Constructing black holes and black hole microstates

String theory and the fuzzball proposal

Clément Ruef, AEI

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Work done with I. Bena, N. Bobev, S. Giusto, N. Warner, G. Dall’Agata.
Work in Progress with G. Bossard

Many different groups

Interesting reviews

- *The fuzzball proposal for black holes: An elementary review*, Mathur, hep-th/0502050,
- *Black holes, black rings and their microstates*, Bena and Warner, hep-th/0701216,
- *The fuzzball proposal for black holes*, Skenderis and Taylor, 0804.0552,
- *Black Holes as Effective Geometries*, Balasubramanian, de Boer, El-Showk and Messamah, 0811.0263.
Motivation: Quantum gravity

But the developed tools are quite general:

- Generation of gravity solutions
- Application to other string theoretical systems:
  - Flux compactifications and Klebanov-Strassler type systems
- Possible applications to cosmology
1. Introduction: black hole issues and entropy counting

2. The fuzzball proposal

3. Constructing three-charge supersymmetric solutions

4. Non-BPS extremal black holes

5. Conclusion and perspectives
1 Introduction: black hole issues and entropy counting

2 The fuzzball proposal

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4 Non-BPS extremal black holes

5 Conclusion and perspectives
Black hole issues

**Fundamental black hole problems:**

- Central singularity
- Microscopic understanding of the BH entropy
- Information paradox

*Cannot be answered in the context of general relativity.*
Black hole issues

**Fundamental black hole problems:**

- Central singularity
- Microscopic understanding of the BH entropy
- Information paradox

Cannot be answered in the context of general relativity.

**What is a black hole?**
Classically, a black hole has a macroscopic entropy:

\[ S = \frac{A}{4G_N} \]

**Uniqueness theorem** → only one single state!
Introduction: black hole issues and entropy counting

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Conclusion and perspectives

Black-hole entropy

Classically, a black hole has a macroscopic entropy:

\[ S = \frac{A}{4G_N} \]

**Uniqueness theorem** \( \rightarrow \) only one single state!

Statistically: \( e^S \) states.

Ex: \( M = M_{\text{center galaxy}} \rightarrow N = e^{10^{90}} \)

Huge discrepancy!
Questions

- Where are the BH microstates?
- What are the BH microstates?
- How do the BH microstates behave?
- What is the correct framework to understand the BH microstates?
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We need a theory of quantum gravity!
Strominger-Vafa counting

String theory provides partial answers:

\[ S_{\text{micro}} = 2\pi \sqrt{Q_1 Q_2 Q_3} \]
\[ S_{\text{macro}} = 2\pi \sqrt{Q_1 Q_2 Q_3} \]
Strominger-Vafa counting

String theory provides partial answers:

$S_{\text{micro}} = 2\pi \sqrt{Q_1 Q_2 Q_3}$

$S_{\text{macro}} = 2\pi \sqrt{Q_1 Q_2 Q_3}$

Open string
Gauge
CFT

Closed string
Gravity
AdS

protected by SUSY

finite $g_s$

$g_s = 0$
How do the "microstates" transform while turning on $g_s$ ?

What about the singularity resolution and the information paradox ?
1 Introduction : black hole issues and entropy counting
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Two descriptions

A macroscopic one, continuous, in terms of thermodynamics and fluid mechanics. Pertinent for long scale effects.

The microscopic one, quantized, in terms of statistical/quantum mechanics. Pertinent for small scale effects.

Macroscopic state = statistical average of microscopic states
Black hole thermodynamics

Two descriptions?

A macroscopic one, continuous, in terms of BH thermodynamics. Pertinent for long scale effects, like gravitational scattering, gravitational lensing...
General features

- Macroscopic state = statistical average of microscopic states
- Same long range behaviour as the BH $\rightarrow$ same mass and charges
- Have to grow with $g_s$, as the BH. Non trivial statement!
- Horizon = Entropy $\rightarrow$ no horizon

**Modification at the horizon scale!**
Key idea

QG effects: \( l \sim l_P \)  
QG effects: \( l \sim N^\alpha l_P \sim r_S \)

\[
l_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} \text{m}
\]

