Full details can be found in Appendix B.

•`bsmumu_() [ bsmumu() ]`
returns the MSSM contribution to $B_s \to \mu^+\mu^-$. Our calculation is based on [30] and agrees with [33]. It includes the loop contributions due to chargino, sneutrino, stop and Higgs exchange. The Δm_b effect relevant for high $\tan\beta$ is taken into account. The current bound from CDF experiment at Fermilab is B.R.($B_s \to \mu^+\mu^- < 9 \times 10^{-6}$) [31] and the expected bound from RunIIa should reach B.R.($B_s \to \mu^+\mu^- < 2 \times 10^{-7}$) [32].

•`gmuon_() [ gmuon() ]`
returns the value of the supersymmetric contribution to the anomalous magnetic moment of the muon [34]. The result depends only on the parameters of the chargino/neutralino sector as well as on the smuon parameters, in particular the trilinear coupling A_μ (`Am`). Our formulas agree with [35]. The latest experimental data on the $(g-2)_\mu$ measurement using μ^-[36], brings the average to $a_\mu^{\mathrm{exp.}} = 11659208 \pm 6 \times 10^{-10}$. The quantity a_μ includes both electroweak and hadronic contributions and is still subject to large theoretical errors, the allowed range for $\delta a_\mu = a_\mu^{\mathrm{exp.}} - a_\mu^{\mathrm{theo.}}$ then has also large errors. We estimate the 3σ range to be $5.1 < \delta a_\mu \times 10^{10} < 64.1$ [37].

•`masslimits_() [ masslimits() ]`
returns a positive value and prints a WARNING when the choice of parameters conflicts with a direct accelerator limits on sparticle masses. The constraint on the light Higgs mass is not implemented and must be added by the user.

Among the routines that calculate constraints, only `masslimits` issues a warning if the chosen model gives a value outside the experimentally allowed range. All other constraints must be checked by the user.

4.5 QCD auxiliary routines.

• `alphaQCD(Q)`
calculates the running α_s at the scale `Q` in the $\overline{MS}$ scheme. The calculation is done using the `NNLO` formula in [20]. Thresholds for b-quark and t-quark are included in n_f at the scales $m_b(m_b)$ and $m_t(m_t)$ respectively. Implicit input parameters are `AlfSMZ`, `Mtp`, and `MbMb` defined in Table 1.

• `MbRun(Q), MtRun(Q)`
calculates top and bottom running masses evaluated at NNLO.

• `MbEff(Q), MtEff(Q)`
calculates effective t- and b-quark masses as in Eq. 2.11.

•`deltaMb()`
calculates the SUSY corrections to Δm_b (Appendix B).

4.6 Partial widths and cross sections

• `decay2(pName,k, out1, out2)`
calculates the decay widths (in GeV) for any $1 \to 2$ processes. The input parameters are *pName*, the name of the decaying particle and *k*, the channel number. *out1* and *out2* are the names of outgoing particles for channel *k*. If *k* exceeds the total number of channels, then *out1* and *out2* are filled as empty strings.

• `newProcess(procName, libName) [ newProcess(procName, libName, address) ]`
prepares and compiles the codes for any $2 \to 2$ reaction in the MSSM. The result of the compilation is stored in the library

`source/2-2/`*libName*`.os`.

If this library already exists, it is not recompiled and the correspondence between the contents of the library and the *procName* parameter is not checked. *libName* is also attached to the names of routines in the *libName*.so library. Therefore *libName* should not contain symbols such as $+, -, *, /,$ which are not legal as identifiers. Library names should not start with *omglib*, these are reserved for the libraries used to evaluate Ωh^2.

The process should be specified in CalcHEP notations, for example

```
"e,E->~1+,~1-"
```

Table 5: Higgs particles.

Name	symbol	mass	width	Name	symbol	mass	width
Light Higgs	h	Mh	wh	CP-odd Higgs	H3	MH3	wH3
Heavy higgs	H	MHH	wHh	Charged Higgs	H+,H-	MHc	wHc

without any blank space. One can find all symbols for MSSM particles in Tables 5,6,7. Multi-process generation is also possible using the command

```
"e,E->2*x"
```

where `x` means arbitrary final states.

The `newProcess` routine returns the *address* of the static structure with contains, for further use, the code for the processes. If the process can not be compiled, then a NULL address is returned (address[1]=0 in Fortran). `newProcess` can also return the address of a library that was already generated, for example, `newProcess("","omglib_o1_o1")` returns the address of the library for neutralino annihilation.

•`infor22(address,nsub,n1,n2,n3,n4,&m1,&m2,&m3,&m4)`
`[ infor22(address,nsub,n1,n2,n3,n4,m1,m2,m3,m4) ]`
allows to check the contents of the library produced by `newProcess`. Here *address* is the returned value of `newProcess` call and *nsub* the subprocess number. The parameters returned correspond to the names of particles for a given subprocess (*n1, n2, n3, n4*) as well as their masses (*m1, m2, m3, m4*). The function returns 2 if the *nsub* parameters exceed the limits and 0 otherwise.

• `cs22(address, nsub, P, c1, c2 , &err)`
evaluates the cross section for a given $2 \to 2$ process with center of mass momentum P(GeV). The differential cross section is integrated from $c1 < cos(\theta) < c2$ and θ is the angle between $\bar{p}_1, \bar{p}_3$ in the center-of-mass frame. If *nsub* exceeds the maximum value for the number of subprocesses then *err* contains a non zero error code.

5 Work with the `micrOMEGAs` package.

5.1 Installation and link with RGE packages.

`micrOMEGAs` can be obtained at

`http://wwwlapp.in2p3.fr/lapth/micromegas`

The name of the file downlaoded should be `micromegas_1.3.0.tar.gz`. After unpacking the file, the root directory of the package, `micromegas_1.3.0`, will be created. This directory contains the `micro_make` file, some sample main programs, a directory for the

Table 6: Names, masses and widths of supersymmetric particles.

Name	symbols		mass	width	Name	symbols		mass	width
chargino 1	~1+	~1-	MC1	wC1	mu-sneutrino	~nm		MSnm	wSnm
chargino 2	~2+	~2-	MC2	wC2	tau-sneutrino	~nl		MSnl	wSnl
neutralino 1	~o1		MNE1	wNE1	u-squark L	~uL	~UL	MSuL	wSuL
neutralino 2	~o2		MNE2	wNE2	u-squark R	~uR	~UR	MSuR	wSuR
neutralino 3	~o3		MNE3	wNE3	c-squark L	~cL	~CL	MScL	wScL
neutralino 4	~o4		MNE4	wNE4	c-squark R	~cR	~CR	MScR	wScR
gluino	~g		MSG	wSG	t-squark 1	~t1	~T1	MSt1	wSt1
selectron L	~eL	~EL	MSeL	wSeL	t-squark 2	~t2	~T2	MSt2	wSt2
selectron R	~eR	~ER	MSeR	wSeR	d-squark L	~dL	~DL	MSdL	wSdL
smuon L	~mL	~ML	MSmL	wSmL	d-squark R	~dR	~DR	MSdR	wSdR
smuon R	~mR	~MR	MSmR	wSmR	s-squark L	~sL	~SL	MSsL	wSsL
stau 1	~l1	~L1	MSl1	wSl1	s-squark R	~sR	~SR	MSsR	wSsR
stau 2	~l2	~L2	MSl2	wSl2	b-squark 1	~b1	~B1	MSb1	wSb1
e-sneutrino	~ne		MSne	wSne	b-squark 2	~b2	~B2	MSb2	wSb2

Table 7: Designations for the Standard Model particles

Name	symbols		Mass	Width	Name	symbols		Mass	Width
photon	A		0	0	tau-neutrino	nl	Nl	0	0
Z boson	Z		MZ	wZ	tau-lepton	l	L	Ml	0
W boson	W+	W-	MW	wW	s-quark	s	S	0	0
gluon	G		0	0	c-quark	c	C	0	0
electron	e	E	0	0	u-quark	u	U	0	0
muon	m	M	0	0	d-quark	d	D	0	0
e-neutrino	ne	Ne	0	0	t-quark	t	T	Mt	wt
mu-neutrino	nm	Nm	0	0	b-quark	b	B	Mb	0

source code, a directory for `CalcHEP` interactive sessions and a directory containing data files. To compile, type either

```
./micro_make
```

This command is a Unix script, which detects the operating system and its version, sets the corresponding compiler options, and compiles the code. Being launched without arguments, `micro_make` compiles only auxiliary libraries needed for relic density evaluation. Otherwise, the first argument is treated as a `C` or `Fortran` main program which should be compiled and linked with these libraries. The executable file created has the same name as the main program without the `.c`/`.f` extension.

It is interesting to investigate the relic density in the framework of some scenario of supersymmetry breaking. We rely on the public codes that evaluate the supersymmetric spectrum in the context of models defined at the GUT scale such as the mSUGRA model.

One of these packages, `Suspect` [10], is included into the `micrOMEGAs` package. We also support an interface with `SOFTSUSY` [11], `Spheno` [12] and `Isajet` [13].

To use `Isajet`, the corresponding library should be attached to the code. It can be done via the variable `EXTLIB` to be defined in the `micro_make` file. For example, to use `Isajet` located in the `~/isajet769` directory the definition should be

```
EXTLIB="$HOME/isajet769/libisajet.a"
```

If `mathlib` from CERNLIB is not included in libisajet.a it should be specified in EXTLIB, for example

```
EXTLIB="$HOME/isajet769/libisajet.a -L/cern/pro/lib -lmathlib"
```

The interface with `SOFTSUSY` and `Spheno` is realized in the framework of the *SUSY Les Houches accord*[9] by direct execution of the corresponding programs. In both cases, the user has to define in the `micro_make` file, the variables `SOFTSUSY` or `SPHENO` which identifies the directory where the corresponding executable file is located. For example,

```
SOFTSUSY=$HOME/softsusy_1.8
```

or

```
SPHENO=$HOME/SPheno2.2.0
```

To install the package, one needs initially about 20MB of disk space. As the program generates libraries for annihilation processes only at the time they are required, the total disk space necessary can double after running the program for different models as described in the next section.

