
Lecture 2: Gravitational-Wave Astronomy & and an Update on LIGO/VIRGO
Stellar Collapse, Core-Collapse Supernovae and The Formation of Black Holes: The Big Picture

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The Core-Collapse Scenario

Evolved Massive Star

$M > 7-10 \, M_{\odot}$

Fe-group nuclei

Si

O/Ne/Mg

C

$10^7 \, \text{km}$

$10^9 \, \text{km}$
The Core-Collapse Scenario

Protonutron Star, R ~30 km

“Core Bounce” at nuclear density.

Iron Core

M > ~1.3 – ~2.2 M\textsubscript{SUN}
The Core Collapse Scenario

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M > 1.3 – 2.2 M\(_{\odot}\)

This causes core bounce.

\[ \Gamma = \frac{d \ln P}{d \ln \rho} \]

\( s = 1.2 \) k\(_B\)/baryon

\( Y_e = 0.3 \)
The Core-Collapse Scenario

Iron Core

Protonutron Star, $R \sim 30$ km

Shock

Accretion

2000 km
The Core-Collapse Supernova Problem

• Shock always stalls:
  Dissociation of Fe-group nuclei @ ~8.8 MeV/baryon (~17 B/M_{Sun}).
  Neutrino losses @ 10 B/s (1 [B]ethe = 10^{51} ergs).

Animation by Evan O’Connor
The Core-Collapse Scenario

Protoneutron Star, $R \sim 30$ km

Accretion

Shock

$L_\nu$

$L_\nu$
The Core-Collapse Scenario

Proton-Neutron Star, $R \sim 30$ km

Supernova Explosion

Accretion

Shock

Shock is revived.
The Core-Collapse Scenario

This talk

Protonutron Star, R ~30 km

Shock

Accretion

\( L_\nu \)

\( L_\nu \)

Shock is revived.

Shock is not revived.

Supernova Explosion

Collapse to Black Hole (Collapsar)
The Core-Collapse Scenario

Protonutron Star, $R \sim 30$ km

Supernova Explosion

Accretion

Shock

$L_\nu$

$L_\nu$

Digression:
What is the “Supernova Mechanism”?
Neutrinos: Carry away $\sim 99\%$ of the gravitational energy of the newborn neutron star ($\sim 300$ B).
The Neutrino Mechanism
Imshenik & Nadyozhin 1964
Arnett 1966
Colgate & White 1966
Bethe & Wilson 1985

The Problem:
Doesn’t quite work...
Fails in 1D, marginal in 2D.
Key may be 3D.
(→ Nordhaus et al. 2010)
Core-Collapse Supernova Mechanisms

**Neutrino Mechanism**
Introduced by:

**Magnetorotational Mechanism**
Alternatives:
[Bisnovaty-Kogan ’70, LeBlanc & Wilson ‘70, Meier et al. ‘76, Symbalisty ‘84, Burrows et al. ‘07]

**Acoustic Mechanism**
[proposed by Burrows et al. ‘06, ‘07; not yet confirmed by other groups/codes]

**Phase-Transition-Induced Mechanism**
[Takahara & Sato ‘88, Gentile et al. ‘93, Sagert et al. ’09, Fischer et al. ’10]
Magnetorotational Explosions

[Burrows et al. 2007, Dessart et al. 2008]

- **Rapid rotation:**
  \[ P_0 < 4-6 \text{ s} \rightarrow \text{millisecond PNS} \]

- PNS rotational energy:
  \( \sim 10^5 \text{ B} = 10^{52} \text{ erg} \)

- Amplification of B fields up to equipartition:
  - compression
  - dynamos
  - **magneto-rotational instability** (MRI)

- Jet-driven outflows.

- MHD-driven explosion may be GRB precursor.
Newtonian Radiation-MHD Simulations with VULCAN/2D

Magnetic field lines in M15B11UP2A1H of Burrows, Dessart, Livne, Ott, Murphy ‘07.
The Making of: Stellar-Mass Black Holes
The Making of Stellar-Mass Black Holes

Route 1 to BH formation:
Collapse to protoneutron star (PNS),
explosion fails -> “Collapsar Type I”

Route 2 to BH formation:
Collapse to PNS, explosion succeeds, but does not unbind entire stellar envelope.
Fallback accretion -> BH formation O(many) seconds.
“Collapsar Type II”

Route 3 to BH formation:
BH formation triggered by Hadron-Quark Phase transition in the PNS cooling phase.

Important aside:
There is no such thing as direct collapse to a black hole for ordinary massive stars (10—130 M\textsubscript{Sun}).
Strong observational evidence for connection between type-Ibc core-collapse supernovae and long GRBs.