**Fuzzball proposal**: Quantum gravity effects extend until the horizon size

Mathur
The fuzzball proposal

**BH microstates** = a horizonless configuration with the same asymptotics as the BH

- Very fuzzy? Fully stringy or only geometric?
- Can the geometric solutions sample the space of microstates?
Back to black hole issues

The fuzzball proposal could solve all the BH issues

- Central singularity resolved
- Microscopic understanding of the BH entropy
The fuzzball proposal could solve all the BH issues

- Central singularity resolved
- Microscopic understanding of the BH entropy
- Hypothesis leading to the information paradox do not hold anymore
Two charge story

A very large body of work for two-charge black holes
Two charge story

A very large body of work for two-charge black holes

- Microscopic, CFT, counting \( S = 4\pi \sqrt{N_1 N_2} \) Sen
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- In supergravity, $S = 0$. Beyond SUGRA
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  Lunin, Mathur;...
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Framework

- String theory: theory of quantum gravity (10D)
- Low energy limit: supergravity (10D or 11D)
String theory: theory of quantum gravity (10D)

Low energy limit: supergravity (10D or 11D)

We will physically describe 4D or 5D black holes. Other dimensions compactified

Keep in mind stringy nature of the objects and interactions

I will switch between 11D/IIA/4D/5D supergravities.
11D Supergravity

The supergravity action:

$$2\kappa_{11}^2 S = \int d^{11}x \sqrt{-G} \left( R - \frac{1}{2} |F^{(4)}|^2 \right) - \frac{1}{6} \int A^{(3)} \wedge F^{(4)} \wedge F^{(4)}.$$ 

Field content:

- $g_{\mu\nu} \leftrightarrow \text{spacetime}$
- $A^{(3)} \leftrightarrow \text{M2 and M5 branes}$

In 4D/5D, gravity coupled to Maxwell and scalar fields.
From a 11D point of view, the charges come from M branes wrapping cycles along the compact $T^6$:

$$\mathcal{M}_{11D} = \mathbb{R}^{4,1} \times T^6 = \mathbb{R}^{4,1} \times T^2 \times T^2 \times T^2$$

M2 branes $\leftrightarrow$ electric charges    M5 branes $\leftrightarrow$ magnetic charges
M Branes

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M2 branes ↔ electric charges  M5 branes ↔ magnetic charges
We want to describe five-dimensional solutions:

\[
\begin{align*}
\text{ds}^2 &= -Z^{-2}(dt + k)^2 + Z \, ds_4^2 + \sum_{I=1}^{3} X_I(dy_{I1}^2 + dy_{I2}^2), \\
A^{(3)} &= \sum_{I=1}^{3} \left(-Z_I^{-1}(dt + k) + B^{(I)} \right) \wedge dy_{I1} \wedge dy_{I2}
\end{align*}
\]

with \( Z = (Z_1 Z_2 Z_3)^{1/3} \) and \( X_I = Z/Z_I \).

This can describe either \textit{black holes, black rings or regular, BPS or non-BPS}, solutions.
Fields and content

- $ds_2^4$: base space
- $B^{(I)}$: magnetic charges
- $Z_I$: electric charges
- $k$: angular momentum
Metric Ansatz

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with $Z = (Z_1 Z_2 Z_3)^{1/3}$ and $X_I = Z / Z_I$.

This will describe either black holes, black rings or regular, BPS or non-BPS, solutions.

“Floating brane” Ansatz
Floating brane Ansatz

No global force, the branes are **mutually BPS**. If SUSY, ansatz imposed
BPS equations

Supersymmetry reduces Einstein equations to a first order system.
BPS equations

Supersymmetry reduces Einstein equations to a first order system.

A four step procedure:
Supersymmetry reduces Einstein equations to a \textbf{first order system}.

A four step procedure:

1. \textbf{Hyperkähler Euclidean 4D base space}
BPS equations

Supersymmetry reduces Einstein equations to a first order system.

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1. Hyperkähler Euclidean 4D base space
2. $\Theta^{(I)} = *_4 \Theta^{(I)}$, where $\Theta^{(I)} = dB^{(I)} \rightarrow \Theta^{(I)}$
BPS equations

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1. Hyperkähler Euclidean 4D base space
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3. $\nabla^2 Z_I = \frac{C_{JK}}{2} *_4 [\Theta^{(J)} \wedge \Theta^{(K)}] \rightarrow Z_I$
BPS equations

Supersymmetry reduces Einstein equations to a first order system.