5.2 Dynamic generation of matrix elements and their loading.

In order to take into account all possible processes of annihilation of superparticles into SM particles, we need matrix elements for about 2800 different subprocesses. However, for a given set of parameters, usually only a few processes contribute, other subprocesses are suppressed by the Boltzmann factor.

The `micrOMEGAs` package just after compilation does not contain the code for matrix elements. They are generated and linked in runtime when needed. To generate the matrix elements we use the CalcHEP program [16] in *batch* mode [38]. The compiled matrix elements are stored as *shared* libraries in the subdirectory

```
sources/2-2/
```

The name of the library created corresponds to the names of initial superparticles. Say, the library containing $\tilde\chi_1^0\tilde\chi_1^0$ annihilation processes is `omglib_o1_o1.so`.

On the first few calls, `micrOMEGAs` works slowly because it compiles matrix elements. After being compiled once, the code for matrix elements is stored on the disk and is accessible for all subsequent calls. Each process is generated and compiled only once.

In case several jobs are submitted simultaneously, a problem occurs when CalcHEP receives a new request to generate a matrix element when it has not completed the previous one. We delay the operation of the second program. The warning that CalcHEP is busy signals the presence of a `LOCK` file in the directory

```
sources/work/tmp
```

If for some reason this file is not removed after the CalcHEP session, the user should remove it.

The executable file generated by `micro_make` can be moved and executed in other directories. However it will always use and update the matrix elements stored in
`micromegas_1.3.0/sources/2-2`

5.3 Linking with other codes and including `micrOMEGAs` into other packages.

One can easily add other libraries to the `micrOMEGAs` package similarly to the implementation of `Isajet` described in Section 5.1. One needs to pass the library name to the linker via the EXTLIB variable defined in `micro_make`, by specifying the complete path to the library. One can include the `micrOMEGAs` package into other C, C++, or Fortran projects. The function prototypes for C and C++ projects are stored in the `sources/micromegas.h` file. All the routines of our package as well as `Suspect` and `FeynHiggsFast` routines are stored in

```
sources/micromegas.a
```

which in turn needs the functions of

```
sources/decay2.a
```

to calculate the widths. The user must pass to the linker the library that supports dynamic loading. The name of this library depends on the Unix platform. One can find this name in the `micro_make` file, it is assigned to the `LDDL` environment variable.

To attach `micrOMEGAs` to a C or C++ project, the user should make sure that the library of Fortran functions are also passed to the linker. In the `micro_make` file this library is described by the `LDF` variable.

5.4 Running `micrOMEGAs1.3`: examples.

The directory `micromegas_1.3.0` contains several examples of *main* programs. The files `sugomg.c` and `sugomg_f.f` are *main* programs for the evaluation of the relic density in the `mSUGRA` scenario.

```
./micro_make sugomg.c
```
generates the executable `sugomg` which needs 5 parameters
```
./sugomg <m0> <mhf> <a0> <tb> <sgn>
```
The `sugomg` executable also understands three additional input parameters as m_t, $m_b(m_b)$, $\alpha_s(M_Z)$. The output contains the SUSY and Higgs mass spectrum, the value of the relic density, the relative contributions of different processes to $1/\Omega$ as well as the constraints mentioned in Section 4.4. The list of necessary parameters are written on the screen when `sugomg` is called without specifying parameters.
```
./micro_make sugomg_f.f
```
compiles the corresponding Fortran code. In this case the input parameters are requested after launching the program:

```
> ./sugomg_f
Enter  m0   mhf a0   tb  sgn
>
```

By default these programs call `Suspect` for solving the RGE equations. One can easily change the RGE code by replacing the `suspectSUGRA` call by the appropriate one in `sugomg.c` or `sugomg_f.f`.

The program `s_cycle.c` performs the calculation over 10 `mSUGRA` test points [39]. Results for these points for all RGE programs mentioned in our paper are presented in the file `data/s_cycle.res`.

Finally the `omg.c` and `omg_f.f` programs evaluated the relic density in the case of the unconstrained MSSM. The input parameters are read from a text file written in the format of the `ewsbInitFile` routine. In the C-version the file should be passed as a parameter, for example
```
./omg data/data03
```
If several sets of parameters are passed to the program, the calculation will be done in a cycle. The Fortran version also works in a cycle, waiting for a file name as input and finishes after an empty line input.

The directory `data` contains 22 `"data*"` test input files for this routine. These parameter sets were chosen to check the program in special difficult cases where either strong co-annihilation and/or Higgs pole contribute significantly in relic density evaluation. Results of relic density calculation for all these 22 test points using the option when all masses are evaluated at tree-level are stored in file `data\omg.res`.

5.5 CalcHEP interactive session.

The CalcHEP [16] program used for matrix element generation is included in the `micrOMEGAs` package. The user can calculate interactively various cross sections both in the general MSSM and in SUGRA models. To realize this option the user has to move to the `calchep` subdirectory and launch

```
./calchep
```

The implementation of the MSSM and SUGRA models in `CalcHEP` is identical to the one in `micrOMEGAs` described in previous sections. There are two auxiliary parameters, `LCOn` and `dMbOn` which switch ON/OFF loop corrections to the MSSM particle spectrum and SUSY-QCD correction to $h, H, A \to b\bar{b}$ decays respectively. If `LCOn>0` or `dMbOn>0` the corresponding correction is taken into account.

The list of parameters contains also the scale parameter `Q` which should be set depending of the scale of the process under consideration. This parameter contributes to the running of α_s and to the running masses of `t` and `b` quarks. Here we use the standard $\overline{MS}$ formulae without including the higher order QCD corrections [5] presented in Section 2.1.

For the SUGRA model, all four RGE packages presented in `micrOMEGAs` can be used, `Suspect` is defined by default. External RGE packages are available for CalcHEP if they were already properly installed in the `micrOMEGAs` package as described in section 5.1. To include another RGE package one has to edit the model in `CalcHEP` (in the *Edit model* menu). The `suspectSUGRA` call should be commented in the *Constraints* menu while the line corresponding to the call for another routine should be uncommented. The symbol for comment is %. In the *Edit model* menu one can also defined the non-universal SUGRA model. By default, mSUGRA boundary conditions are implemented. To modify this, first comment the lines in the *Constraints* menu which express the GUT scale parameters in Table 3 in terms of the mSUGRA parameters. The corresponding non-universal parameters should then be introduced as new variables in the *Variables* menu.

In this realization of MSSM/SUGRA all widths of super-partners are evaluated automatically at tree-level including all `1->2` decay modes generated in the model. The relic density and other constrains mentioned in section 4.4 are included in the list of *Constrains* and automatically attached to CalcHEP numerical sessions.

In CalcHEP numerical sessions for `2->2` processes we provide an option to construct a plot for the $v \cdot \sigma$ dependence on the incoming momentum. This option is found under

[5] These corrections can be simulated by decreasing of scale Q.

the *Simpson* menu function.

5.6 Sample output file

Running `micrOMEGAs1.3` with the default values of the standard parameters and choosing the `Suspect` RGE package with the mSUGRA input parameters

```
sugomg 107 600 0 5 1
```

will produce the following output:

```
Higgs masses and widths
h   : Mh    =  116.0 (wh    =2.5E-03)
H   : MHH   =  899.2 (wHh   =1.9E+00)
H3  : MH3   =  898.5 (wH3   =2.2E+00)
H+  : MHc   =  902.0 (wHc   =2.3E+00)

Masses of SuperParticles:
~o1 : MNE1  =   249.1 || ~l1 : MSl1  =   254.2 || ~eR : MSeR  =   256.0
~mR : MSmR  =   256.0 || ~nl : MSnl  =   413.1 || ~ne : MSne  =   413.4
~nm : MSnm  =   413.4 || ~eL : MSeL  =   420.2 || ~mL : MSmL  =   420.2
~l2 : MSl2  =   420.4 || ~1+ : MC1   =   468.3 || ~o2 : MNE2  =   468.5
~o3 : MNE3  =   780.0 || ~2+ : MC2   =   793.2 || ~o4 : MNE4  =   794.3
~t1 : MSt1  =   946.7 || ~b1 : MSb1  =  1153.1 || ~b2 : MSb2  =  1187.8
~dR : MSdR  =  1188.4 || ~sR : MSsR  =  1188.4 || ~t2 : MSt2  =  1190.6
~uR : MSuR  =  1194.8 || ~cR : MScR  =  1194.8 || ~uL : MSuL  =  1248.2
~cL : MScL  =  1248.2 || ~dL : MSdL  =  1250.5 || ~sL : MSsL  =  1250.5
~g  : MSG   =  1358.1 ||
Xf=2.67e+01 Omega=8.87e-02

Channels which contribute to 1/(omega) more than 1%.
Relative contrubutions in % are displyed
  1% ~o1 ~o1 -> l L
  3% ~o1 ~l1 -> Z l
 12% ~o1 ~l1 -> A l
  2% ~o1 ~eR -> Z e
  8% ~o1 ~eR -> A e
  2% ~o1 ~mR -> Z m
  8% ~o1 ~mR -> A m
 11% ~l1 ~l1 -> l l
  2% ~l1 ~L1 -> A Z
  3% ~l1 ~L1 -> A A
  8% ~eR ~l1 -> e l
  6% ~eR ~eR -> e e
  1% ~eR ~ER -> A Z
  2% ~eR ~ER -> A A
  6% ~eR ~mR -> e m
  8% ~mR ~l1 -> m l
  6% ~mR ~mR -> m m
```

```
  1% ~mR ~MR -> A Z
  2% ~mR ~MR -> A A
deltartho=9.11E-06
gmuon=3.12E-10
bsgnlo=3.85E-04
bsmumu=3.13E-09
MassLimits OK
```

