IRAP PhD and Erasmus Mundus School/Workshop
Les Houches, April 2011

Lecture 1: Stellar Collapse, Core-Collapse Supernovae, and the Formation of Black Holes.

Lecture 2: Gravitational-Wave Astronomy & and an Update on LIGO/VIRGO

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Stellar Collapse, Core-Collapse Supernovae and The Formation of Black Holes:

The Big Picture

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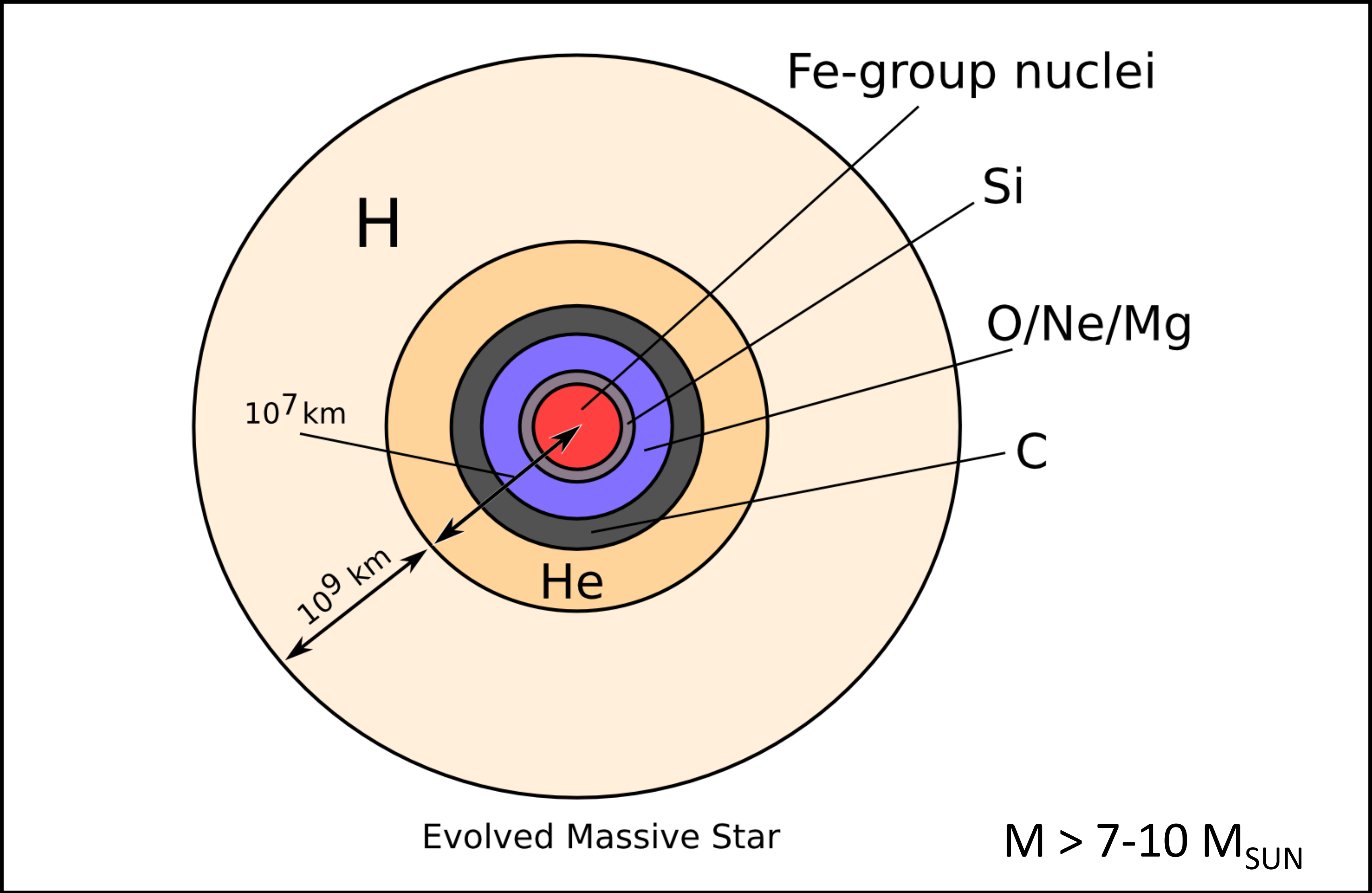
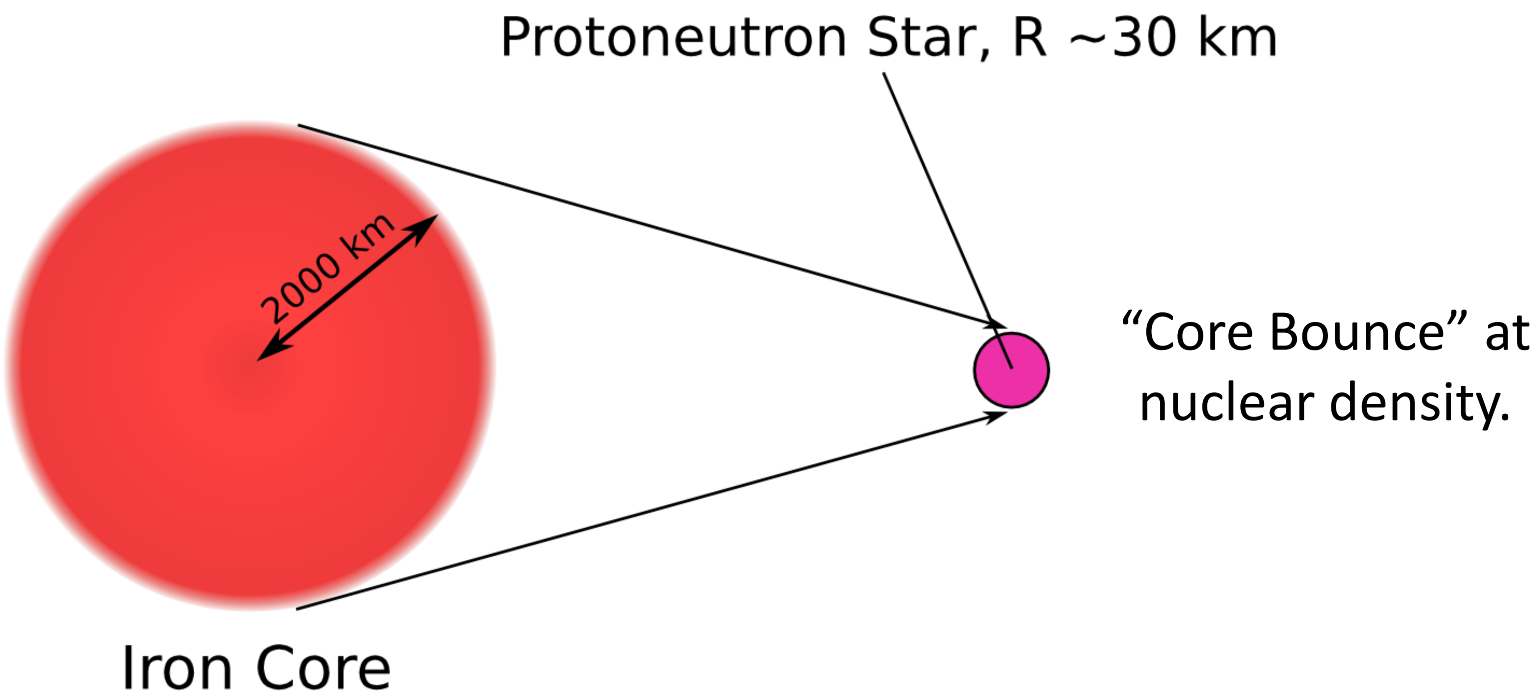
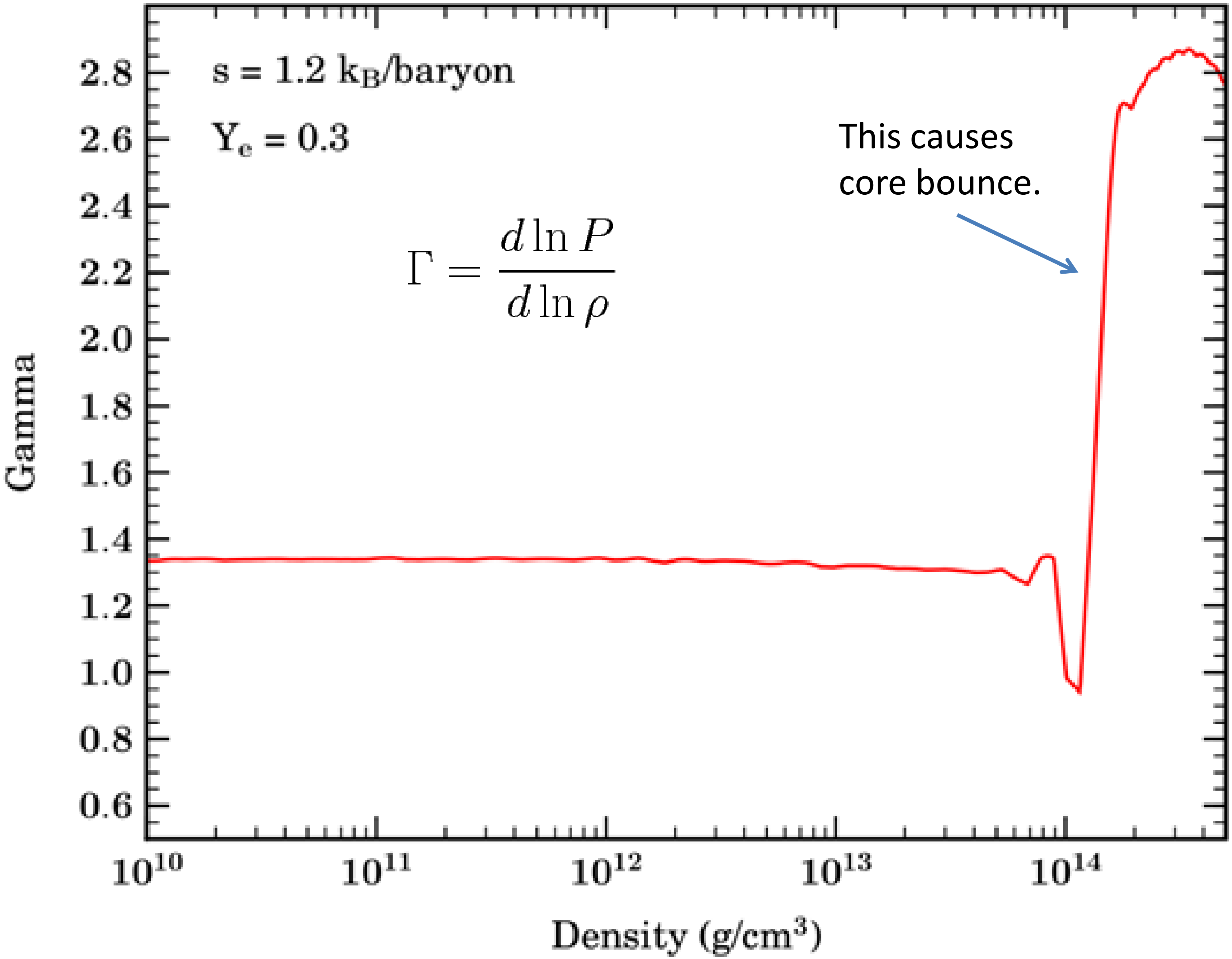
The Core-Collapse Scenario



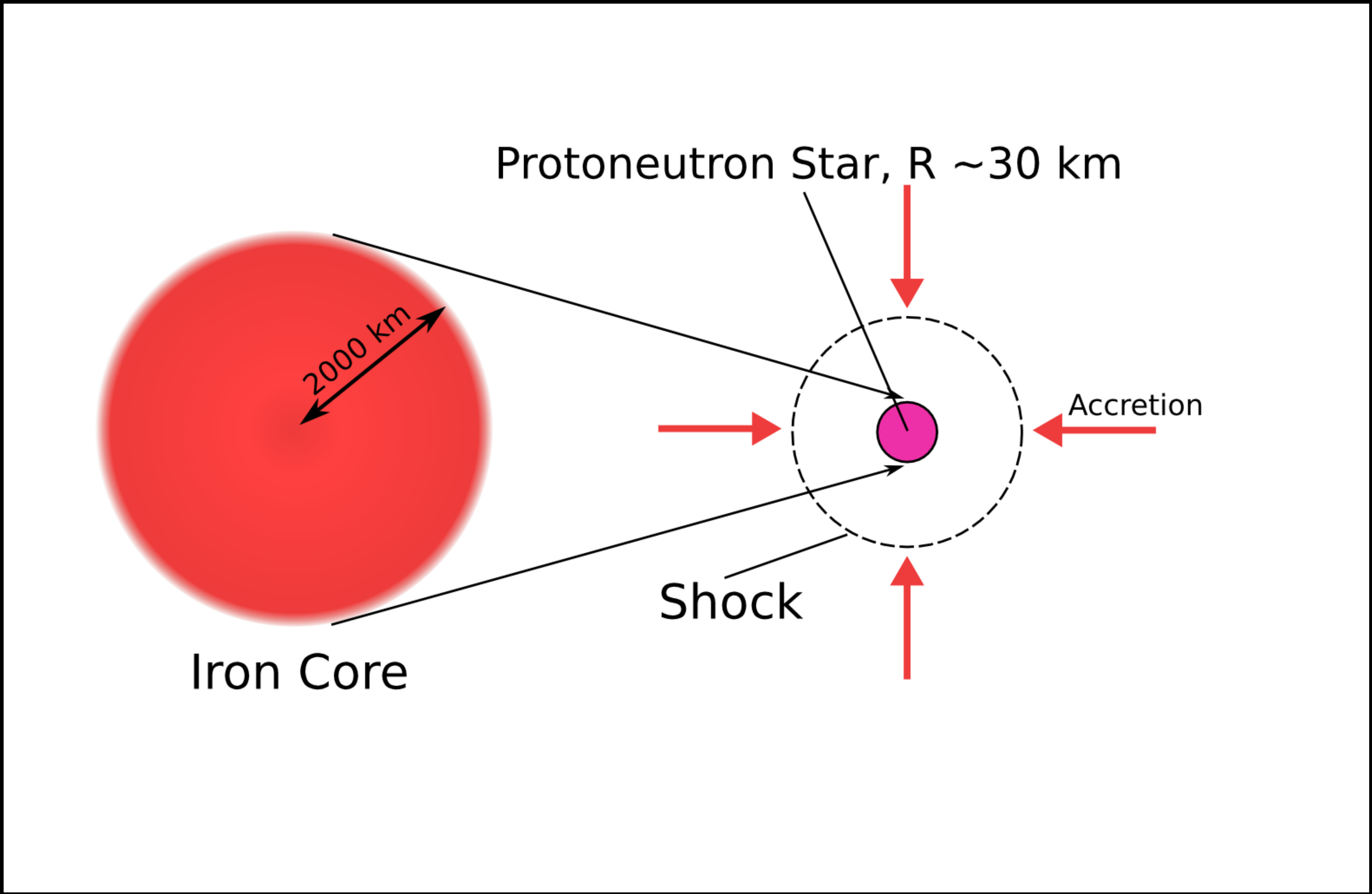
The Core-Collapse Scenario



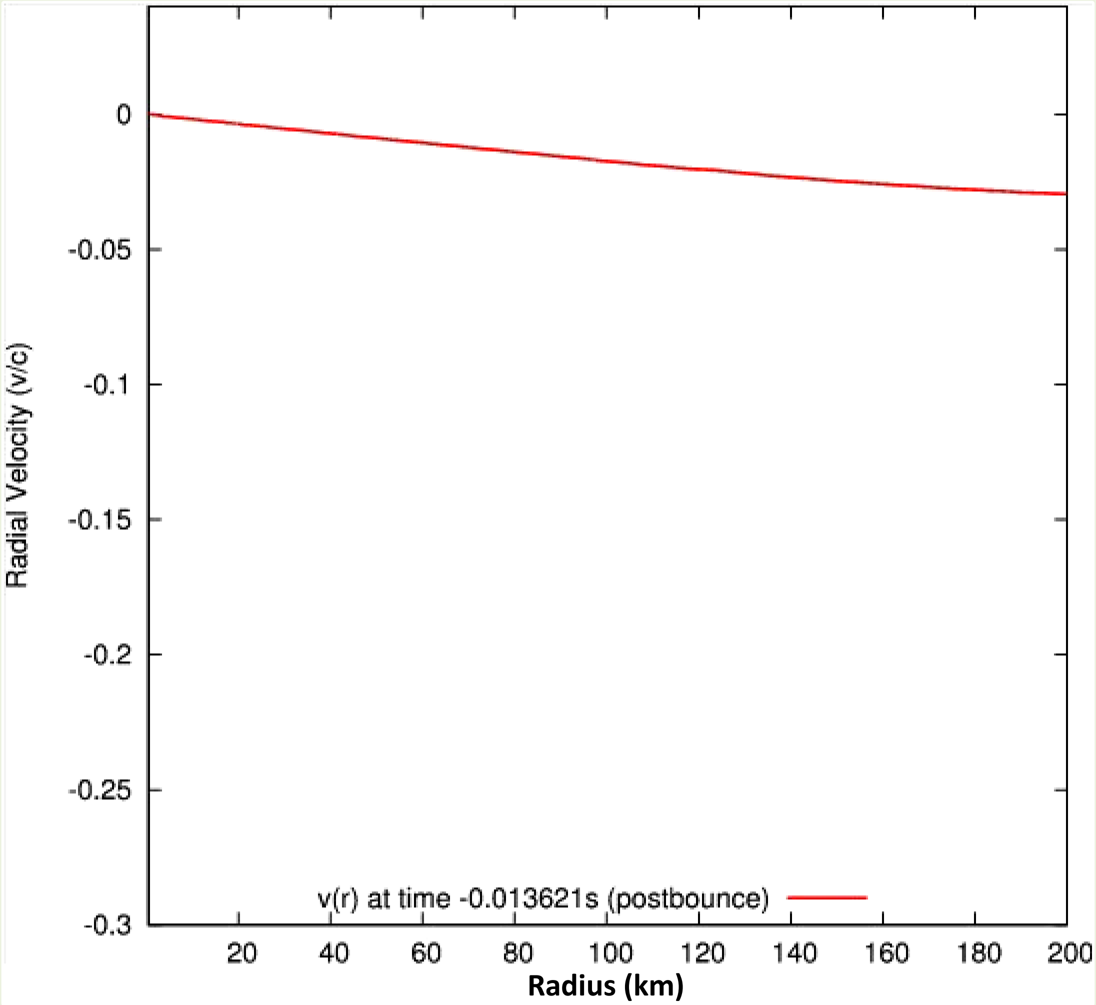
$M > \sim 1.3 - \sim 2.2\ M_{SUN}$


s = 1.2 k$_B$/baryon
Y$_e$ = 0.3
$\Gamma = \frac{d \ln P}{d \ln \rho}$
This causes core bounce.
Gamma
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
Density (g/cm^3)