A four step procedure:

1. Hyperkähler Euclidean 4D base space
2. $\Theta^{(I)} = \ast_4 \Theta^{(I)}$, where $\Theta^{(I)} = dB^{(I)} \rightarrow \Theta^{(I)}$
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4. $dk + \ast_4 dk = Z_I \Theta^{(I)} \rightarrow k$
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Linear system of equations! Bena, Warner
Gibbons-Hawking metrics

Assuming a triholomorphic $U(1)$ isometry, an hyperkähler space is Gibbons-Hawking:

\[ ds_4^2 = V^{-1}(d\psi + A)^2 + Vds_3^2 \]

$V$ harmonic, $dV = \ast_3 dA$.

Ex: $V = \frac{1}{r}$, flat $\mathbb{R}^4$
Gibbons-Hawking metrics

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$$\begin{align*}
ds_4^2 &= V^{-1} (d\psi + A)^2 + V ds_3^2 \\
V \text{ harmonic, } dV &= \ast_3 dA.
\end{align*}$$

Ex: $V = 1 + \frac{1}{r}$, Taub-NUT space, interpolates between $\mathbb{R}^4$ and $\mathbb{R}^3 \times S^1$
Gibbons-Hawking metrics

Assuming a triholomorphic $U(1)$ isometry, an hyperkähler space is Gibbons-Hawking:

$$ds^2_4 = V^{-1}(d\psi + A)^2 + Vds_3^2$$
$$V \text{ harmonic, } dV = *_3dA.$$

Ex: $V = 1 + \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|}$, Multi Taub-NUT space
Assuming this Ansatz, all BPS solutions have been found. They are given by 8 harmonic functions: Gauntlett, Gutowski, Hull, Pakis, Reall.
BPS Solutions

Assuming this Ansatz, all BPS solutions have been found.

- black holes  \( S = 2\pi \sqrt{Q_1 Q_2 Q_3} \)
- black rings, horizon  \( S^2 \times S^1 \)
- multicentered black holes
- smooth, regular solutions
BPS Solutions

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- smooth, regular solutions
Smooth solutions

How to build smooth solutions?
Start from a multi-centered Taub-NUT space

\[ ds_4^2 = V^{-1}(d\psi + A)^2 + V ds^2_3 \]

\[ V = 1 + \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|} \]

The \( S^1 \) fiber shrinks at the each GH point \( \rightarrow \) bubbles
Smooth solutions

- Bubbles stabilized by magnetic fluxes
- No localized sources, no singularity
- **bf Integrability, or bubble equation** Denef; Bena, Warner:

\[ \sum_j \frac{\langle \Gamma_i, \Gamma_j \rangle}{r_{ij}} = \langle \Gamma_i, h \rangle \]

- Fluxes create the charges seen at infinity
Smooth solutions and ambipolar spaces

Need to start from an 4D ambipolar base. Signature switches from $(+,+,+,+)$ to $(-,-,-,-)$ seems to be highly singular!
Smooth solutions and ambipolar spaces

Need to start from an 4D ambipolar base. Signature switches from 
(+, +, +, +) to (−, −, −, −) → seems to be highly singular!

**Complete 11D (5D) solutions completely regular** Giusto, Mathur, Saxena
Giving up the $U(1)$ isometry

One can count the entropy coming from the microstates \(\rightarrow\) **not enough**

It was expected :

$U(1)$-isometry : cuts all the modes along the fiber!

**Two-charge case : entropy comes from these modes**
Giving up the $U(1)$ isometry

Look at a wiggling supertube dual to a smooth GH center

Problem: we need the Green function on an ambipolar GH space

- known for $(+,+,\ldots,+)\text{ centers} \quad \text{Page}
- can be found for ambipolar two centers $(+,−)$ from $AdS_3 × S^2$
- Very hard problem in general

New solutions with a function $f(\theta)$ as parameter $\rightarrow$ **Infinite dimensional moduli space** Bena, Bobev, Giusto, CR, Warner
Entropy enhancement mechanism

- Entropy of the supertube in flat space
  \[ S \sim \sqrt{Q_1 Q_2} \]

- Entropy of the supertube in dipole-charged background
  \[ S \sim \sqrt{Q_{1 \text{eff}} Q_{2 \text{eff}}} \]
Entropy enhancement mechanism

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Much more entropy than naively expected!