Under the same conditions and for the same set of parameters, running the cross-section and branching ratios routines

```
cs_br
```

will produce the following output:

```
Example of some cross sections and widths calculation
  for mSUGRA point m0=107.0,mhf=600.0,a0=0.0,tb=5.0

 Z partial widths
  b   B - 3.684E-01 GeV
  d   D - 3.703E-01 GeV
  u   U - 2.873E-01 GeV
  c   C - 2.873E-01 GeV
  s   S - 3.703E-01 GeV
  l   L - 8.378E-02 GeV
 nl  Nl - 1.670E-01 GeV
 nm  Nm - 1.670E-01 GeV
 ne  Ne - 1.670E-01 GeV
  m   M - 8.397E-02 GeV
  e   E - 8.397E-02 GeV
Total 2.436E+00 GeV

 h partial widths
  b   B - 2.460E-03 GeV
  l   L - 2.552E-04 GeV
Total 2.716E-03 GeV

Cross sections at Pcm=500.0 GeV
e,E->~1+,~1-
e,E->~1+(468),~1-(468)  is 7.135E-03 pb
e,E->~o1,~o2
e,E->~o1(249),~o2(468)  is 1.130E-02 pb
```

6 Results

We have compared the results obtained with `micrOMEGAs1.3` and those obtained with `DarkSUSY`4.0 for 10 benchmarks mSUGRA points[39]. For this check, we have used `Isajet`7.69, $m_t^{pole} = 174.3$GeV, $\alpha_s(M_Z) = .1172, m_b(m_b) = 4.23$GeV. The latter is only relevant for the calculation of the Higgs widths.

As seen in Table 8, the two programs agree at the 3% level except at large $\tan\beta$. This discrepancy is due to a difference in the width of the pseudoscalar. We recover good agreement with `DarkSUSY` (below 3%) if we substitute their value for the pseudoscalar width.

Table 8: Comparison between `micrOMEGAs1.3` and `DarkSUSY`4.0

name	M_0	$M_{1/2}$	A_0	$\tan\beta$	$sgn(\mu)$	`micrOMEGAs1.3`	`DarkSUSY4.0`
A	107	600	0	5	1	0.0944	0.0929
B	57	250	0	10	1	0.124	0.121
C	80	400	0	10	-1	0.117	0.115
D	101	525	0	20	1	0.0876	0.0864
G	113	375	0	20	1	0.133	0.129
H	244	935	0	20	1	0.166	0.163
I	181	350	0	35	1	0.142	0.132
J	299	750	0	35	1	0.102	0.0975
K	1001	1300	0	46	-1	0.0893	0.0870
L	303	450	0	47	1	0.114	0.0982

7 Conclusion

`micrOMEGAs1.3` solves with an accuracy at the percent level the evolution equation for the density of supersymmetric particles and calculates the relic density of dark matter. All possible channels for annihilation and coannihilations are included and all matrix elements are calculated exactly in an improved tree-level approximation that uses pole masses and loop-corrected mixing matrices for supersymmetric particles. Loop corrections to the masses of Higgs particles and to the partial widths of the Higgs (QCD and SUSY) are implemented. These higher-order corrections are essential since the annihilation cross-section can be very sensitive to the mass of the particles that contribute to the various annihilation processes, in particular near a resonance or in regions of parameter space where coannihilations occur. Furthermore, both these processes are often the dominant ones in physically interesting supersymmetric models, that is in models where the relic density is below the WMAP upper limit.

The relic density can be calculated starting from a set of MSSM parameters defined at the weak scale or at the GUT scale. We provide an interface to the four major codes that calculate the supersymmetric spectrum using renormalization group equations. Within the context of the mSUGRA model, there are still large uncertainties in the computation of the supersymmetric spectrum [42], this of course will have a strong impact on the prediction for the relic density [14]. An accurate prediction of the relic density within SUGRA models then presupposes a precise knowledge of the supersymmetric spectrum.

New features of the package also include the computation of cross-sections and decay widths for any process in the MSSM with two-body final states as well as an improved NLO calculation of the $b \to s\gamma$ branching ratio and a new routine for the $B_s \to \mu^+\mu^-$ decay rate.

8 Acknowledgements

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Appendix

A List of functions

A.1 `micrOMEGAs` functions in C.

```
int assignVal(char * name, double val)
void assignValW(char * name, double val)
int readLesH(char *fname)
int ewsbInitFile(char * fname,int LC)}
int ewsbMSSM(tb,MG1,MG2,MG3,Am,Al,At,Ab,MH3,mu,Ml1,Ml2,Ml3,Mr1,Mr2,Mr3,Mq1,Mq2,Mq3,
    Mu1,Mu2,Mu3,Md1,Md2,Md3,LC); int LC; all other parameters are 'double'
int xxxxxSUGRA(tb,MG1,MG2,MG3,Al,At,Ab,sgn,MHu,MHd,Ml1,Ml3,Mr1,Mr3,Mq1,Mq3,
                Mu1,Mu3,Md1,Md3)
```

All parameters are ‘double’. `'xxxx'` is `'suspect'`,`'isajet'`,`'softSusy'`,or `'spheno'`

```
int calcDep(int dMbOn)
int findVal(char * name, double * val)
double findValW(char * name)
void printVar(FILE *f, int N)
void printMasses(FILE * f, int sort)
char * lsp(void)
double lspmass_()
double darkOmega(double *Xf,int fast, double Beps)
double darkOmegaFO(double *Xf,int fast, double Beps)
double printChannels(double Xf, double cut, double Beps, int prcnt, FILE *f )
double deltarho_(void)
double bsgnlo_(void)
double bsmumu_(void)
double gmuon_(void)
int    masslimits_(void)
double MbRun(double Q)
double MtRun(double Q)
double MbEff(double Q)
double MtEff(double Q)
double deltaMb(void)
double decay2(char*pIn, int k, char*pOut1, char*pOut2)
void* newProcess(char* procName, char*libName)
int infor22(void*address,int nsub, char*pIn1,char*pIn2,char*pOut1,char*pOut2,
             double*m1,double*m2,double*m3,double*m4)
double cs22(void*address, int nsub, double Pcm, double c1, double c2, int*err)
double  annihilation(double v, int k, char * pOut1, char pOut2)
```

`micrOMEGAs`functions in Fortran.

```
  INTEGER FUNCTION assignVal(name,val)
  SUBROUTINE       assignValW(name,val)
  INTEGER FUNCTION readLesH(fname)
  INTEGER FUNCTION ewsbInitFile(fname,LC)
  INTEGER FUNCTION ewsbMSSM(tb,MG1,MG2,MG3,Am,Al,At,Ab,MH3,mu,Ml1,Ml2,Ml3,
         Mr1,Mr2,Mr3,Mq1,Mq2,Mq3, Mu1,Mu2,Mu3,Md1,Md2,Md3,LC)
  INTEGER FUNCTION xxxxSUGRA(tb,MG1,MG2,MG3,Al,At,Ab,sgn,MHu,MHd,
         Ml1,Ml3,Mr1,Mr3,Mq1,Mq3,Mu1,Mu3,Md1,Md3)
```

All parameters are ‘double’. `'xxxx'` is `'suspect'`,`'isajet'`,`'softSusy'`,or `'spheno'`

```
  INTEGER FUNCTION calcDep(dMbOn)
  INTEGER FUNCTION findVal(name, val)
  REAL*8  FUNCTION findValW(name)
  SUBROUTINE       printVar(n)
  SUBROUTINE       printMasses(sort)
  SUBROUTINE       LSP(name)
  REAL*8  FUNCTION lspMass()
  REAL*8  FUNCTION darkOmega(Xf,fast,Beps)
  REAL*8  FUNCTION darkOmegaFO(Xf,fast,Beps)
```

```
REAL*8  FUNCTION printChannels(Xf,cut,Beps,prcnt)
REAL*8  FUNCTION deltarho()
REAL*8  FUNCTION bsgnlo()
REAL*8  FUNCTION bsmumu()
REAL*8  FUNCTION gmuon()
INTEGER FUNCTION MassLimits()
REAL*8  FUNCTION MbRun(Q)
REAL*8  FUNCTION MtRun(Q)
REAL*8  FUNCTION MbEff(Q)
REAL*8  FUNCTION MtEff(Q)
REAL*8  FUNCTION deltaMb()
REAL*8  FUNCTION decay2(pIn, k, pOut1,pOut2)
SUBROUTINE newProcess(procName,libName,address)
INTEGER FUNCTION infor22(address,nsub, pIn1,pIn2,pOut1,pOut2,m1,m2,m3,m4)
REAL*8  FUNCTION cs22(address, nsub, Pcm, c1, c2 , ERR)
REAL*8  FUNCTION annihilation(v,k, pOut,pOut2)
```

The types of the parameters are:

```
 CHARACTER  pIn*(*),pIn1*(*),pIn2*(*),pOut1*(*),pOut2*(*),
>  name*(*),fname*(*),procName,*(*),libName(*,*)
 REAL*8   val,Xf,Beps,cut,Pcm,c1,c2,v,Q,m1,m2,m3,m4
 REAL*8   tb,MG1,MG2,MG3,Am,Al,At,Ab,MH3,mu,Ml1,Ml2,Ml3,
>       Mr1,Mr2,Mr3,Mq1,Mq2,Mq3, Mu1,Mu2,Mu3,Md1,Md2,Md3
 INTEGER    n,k,sort,prcnt,ERR,LC,dmbOn,fast, address[2]
```

B Implementation of $B(B \to s\gamma)$ in micrOMEGAs

The calculation for $B(B \to s\gamma)$ in the MSSM is quite involved and requires that one goes beyond one-loop. Most of what is described below, as implemented in micrOMEGAs, is in fact just a, unified, compendium of different contributions that have appeared in the literature. There is no claim of originality, most expressions are taken verbatim. However care has been taken in carefully checking all formulae that have appeared in the literature. This has helped, for example, identify a few misprints and typos and allowed to generalise some results. By giving the detail of the implementation, it is possible to easily modify this routine of the micrOMEGAs code in order to include future new contributions both to the SM and the MSSM. Note that we redefine in this routine many parameters used in micrOMEGAs1.3, for example the running quark masses, this routine can then be used as a stand-alone routine.