The Core-Collapse Scenario



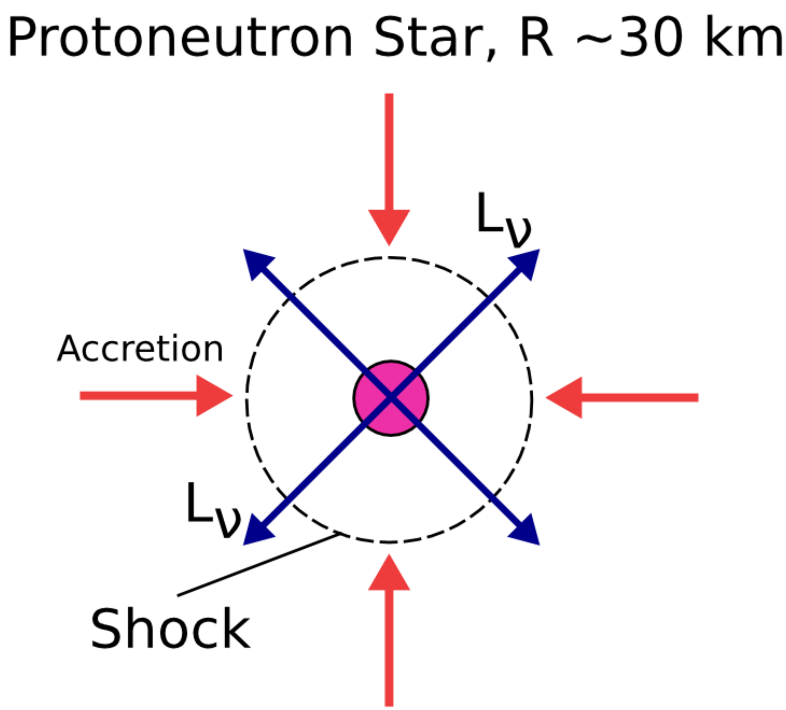
The Core-Collapse Supernova Problem



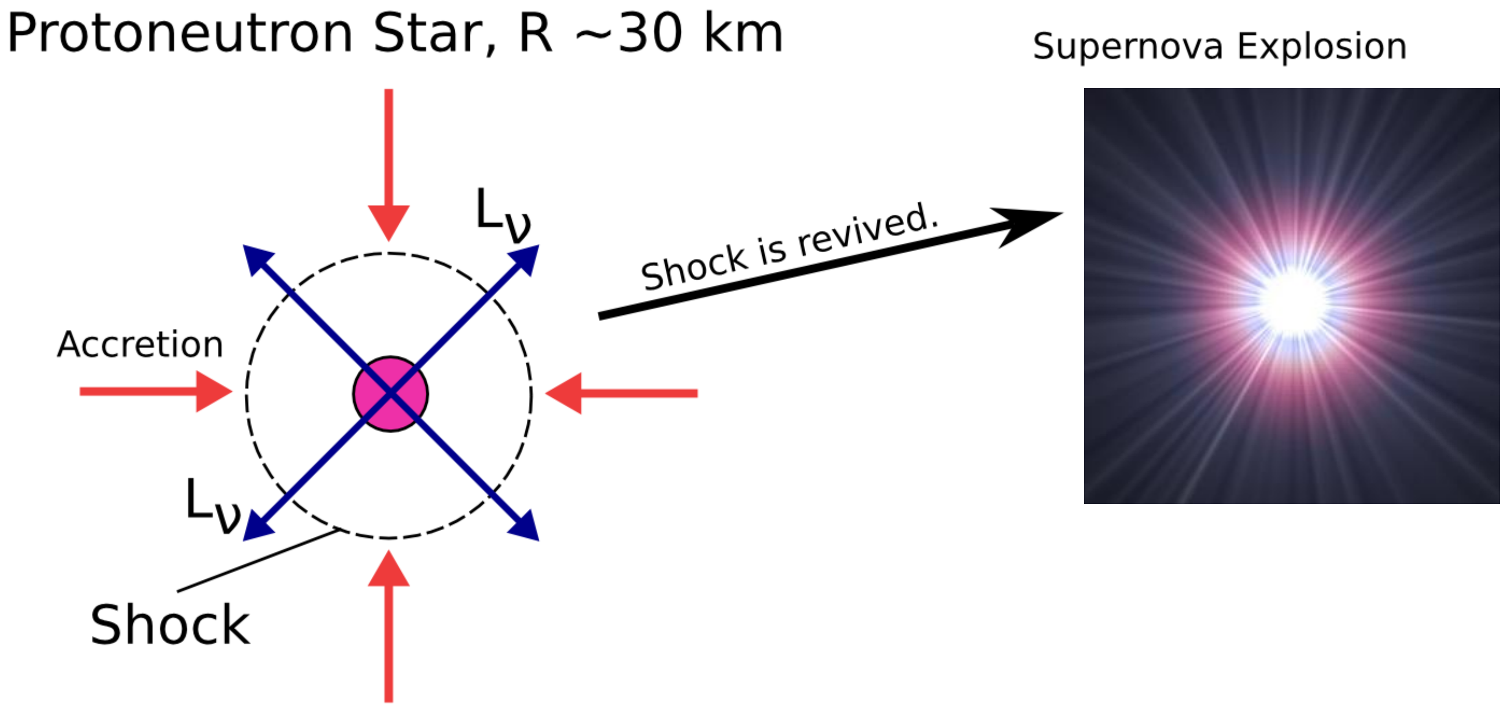
Animation by Evan O'Connor

- **Shock always stalls:**
  Dissociation of Fe-group nuclei @ ~8.8 MeV/baryon (~17 B/M_{Sun}).
  Neutrino losses @ 10 B/s (1 [B]ethe = 10^{51} ergs).