2 competing central scenarios:

1. **Millisecond Magnetar** (= $10^{15}$ gauss Neutron Star) Model
   - (Bucciantini, Quatert, Metzger et al. '07-'10, supported by: Dessart, Burrows, Ott et al. '08)

2. **Collapsar Model** (Woosley 1993)
   - BH + accretion disk formed in a *failing core-collapse supernova* or after a core-collapse supernova explosion via *fallback accretion*.
Studying Black Hole Formation

Key ingredient: General Relativity

Einstein Equations:

\[ G_{ab} = \frac{8\pi G}{c^4} T_{ab} \]

Curvature of spacetime

Stress-Energy sourcing Curvature
In Newtonian Gravity, black holes cannot form.
Effects of General Relativity

Woosley & Heger '07, 40 $M_\odot$, solar metallicity
Einstein -> Computer: Numerical Relativity

Arnowitt
Deser
Misner (ADM)

\[ G_{ab} = \frac{8\pi G}{c^4} T_{ab} \]

• 12 first-order hyperbolic \textit{evolution} equations.
• 4 elliptic \textit{constraint} equations
• 4 coordinate gauge degrees of freedom: \( \alpha, \beta^i \).

ADM
3+1 split of spacetime

Figure: C. Reisswig

Foliation of spacetime

3-hyper-surface

\( t=0 \)
\( t=dt \)
\( n \)
\( \gamma_{ab} \)
\( K_{ab} \)

\( \alpha dt \)
\( \beta dt \)
The GR1D Code [O’Connor & Ott 2010 CQG]

- **GR1D**: Open-Source 1.5D GR (1D + rotation) GR hydrodynamics code.
- **Eulerian Radial-gauge, polar-slicing** (→ Schwarzschild-like coordinates).

\[ ds^2 = -\alpha(r, t)^2 dt^2 + X(r, t)^2 dr^2 + r^2 d\Omega^2 \]

[Gourgoulhon ‘91, Romero et al. ‘96]

- Choice of coordinates greatly simplifies GR hydro equations (zero shift).
- Disadvantage: Cannot evolve past horizon formation (like May & White, Misner & Sharp, van Riper formulations).

**GR Hydro equations in GR1D**

Implemented as semi-discrete finite-volume scheme with PPM reconstruction, HLLE Riemann solver and Runge-Kutta time integration.

\[ \partial_t \vec{U} + \frac{1}{r^2} \partial_r \left[ \frac{\alpha r^2}{X} \vec{F} \right] = \vec{S} \]

- **Rotation**: Assume constant angular velocity on spherical shells → effective centrifugal force.
GR1D: EOS & Microphysics

- Low-density EOS: Timmes EOS (tabulated & matched).
- Neutrinos during collapse: effective $Y_e(\rho)$ approx. [Liebendörfer ’05].
- Postbounce: 3-flavor, energy-averaged (gray) neutrino leakage scheme (tuned to reproduce luminosities from full radiation-hydro simulations to ~20%).
- Approximate neutrino heating:

$$Q_{\nu_i}^{\text{heat}}(r) = f_{\text{heat}} \frac{L_{\nu_i}^{\text{FRF}}(r)}{4\pi r^2} \sigma_{\text{heat},\nu_i} \frac{\rho}{m_u} X_i \left\langle \frac{1}{F_{\nu_i}} \right\rangle e^{-2\tau_{\nu_i}}$$
Selected Results from 1.5D Black Hole Formation Simulations

Parameter Study: > 700 Model Calculations

Protoneutron Star Collapse

O’Connor & Ott 2010, open-source GR1D code @ http://www.stellarcollapse.org

![Graph showing times vs. densities and mass for protoneutron star collapse](image)
Influence of Rotation

\[ f_{\text{cent}} = \frac{2}{3} \omega^2 r \]

\[ j(r) = j_{16,\infty} \left[ 1 + \left( \frac{A_{\odot}}{r} \right)^2 \right]^{-1} \times 10^{16} \text{ cm}^2 \text{ s}^{-1} \]

\[ \Delta j_{16,\infty} = 0.25 \text{ between models} \]

\[ u40WHW02 \text{ LS180} \]

\[ j_{16,\infty} = 0 \]
\[ j_{16,\infty} = 1 \]
\[ j_{16,\infty} = 2 \]
\[ j_{16,\infty} = 3 \]

[O’Connor & Ott 2011]
Postbounce Spin Evolution

\[ \frac{T}{|W|} \approx 0.27, \quad \frac{T}{|W|} \approx 0.14. \]

- At lower \( \frac{T}{|W|} \):
  - Dynamical shear instability.

\[ \Delta j_{16,\infty} = 0.25 \text{ between simulations} \]

\[ T/|W| = 0.27 \]

\[ u40WHW02 LS180 \]