B.1 General set-up: From M_W to μ_b, QED corrections

Our implementation of the Standard Model contribution follows the work of Kagan and Neubert [43] very closely. We however include the effect of a running c quark mass heuristically so that our results take into account the latest calculations of Gambino and Misiak[44] who advocate the use of the $\overline{MS}$ charm mass, $m_c(m_b)$. The (*relevant*) operator basis is

$$\begin{aligned} O_2 &= \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L \,, \\ O_7 &= \frac{e\, m_b}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R \,, \\ O_8 &= \frac{g_s m_b}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} G_a^{\mu\nu} t_a b_R \,. \end{aligned} \tag{B.13}$$

which defines

$$H_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\, V_{ts}^* V_{tb} \sum_i C_i(\mu_b) O_i(\mu_b) \,. \tag{B.14}$$

The renormalisation scale μ_b in (Eq. B.14) is of order m_b and is usually let to vary in the range $(m_b/2, 2m_b)$. The default value in the code is m_b. Varying μ_b is one measure of the theoretical error. The branching fraction writes

$$\mathrm{B}(B \to X_s \gamma) = \frac{6\alpha}{\pi f(z_0)} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 K_{\rm NLO}(\delta) \;\times\; \mathrm{B}(B \to X_c\, e\, \bar{\nu}) \,. \tag{B.15}$$

By default we take

$$\mathrm{B}(B \to X_c\, e\, \bar{\nu}) = 0.1045 \tag{B.16}$$

The kinematical function, $f(z_0)$, is defined as

$$f(z_0) = 1 - 8z_0 + 8z_0^3 - z_0^4 - 12 z_0^2 \ln z_0 \approx 0.542 - 2.23(\sqrt{z}_0 - 0.29) \tag{B.17}$$

with $z_0 = (m_c/m_b)^2$ defined in terms of the *pole* masses, giving a value in the range,$\sqrt{z}_0 = 0.29 \pm 0.02$. For the radiative photon we take $\alpha = 1/137.036$.

The factor $K_{\rm NLO}(\delta)$ involves the photon energy cut-off parameter δ that shows up at the NLO. In `micrOMEGAs` this value is set to $\delta = 0.9$ as is generally assumed in order to describe the "total" (fake) branching ratio. With the formulae given below the code can be modified in a very straightforward way to take into account the full δ dependence. $K_{\rm NLO}(\delta)$ is decomposed in terms of the Wilson coefficients with leading (LO) and next-to-leading (NLO) contributions as

$$K_{\rm NLO}(\delta) \;=\; \sum_{\substack{i,j=2,7,8 \\ i\le j}} k_{ij}(\delta,\mu_b)\, \mathrm{Re}\Big[C_i^{(0)}(\mu_b)\, C_j^{(0)*}(\mu_b)\Big] + S(\delta)\, \frac{\alpha_s(\mu_b)}{2\pi}\, \mathrm{Re}\Big[C_7^{(1)}(\mu_b)\, C_7^{(0)*}(\mu_b)\Big]$$

$$+ S(\delta)\, \frac{\alpha}{\alpha_s(\mu_b)} \Big(2\,\mathrm{Re}\Big[C_7^{(\mathrm{em})}(\mu_b)\, C_7^{(0)*}(\mu_b) \Big] - k_{\mathrm{SL}}^{(\mathrm{em})}(\mu_b)\, |C_7^{(0)}(\mu_b)|^2 \Big) , \qquad \text{(B.18)}$$

and

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi}\, C_i^{(1)}(\mu_b) + \frac{\alpha}{\alpha_s(\mu_b)}\, C_i^{(\mathrm{em})}(\mu_b) + \ldots . \qquad \text{(B.19)}$$

The leading-order coefficients at the low scale μ_b of order m_b are given by

$$\begin{aligned}
C_2^{(0)}(\mu_b) &= \frac{1}{2}\left(\eta^{-\frac{12}{23}} + \eta^{\frac{6}{23}}\right) , \\
C_7^{(0)}(\mu_b) &= \eta^{\frac{16}{23}}\, C_7^{(0)}(m_W) + \frac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_8^{(0)}(m_W) + \sum_{i=1}^{8} h_i\, \eta^{a_i} , \\
C_8^{(0)}(\mu_b) &= \eta^{\frac{14}{23}}\left(C_8^{(0)}(m_W) + \frac{313063}{363036}\right) + \sum_{i=1}^{4} h_i^{(8)}\, \eta^{b_i} ,
\end{aligned} \qquad \text{(B.20)}$$

where $\eta = \alpha_s(m_W)/\alpha_s(\mu_b)$, and h_i, $h_i^{(8)}$ and a_i are known *numerical* coefficients [46].

$$\begin{aligned}
h_i &= (\frac{626126}{272277}, -\frac{56281}{51730}, -\frac{3}{7}, -\frac{1}{14}, -0.6494, -0.0380, -0.0186, -0.0057) \\
a_i &= (\frac{14}{23}, \frac{16}{23}, \frac{6}{23}, -\frac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456) \\
h_i^{(8)} &= (-0.9135, 0.0873, -0.0571, 0.0209) \\
b_i &= (0.4086, -0.4230, -0.8994, 0.1456)
\end{aligned} \qquad \text{(B.21)}$$

For the running of α_s between the scale M_W and μ_b we use the SM running with 5 flavours which, to a very good precision, can be implemented as:

$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{v_s(\mu)} \left(1 - \frac{116}{23} \frac{\alpha_s(M_Z)}{4\pi} \frac{\ln(v_s(\mu))}{v_s(\mu)} \right) \qquad v_s(\mu) = 1 - \frac{23}{3} \frac{\alpha_s(M_Z)}{2\pi} \ln(M_Z/\mu) \qquad \text{(B.22)}$$

The value of $\alpha_s(M_Z)$ is read in by the main code `micrOMEGAs`. For the numerical values that we quote in this note, we take the default $\alpha_s(M_Z) = 0.1185$.

The next-to-leading Wilson coefficient at μ_b, $C_7^{(1)}(\mu_b)$ is implemented according to [46],

$$\begin{aligned}
C_7^{(1)eff}(\mu_b) &= \eta^{\frac{39}{23}} C_7^{(1)eff}(M_W) + \frac{8}{3}\left(\eta^{\frac{37}{23}} - \eta^{\frac{39}{23}}\right) C_8^{(1)eff}(M_W) \\
&+ \left(\frac{297664}{14283}\eta^{\frac{16}{23}} - \frac{7164416}{357075}\eta^{\frac{14}{23}} + \frac{256868}{14283}\eta^{\frac{37}{23}} - \frac{6698884}{357075}\eta^{\frac{39}{23}} \right) C_8^{(0)}(M_W) \\
&+ \frac{37208}{4761}\left(\eta^{\frac{39}{23}} - \eta^{\frac{16}{23}}\right) C_7^{(0)}(M_W) + \sum_{i=1}^{8} (e_i \eta E(x) + f_i + g_i \eta)\eta^{a_i} ,
\end{aligned} \qquad \text{(B.23)}$$

$$
\begin{array}{lllllllll}
e_i = (& \frac{4661194}{816831}, & -\frac{8516}{2217}, & 0, & 0, & -1.9043, & -0.1008, & 0.1216, & 0.0183) \\
f_i = (& -17.3023, & 8.5027, & 4.5508, & 0.7519, & 2.0040, & 0.7476, & -0.5385, & 0.0914) \\
g_i = (& 14.8088, & -10.8090, & -0.8740, & 0.4218, & -2.9347, & 0.3971, & 0.1600, & 0.0225)
\end{array}
$$

and

$$
E(x) = \frac{x(18 - 11x - x^2)}{12(1-x)^3} + \frac{x^2(15 - 16x + 4x^2)}{6(1-x)^4} \ln x - \frac{2}{3} \ln x. \tag{B.24}
$$

The QED coefficients $C_7^{(\mathrm{em})}(\mu_b)$ and $k_{\mathrm{SL}}^{(\mathrm{em})}(\mu_b)$ are

