Applied Quantum Field Theory

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2 Participants

2.1 ICRANet participants

- Hagen Kleinert
- Remo Ruffini
- She-Sheng Xue

2.2 Ongoing collaborations

- Alexander Chervyakov (FU-Berlin, Germany)
- Patrick Haener (Nomura Bank, London, Great Britain)
- Petr Jizba (FU-Berlin, Germany)
- Flavio Nogueira (FU-Berlin, Germany)
- Axel Pelster (Universität Essen-Duisburg, Germany)

2.3 Students

- Tim X.J. Chen (FU-Berlin, Germany)
- Konstantin Glaum (FU-Berlin, Germany)
- Sonja Overesch (FU-Berlin, Germany)
- Walja Korolevski (FU-Berlin, Germany)
- Mathias Ohlinger (FU-Berlin, Germany)
- Moritz Schütte (FU-Berlin, Germany)
- Steffen Röthel (FU-Berlin, Germany)
- Matthias Ohliger (FU-Berlin, Germany)
- Pascal Mattern (FU-Berlin, Germany)

- Ednilson Santos (FU-Berlin, Germany)
- Alexander Hoffmann (FU-Berlin, Germany)
- Parvis Soltan-Panahi: Thermodynamic Properties of F=1 Spinor Bose-Einstein Condensates; Diplomarbeit bei Prof. Dr. Dr. h.c. mult. Hagen Kleinert, Freie Universitt Berlin (2006)
- Markus Dttmann: *Variational Methods in Disorder Problems – Testing Approximation Techniques with and without Replicas on a Zero-Dimensional Disorder Model*; (2009)
- Oliver Gabel: *Non-Equilibrium Quantum Statistics of Trapped Ideal Bose Gases*; (2009)
- Tobias Grass: *Real-Time Ginzburg-Landau Theory for Bosonic Gases in Optical Lattices*; (2009)
- Pascal Mattern: *Quench Dynamics of Bosons in Optical Lattices*; (2009)
- Lance Labun (USA): *Dipolar Bose Gases*; DAAD-RISE-Program
- Henrik Enoksen (Norway): *Green's Function of Bosons in Optical Lattices*; DAAD-IAESTE-Program (2007)
- Kiel Howe (USA): *Rotating Spinor-Fermi Gases*; DAAD-RISE-Program (2008)
- Barry Bradlyn (USA): *Effective Action of Bosons in Optical Lattices*; DAAD-RISE-Program (2008)
- Isaac Abban-Mensah (Ghana): *Hanbury Brown-Twiss-Effect of Bosons in Optical Lattices*; DAAD-IAESTE-Program (2008)
- Melek Kücüktalaslio (Turkey): *Green's Function of Bose-Fermi Mixture in Optical Lattices*; DAAD-IAESTE-Program (2008)
- Avinash Kumar (India): *Fidelity of a Quantum Mechanical Particle in Random Potential*; SFB/TR 12 (2008)
- Srinivas Kumar (India): *Vortices in Bose-Einstein Condensates* (2009)
- Bridget Bertoni (USA): *Dipolar Spinor Fermi Gases*; DAAD-RISE-Program (2009)
- Jerome Simons (USA): *Frustation of Bosons in Triangular Optical Lattice*; DAAD-RISE-Program (2009)
- Artem Gryshchuk (Ukraine): *Bose-Gas in Random Potential*; DAAD-IAESTE-Program (2009)

- Eduardo Paulo Jorge da Costa Alves (Portugal): *Two Weakly Coupled Bose-Gases*; DAAD-IAESTE-Program (2009)

3 Brief Description

This subgroup of ICRANet is occupied with various physical fields. Most closely related to Astrophysics is the collaboration with Ruffini and Xue (1), where we investigate the different rates of pair production in space-dependent electric fields. In a subsequent work with Alexander Chervyakov (2), an *exact* formula was found for the potential step of the Sauter type ($V \propto tanhx$).

The work on phase transition (3; 4) is linked to astrophysics by the possible phase transitions in neutron stars. In this context the question whether there exists color superconductivity in nuclear matter at high pressure will ultimately have to be answered.

The work on Bose-Einstein condensation (5; 6) is preparatory to ultimately understand the problem of stability of Bose stars, tackled by Ruffini and Bonazzola many years ago (7).

The work on the relation between geometry and defects is of interest on fundamental and pedagogical grounds. On the one hand, we have developed the most pedagogical method of teaching the properties of spacetimes with curvature and torsion (Einstein-Cartan spacetimes). On the other hand we have discovered a new principle according to which the physics in spaces with curvature and torsion can be predicted from the well-known physical laws in flat spacetime by performing simply a multivalued coordinate transformation. This technique is successful not only in the realm of gravity (8), but also in the theory of vortex lines in superfluids and the theory of plastic deformation of materials, as explained in the textbook (9). From the work done in this project, a new textbook (4) has emerged.