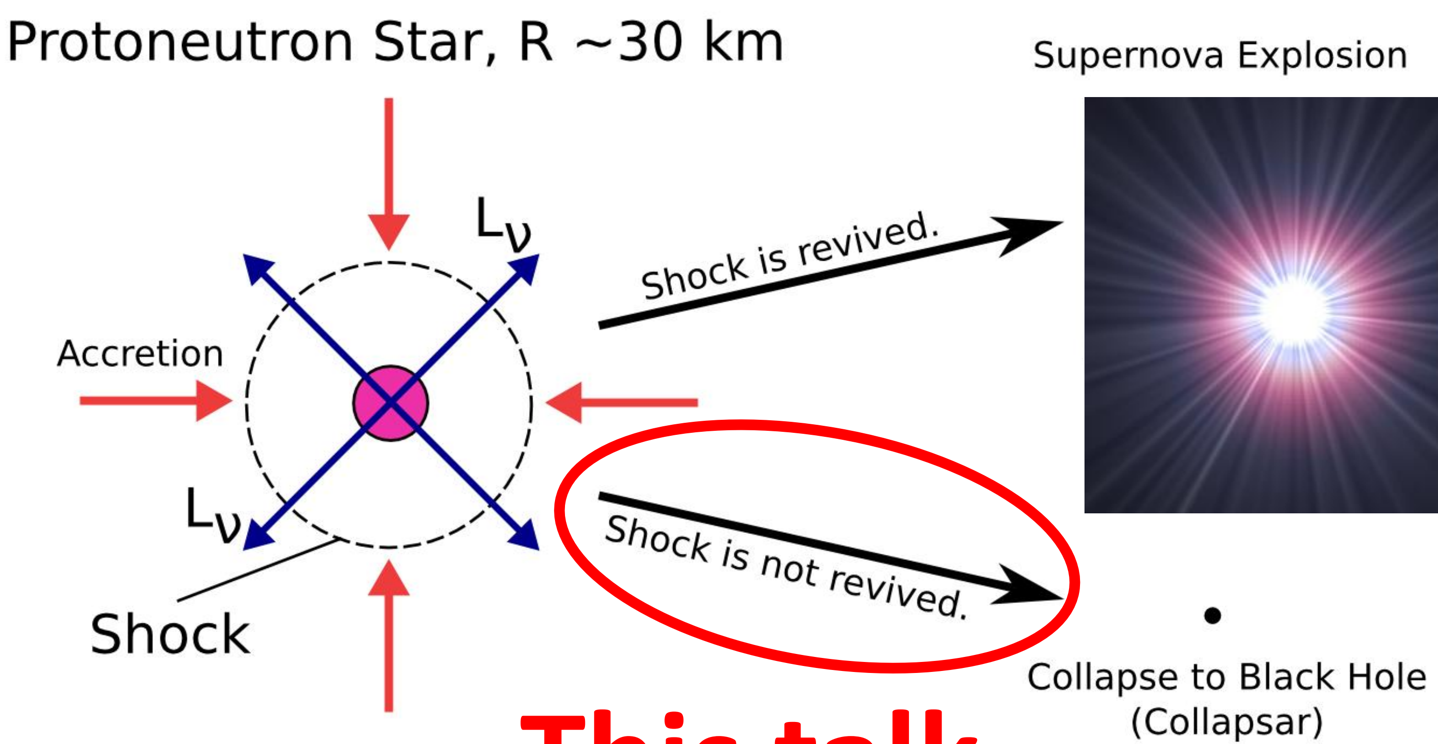
The Core-Collapse Scenario



The Core-Collapse Scenario

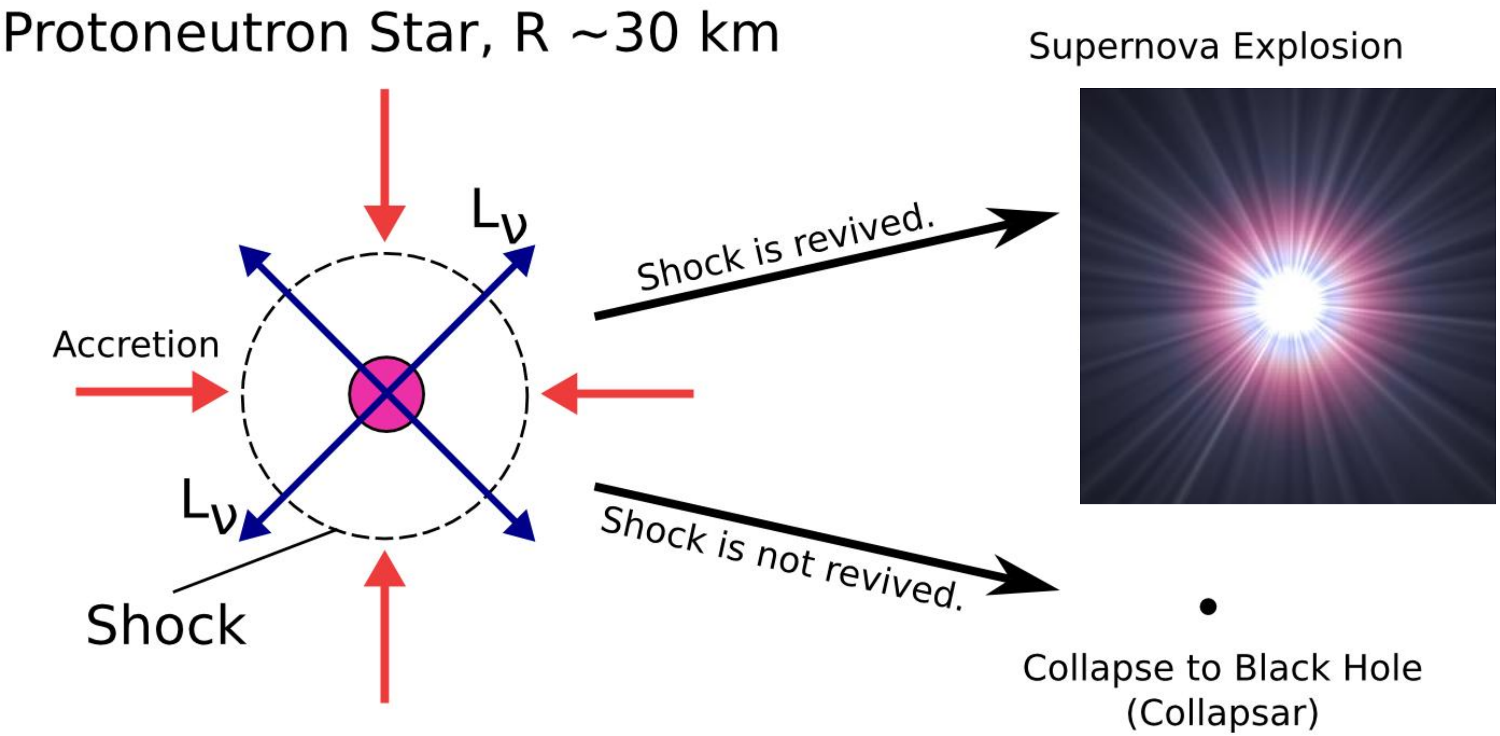


The Core-Collapse Scenario



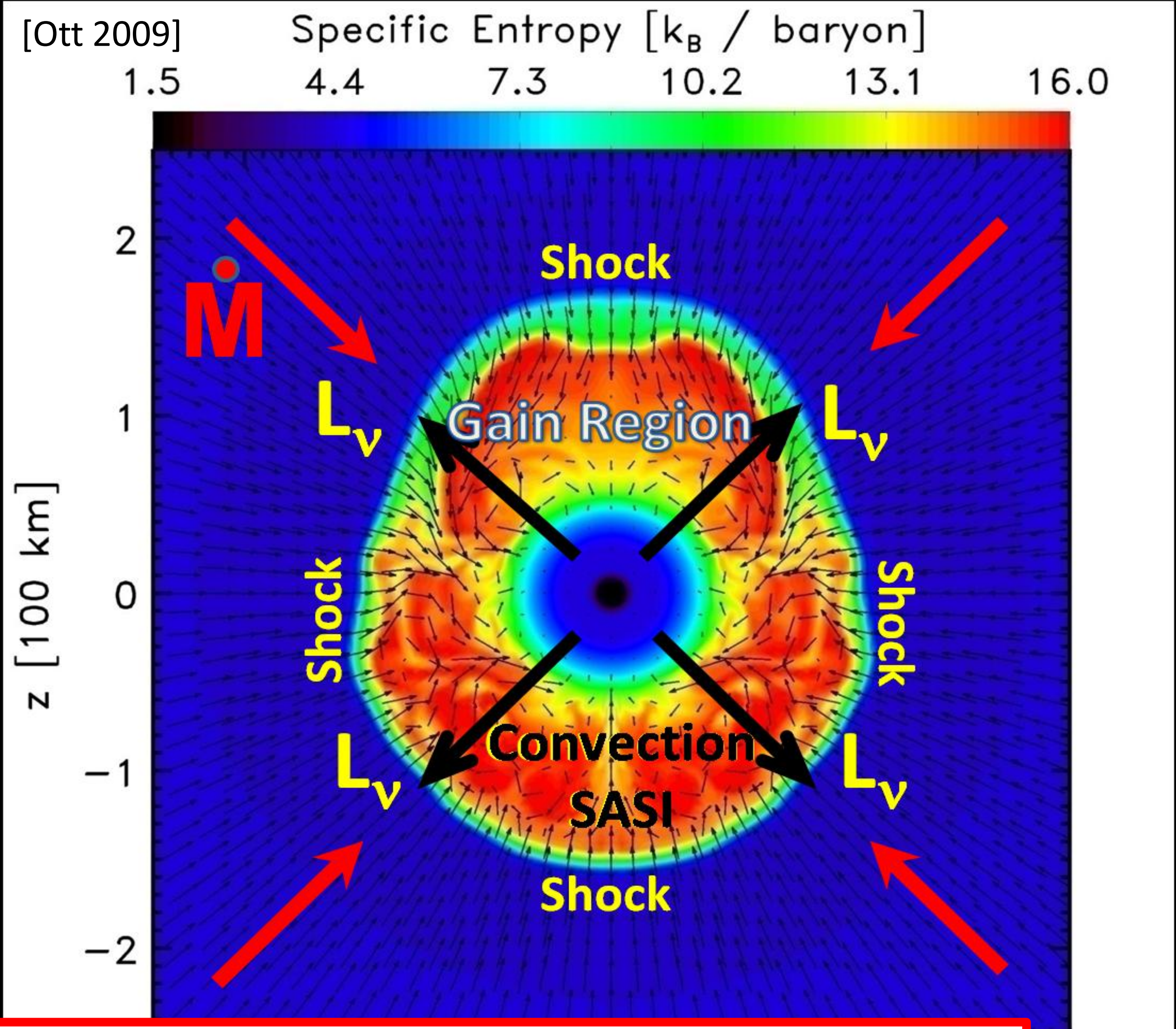
This talk

The Core-Collapse Scenario

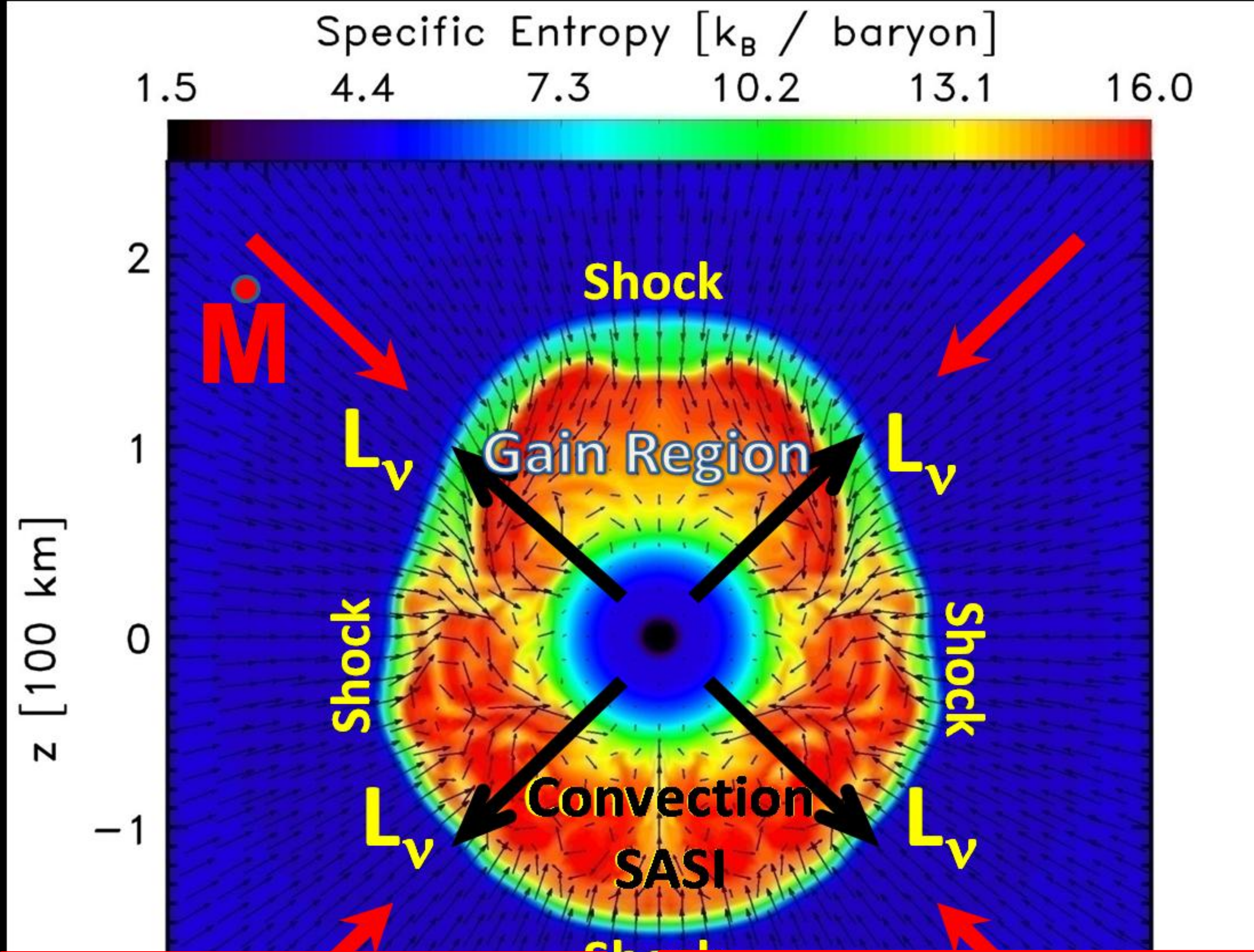


Digression:

What is the "Supernova Mechanism"?



Neutrinos: Carry away ~99% of the gravitational energy of the newborn neutron star (~300 B).



The Neutrino Mechanism

Imshenik & Nadyozhin 1964
Arnett 1966
Colgate & White 1966
Bethe & Wilson 1985

The Problem:

Doesn't quite work...
Fails in 1D, marginal in 2D.
Key may be 3D.
(->Nordhaus et al. 2010)

Core-Collapse Supernova Mechanisms

Introduced by:

Neutrino Mechanism

[Imshenik & Nadyozhin '64 Colgate & White '66, Arnett '66, Wilson '85, Bethe & Wilson '85]

Alternatives:

Magnetorotational Mechanism

[Bisnovaty-Kogan '70, LeBlanc & Wilson '70, Meier et al. '76, Symbalisty '84, Burrows et al. '07]

Acoustic Mechanism

[proposed by Burrows et al. '06, '07; not yet confirmed by other groups/codes]

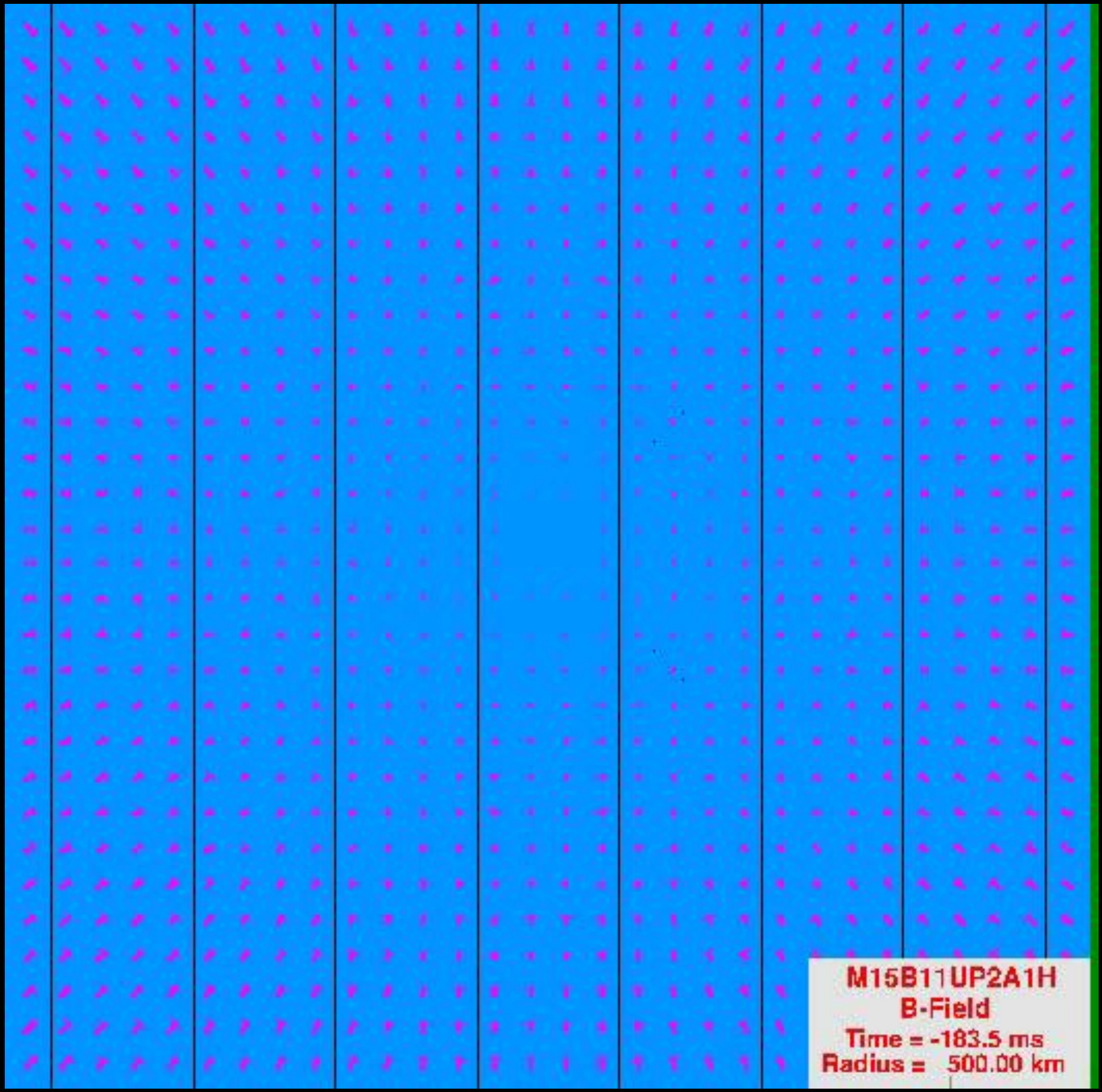
Phase-Transition-Induced Mechanism

[Takahara & Sato '88, Gentile et al. '93, Sagert et al. '09, Fischer et al. '10]

Magnetorotational Explosions

[Burrows et al. 2007, Dessart et al. 2008]

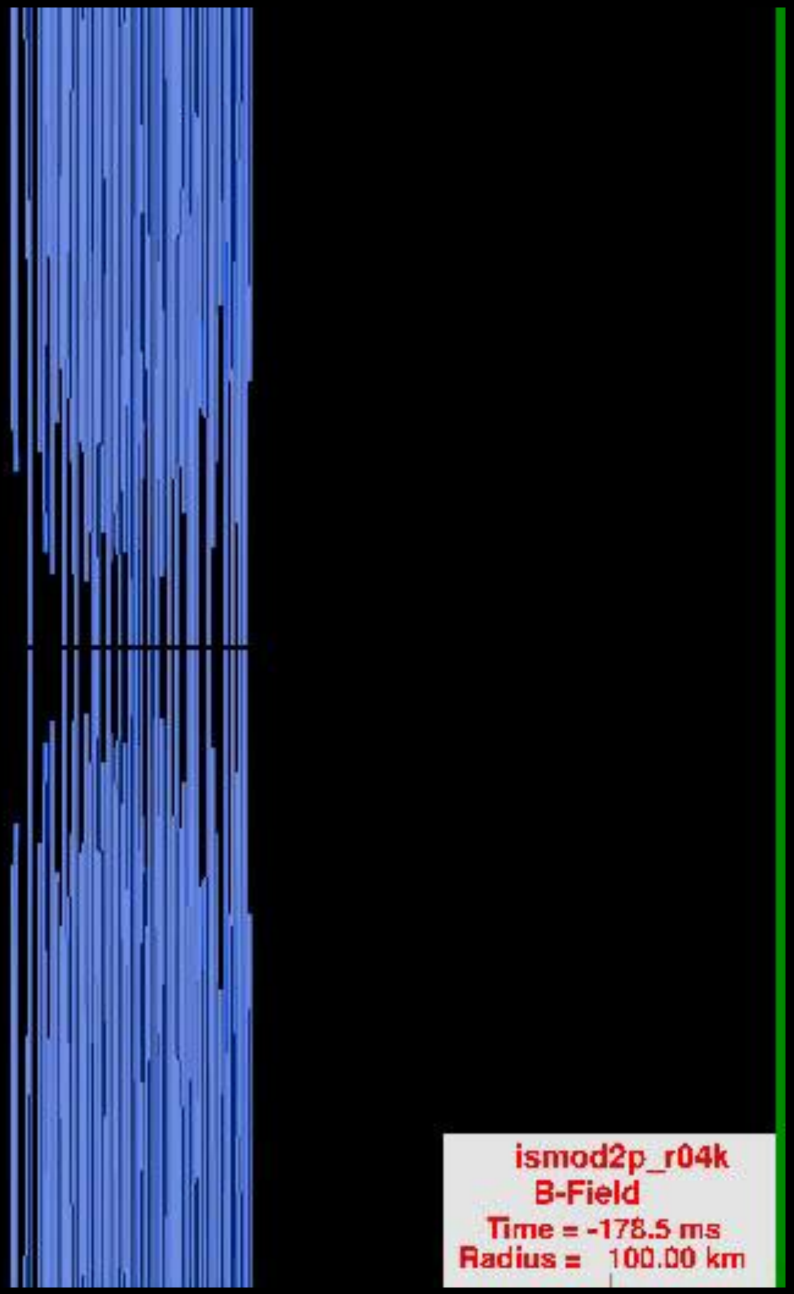
- **Rapid rotation**: P_0<4-6 s -> millisecond PNS
- PNS rotational energy: ~10 B = 10^{52} erg
- Amplification of B fields up to equipartition:
  - compression
  - dynamos
  - **magneto-rotational instability** (MRI)
- Jet-driven outflows.
- MHD-driven explosion may be GRB precursor.