Bena, Bobev, CR, Warner
Other approaches to the fuzzball proposal

One can make use of the AdS/CFT correspondence in the context of the fuzzball proposal

- Identification of the microstates on the CFT side Skenderis, Taylor
- Computation of perturbative corrections of fuzzballs to the flat metric from a pure worldsheet point of view Giusto, Morales, Russo
- Precision counting on both sides of the correspondence, using indices and partition functions Sen

Gravity macroscopic
Gauge microscopic

All approaches, despite being very different, seem to confirm the conjecture.
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Non-BPS extremal black holes

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Non-BPS black holes

What can we do without supersymmetry?

Recent years: a lot of progress for extremal non-BPS black holes, through different approaches

- Fake superpotential and first order formalism: Ceresole, Dall’Agata et al.; Andrianopoli, D’Auria, Trigiante et al.; Gimon, Larsen, Simon; Perz, Galli, Jansen, Smyth, Van Riet, Vercnocke;...
- Almost BPS equations: Goldstein, Katmadas; Bena, Giusto, CR, Warner
- Integrability conditions: Andrianopoli, D’Auria, Orazi, Trigiante et al
- Reduction to three dimensions: Clement, Galt’sov, Scherbluk et al.; Bossard et al.; Virmani et al.; Chemissany, Rosseel, Trigiante, Van Riet et al.; ...
Non-BPS black holes

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Supersymmetry $\rightarrow$ extremality
Almost BPS solutions

**Fondamental idea:** SUSY broken by the relative orientation of the branes

Solve *almost* the same system of equations

**BPS system**

\[
\begin{align*}
    dV &= *_3 dA \\
    \Theta^{(I)} &= *_4 \Theta^{(I)} \\
    \nabla^2 Z_I &= \frac{C_{IJK}}{2} *_4 [\Theta^{(J)} \wedge \Theta^{(K)}] \\
    dk + *_4 dk &= Z_I \Theta^{(I)}
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**Ex:** BPS 4-charge black hole D6-D2-D2-D2
Almost BPS solutions

Fondamental idea: SUSY broken by the relative orientation of the branes

Solve almost the same system of equations

non-BPS system: Goldstein, Katmadas

\[
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dV &= -\ast_3 dA \\
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Ex: non-BPS 4-charge black hole \(\overline{D6}-D2-D2-D2\)
Almost BPS solutions

Tools developed in the SUSY context can be used
Large class of new solutions: Bena, Dall’Agata, Giusto, CR, Warner

- Black holes
- Black rings
- Multicentered black holes
- No microstates

One recovers all solutions found with the fake superpotential approach, by solving linear systems.

Further generalization of the system of equations, and the solutions.
Floating brane vs extremality

First assumption: Floating brane ansatz $\sim$ extremality
**Floating brane vs extremality**

**First assumption**: Floating brane ansatz $\sim$ extremality

- Dualities: map solutions to solutions.
  - **BPS case**: solution space is closed, all in (the closure of) the floating brane ansatz
  - **Almost BPS case**: solution space not closed. New solutions obtained by duality, not floating brane

Ex: New non-BPS doubly spinning black ring in Taub-NUT with dipole charges [Dall'Agata, Giusto, CR; Bena, Giusto, CR]
Floating brane vs extremality

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  - Ex: **New non-BPS doubly spinning black ring in Taub-NUT with dipole charges** Dall’Agata, Giusto, CR; Bena, Giusto, CR
- Possible to obtain non-extremal microstates within the floating brane ansatz
Floating brane Ansatz

Floating brane ansatz

BPS solutions

non-BPS extremal solutions

non-BPS non-extremal microstates
Key point: the equations are solved in a linear way.

\[ dV = *_3 dA \]
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This is a graded system.
Underlying structure behind. How can we make it explicit, and use it?
Key point: **the equations are solved in a linear way.**

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This is a graded system.
Underlying structure behind. **How can we make it explicit, and use it?**