The result for $C_7^{(\mathrm{em})}(\mu_b)$ is

$$
\begin{aligned}
C_7^{(\mathrm{em})}(\mu_b) \;\; = \;\; & \left(\frac{32}{75}\, \eta^{-\frac{9}{23}} - \frac{40}{69}\, \eta^{-\frac{7}{23}} + \frac{88}{575}\, \eta^{\frac{16}{23}} \right) C_7^{(0)}(m_W) \\
& + \left(-\frac{32}{575}\, \eta^{-\frac{9}{23}} + \frac{32}{1449}\, \eta^{-\frac{7}{23}} + \frac{640}{1449}\, \eta^{\frac{14}{23}} - \frac{704}{1725}\, \eta^{\frac{16}{23}} \right) C_8^{(0)}(m_W) \\
& - \frac{190}{8073}\, \eta^{-\frac{35}{23}} - \frac{359}{3105}\, \eta^{-\frac{17}{23}} + \frac{4276}{121095}\, \eta^{-\frac{12}{23}} + \frac{350531}{1009125}\, \eta^{-\frac{9}{23}} \\
& + \frac{2}{4347}\, \eta^{-\frac{7}{23}} - \frac{5956}{15525}\, \eta^{\frac{6}{23}} + \frac{38380}{169533}\, \eta^{\frac{14}{23}} - \frac{748}{8625}\, \eta^{\frac{16}{23}} \, .
\end{aligned} \tag{B.25}
$$

$$
k_{\mathrm{SL}}^{(\mathrm{em})}(\mu_b) = \frac{12}{23} \left(\eta^{-1} - 1 \right) = \frac{2\alpha_s(\mu_b)}{\pi} \ln \frac{m_W}{\mu_b} \, . \tag{B.26}
$$

The coefficient functions $k_{ij}(\delta, \mu_b)$ in (B.18) are given by [43]

$$
\begin{aligned}
k_{77}(\delta, \mu_b) \;\; &= \;\; S(\delta) \left\{ 1 + \frac{\alpha_s(\mu_b)}{2\pi} \left(r_7 + \gamma_{77} \ln \frac{m_b}{\mu_b} - \frac{16}{3} \right) + \left[\frac{(1-z_0)^4}{f(z_0)} - 1 \right] \frac{6\lambda_2}{m_b^2} \right\} \\
& \quad + \frac{\alpha_s(\mu_b)}{\pi}\, f_{77}(\delta) + S(\delta)\, \frac{\alpha_s(\bar{\mu}_b)}{2\pi}\, \bar{\kappa}(z) \, , \\
k_{27}(\delta, \mu_b) \;\; &= \;\; S(\delta) \left[\frac{\alpha_s(\mu_b)}{2\pi} \left(\mathrm{Re}(r_2) + \gamma_{27} \ln \frac{m_b}{\mu_b} \right) - \frac{\lambda_2}{9\, m_b^2 z_0} \right] + \frac{\alpha_s(\mu_b)}{\pi}\, f_{27}(\delta) \, , \\
k_{78}(\delta, \mu_b) \;\; &= \;\; S(\delta)\, \frac{\alpha_s(\mu_b)}{2\pi} \left(\mathrm{Re}(r_8) + \gamma_{87} \ln \frac{m_b}{\mu_b} \right) + \frac{\alpha_s(\mu_b)}{\pi}\, f_{78}(\delta) \, , \\
k_{ij}(\delta, \mu_b) \;\; &= \;\; \frac{\alpha_s(\mu_b)}{\pi}\, f_{ij}(\delta) \, ; \qquad \{i, j\} = \{2,2\},\, \{8,8\},\, \{2,8\} \, ,
\end{aligned} \tag{B.27}
$$

with the Sudakov factor

$$
S(\delta) = \exp\left[-\frac{2\alpha_s(\mu_b)}{3\pi} \left(\ln^2\delta + \frac{7}{2} \ln \delta \right) \right] \tag{B.28}
$$

and $\gamma_{77} = \frac{32}{3}$, $\gamma_{27} = \frac{416}{81}$ and $\gamma_{87} = -\frac{32}{9}$ are entries of the anomalous dimension matrix. The value of the hadronic parameter is $\lambda_2 = 0.12 \mathrm{GeV}^2$.

For these functions we deviate slightly from KN[43] in the sense that we define z_0 in terms of the pole masses in the kinematics factor and also z that differs from z_0 by the use of the $\overline{MS}$ running charm mass, $m_c(m_b)$, as advocated recently by Gambino and Misiak[44] in order to reduce the NNLO uncertainty. We then take

$$\sqrt{z} = 0.22 \pm .04 \tag{B.29}$$

everywhere else in Eq. B.27.

The other coefficients are given by

$$\begin{aligned} r_7 &= -\frac{10}{3} - \frac{8\pi^2}{9}, \qquad \mathrm{Re}(r_8) = \frac{44}{9} - \frac{8\pi^2}{27}, \\ \mathrm{Re}(r_2) &\approx -4.987 + 12.78(\sqrt{z} - 0.22), \qquad \bar{\kappa}(z) \approx 3.672 - 4.14(\sqrt{z} - 0.22) \end{aligned} \tag{B.30}$$

Note that the scale $\bar{\mu}_b$, in Eq. B.27, of relevance in semileptonic B decays is in principle different from the one in the radiative decay.

The real-gluon radiation functions $f_{ij}(\delta)$ can be coded for a general photon energy cut-off. They are taken from [43] and write as

$$\begin{aligned} f_{77}(\delta) &= \frac{1}{3}\left[10\delta + \delta^2 - \frac{2\delta^3}{3} + \delta(\delta - 4)\ln\delta\right], \\ f_{88}(\delta) &= \frac{1}{27}\left\{4Li_2(1-\delta) - \frac{2\pi^2}{3} + 8\ln(1-\delta) - \delta(2+\delta)\ln\delta \right. \\ &\qquad \left. + 7\delta + 3\delta^2 - \frac{2\delta^3}{3} - 2\left[2\delta + \delta^2 + 4\ln(1-\delta)\right] ln_{bs}\right\}, \\ ln_{bs} &= \ln\frac{m_b}{m_s} \quad \text{we take } \frac{m_b}{m_s} \simeq 50, \\ f_{78}(\delta) &= \frac{8}{9}\left[Li_2(1-\delta) - \frac{\pi^2}{6} - \delta\ln\delta + \frac{9\delta}{4} - \frac{\delta^2}{4} + \frac{\delta^3}{12}\right], \\ f_{22}(\delta) &= \frac{16}{27}\int_0^1 \mathrm{d}x\,(1-x)(1-x_\delta)\left|\frac{z}{x}G\left(\frac{x}{z}\right) + \frac{1}{2}\right|^2, \\ f_{27}(\delta) &= -3f_{28}(\delta) = -\frac{8z}{9}\int_0^1 \mathrm{d}x\,(1-x_\delta)\,\mathrm{Re}\left[G\left(\frac{x}{z}\right) + \frac{x}{2z}\right], \end{aligned} \tag{B.31}$$

where $x_\delta = \max(x, 1-\delta)$, and

$$G(t) = \begin{cases} -2\arctan^2\sqrt{t/(4-t)} & ;\ t < 4, \\ 2\left(\ln\left[(\sqrt{t} + \sqrt{t-4})/2\right] - \frac{i\pi}{2}\right)^2 & ;\ t \geq 4. \end{cases} \tag{B.32}$$

Since we will specialise to the case $\delta = 0.9$, it is more efficient to quote the corresponding values of f_{ij}, and give approximations to f_{27}, f_{22} that we use in the code:

$$
\begin{aligned}
f_{77}(0.9) &= 3.20599\,, \\
f_{88}(0.9) &= 1.31742\,, \\
f_{78}(0.9) &= 0.387341\,, \\
f_{22}(0.9) &\approx 0.107636 \;-\; 0.208484\sqrt{z} \;-\; 0.156146 z = 0.05421 - 0.2772\epsilon_z - 0.156146\epsilon_z^2\,, \\
f_{27}(0.9) &= -3 f_{28}(0.9) \approx -0.190805 \;+\; 0.948865\sqrt{z} \;-\; 0.787805 z \\
&= -0.02023 + .6020\epsilon_z - 0.7878\epsilon_z^2\,. \\
\epsilon_z &= \sqrt{z} - 0.22
\end{aligned}
\qquad (B.33)
$$

Taking $z = 0.22^2$, Eq.B.29, rather than $z = 0.29^2$ mainly affects K_{27} especially through $Re(r_2)$. Note the large coefficient of ϵ_z in $Re(r_2)$ in Eq. B.30.

B.2 Standard Model contribution

This contribution we take from [47]. We first define the functions

$$F_7^{(1)}(x) = \frac{x(7-5x-8x^2)}{24(x-1)^3} + \frac{x^2(3x-2)}{4(x-1)^4}\ln x \qquad (B.34)$$

$$F_8^{(1)}(x) = \frac{x(2+5x-x^2)}{8(x-1)^3} - \frac{3x^2}{4(x-1)^4}\ln x \qquad (B.35)$$

B.2.1 LO at M_W

We have

$$C_{7,8}^{(0)\mathrm{SM}}(\mu_W) = F_{7,8}^1(x_{tw}) \qquad (B.36)$$

where x_{tw} is defined in terms of the running top mass at the weak scale, $\mu_W = M_W$.

$$x_{tw} = \frac{\overline{m}_t^2(\mu_W)}{M_W^2}. \qquad (B.37)$$

For the NLO top-quark running mass at the scale μ_W we follow [47]

$$
\begin{aligned}
\overline{m}_t(\mu_W) &= \overline{m}_t(m_t)\left[\frac{\alpha_s(\mu_W)}{\alpha_s(m_t)}\right]^{\frac{12}{23}}\left[1 + \frac{\alpha_s(m_t)}{4\pi}\frac{\gamma_0^m}{2\beta_0}\left(\frac{\gamma_1^m}{\gamma_0^m} - \frac{\beta_1}{\beta_0}\right)\left(\frac{\alpha_s(\mu_W)}{\alpha_s(m_t)} - 1\right)\right] \\
\overline{m}_t(m_t) &= m_t\left[1 - \frac{4}{3}\frac{\alpha_s(m_t)}{\pi}\right] \quad , \quad \overline{m}_t(m_t)^2 = m_t^2\left[1 - \frac{8}{3}\frac{\alpha_s(m_t)}{\pi}\right]
\end{aligned}
\qquad (B.38)
$$

m_t is the pole mass which in this note we take as $m_t = 174.3 \pm 5.1$GeV for comparison with other authors.