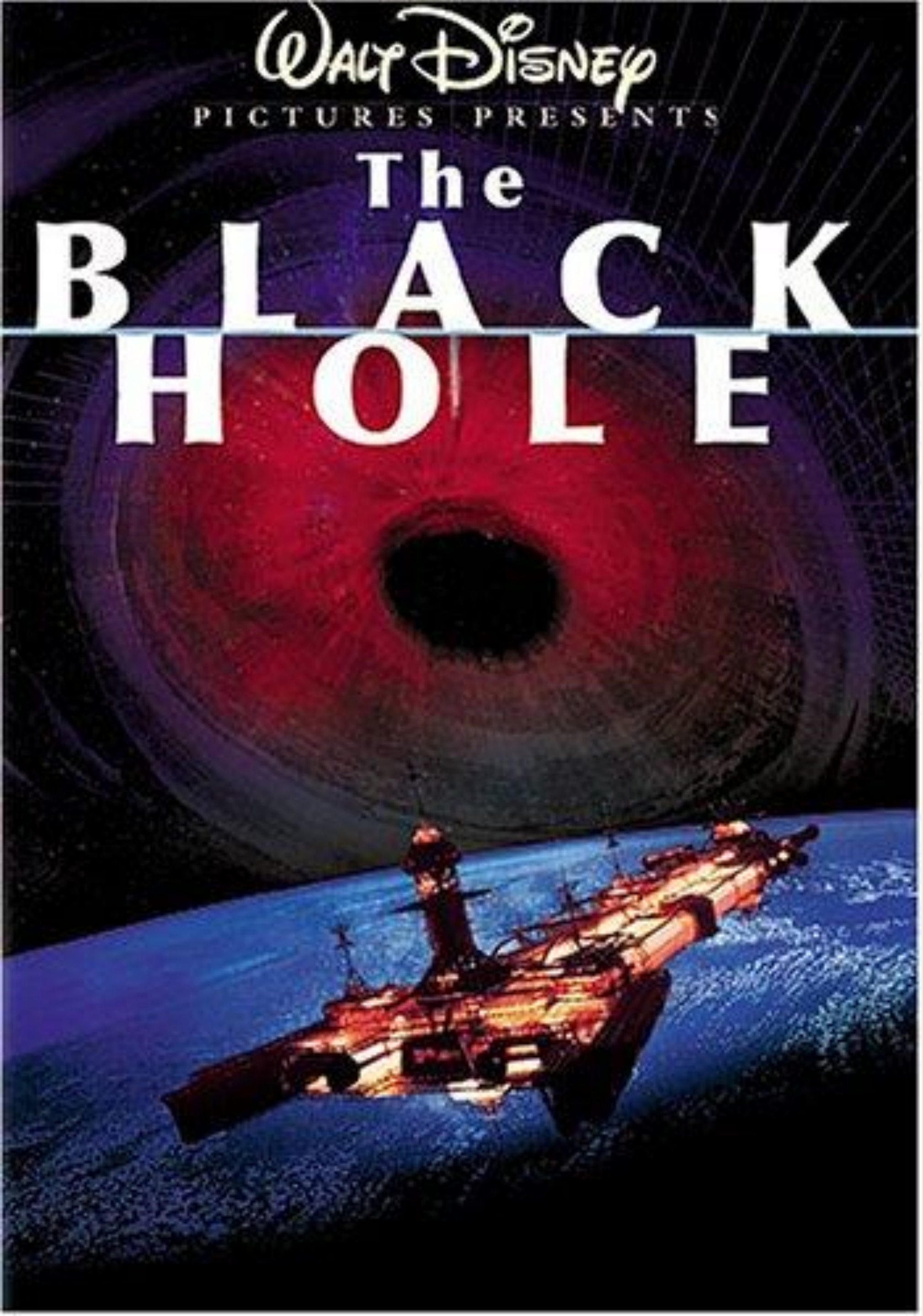


VULCAN 2D R-MHD code, Livne et al. 2007, Burrows et al. 2007.

Newtonian Radiation-MHD Simulations with VULCAN/2D

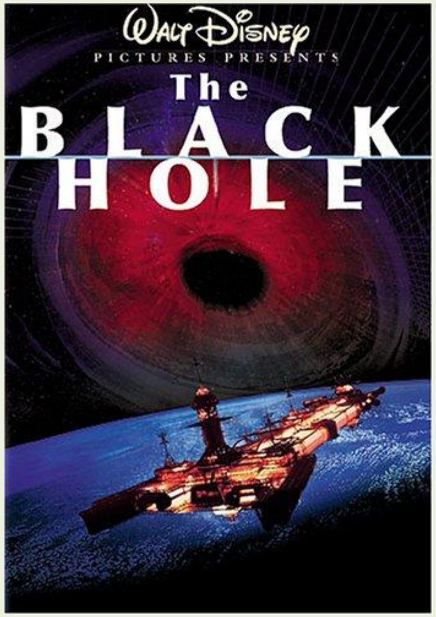
Magnetic field lines in M15B11UP2A1H of Burrows, Dessart, Livne, Ott, Murphy ‘07.





The Making of: Stellar-Mass Black Holes

The Making of Stellar-Mass Black Holes



Route 1 to BH formation:

Collapse to protoneutron star (PNS),
explosion fails -> “Collapsar Type I”

Route 2 to BH formation:

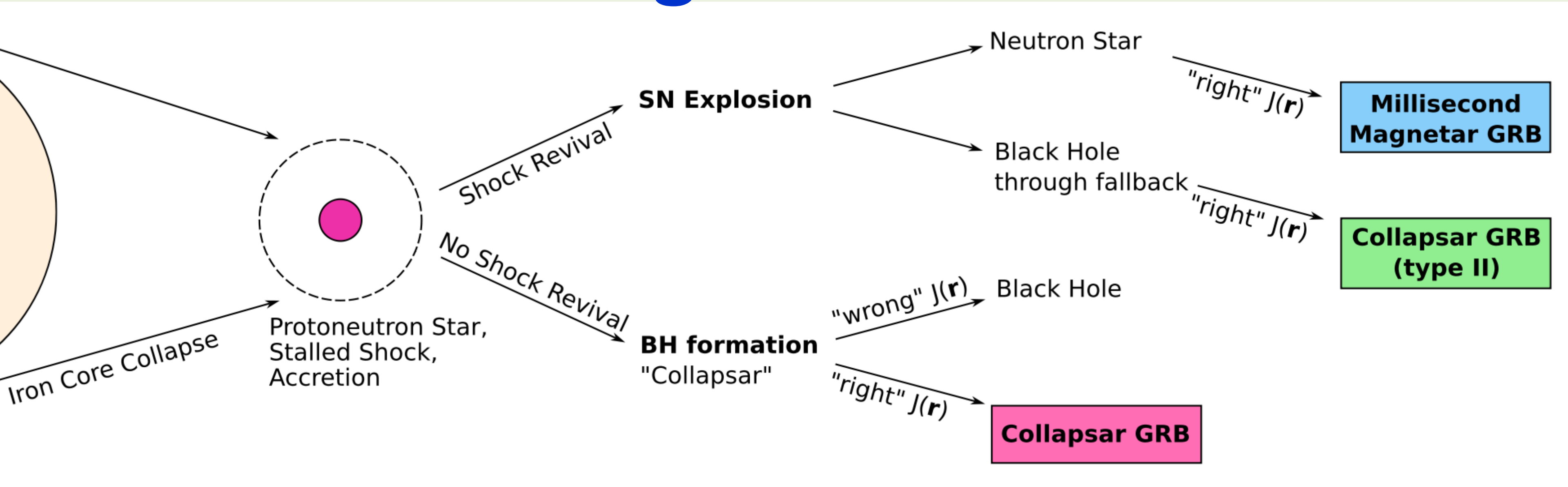
Collapse to PNS, explosion succeeds, but does not unbind entire stellar envelope.
Fallback accretion -> BH formation O(many) seconds.
“Collapsar Type II”

Route 3 to BH formation:

BH formation triggered by Hadron-Quark Phase transition in the PNS cooling phase.

Important aside:
There is no such thing as *direct collapse* to a black hole for ordinary massive stars (10—130 M_{Sun}).

CCSN – Long-GRB Connection



- Strong observational evidence for connection between type-Ibc core-collapse supernovae and long GRBs.
- 2 competing central scenarios:

  (1) **Millisecond Magnetar** (= 10^{15} gauss Neutron Star) **Model**

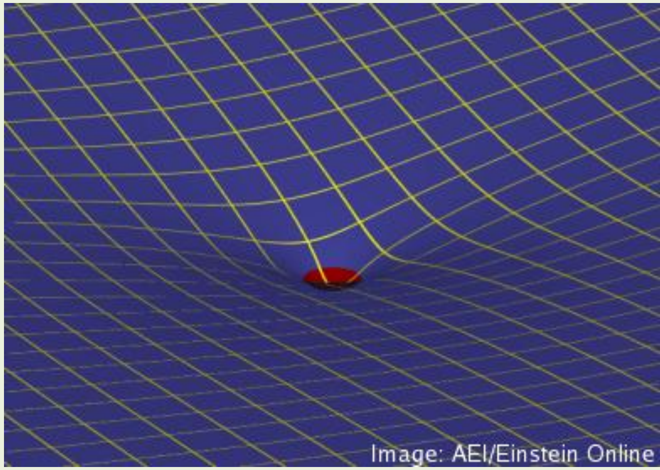
  (Bucciantini, Quatert, Metzger et al. '07-'10, supported by: Dessart, Burrows, Ott et al. '08)

  (2) **Collapsar Model** (Woosley 1993)

  BH + accretion disk formed in a *failing core-collapse supernova* or after a core-collapse supernova explosion via *fallback accretion*.

Studying Black Hole Formation

Key ingredient: **General Relativity**



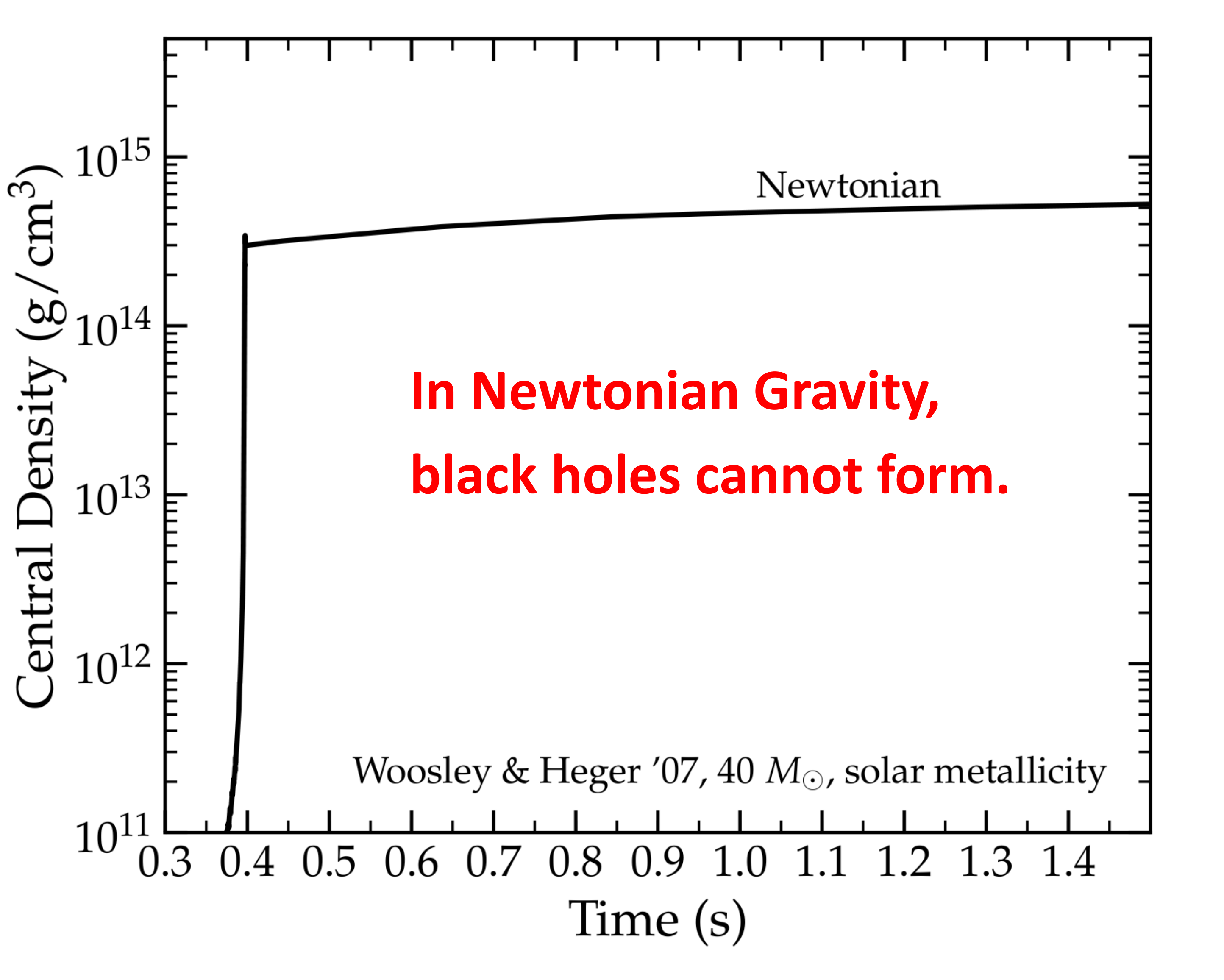
Einstein Equations:

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

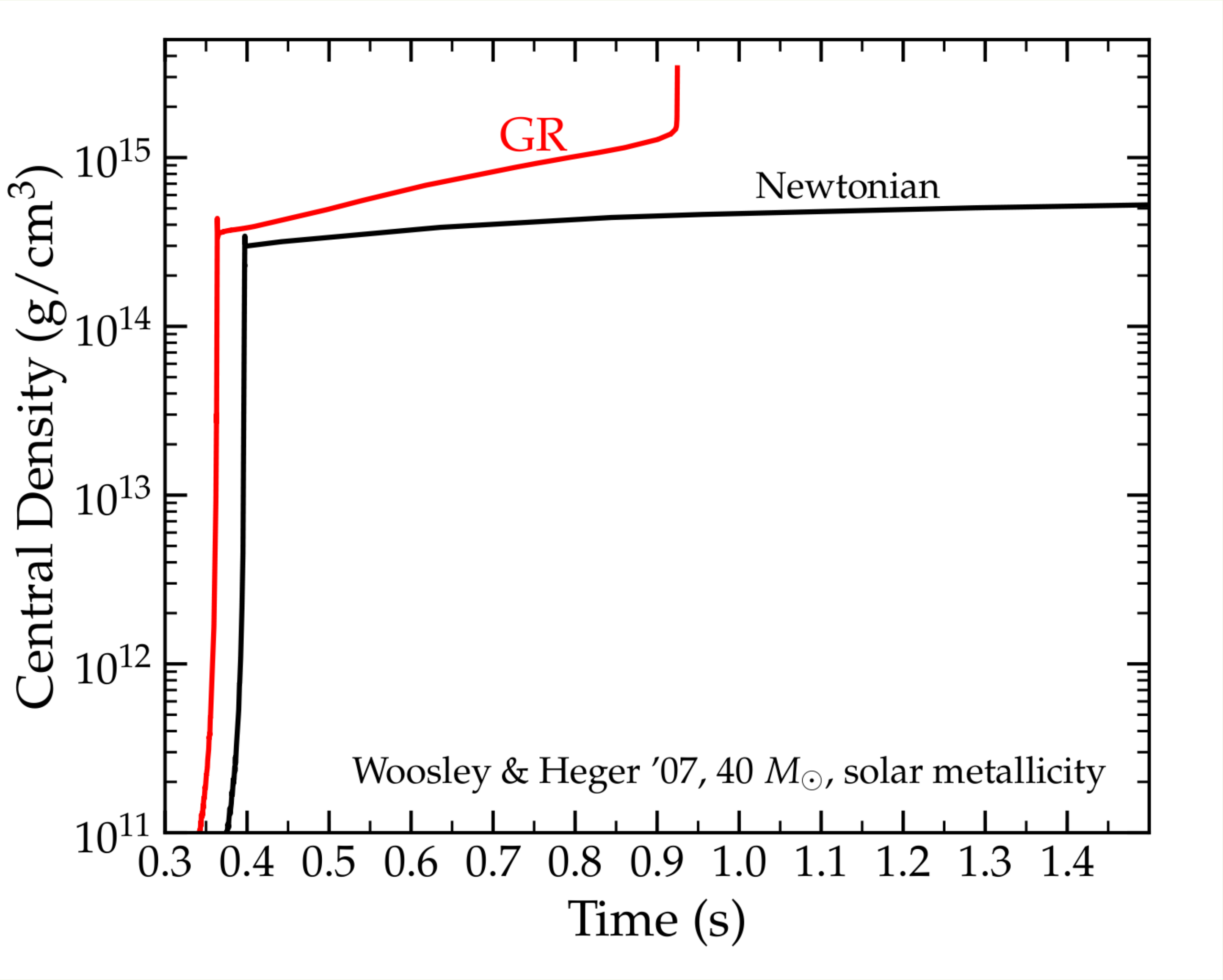
Curvature of spacetime

Stress-Energy sourcing Curvature

Effects of General Relativity



Effects of General Relativity



Einstein -> Computer: Numerical Relativity

Arnowitt
Deser
Misner
(**ADM**)