- \( j_{16,\infty} = 1.0 \)
- \( j_{16,\infty} = 2.0 \)
- \( j_{16,\infty} = 3.0 \)

\[ \left( \frac{T}{|W|} \right)_{\text{dynamical}} \approx 0.27, \quad \left( \frac{T}{|W|} \right)_{\text{secular}} \approx 0.14. \]

[O’Connor & Ott 2011]
Rotational Nonaxisymmetric Instability

\[
(T/|W|)_{\text{dynamical}} > \approx 0.27.
\]

[Ott '08, unpublished]
Rotational Nonaxisymmetric Instability

[Ott '08, unpublished]

- Limit on PNS spin by redistribution / radiation of angular momentum.
Kerr Solution and Black Hole Spin

\[ a^* = \frac{J}{M^2} \]

\[ R_{\text{Horizon}} = \frac{2M + \sqrt{4M^2 - 4a^*^2M^2}}{2} \]

[Wikipedia]
Black Hole Birth Spin

Spin of the nascent BH is limited to $a^* < 1$ by nonaxisymmetric instabilities in the protoneutron star.

\[ \Omega_c \text{ [ rad s}^{-1}] \]

\[ \frac{\dot{a}^*_{\text{PNS}}}{M_{\text{e, PNS}}} \]  

\[ [\text{O’Connor & Ott 2011}] \]
3+1 GR Simulations of BH Formation
3+1 GR Computational Framework

- **Cactus**: Open-source software framework for HPC, developed at the Center for Computation & Technology at LSU.

- **Einstein Toolkit** ([http://einstein Toolkit.org](http://einstein Toolkit.org))
  Set of open-source codes for numerical relativity and computational relativistic astrophysics. (LSU, Caltech, RIT, GATech)
  - Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation of numerical GR (code: McLachlan).
  - General-Relativistic Hydrodynamics (code: GRHydro).
  - Adaptive Mesh Refinement (code: Carpet driver).
  - Analysis: Horizon finder, GW extraction etc.

- Nuclear equation of state and neutrino leakage from open-source GR1D code: [http://stellarcollapse.org](http://stellarcollapse.org)
Computational Framework: AMR

11 levels of AMR

Finest level: $dx \sim 100$ m

Buras et al. 2006ab, Scheck et al. 2007, Burrows et al. 2006,
Ott et al. '11
Physical Review Letters (in press)
The End
Supplemental Slides
GR Hydrodynamics

Local conservation laws:

\[ J^\mu_{\ ;\mu} = 0 \]  
\[ T^{\mu\nu}_{\ ;\mu} = 0 \]

- mass
- stress-energy

Covariant derivative; here: divergence takes into account that space is curved.

\[ \frac{1}{\sqrt{-g}} \left( (\sqrt{\gamma} U)_{,0} + (\sqrt{\gamma} F^i)_{,i} \right) = s \]

\[ D = J^\mu n_\mu = \alpha \rho u^0 = \rho W, \]

\[ S^i = \perp^i_v T^{\mu\nu} n_\mu = \rho h W^2 v^i, \]

\[ \tau = T^{\mu\nu} n_\mu n_\nu - J^\mu n_\mu = \rho h W^2 - P - D \]

\[ F^i = \begin{pmatrix} \alpha v^i - \beta^i \end{pmatrix} D \\
\begin{pmatrix} \alpha v^i - \beta^i \end{pmatrix} S_j + \alpha p \delta^i_j \\
\begin{pmatrix} \alpha v^i - \beta^i \end{pmatrix} \tau + \alpha p v^i \]

\[ s = \begin{pmatrix} 0 \\
T^{\mu\nu} (g_{\nu j, \mu} + \Gamma^\delta_{\mu\nu} g_{\delta j}) \\
\alpha (T^{\mu 0}_{\nu} 1 + \alpha, \mu - T^{\mu\nu} \Gamma^0_{\mu\nu}) \end{pmatrix} \]

\[ h = 1 + \epsilon + P/\rho \]

\[ \gamma = \det(\gamma_{ik}) \]

\[ \sqrt{-g} = \alpha \sqrt{\gamma} \]

\[ W = \alpha u^0 = 1/\sqrt{1 - \gamma_{ij} v^i v^j} \]

\[ u^\alpha 4 - velocity \]

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Some more Details:

ADM Line Element:

\[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \]

\[ \gamma_{ij} = 3 \text{ - metric} \]
\[ \alpha = \text{lapse function} \]
\[ \beta^i = \text{shift vector} \]