$$\beta_0 = \frac{23}{3}\ ,\ \beta_1 = \frac{116}{3}\ ,\ \gamma_0^m = 8\ ,\ \gamma_1^m = \frac{1012}{9} \tag{B.39}$$

B.2.2 NLO SM

We have

$$C_{7,8}^{(1)\mathrm{SM}}(M_W) = G_{7,8}^1(x_{tw}) \tag{B.40}$$

$$\begin{aligned}
G_7(x) \;&=\; \frac{-16x^4-122x^3+80x^2-8x}{9(x-1)^4}\mathrm{Li}_2\left(1-\frac{1}{x}\right)+\frac{6x^4+46x^3-28x^2}{3(x-1)^5}\ln^2 x \\
&+\frac{-102x^5-588x^4-2262x^3+3244x^2-1364x+208}{81(x-1)^5}\ln x \\
&+\frac{1646x^4+12205x^3-10740x^2+2509x-436}{486(x-1)^4} && \text{(B.41)} \\
G_8(x) \;&=\; \frac{-4x^4+40x^3+41x^2+x}{6(x-1)^4}\mathrm{Li}_2\left(1-\frac{1}{x}\right)+\frac{-17x^3-31x^2}{2(x-1)^5}\ln^2 x \\
&+\frac{-210x^5+1086x^4+4893x^3+2857x^2-1994x+280}{216(x-1)^5}\ln x \\
&+\frac{737x^4-14102x^3-28209x^2+610x-508}{1296(x-1)^4} && \text{(B.42)}
\end{aligned}$$

B.2.3 Results and Comparisons

To check the different components of the SM part, we have also introduced the B_{ij} functions [43]. These can be very useful if one wants to introduce the effects of New Physics through the Wilson coefficients defined at the scale M_W. Dismissing any right-handed light quark operator and assuming purely real contributions, the New Physics contribution can be written as

$$C_{7,8}^{(0,1)} = x_{7,8}\, C_{7,8}^{(0,1),\mathrm{SM}}(M_W) \tag{B.43}$$

then the B_{ij} are the coefficients of the different x_i factors (linear and quadratic, and a x_i independent term). In other words, the contribution of the New Physics can be expressed as

$$B_{s\gamma}^{NLO} \;=\; B_{22} \;+\; B_{27}\, x_7 \;+\; B_{28}\, x_8 \;+\; B_{77}\, x_7^2 + \; B_{88}\, x_8^2 \;+\; B_{78}\, x_7 x_8 \tag{B.44}$$

Note the assumption in [43] that the proportionality factor is the same for the LO and NLO. In our case we define a larger set of B_{ij} by allowing

$$C_{7,8}^{(0,1)} = x_{7,8}^{0,1}\, C_{7,8}^{(0,1,)\mathrm{SM}} \tag{B.45}$$

As a check on the SM results note that we exactly recover the results for the Wilson coefficients at M_W for both $C_{7,8}$ and at both LO and NLO (this also agrees with [47]. More satisfying is that we recover all the results of [43]. Below, see Table 9, we show our results for the coefficients B_{ij}. Here we switch to the default values of [43] ($\sqrt{z} = m_c/m_b = 0.29$, $m_b = 4.80\,\mathrm{GeV}$, $m_t = 175\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.118$, $|V_{ts}^* V_{tb}|/|V_{cb}| = 0.976$) and $\mu_b = \bar{\mu}_b$

Table 9: *Values of the coefficients $B_{ij}(\delta)$ in units of 10^{-4}, for different choices of μ_b*

μ_b	δ	B_{22}	B_{77}	B_{88}	B_{27}	B_{28}	B_{78}	$\sum B_{ij}$
$m_b/2$	0.90	1.322	0.335	0.015	1.265	0.179	0.074	3.190
	0.30	1.169	0.322	0.005	1.196	0.136	0.070	2.898
	0.15	1.081	0.309	0.004	1.144	0.126	0.067	2.730
m_b	0.90	1.258	0.382	0.015	1.395	0.161	0.083	3.293
	0.30	1.239	0.361	0.005	1.387	0.137	0.080	3.210
	0.15	1.200	0.347	0.004	1.354	0.132	0.077	3.114
$2m_b$	0.90	1.023	0.428	0.015	1.517	0.132	0.092	3.206
	0.30	1.041	0.402	0.004	1.552	0.118	0.091	3.209
	0.15	1.021	0.386	0.004	1.534	0.115	0.088	3.149

Switching back to our default central values, with $m_b = 4.8\mathrm{GeV}$, $|V_{ts}^* V_{tb}|/|V_{cb}|^2 = 0.971$, $m_c = 1.25\mathrm{GeV}$, $m_t = 174.3$, $\alpha_s(M_Z^2) = 0.1185$ and with $\mu_b = \bar{\mu}_b = m_b = m_b^{1s} = 4.80\mathrm{GeV}$ we find

$$\begin{aligned} 10^4\, B_{s\gamma}^{NLO}(\delta = 0.9, \sqrt{z} = 0.22) &= 1.512 + 1.417\, x_7^0 + 0.155\, x_8^0 \\ &+ 0.136\, x_7^1 + 0.017\, x_8^1 + 0.283\, (x_7^0)^2 + 0.014\, (x_8^0)^2 + 0.064\, x_7^0 x_8^0 \\ &+ 0.103\, x_7^0 x_7^1 + 0.013\, x_7^0 x_8^1 + 0.007\, x_8^0 x_7^1 + 0.001\, x_8^0 x_8^1 \end{aligned} \tag{B.46}$$

whereas in the assumption of [43] we get

$$\begin{aligned} 10^4\, B_{s\gamma}^{NLO}(\delta = 0.9, \sqrt{z} = 0.22) &= 1.512 + 1.553\, x_7 + 0.173\, x_8 + 0.386\, x_7^2 \\ &+ 0.015\, x_8^2 + 0.084\, x_7 x_8 \end{aligned} \tag{B.47}$$

leading to

$$B_{s\gamma}^{NLO,SM}(\delta = 0.9, \sqrt{z} = 0.22) = 3.723\; 10^{-4} \tag{B.48}$$

This result agrees at the 2 per-mil with the more sophisticated analysis of [44] and is in very good agreement with the current experimental value.

B.3 Charged Higgs contribution

The charged Higgs contribution in our code is estimated at the M_W scale and is based on [47], therefore we neglect any small running from $M_{H^\pm}$ to M_W. Indeed either $M_{H^\pm}$ is very large in which case the Higgs contribution is too small, or there is little running in a region where α_s is not too large.

B.3.1 LO

We have

$$C_{7,8}^{(0)H^\pm}(\mu_W) = F_{7,8}^2(x_{Ht}) \ + \ \frac{1}{\tan\beta^2}\frac{1}{3}F_{7,8}^1(x_{Ht}) \tag{B.49}$$

with

$$x_{Ht} = \frac{\overline{m}_t^2(\mu_W)}{M_H^2} \ , \tag{B.50}$$

and

$$\begin{aligned} F_7^{(2)}(x) &= \frac{x(3-5x)}{12(x-1)^2} + \frac{x(3x-2)}{6(x-1)^3}\ln x \\ F_8^{(2)}(x) &= \frac{x(3-x)}{4(x-1)^2} - \frac{x}{2(x-1)^3}\ln x \end{aligned} \tag{B.51}$$

B.3.2 NLO

We have

$$\begin{aligned} C_7^{(1)H^\pm}(\mu_W) &= G_7^H(x_{Ht}) + \Delta_7^H(x_{Ht})\ln\frac{\mu_W^2}{M_H^2} - \frac{4}{9}E^H(x_{Ht}) \\ C_8^{(1)H^\pm}(\mu_W) &= G_8^H(x_{Ht}) + \Delta_8^H(x_{Ht})\ln\frac{\mu_W^2}{M_H^2} - \frac{1}{6}E^H(x_{Ht}) \end{aligned} \tag{B.52}$$

$$\begin{aligned} G_7^H(x) = & -\frac{4}{3}x\left[\frac{4(-3+7x-2x^2)}{3(x-1)^3}\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{8-14x-3x^2}{3(x-1)^4}\ln^2 x\right. \\ & \left. + \frac{2(-3-x+12x^2-2x^3)}{3(x-1)^4}\ln x + \frac{7-13x+2x^2}{(x-1)^3}\right] \\ & + \frac{1}{\tan\beta^2}\frac{2}{9}x\left[\frac{x(18-37x+8x^2)}{(x-1)^4}\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{x(-14+23x+3x^2)}{(x-1)^5}\ln^2 x\right. \\ & + \frac{-50+251x-174x^2-192x^3+21x^4}{9(x-1)^5}\ln x \end{aligned}$$

$$
+\frac{797-5436x+7569x^2-1202x^3}{108(x-1)^4}\Bigg] \tag{B.53}
$$

$$
\begin{aligned}
\Delta_7^H(x) &= -\frac{2}{9}x\left[\frac{21-47x+8x^2}{(x-1)^3}+\frac{2(-8+14x+3x^2)}{(x-1)^4}\ln x\right] \\
&\quad +\frac{1}{\tan\beta^2}\frac{2}{9}x\left[\frac{-31-18x+135x^2-14x^3}{6(x-1)^4}+\frac{x(14-23x-3x^2)}{(x-1)^5}\ln x\right]
\end{aligned} \tag{B.54}
$$