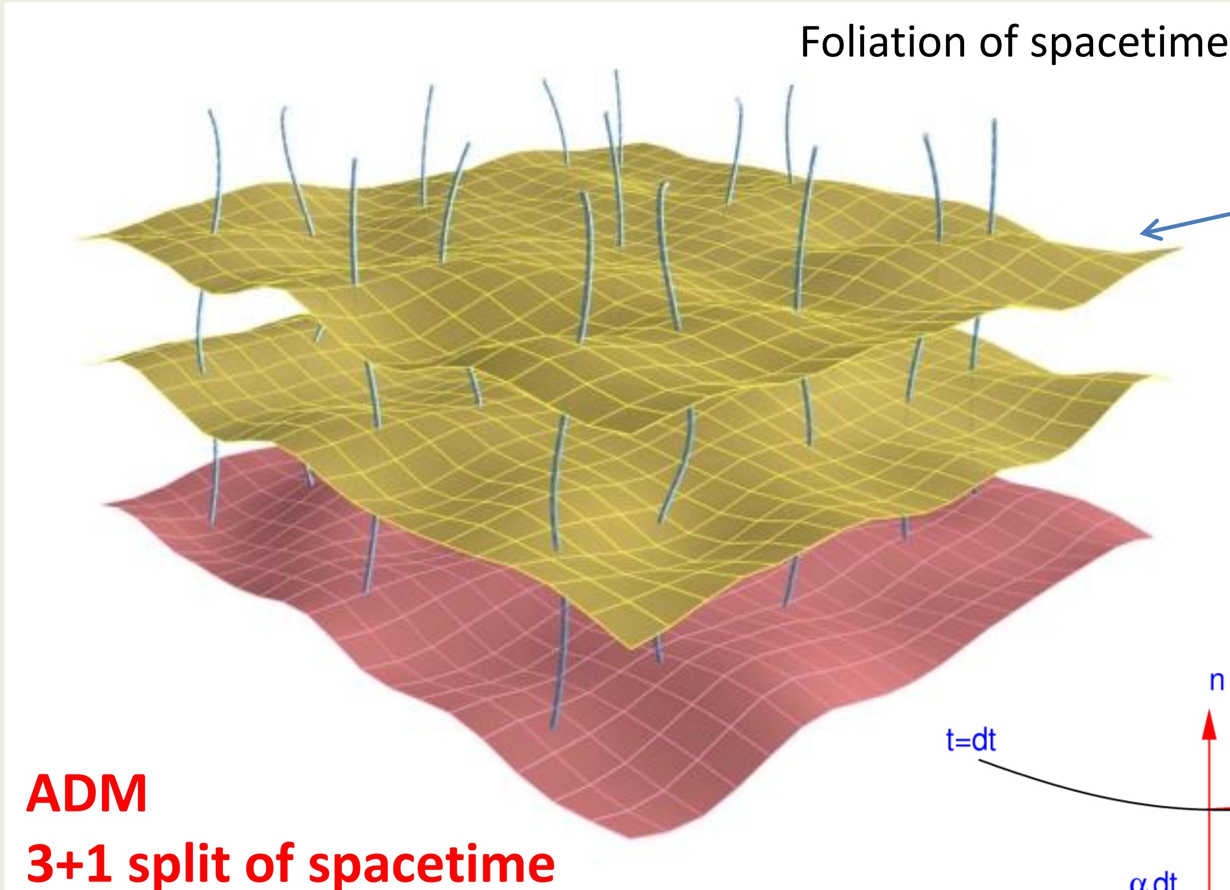
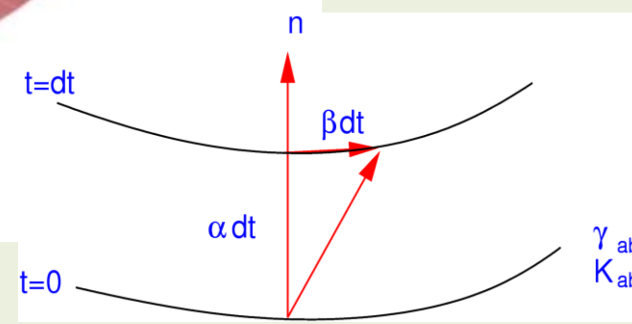


Figure: C. Reisswig



$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

- 12 first-order hyperbolic *evolution* equations.
- 4 elliptic *constraint* equations
- 4 coordinate gauge degrees of freedom: α, β^i.

The GR1D Code [O'Connor & Ott 2010 CQG]

- **GR1D**: ***Open-Source 1.5D GR (1D + rotation) GR hydrodynamics code.***
- Available from http://www.stellarcollapse.org .
- **Eulerian Radial-gauge, polar-slicing** (-> Schwarzschild-like coordinates).

$$ds^2 = -\alpha(r,t)^2 dt^2 + X(r,t)^2 dr^2 + r^2 d\Omega^2$$

[Gourgoulhon '91, Romero et al. '96]

- **Choice of coordinates greatly simplifies GR hydro equations (zero shift).**
- **Disadvantage: Cannot evolve past horizon formation**
  (like May & White, Misner & Sharp, van Riper formulations).
- **GR Hydro equations in GR1D**
  Implemented as semi-discrete finite-volume scheme with PPM reconstruction, HLLE Riemann solver and Runge-Kutta time integration.

$$\partial_t \vec{U} + \frac{1}{r^2}\partial_r \left[\frac{\alpha r^2}{X}\vec{F}\right] = \vec{S}$$

- **Rotation**: Assume constant angular velocity on spherical shells -> effective centrifugal force.

GR1D: EOS & Microphysics

- Multiple finite-temperature microphysical nuclear EOS:
  H. **Shen** et al. 1998, **Lattimer & Swesty** 1991 with K={180,220,375} MeV.
  EOS tables available in HDF5 format on http://www.stellarcollapse.org .
- Low-density EOS: Timmes EOS (tabulated & matched).
- Neutrinos during collapse: effective $Y_e(\rho)$ approx. [Liebendörfer '05].
- Postbounce:
  3-flavor, energy-averaged (gray) neutrino leakage scheme
  (tuned to reproduce luminosities from full radiation-hydro simulations to ~20%).
- Approximate neutrino heating:

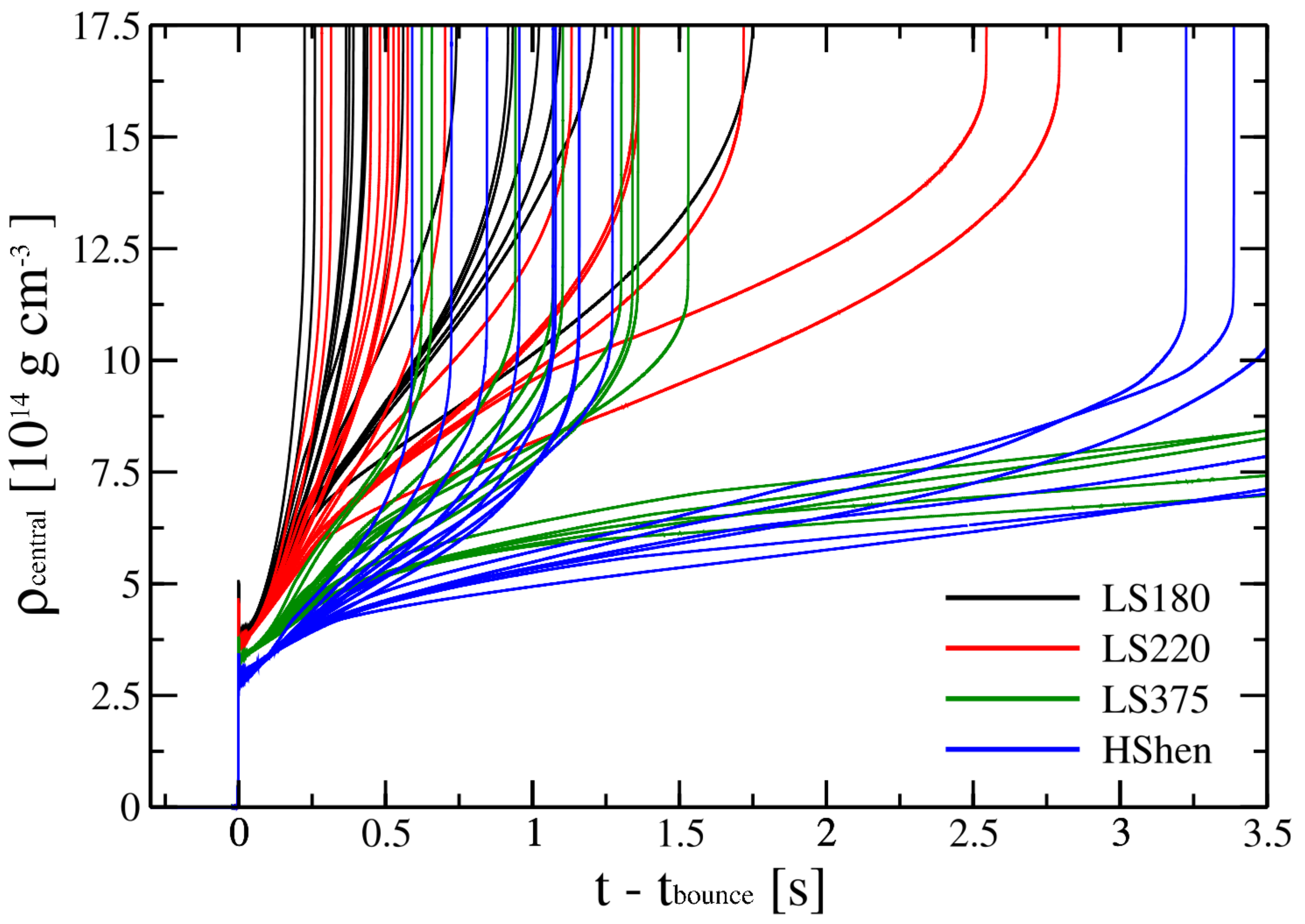
$$Q^{\mathrm{heat}}_{\nu_i}(r) = f_{\mathrm{heat}} \frac{L^{\mathrm{FRF}}_{\nu_i}(r)}{4\pi r^2} \sigma_{\mathrm{heat},\nu_i} \frac{\rho}{m_u} X_i \left\langle \frac{1}{F_{\nu_i}} \right\rangle e^{-2\tau_{\nu_i}}$$

Selected Results from
1.5D Black Hole Formation Simulations

O’Connor & Ott 2011, ApJ 730, 70

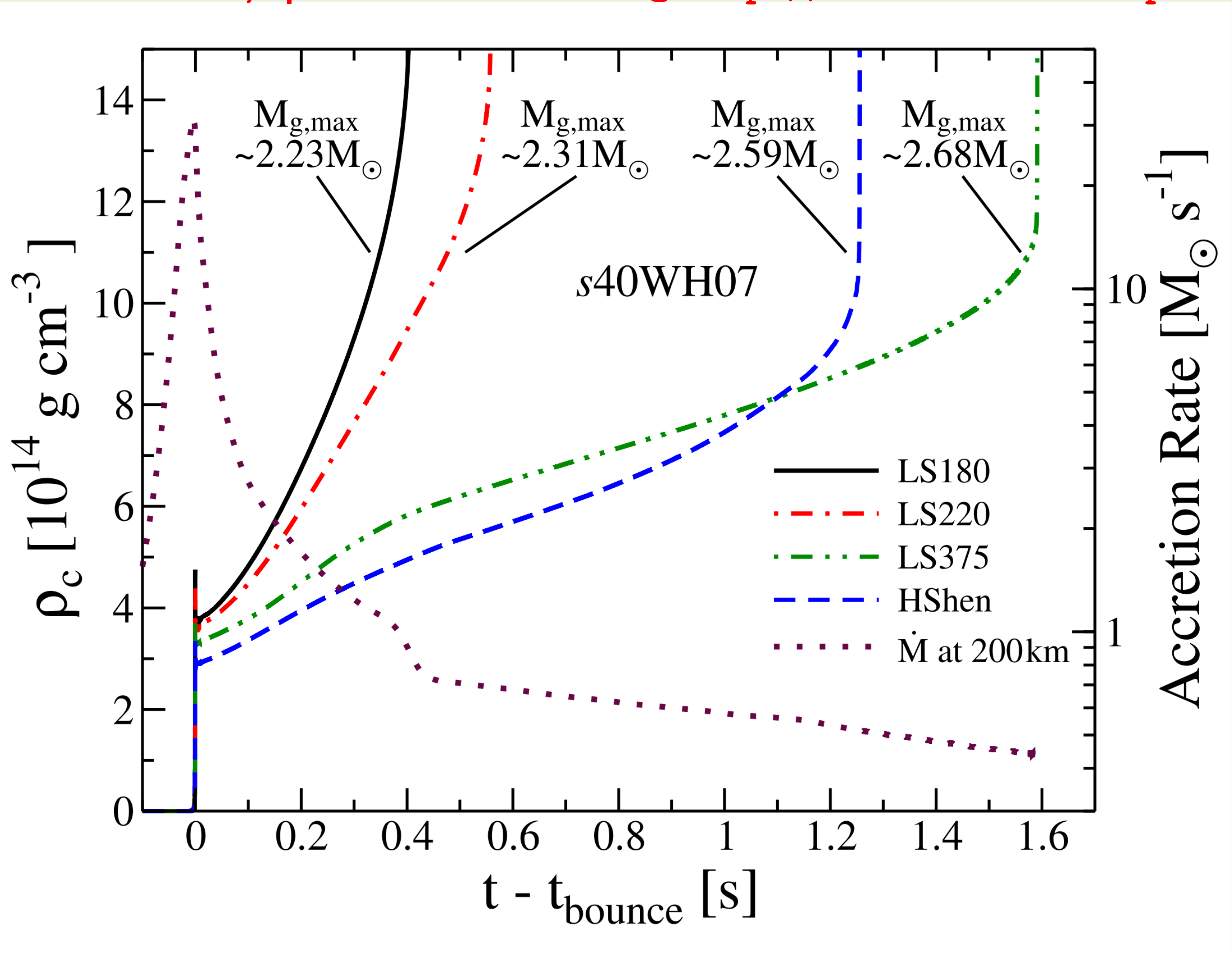
Parameter Study: > 700 Model Calculations

[O'Connor & Ott 2010, ApJ]