Reduction to a three-dimensional problem
Three-dimensional approach

- In 3D, electric-magnetic duality $\rightarrow$ gravity coupled to scalars

Breitenlohner, Gibbons, Maison
Three-dimensional approach

- In 3D, electric-magnetic duality $\rightarrow$ gravity coupled to scalars
  - Breitenlohner, Gibbons, Maison
- Moduli space is a coset $\mathcal{M} = G/K$.
  - Ex: In our case $\mathcal{M} = SO(4,4)/SL(2)^4$. 
Three-dimensional approach

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- Use the algebraic structure of the space
  - Dualizing Clement, Galt’sov et al; Jamsin, Virmani et al
  - Solving equations Bossard et al
  - Using the integrability properties of the theory Figueras et al
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Extremal solutions $\leftrightarrow$ Nilpotent orbits in $\mathcal{M}$

Graded system $\leftrightarrow$ Lie algebra graded decomposition
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Three-dimensional approach

**Multicenter solutions**

- Recover all BPS solutions

  Bossard, Nicolai, Stelle
**Multicenter solutions**

- **Recover all BPS solutions**
  Bossard, Nicolai, Stelle

- **Recover almost BPS solutions**
  Perz, Galli; Bossard; Bossard, CR
Three-dimensional approach

**Multicenter solutions**

- **Recover all BPS solutions**
  Bossard, Nicolai, Stelle

- **Recover almost BPS solutions**
  Perz, Galli; Bossard; Bossard, CR

- **Find new solutions**
  Bossard, CR
Wider generalisation of the system of equations: Bena, Giusto, CR, Warner

\[
\begin{align*}
R_{ab} &= 0 \\
\Theta^{(l)} &= *_{4}\Theta^{(l)} \\
\nabla^{2}Z_{l} &= \frac{C_{IJK}}{2} *_{4} [\Theta^{(J)} \wedge \Theta^{(K)}] \\
dk + *_{4}dk &= Z_{l}\Theta^{(l)}
\end{align*}
\]

This system allows for microstates!
Bolt solutions

\[ R_{ab} = 0 \rightarrow \text{Why not start with an Euclidean black hole?} \]
\( R_{ab} = 0 \rightarrow \) Why not start with an Euclidean black hole?

Lorentzian \( \rightarrow \) Euclidean:
- Event horizon becomes a bolt, a non-trivial \( S^2 \)
- The space ends smoothly at \( r = r_+ \), and interpolates between \( \mathbb{R}^2 \times S^2 \) and \( \mathbb{R}^3 \times S^1 \)
**Bolt solutions**

\[ R_{ab} = 0 \rightarrow \text{Why not start with an Euclidean black hole?} \]

Lorentzian $\rightarrow$ Euclidean:
- Event horizon becomes a bolt, a non-trivial $S^2$
- The space ends smoothly at $r = r_+$, and interpolates between $\mathbb{R}^2 \times S^2$ and $\mathbb{R}^3 \times S^1$

**No singularity**
Putting fluxes on the bolt

The bolt gives us an $S^2$ to put magnetic fluxes. As in the BPS case, this fluxes create the charges seen from infinity.

“Charges dissolved in fluxes”
Putting fluxes on the bolt

The bolt gives us an $S^2$ to put magnetic fluxes. As in the BPS case, this fluxes create the charges seen from infinity. “Charges dissolved in fluxes”

Regular solutions, no singularity, no horizon \cite{Bena, Giusto, Warner, Bobev, CR}

Have the same asymptotics as a non-extremal black hole

\[ M = M_{\text{sol}} + \sum Q_I \]
1 Introduction: black hole issues and entropy counting

2 The fuzzball proposal

3 Constructing three-charge supersymmetric solutions

4 Non-BPS extremal black holes

5 Conclusion and perspectives
Conclusion

- Black hole issues yet to be solved
- Fuzzball proposal:
  - physically intuitive motivated
  - rigorously defended, from various point of views
  - works in the two charge case
- Microstates built by putting magnetic fluxes on non-trivial two-cycles
- Entropy enhancement mechanism
- New non-BPS microstates
Perspectives

- Need to find more general solutions
- Extremal non-BPS microstates from 3D approach
- How much entropy can be obtained by the entropy enhancement mechanism?
- Study of the (in-)stability of the non extremal microstates Mathur, Chowdhury
- Application to very early universe cosmology Mathur, Chowdhury
Thank you for your attention