$$
\begin{aligned}
G_8^H(x) &= -\frac{1}{3}x\left[\frac{-36+25x-17x^2)}{2(x-1)^3}\mathrm{Li}_2\left(1-\frac{1}{x}\right)+\frac{19+17x}{(x-1)^4}\ln^2 x\right. \\
&\quad \left.+\frac{-3-187x+12x^2-14x^3)}{4(x-1)^4}\ln x+\frac{3(143-44x+29x^2)}{8(x-1)^3}\right] \\
&\quad +\frac{1}{\tan\beta^2}\frac{1}{6}x\left[\frac{x(30-17x+13x^2)}{(x-1)^4}\mathrm{Li}_2\left(1-\frac{1}{x}\right)-\frac{x(31+17x)}{(x-1)^5}\ln^2 x\right. \\
&\quad +\frac{-226+817x+1353x^2+318x^3+42x^4}{36(x-1)^5}\ln x \\
&\quad \left.+\frac{1130-18153x+7650x^2-4451x^3}{216(x-1)^4}\right]
\end{aligned} \tag{B.55}
$$

$$
\begin{aligned}
\Delta_8^H(x) &= -\frac{1}{3}x\left[\frac{81-16x+7x^2}{2(x-1)^3}-\frac{19+17x}{(x-1)^4}\ln x\right] \\
&\quad +\frac{1}{\tan\beta^2}\frac{1}{6}x\left[\frac{-38-261x+18x^2-7x^3}{6(x-1)^4}+\frac{x(31+17x)}{(x-1)^5}\ln x\right]
\end{aligned} \tag{B.56}
$$

$$
E^H(x) = \frac{1}{\tan\beta^2}\left[\frac{x(16-29x+7x^2)}{36(x-1)^3}+\frac{x(3x-2)}{6(x-1)^4}\ln x\right]\ . \tag{B.57}
$$

As a check we recover exactly the LO and NLO values quoted in Table 1 of [47].

B.4 SUSY contributions

We only consider the contribution from charginos (and accompanying squarks). Here we follow [45] rather closely but adapt the expressions for the $\epsilon_b, \epsilon_b(t)$.

B.4.1 LO

We first consider the LO SUSY contribution at the SUSY scale. Though the code allows to choose this scale, our default value is set at the gluino mass $\mu_{\rm susy} = m_{\tilde{g}}$. This is also the scale we take for the SUSY Δm_b corrections.

With

$$F_7^{(3)}(x) = \frac{5-7x}{6(x-1)^2} + \frac{x(3x-2)}{3(x-1)^3}\ln x, \tag{B.58}$$

$$F_8^{(3)}(x) = \frac{1+x}{2(x-1)^2} - \frac{x}{(x-1)^3}\ln x \ . \tag{B.59}$$

$$\begin{aligned}
C_{7,8}^{\chi}(\mu_{SUSY}) &= \sum_{a=1,2}\Bigg\{\frac{2}{3}\frac{M_W^2}{\tilde{m}^2}\tilde{V}_{a1}^2 F_{7,8}^{(1)}(x_{\tilde{q}\,\chi_a^+}) \\
&- \frac{2}{3}\left(c_{\tilde{t}}\tilde{V}_{a1} - s_{\tilde{t}}\tilde{V}_{a2}\frac{m_t}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m_{\tilde{t}_1}^2} F_{7,8}^{(1)}(x_{\tilde{t}_1\,\chi_a^+}) \\
&- \frac{2}{3}\left(s_{\tilde{t}}\tilde{V}_{a1} + c_{\tilde{t}}\tilde{V}_{a2}\frac{m_t}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m_{\tilde{t}_2}^2} F_{7,8}^{(1)}(x_{\tilde{t}_2\,\chi_a^+}) \\
&+ \frac{1}{\cos\beta}\Bigg(\frac{\tilde{U}_{a2}\tilde{V}_{a1}M_W}{\sqrt{2}m_{\chi_a^+}}\left[F_{7,8}^{(3)}(x_{\tilde{q}\,\chi_a^+}) - c_{\tilde{t}}^2\, F_{7,8}^{(3)}(x_{\tilde{t}_1\,\chi_a^+}) - s_{\tilde{t}}^2\, F_{7,8}^{(3)}(x_{\tilde{t}_2\,\chi_a^+})\right] \\
&+ s_{\tilde{t}}\, c_{\tilde{t}}\frac{\tilde{U}_{a2}\,\tilde{V}_{a2}\, m_t}{2\sin\beta\, m_{\chi_a^+}}\left[F_{7,8}^{(3)}(x_{\tilde{t}_1\,\chi_a^+}) - F_{7,8}^{(3)}(x_{\tilde{t}_2\,\chi_a^+})\right]\Bigg)\Bigg\} \ .
\end{aligned} \tag{B.60}$$

with obvious notations for the sparticles. $x_{ij} = m_i^2/m_j^2$ here and in the following. Our diagonalising matrices for the chargino is as in [45] (as well as our convention for the sign of μ)

$$\tilde{U}\begin{pmatrix} M_2 & M_W\sqrt{2}\sin\beta \\ M_W\sqrt{2}\cos\beta & \mu \end{pmatrix}\tilde{V}^{-1} \tag{B.61}$$

The squark mixing and definitions are also as in [45]

$$\begin{aligned}
c_{\tilde{q}} &\equiv \cos\theta_{\tilde{q}} \ , \ \ s_{\tilde{q}} \equiv \sin\theta_{\tilde{q}} \ \text{ with} \\
\tilde{q}_1 &= c_{\tilde{q}}\,\tilde{q}_L + s_{\tilde{q}}\,\tilde{q}_R \ , \ \tilde{q}_2 = -s_{\tilde{q}}\,\tilde{q}_L + c_{\tilde{q}}\,\tilde{q}_R \ , \ \text{and} \ \ m_{\tilde{q}_1} > m_{\tilde{q}_2}
\end{aligned} \tag{B.62}$$

B.4.2 Δm_b corrections and large $\tan\beta$ effects

$$m_b = \sqrt{2}M_W\ \frac{y_b}{g}\ \cos\beta\,(1+\epsilon_b\tan\beta) \ \ , \ \ \delta m_b = \frac{\Delta m_b}{m_b} = \epsilon_b\tan\beta. \tag{B.63}$$

We implement δm_b in our code as follows. First define

$$H_2(x,y) = \frac{x\ln\, x}{(1-x)(x-y)} + \frac{y\ln\, y}{(1-y)(y-x)} \qquad H(i,j,k) = H_2(x_{ik}, y_{jk}) \tag{B.64}$$

where $(x,y)_{ij} = m_i^2/m_j^2$.

$$B(m_1,m_2,Q^2) = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{1-x} + \frac{\ln x}{(1-x)^2} - \ln(m_2^2/Q^2)\right) \ \ , \ \ x = m_2^2/m_1^2 \tag{B.65}$$

Then

$$
\begin{aligned}
\delta m_b &= \frac{2\,\alpha_s(\mu_{susy})}{3\,\pi}\left(\frac{A_b-\mu\tan\beta}{m_{\tilde g}}H(\tilde b_1,\tilde b_2,\tilde g)-\frac{1}{2}(B(\tilde g,\tilde b_1,\mu^2_{susy})+B(\tilde g,\tilde b_2,\mu^2_{susy}))\right)\\
&+\frac{\tilde y_t^2(\mu_{susy})}{16\,\pi^2}\sum_{a=1,2}\tilde U_{a2}\frac{\mu-A_t\tan\beta}{m_{\chi_a^+}}H(\tilde t_1,\tilde t_2,\chi_a^+)\,\tilde V_{a2}\\
&+\frac{\alpha(M_Z)}{4s_W^2\pi}\mu M_2\tan\beta\left(\frac{c_{\tilde t}^2}{m_{\tilde t_1}^2}H(M_2,\mu,\tilde t_1)+\frac{s_{\tilde t}^2}{m_{\tilde t_2}^2}H(M_2,\mu,\tilde t_2)\right.\\
&\qquad\left.+\frac{c_{\tilde b}^2}{2m_{\tilde b_1}^2}H(M_2,\mu,\tilde b_1)+\frac{s_{\tilde b}^2}{2m_{\tilde b_2}^2}H(M_2,\mu,\tilde b_2)\right)
\end{aligned}
\tag{B.66}
$$

Note that we have included the electroweak contributions with enhanced $\tan\beta$. For this part we make the approximation that the masses of the charginos are given by M_2 and μ, and neglect the mixing matrices, we have also neglected the $U(1)$ contribution which is even smaller. Although the formulae above include non-leading $\tan\beta$ effects, for consistency we have not coded these contributions in our program. The electroweak corrections agree with those in [49] whereas the sbottom term is missing in [50]. However we find the $SU(2)$ gauge contribution to be rather small compared to the strong and Yukawa contributions.