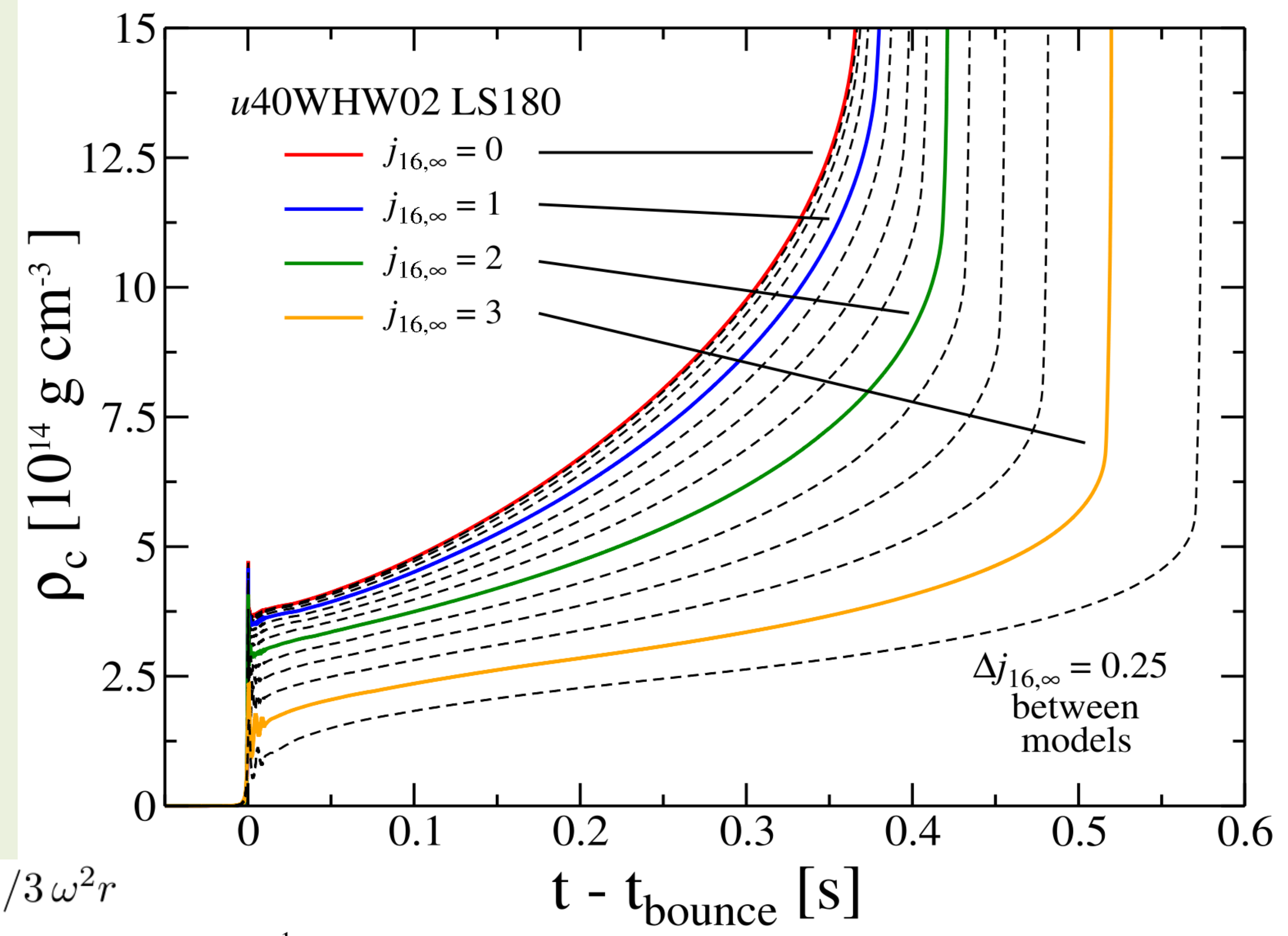


Protoneutron Star Collapse

O'Connor & Ott 2010, open-source GR1D code @ http://www.stellarcollapse.org



Influence of Rotation

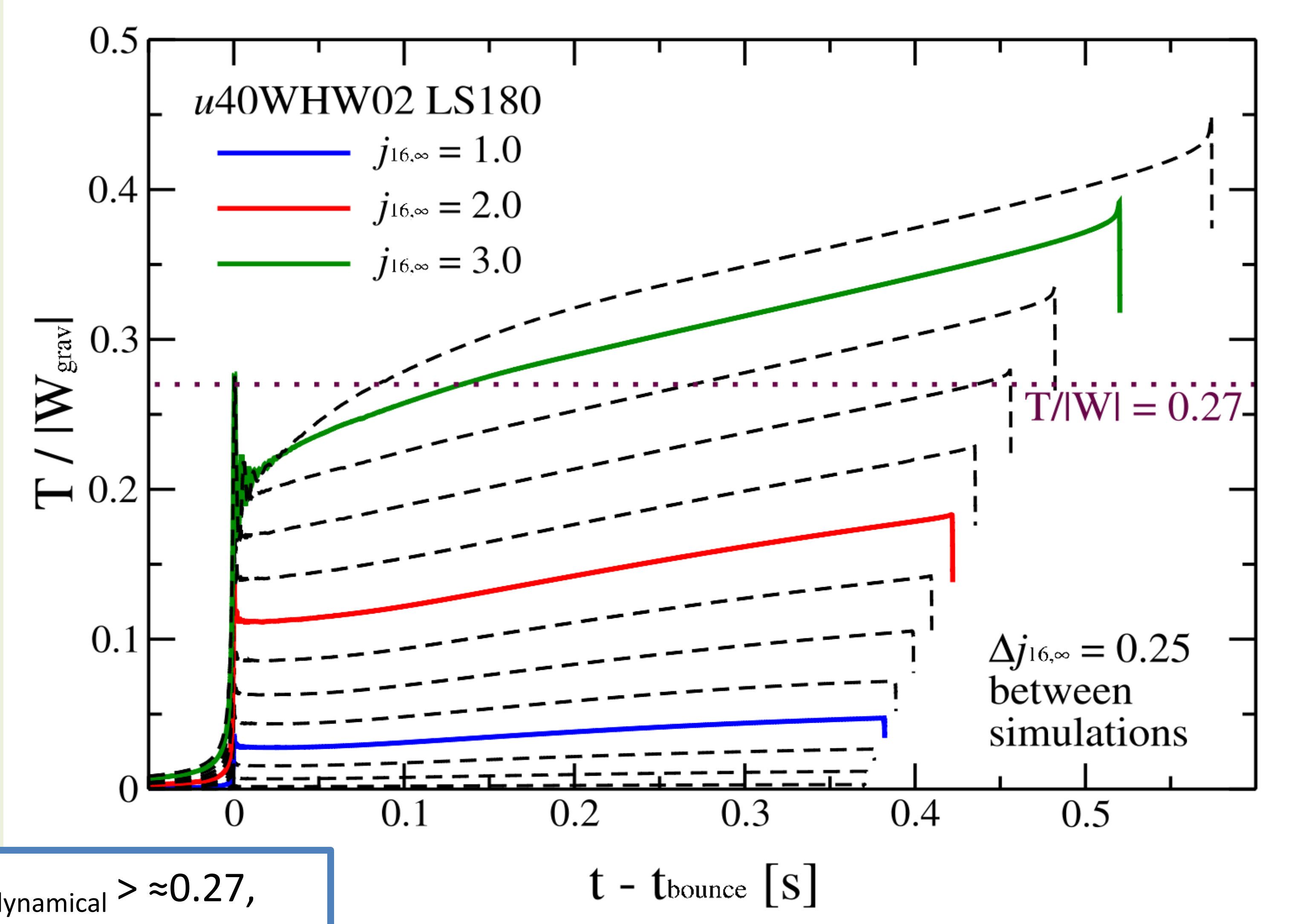


$$f_{\text{cent}} = 2/3\,\omega^2 r$$

$$j(r) = j_{16,\infty}\left[1+\left(\frac{A_{M_\odot}}{r}\right)^2\right]^{-1} 10^{16}\ \mathrm{cm^2\ s^{-1}}$$

[O'Connor & Ott 2011]

Postbounce Spin Evolution

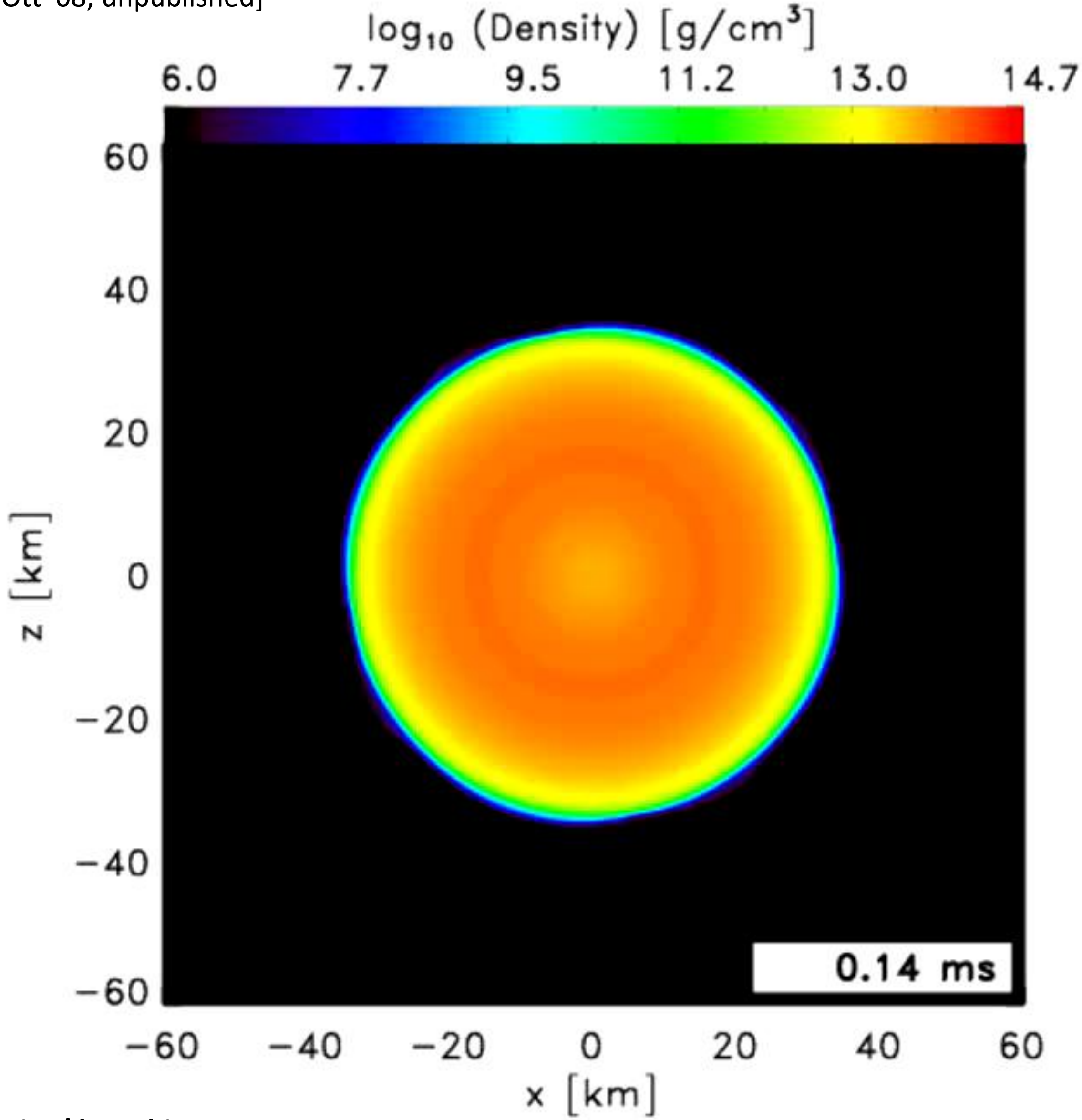


- $(T/|W|)_{dynamical} > \approx 0.27$,
  $(T/|W|)_{secular} > \approx 0.14$.
- **At lower T/|W|:**
  Dynamical shear instability.

[O'Connor & Ott 2011]

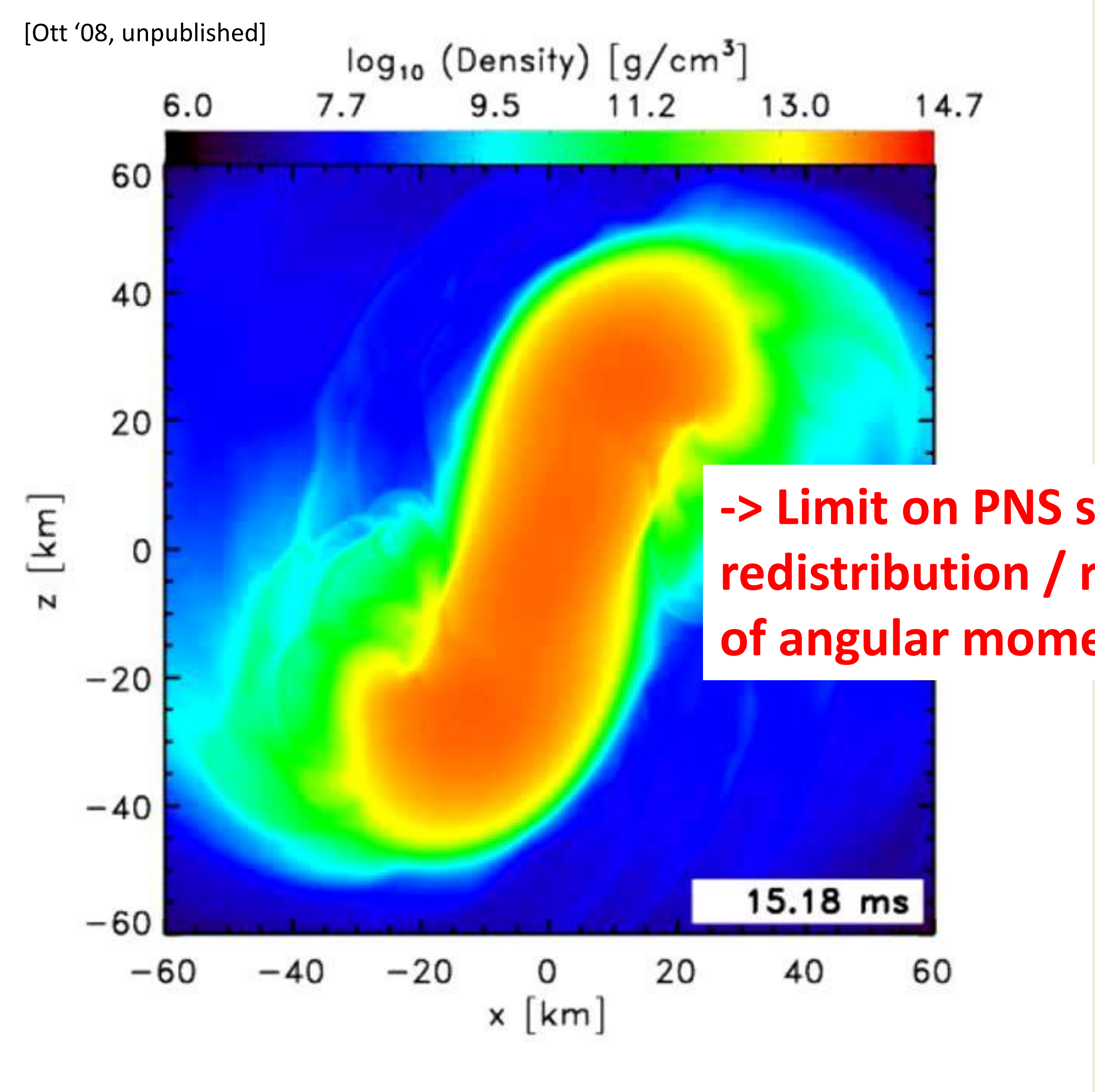
Rotational Nonaxisymmetric Instability

[Ott '08, unpublished]



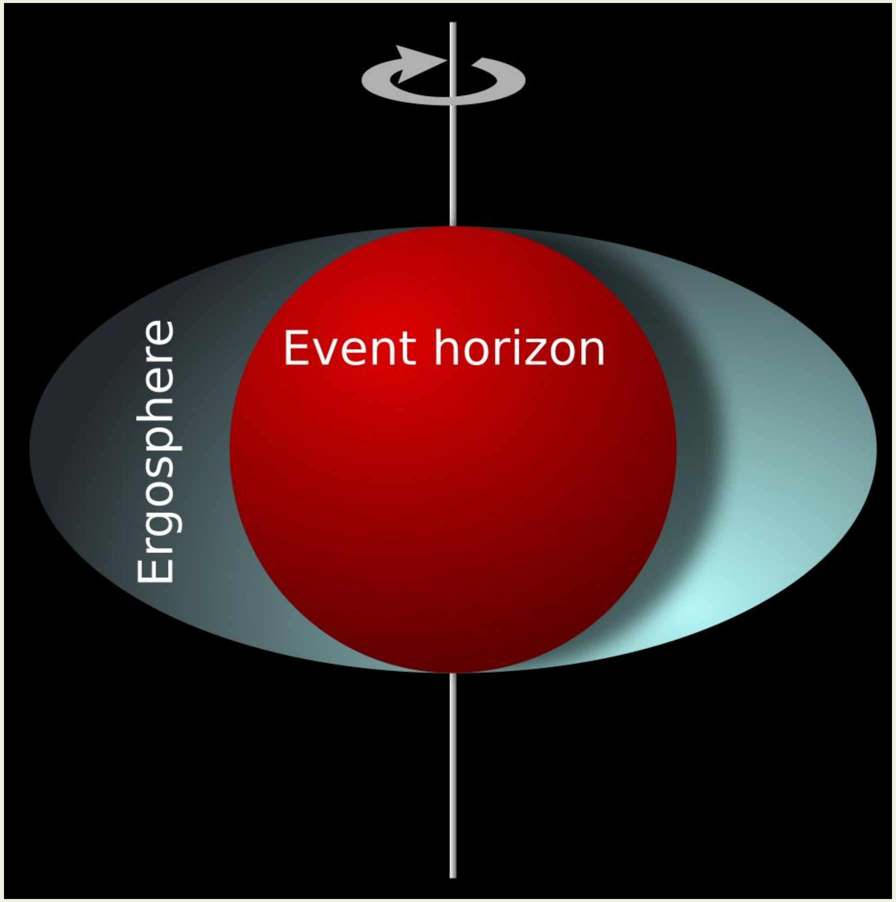
$(T/|W|)_{\text{dynamical}} > \approx 0.27$.

Rotational Nonaxisymmetric Instability



-> Limit on PNS spin by redistribution / radiation of angular momentum.