ADM Evolution Equations:

\[ K_{ij} \text{ \textquotedblleft Extrinsic Curvature\textquotedblright} \]
\[ S_{ij} \text{ \textquotedblleft Projection of stress-energy tensor.\textquotedblright} \]
\[ \rho_{ADM} \text{ \textquotedblleft ADM energy density.\textquotedblright} \]

\[ \partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \]
\[ \partial_t K_{ij} = -\nabla_i \nabla_j \alpha + \alpha \left[ R_{ij} + K K_{ij} - 2 K_{im} K^m_j \right. \]
\[ - 8\pi \left( S_{ij} - \frac{1}{2} \gamma_{ij} S \right) - 4\pi \rho_{ADM} \gamma_{ij} \left. \right] + \beta^m \nabla_m K_{ij} \]
\[ + K_{im} \nabla_j \beta^m + K_{mj} \nabla_i \beta^m. \]

ADM Constraint Equations:

\[ (3) R + K^2 - K_{ij} K^{ij} - 16\pi \rho_{ADM} = 0, \quad \nabla_j K^{ij} - \gamma^{ij} \nabla_j K - 8\pi j^i = 0. \]
Some more Details:

ADM Line Element:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\gamma_{ij} = \text{3 - metric}$$

Problem:

ADM equations unstable in multi-D simulations in the presence of strong curvature (e.g. a black hole).

Solution (late 1990s):

Use stable re-formulation of ADM equations, e.g., the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation.

$$+ K_{im}\nabla_j\beta^m + K_{mj}\nabla_i\beta^m.$$
Special Case: Spherical Symmetry

(see, e.g., O’Connor & Ott 2010)

Equations simplify significantly; may use particularly convenient choice of gauge conditions: “Radial Gauge, Polar Slicing”

\[ ds^2 = -\alpha(r,t)^2 dt^2 + X(r,t)^2 dr^2 + r^2 d\Omega^2 \]

\[ \alpha(r,t) = \exp[\phi(r,t)], \quad X(r,t) = \left(1 - \frac{2m(r,t)}{r}\right)^{-1/2} \]

\[ \phi(r,t) = \int_0^r X^2 \left[ \frac{m(r',t)}{r'^2} + 4\pi r'(P + \rho X^2 u' u') \right] dr' \]

-> Schwarzschild-like line element

-> Solving for the spacetime metric becomes an ODE that can just be integrated.

Hydrodynamics:

\[ \partial_t \vec{U} + \frac{1}{r^2} \partial_r \left[ \frac{\alpha r^2}{X} \vec{F} \right] = \vec{S} \]

U, F, S similar to general case, but total energy conserved by construction. Additional source terms due to neutrinos.
Another Twist: Rotation & Convection

\[ N^2 + \frac{1}{r^3} \frac{d}{dr} j^2 < 0 \]

\[ j = \Omega r^2 \]

\[ N^2 = \frac{g}{\rho \gamma} \left( \frac{\partial p}{\partial s}_{\rho, Y_l} \frac{ds}{dr} + \frac{\partial p}{\partial Y_l}_{\rho, s} \frac{dY_l}{dr} \right) \]
Preollapse Stellar Structure

[Image of a graph showing the relationship between density and radius for different stellar masses. The graph is labeled with various curves for densities of 12 M☉ solar, 12 M☉ zero, 20 M☉ solar, 20 M☉ zero, 40 M☉ solar, 40 M☉ zero, 75 M☉ solar, and 75 M☉ 10⁻⁴ solar.]

Woosley, Heger, Weaver 2002

[O’Connor & Ott 2010 in prep.]
The **VULCAN/2D Code**


- **Axisymmetric Newtonian Magnetohydrodynamics** with rotation (2.5D).
- Unsplit 2\textsuperscript{nd} order arbitrary Eulerian/Lagrangian (**ALE**) scheme.
- Newtonian gravity, logically cylindrical coordinates, **arbitrary mesh**.
- Radiation Transport:
  - Multiple energy groups, \( \nu_e, \bar{\nu}_e, \nu_\mu \) species.
  - Slow-motion approximation.
- Multiple finite-temperature nuclear EOS options.
- Efficient parallelization in neutrino species/energy groups.
- Typical run size: 48–96 cores.

![VULCAN/2D Grid](image)

Typical problem sizes:
- 50k zones x O(50) vars (MGFLD)
- x O(50) for angle-dep. transport.
Testing the Acoustic Mechanism

• So far no independent confirmation of the acoustic mechanism.
• Overstable physical g-modes PNS shown to exist. [Ferrari et al. 2003, 2007; Yoshida et al. 2007]
• Questions:
  – Do modes reach amplitudes as high as seen in our calculations?
  – Effects of GR and 3D?
• Fundamental prerequisite for non-linear numerical tests of mode excitation: Grid must be singularity free & allow change of the core’s geometric center.
• Marek & Janka ‘09:
  Modes shown to exist, but don’t reach high amplitudes. But: (1) Amplitudes become high only at t > 0.6 - 0.8 s (not simulated), (2) MJ09 grid not singularity free. -> Acoustic Mech. not yet numerically ruled out.

VULCAN/2D grid