We turn now to $\epsilon_b'(t)$

$$
\begin{aligned}
\epsilon_b'(t) &= +\frac{2\,\alpha_s(\mu_{susy})}{3\,\pi}\frac{A_b/\tan\beta-\mu}{m_{\tilde g}}\left[c_{\tilde t}^2c_{\tilde b}^2\,H(\tilde t_1,\tilde b_2,\tilde g)+c_{\tilde t}^2s_{\tilde b}^2\,H(\tilde t_1,\tilde b_1,\tilde g)+\right.\\
&\qquad\left.s_{\tilde t}^2c_{\tilde b}^2\,H(\tilde t_2,\tilde b_2,\tilde g)+s_{\tilde t}^2s_{\tilde b}^2\,H(\tilde t_2,\tilde b_1,\tilde g)\right]\\
&\quad+\frac{y_t^2(\mu_{susy})}{16\,\pi^2}\sum_{a=1}^{4}N_{a4}^*\frac{A_t-\mu/\tan\beta}{m_{\chi_a^0}}\left[c_{\tilde t}^2c_{\tilde b}^2\,H(\tilde t_2,\tilde b_1,\chi_a^0)+c_{\tilde t}^2s_{\tilde b}^2\,H(\tilde t_2,\tilde b_2,\chi_a^0)+\right.\\
&\qquad\left.s_{\tilde t}^2c_{\tilde b}^2\,H(\tilde t_1,\tilde b_1,\chi_a^0)+s_{\tilde t}^2s_{\tilde b}^2\,H(\tilde t_1,\tilde b_2,\chi_a^0)\right]\,N_{a3}\\
&+\frac{\alpha(M_Z)}{4s_W^2\pi}\mu M_2\left(\frac{c_{\tilde b}^2}{m_{\tilde b_1}^2}H(M_2,\mu,\tilde b_1)+\frac{s_{\tilde b}^2}{m_{\tilde b_2}^2}H(M_2,\mu,\tilde b_2)\right.\\
&\qquad\left.+\frac{c_{\tilde t}^2}{2m_{\tilde t_1}^2}H(M_2,\mu,\tilde t_1)+\frac{s_{\tilde t}^2}{2m_{\tilde t_2}^2}H(M_2,\mu,\tilde t_2)\right)
\end{aligned}
\tag{B.67}
$$

Note that we have added an electroweak contribution, in the same approximation as in δm_b. Most importantly, the sign of the Yukawa contribution and the elements of the diagonalising matrices that appear in Eq. B.67 are different from those in [45]. We have verified this by explicit calculation of $\epsilon_b'(t)$, moreover with our formula, we find that in the decoupling limit we do indeed have $\epsilon_b'(t)\to\epsilon_b$, which would not have been the case had we blindly used the formula of [45]. One of the authors of [45], Paolo Gambino, has recently confirmed our implementation.

As for $\epsilon'_t(s)$, we find that only the QCD contribution remains. Note, as said previously, we shall only keep the $\tan\beta$ enhanced term in our code.

For $\epsilon'_t(s)$

$$\epsilon'_t(s) \;\; = \;\; -\frac{2\,\alpha_s}{3\,\pi}\frac{\mu + A_t/\tan\beta}{m_{\tilde{g}}}\left[c^2_{\tilde{t}}\,H(\tilde{t}_2,\tilde{s},\tilde{g}) \;+\; s^2_{\tilde{t}}H(\tilde{t}_1,\tilde{s},\tilde{g})\right] \qquad \text{(B.68)}$$

We find no $\tan\beta$ enhanced electroweak gauge contribution.

Once more let us stress that, although we have derived, for the ϵ, ϵ' the $\tan\beta$ enhanced and the non $\tan\beta$ enhanced we will only keep the $\tan\beta$ enhanced terms, since these are the ones that can be resummed. Therefore in our numerical analysis that takes this resummation into account we only keep the $\tan\beta$ enhanced terms.

As we mentioned earlier these ϵ's contributions are to be evaluated at the scale $Q^2 = \mu^2_{\rm susy} > m_t^2$ which we associate with the gluino mass. In particular α_s is to be evaluated here taking into account 6 active quarks.

$$\eta_s \equiv \alpha_s(\mu_{SUSY})/\alpha_s(\mu_W) = \left(1 + \frac{7\alpha_s(\mu_W)}{2\pi}\ln(\mu_{SUSY}/\mu_W)\right)^{-1} \qquad \text{(B.69)}$$

Also the Yukawa (and top mass) that is used for the chargino contribution is

$$\begin{aligned}\tilde{y}_t(\mu_{SUSY}) \;\; &= \;\; y_t(\mu_W)\left[\frac{\alpha_s(\mu_{SUSY})}{\alpha_s(m_t)}\right]^{4/7}\left[\frac{\alpha_s(m_t)}{\alpha_s(\mu_W)}\right]^{12/23} \\ &\times \;\; \frac{1}{\sqrt{1 + \frac{9y_t^2(m_t)}{8\pi\alpha_s(m_t)}\left\{\left[\frac{\alpha_s(\mu_{SUSY})}{\alpha_s(m_t)}\right]^{1/7} - 1\right\}}}\end{aligned} \qquad \text{(B.70)}$$

At all scales we relate y_t to m_t as follows

$$y_t^2(Q^2) = \frac{2\pi\alpha(M_Z)}{s_W^2}\frac{1+\tan\beta^2}{\tan\beta^2}\frac{m_t^2(Q^2)}{M_W^2} \qquad \text{(B.71)}$$

We used a fixed $\tan\beta$ at all scales, we neglect the small running of α between M_Z and μ_{susy}.

Though the ϵ effects are to be extracted at $\mu_{\rm susy}$, they are included in the SM and charged Higgs Wilson coefficient at μ_W

$$\delta C_{7,8}^{(SM)}(\text{leading}\tan\beta)(\mu_W) \;\; = \;\; \frac{[\epsilon_b - \epsilon'_b(t)]\tan\beta}{1+\epsilon_b\tan\beta}\,F_{7,8}^{(2)}(x_{tw}) \qquad \text{(B.72)}$$

$$\delta C_{7,8}^{(H^\pm)}(\text{leading}\tan\beta)(\mu_W) \;\; = \;\; -\frac{[\epsilon'_t(s) + \epsilon_b]\tan\beta}{1+\epsilon_b\tan\beta}\,F_{7,8}^{(2)}(x_{Ht}). \qquad \text{(B.73)}$$

For the chargino contribution we first evaluate the Wilson coefficients at the scale $\mu_{\rm SUSY}$ by including the top mass effect and the ϵ effects. In our implementation we assume, as occurs in most cases in mSUGRA, that both stops are heavy, so that they are decoupled together at the same scale $\mu_{\rm SUSY}$,

$$\begin{aligned}
C_{7,8}^{\chi}(\mu_{SUSY}) &= \sum_{a=1,2}\left\{\frac{2}{3}\frac{M_W^2}{\tilde{m}^2}\tilde{V}_{a1}^2 F_{7,8}^{(1)}(x_{\tilde{q}\,\chi_a^+})\right.\\
&- \frac{2}{3}\left(c_{\tilde{t}}\,\tilde{V}_{a1} - s_{\tilde{t}}\,\tilde{V}_{a2}\frac{\overline{m}_t(\mu_{SUSY})}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m_{\tilde{t}_1}^2}F_{7,8}^{(1)}(x_{\tilde{t}_1\,\chi_a^+})\\
&- \frac{2}{3}\left(s_{\tilde{t}}\,\tilde{V}_{a1} + c_{\tilde{t}}\,\tilde{V}_{a2}\frac{\overline{m}_t(\mu_{SUSY})}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m_{\tilde{t}_2}^2}F_{7,8}^{(1)}(x_{\tilde{t}_2\,\chi_a^+})\\
&+ \frac{K_b}{\cos\beta}\left(\frac{\tilde{U}_{a2}\tilde{V}_{a1}M_W}{\sqrt{2}m_{\chi_a^+}}\left[F_{7,8}^{(3)}(x_{\tilde{q}\,\chi_a^+}) - c_{\tilde{t}}^2\,F_{7,8}^{(3)}(x_{\tilde{t}_1\,\chi_a^+}) - s_{\tilde{t}}^2\,F_{7,8}^{(3)}(x_{\tilde{t}_2\,\chi_a^+})\right]\right.\\
&+ \left.\left. s_{\tilde{t}}\,c_{\tilde{t}}\frac{\tilde{U}_{a2}\,\tilde{V}_{a2}\,\overline{m}_t(\mu_{SUSY})}{2\sin\beta\, m_{\chi_a^+}}\left[F_{7,8}^{(3)}(x_{\tilde{t}_1\,\chi_a^+}) - F_{7,8}^{(3)}(x_{\tilde{t}_2\,\chi_a^+})\right]\right)\right\} \quad . \qquad \text{(B.74)}
\end{aligned}$$

with

$$K_b = 1/(1+\epsilon_b\tan\beta) \qquad \text{(B.75)}$$

These are then evolved to μ_W as

$$\begin{aligned}
C_7^{\chi}(\mu_W) &= \eta^{-\frac{16}{3\beta_0'}}C_7^{\chi}(\mu_{SUSY}) + \frac{8}{3}\left(\eta^{-\frac{14}{3\beta_0'}} - \eta^{-\frac{16}{3\beta_0'}}\right)C_8^{\chi}(\mu_{SUSY})\\
C_8^{\chi}(\mu_W) &= \eta^{-\frac{14}{3\beta_0'}}C_8^{\chi}(\mu_{SUSY}) \quad ; \quad \beta_0' = -7 \qquad \text{(B.76)}
\end{aligned}$$

At μ_W all the contributions (SM, $H^\pm$ including $\tan\beta$ effects) are added together with those of the chargino contribution and evolved according to Eq. B.20.

To check on the SUSY part and the implementation of $\tan\beta$ enhanced terms, we have first verified that we had perfect agreement with Fig. 4 of [45]. This in fact is only a check on the implementation of the ϵ's in the SM and H^+ contribution with fixed ϵ. A full check of the SUSY contribution requires a quite large set of inputs. In [45] one can read the effect on $B \to X_s\gamma$ of a SUGRA model. Since the outputs of SUGRA codes can differ quite a bit, we have not used our own RGE but requested the weak scale parameters from Paolo Gambino used for their Fig. 2 and Fig. 3 in [45] , which in passing includes only the α_s contribution to the ϵ. We have found an excellent agreement both for positive and negative μ.

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