Kerr Solution and Black Hole Spin

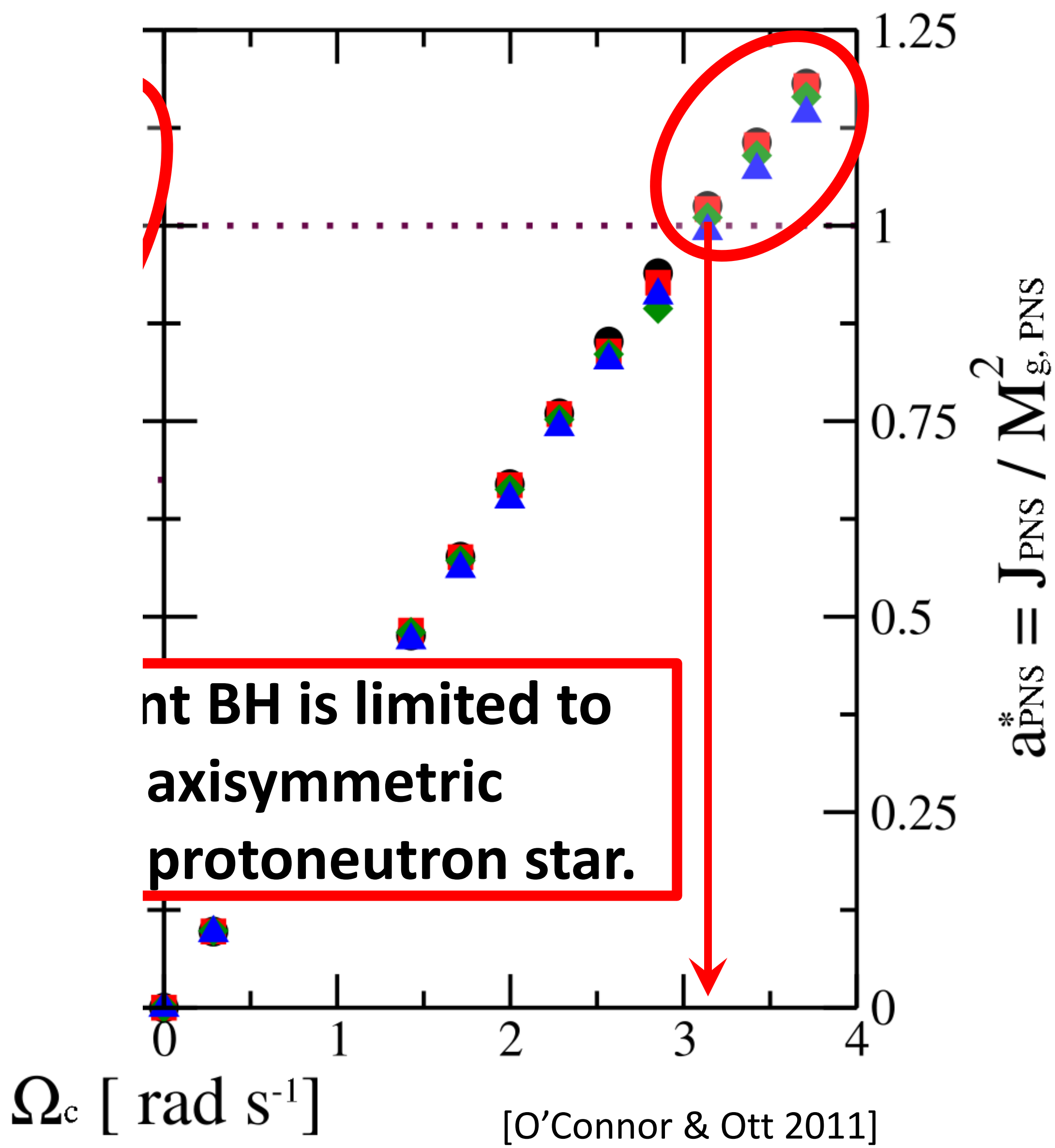


[Wikipedia]

$$a^\star = \frac{J}{M^2}$$

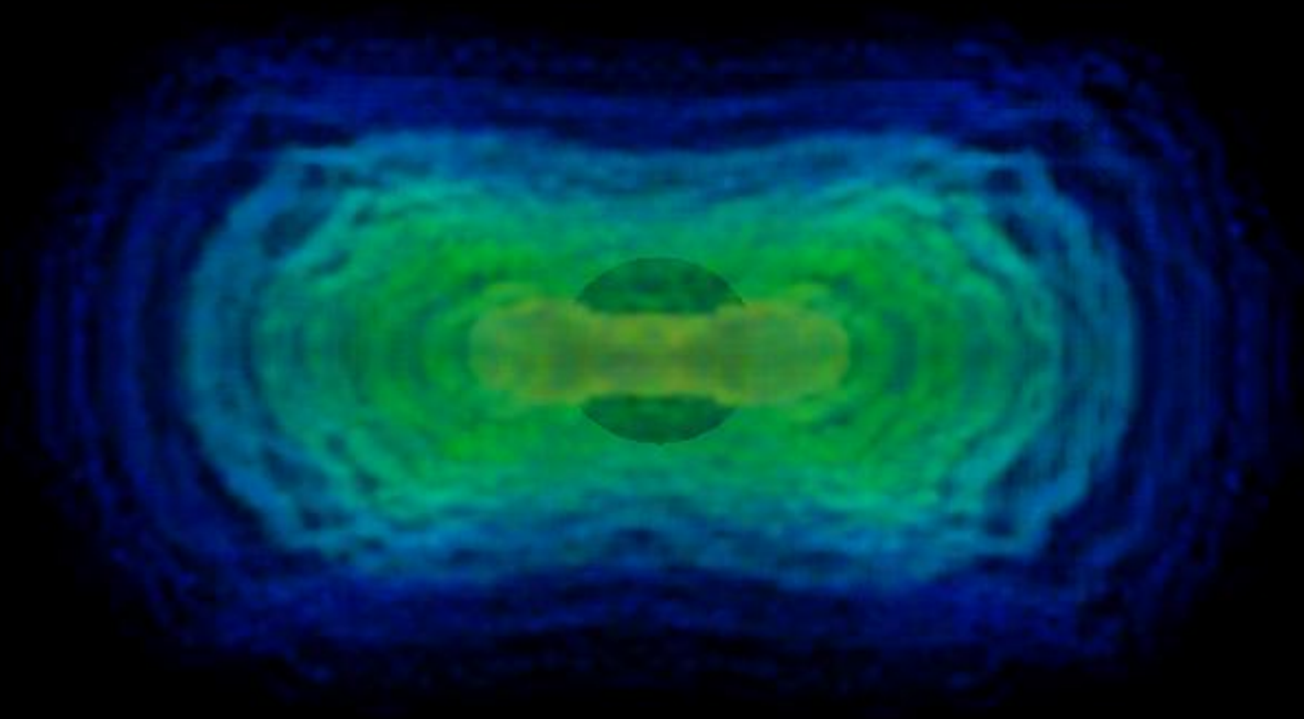
$$R_{\mathrm{Horizon}} = \frac{2M + \boxed{\sqrt{4M^2 - 4a^{\star 2} M^2}}}{2}$$

Black Hole Birth Spin



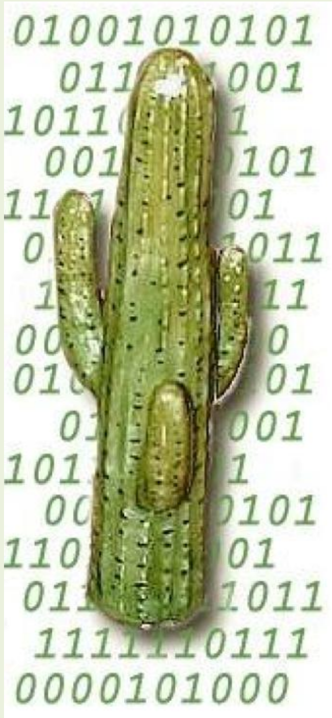
[O'Connor & Ott 2011]

3+1 GR Simulations of BH Formation



3+1 GR Computational Framework

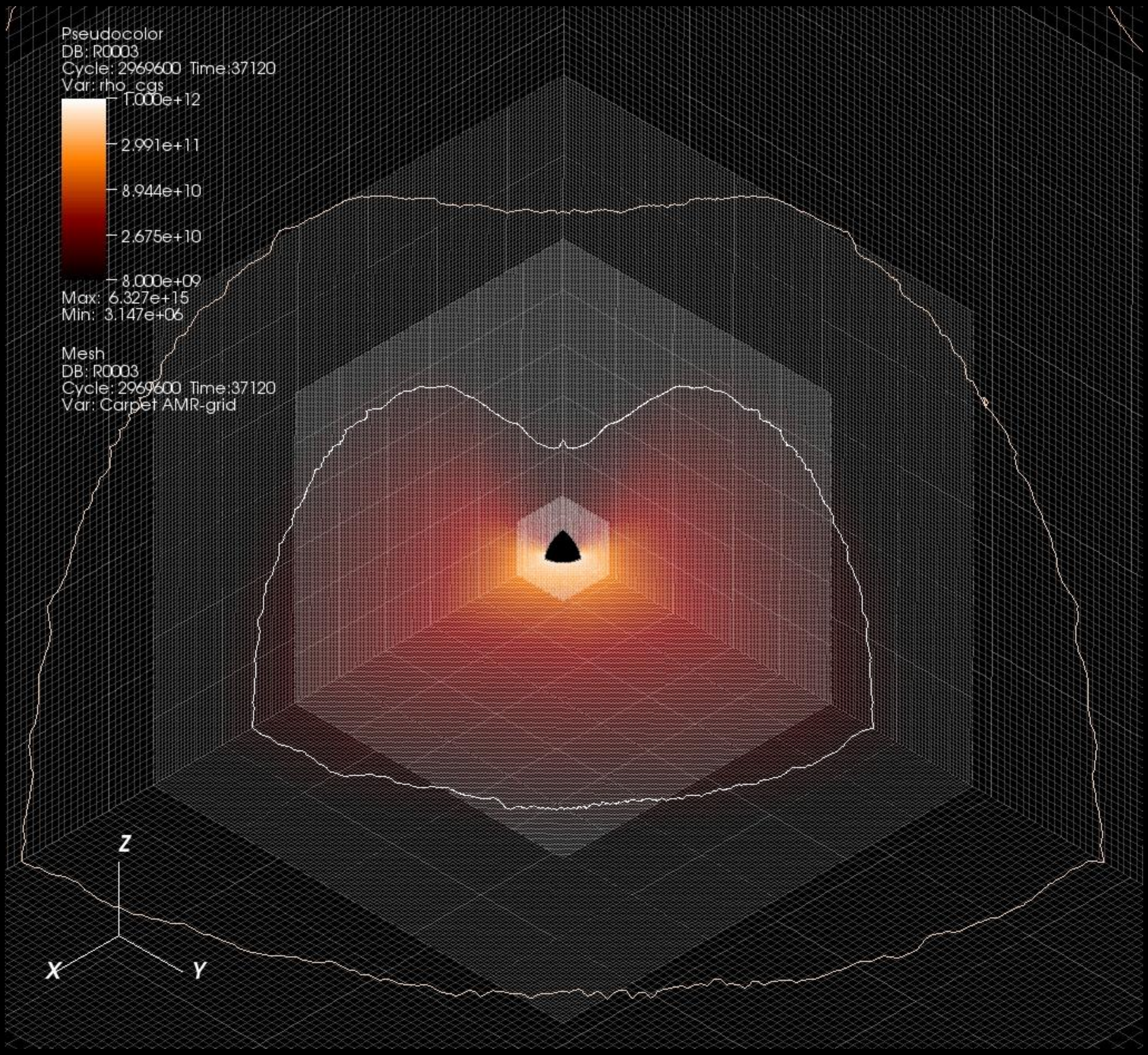
- **Cactus:** Open-source software framework for HPC, developed at the **Center for Computation & Technology** at LSU.



- **Einstein Toolkit** (http://einsteintoolkit.org)
  Set of open-source codes for numerical relativity and computational relativistic astrophysics.
  (LSU, Caltech, RIT, GATech)
  – Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation of numerical GR (code: McLachlan).
  – General-Relativistic Hydrodynamics (code: GRHydro).
  – Adaptive Mesh Refinement (code: Carpet driver).
  – Analysis: Horizon finder, GW extraction etc.

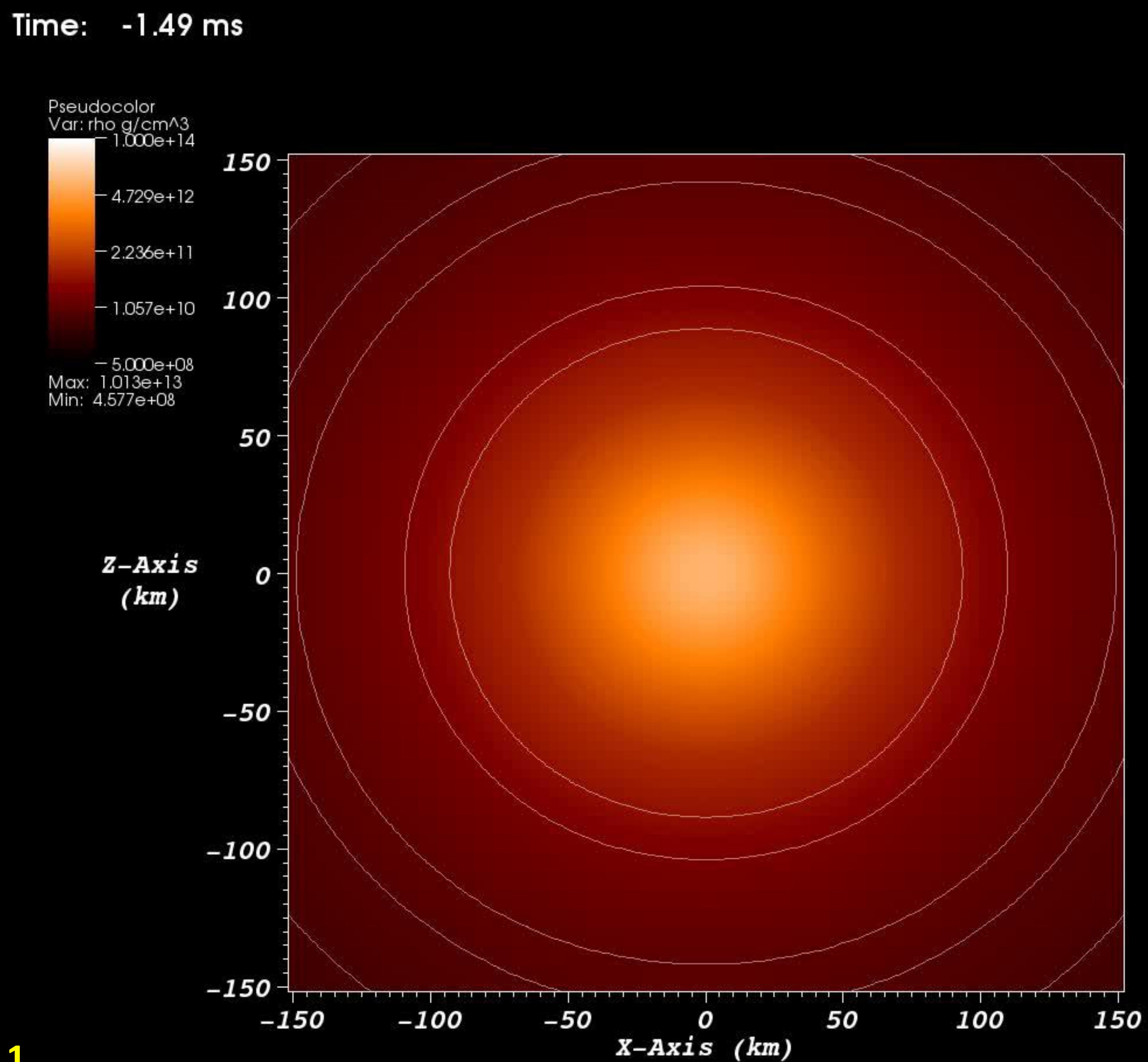
- Nuclear equation of state and neutrino leakage from open-source GR1D code: http://stellarcollapse.org

Computational Framework: AMR



11 levels of AMR

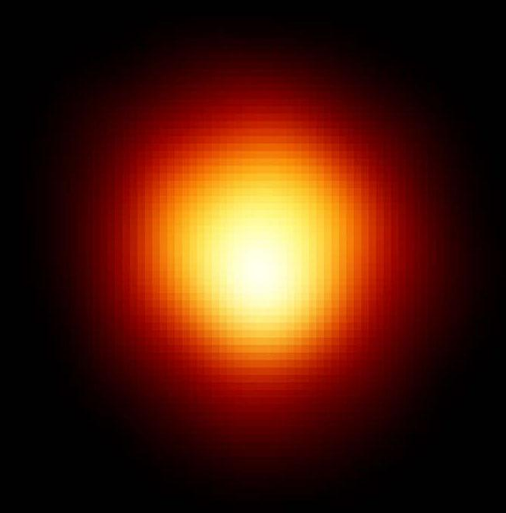
Finest level:
dx ~ 100 m



Ott et al. '11

Physical Review Letters (in press)

The End



Supplemental Slides

GR Hydrodynamics

Local conservation laws:

$$J^{\mu}{}_{;\mu} = 0 \qquad \text{mass}$$

$$T^{\mu\nu}{}_{;\mu} = 0 \qquad \text{stress-energy}$$

$$h = 1 + \epsilon + P/\rho$$

covariant derivative; here: divergence
takes into account that space is curved.

$$\frac{1}{\sqrt{-g}}\left(\left(\sqrt{\gamma}\,\mathbf{U}\right)_{,0} + \left(\sqrt{\gamma}\,\mathbf{F^i}\right)_{,i}\right) = \mathbf{s}$$

$$\gamma = \det(\gamma_{ik})$$

$$\sqrt{-g} = \alpha\sqrt{\gamma}$$

$$\mathbf{U} = (D, S_i, \tau)^T$$

$$D = J^{\mu} n_{\mu} = \alpha\rho u^0 = \rho W\,,$$

$$S^i = \perp^i_\nu\, T^{\mu\nu} n_{\mu} = \rho h W^2 v^i\,,$$

$$\tau = T^{\mu\nu} n_{\mu} n_{\nu} - J^{\mu} n_{\mu} = \rho h W^2 - P - D$$

$$W = \alpha u^0 = 1/\sqrt{1 - \gamma_{ij} v^i v^j}$$

u^{α} 4 – velocity

$$\mathbf{F}^i = \begin{pmatrix} (\alpha v^i - \beta^i)\, D \\ (\alpha v^i - \beta^i)\, S_j + \alpha p \delta^i_j \\ (\alpha v^i - \beta^i)\, \tau + \alpha p v^i \end{pmatrix} \qquad \mathbf{s} = \begin{pmatrix} 0 \\ T^{\mu\nu}\left(g_{\nu j,\mu} + \Gamma^{\delta}_{\mu\nu} g_{\delta j}\right) \\ \alpha\left(T^{\mu 0}\frac{1}{\alpha}\,\alpha_{,\mu} - T^{\mu\nu}\Gamma^{0}_{\mu\nu}\right) \end{pmatrix}$$

Some more Details:

ADM Line Element:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\begin{aligned} \gamma_{ij} &= 3 - \text{metric} \\ \alpha &= \text{lapse function} \\ \beta^i &= \text{shift vector} \end{aligned}$$

ADM Evolution Equations:

K_{ij} "Extrinsic Curvature"

S_{ij} Projection of stress-energy tensor.

ρ_{ADM} ADM energy density

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i,$$

$$\begin{aligned} \partial_t K_{ij} = -\nabla_i \nabla_j \alpha + \alpha \Big[& R_{ij} + K K_{ij} - 2K_{im}K^m_j \\ & - 8\pi\Big(S_{ij} - \frac{1}{2}\gamma_{ij}S\Big) - 4\pi\rho_{\mathrm{ADM}}\gamma_{ij}\Big] + \beta^m \nabla_m K_{ij} \\ & + K_{im}\nabla_j \beta^m + K_{mj}\nabla_i \beta^m. \end{aligned}$$

ADM Constraint Equations:

$${}^{(3)}R + K^2 - K_{ij}K^{ij} - 16\pi\rho_{\mathrm{ADM}} = 0, \quad \nabla_j K^{ij} - \gamma^{ij}\nabla_j K - 8\pi j^i = 0.$$

Some more Details:

ADM Line Element:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\gamma_{ij} = 3 - \text{metric}$$

ADM Evo

Equation

K_{ij} "Extrins

S_{ij} Project

energy

ρ_{ADM} ADM energy

density

$$+ K_{im}\nabla_j\beta^m + K_{mj}\nabla_i\beta^m.$$

Problem:

ADM equations unstable in multi-D simulations in the presence of strong curvature (e.g. a black hole).

Solution (late 1990s):

Use stable re-formulation of ADM equations, e.g., the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation.

ADM Constraint Equations:

$$^{(3)}R + K^2 - K_{ij}K^{ij} - 16\pi\rho_{\mathrm{ADM}} = 0, \quad \nabla_j K^{ij} - \gamma^{ij}\nabla_j K - 8\pi j^i = 0.$$

Special Case: Spherical Symmetry

(see, e.g., O'Connor & Ott 2010)

Equations simplify significantly; may use particularly convenient choice of gauge conditions: "Radial Gauge, Polar Slicing"

$$ds^2 = -\alpha(r,t)^2 dt^2 + X(r,t)^2 dr^2 + r^2 d\Omega^2$$

-> Schwarzschild-like line element

$$\alpha(r,t) = \exp[\phi(r,t)], \quad X(r,t) = \left(1 - \frac{2m(r,t)}{r}\right)^{-1/2}$$

$$\phi(r,t) = \int_0^r X^2 \left[\frac{m(r',t)}{r'^2} + 4\pi r'(P + \rho h X^2 u^{r'} u^{r'})\right] dr'$$

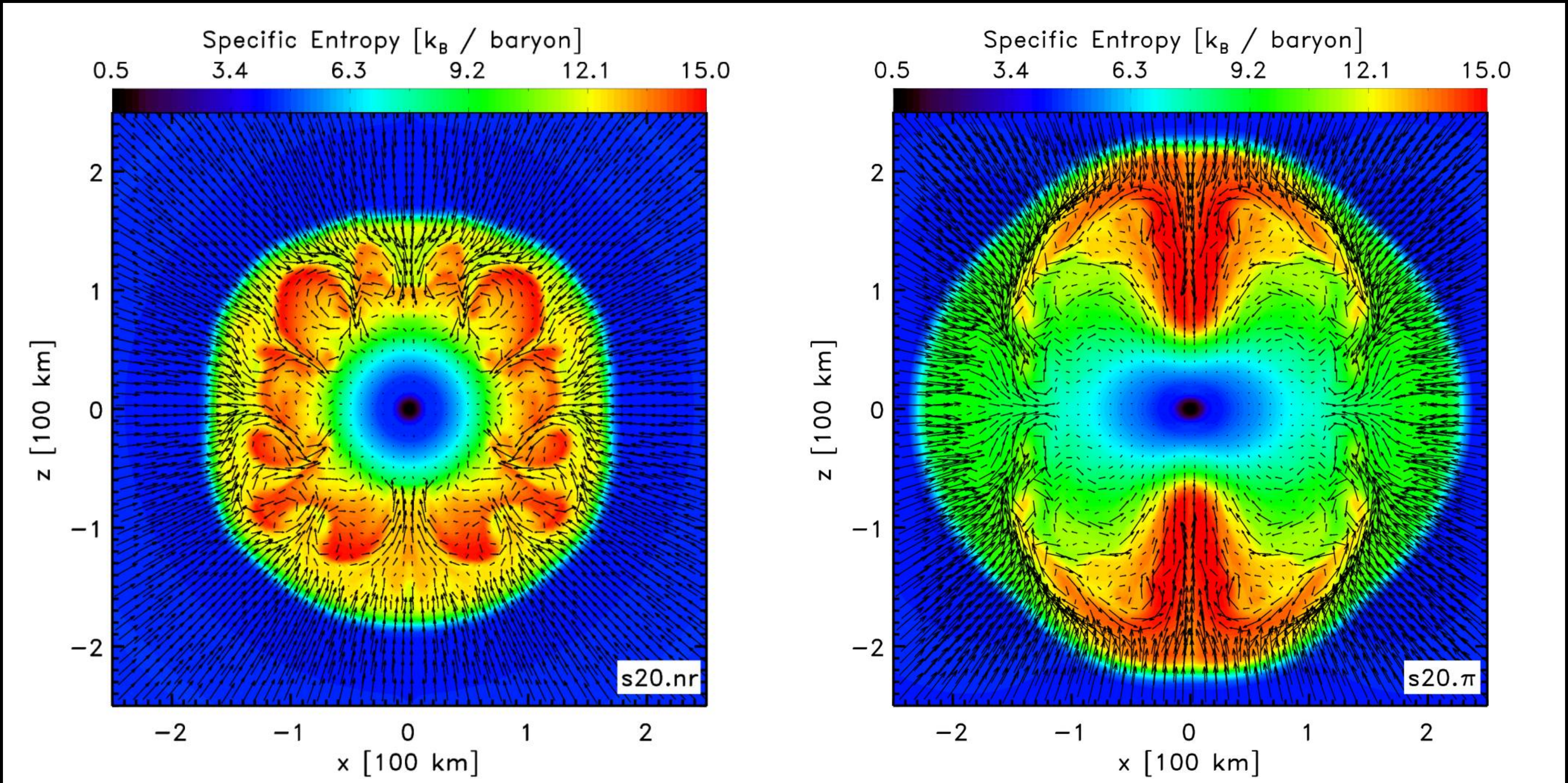
-> Solving for the spacetime metric becomes an ODE that can just be integrated.

Hydrodynamics:

$$\partial_t \vec{U} + \frac{1}{r^2}\partial_r\left[\frac{\alpha r^2}{X}\vec{F}\right] = \vec{S}$$

U, F, S similar to general case,
but total energy conserved by construction.
Additional source terms due to neutrinos.

Another Twist: Rotation & Convection

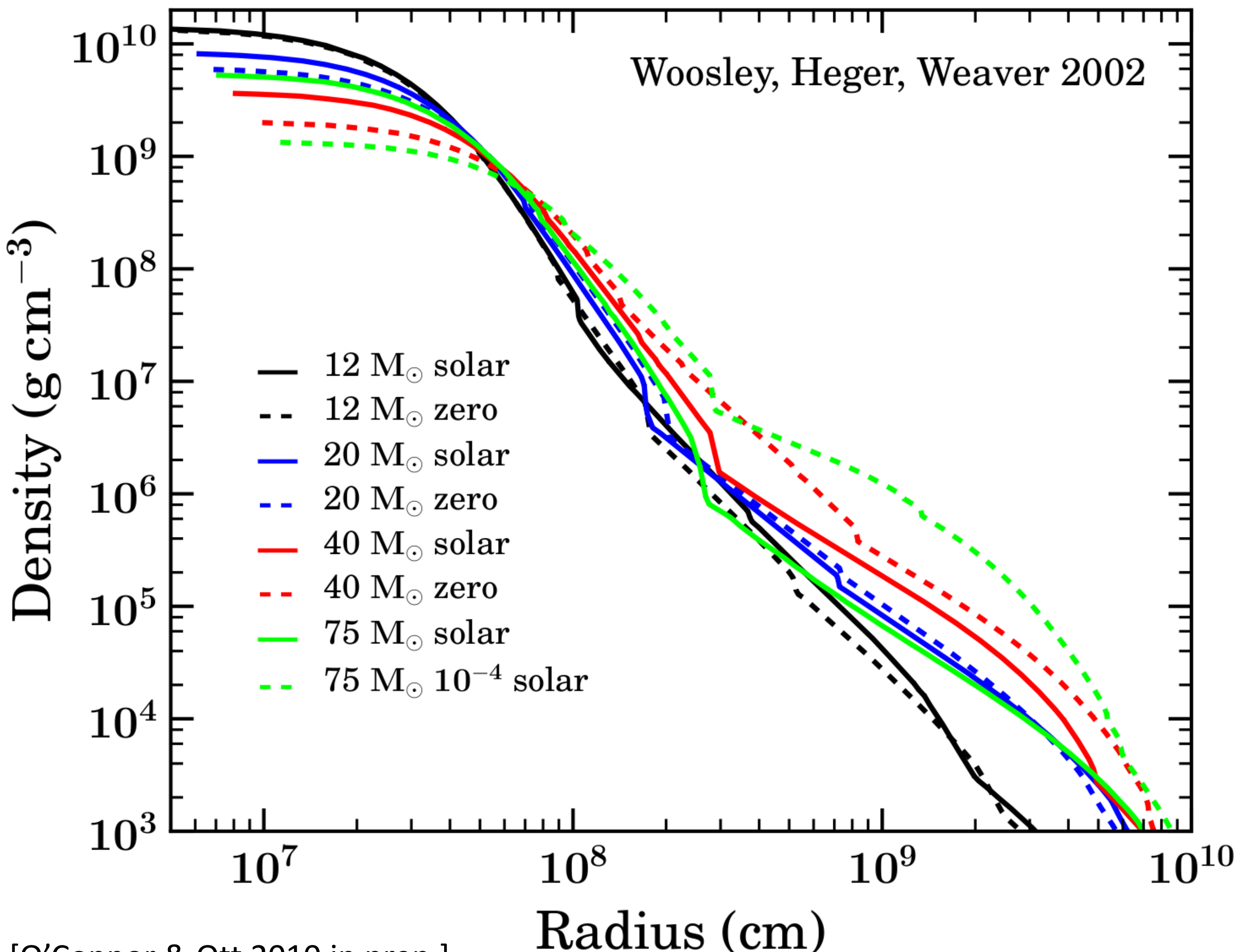


$$N^2 + \frac{1}{r^3}\frac{d}{dr} j^2 < 0$$

$$j = \Omega r^2$$

$$N^2 = \frac{g}{p\gamma}\left(\left.\frac{\partial p}{\partial s}\right|_{\rho, Y_l} \frac{ds}{dr} + \left.\frac{\partial p}{\partial Y_l}\right|_{\rho, s} \frac{dY_l}{dr} \right)$$

Precollapse Stellar Structure

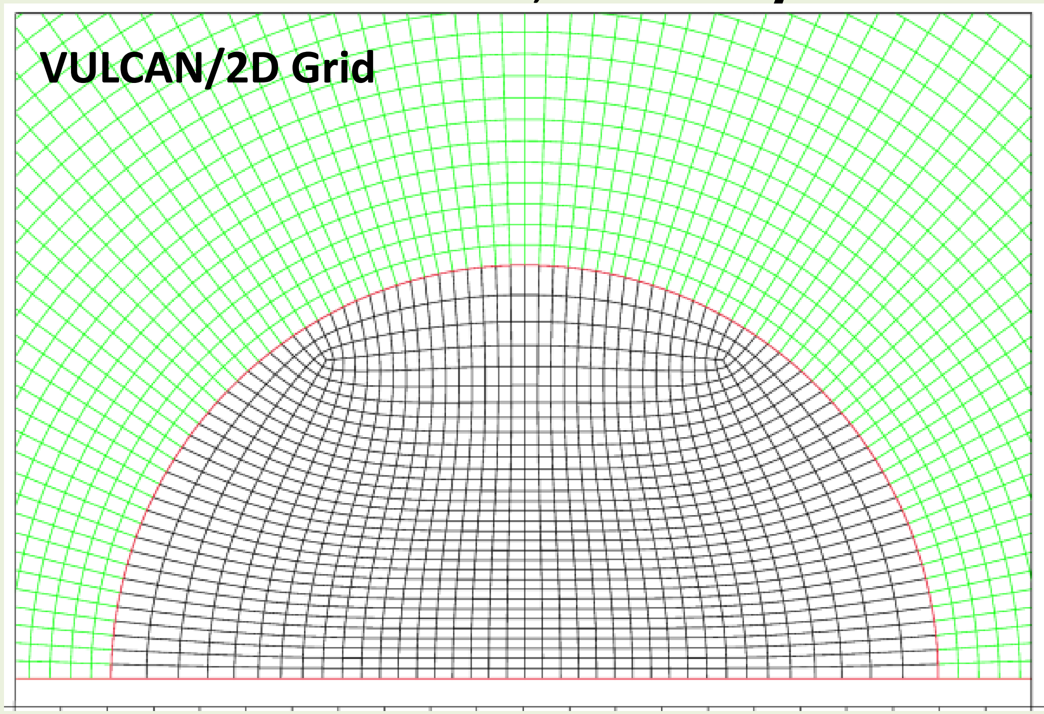


[O'Connor & Ott 2010 in prep.]

The VULCAN/2D Code

[Livne 1993, Livne et al. 2004, Livne et al. 2007, Burrows et al. 2007, **Ott et al. 2008**]

- **Axisymmetric Newtonian Magnetohydrodynamics** with rotation (2.5D).
- Unsplit 2nd order arbitrary Eulerian/Lagrangian (**ALE**) scheme.
- Newtonian gravity, logically cylindrical coordinates, **arbitrary mesh**.



- Radiation Transport:
  - **multi-group flux-lim. diff.** & **angle-dependent transport**.
  - Multiple energy groups, ν_e, $\bar{\nu}_e$, "ν_μ" species.
  - Slow-motion approximation.
- Multiple finite-temperature nuclear EOS options.
- Efficient parallelization in neutrino species/energy groups. Typical run size: 48–96 cores.
- Typical problem sizes: 50k zones x $\mathcal{O}(50)$ vars (MGFLD) x $\mathcal{O}(50)$ for angle-dep. transport.

Testing the Acoustic Mechanism

- So far no independent confirmation of the acoustic mechanism.
- Overstable physical g-modes PNS shown to exist. [Ferrari et al. 2003, 2007; Yoshida et al. 2007]
- Questions:
  - Do modes reach amplitudes as high as seen in our calculations?
  - Effects of GR and 3D?
- Fundamental prerequisite for non-linear numerical tests of mode excitation: Grid must be singularity free & allow change of the core's geometric center.
- Marek & Janka '09:
  Modes shown to exist, but don't reach high amplitudes. But: (1) Amplitudes become high only at t > 0.6 - 0.8 s (not simulated), (2) MJ09 grid not singularity free.
  -> Acoustic Mech. not yet numerically ruled out.