In a recent paper (10) a seimple explanation has been found for the Hawking temperature with which a black hole looses its mass.

The work on econophysics serves to understand fluctuating systems by methods developed in quantum physics. Here we continue to deduce practical consequences of the methods developed in the textbook (6) for financial markets. In this context it was essential to derive the analog of It's formula for non-Gaussian fluctuations and to find from it techniques how to stabilize fluctuating portfolios of investment (hedging) (11).

4 Publications (2005 - 2009)

Books

1. H. Kleinert, "*Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*" World Scientific, Singapore 2006, pp. 1-1547

   This is the fourth, significantly expanded edition of the comprehensive textbook of 1990 on the theory and applications of path integrals. It is the first book to explicitly solve path integrals of a wide variety of nontrivial quantum-mechanical systems, in particular of the hydrogen atom. The solutions have been made possible by two major advances.

   * The first is a new euclidean path integral formula which increases the restricted range of applicability of Feynman's famous formula to include singular attractive $1/r$- and $1/r^2$-potentials.

   * The second is a simple quantum equivalence principle governing the transformation of euclidean path integrals to spaces with curvature and torsion.

   The powerful Feynman-Kleinert variational approach is explained and developed systematically into a variational perturbation expansion. In contrast to ordinary perturbation expansions, divergencies are absent. Instead, there is a uniform convergence from weak to strong couplings, opening a way to precise approximate evaluations of analytically unsolvable path integrals.

   Tunneling processes are treated in detail. The results are used to determine the lifetime of supercurrents, the stability of metastable thermodynamic phases, and the large-order behavior of perturbation expansions. A new variational treatment extends the range of validity of previous tunneling theories from large to small barriers. A corresponding extension of large-order perturbation theory now also applies to small orders.

   Special attention is devoted to path integrals with topological restrictions. These are relevant to the understanding of the statistical properties of elementary particles and the entanglement phenomena in polymer physics and biophysics. The Chern-Simons theory of particles with fractional statistics (anyons) is introduced and applied to explain the fractional quantum Hall effect.

   The relevance of path integral to financial markets is discussed, and improvements of the famous Black-Scholes formula for option prices are developed which account for the fact that large market fluctuation occur much more frequently than in Gaussian distributions.

2. H. Kleinert, "*Multivalued Fields*" World Scientific, Singapore 2008, pp. 1-500 (`http://www.physik.fu-berlin.de/~kleinert/b11`)

   This book lays the foundations of the theory of fluctuating multivalued fields with numerous applications. Most prominent among these are phenomena dominated by the statistical mechanics of line-like objects, such as the phase transitions in superfluids and superconductors as well as the melting process of crystals, and the electromagnetic potential as a multivalued field that can produce a condensate of magnetic

monopoles. In addition, multivalued mappings play a crucial role in deriving the physical laws of matter coupled to gauge fields and gravity with torsion from the laws of free matter. Through careful analysis of each of these applications, the book thus provides students and researchers with supplementary reading material for graduate courses on phase transitions, quantum field theory, gravitational physics, and differential geometry.

3. H. Kleinert, "*Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*" World Scientific, Singapore 2009, pp. 1-1547 (`http://www.physik.fu-berlin.de/~kleinert/b5`)

   This is the fifth, significantly expanded edition of the first textbook.

Articles

1. B. Hamprecht and H. Kleinert "*End-To-End Distribution Function Function of Stiff Polymers for all Persistence Lengths* " Phys. Rev. E **71**, 031803 (2005) (cond-mat/0305226)

   We set up recursion relations for calculating all even moments of the end-to-end distance of a Porod-Kratky wormlike chains in D dimensions. From these moments we derive a simple analytic expression for the end-to-end distribution in three dimensions valid for all peristence lengths. It is in excellent agreement with Monte Carlo data for stiff chains and goes properly over into the Gaussian random-walk distributions for low stiffness.

2. H. Kleinert and V.I. Yukalov "*Highly Accurate Critical Exponents from Self-Similar Variational Perturbation Theory* " Phys. Rev. E **71**, 026131 (2005) (cond-mat/0402163)

   We extend field theoretic variational perturbation theory by self-similar approximation theory, which greatly accelerates convergence. This is illustrated by re-calculating the critical exponents of O(N)-symmetric ϕ^4 theory. From only three-loop perturbation expansions in $4-\epsilon$ dimensions we obtain *analytic results for the exponents, with practically the same accuracy as those derived recently from ordinary field-theoretic variational perturbational theory to seventh order. In particular, the theory explains the best-measured exponent* $\alpha \approx -0.0127$ *of the specific heat peak in superfluid helium, found in a satellite experiment with a temperature resolution of nanoKelvin. In addition, our analytic expressions reproduce also the exactly known large-N behaviour of the exponents* ν *and* $\gamma = \nu(2-\eta)$ *with high precision.*

3. S.F. Brandt, H. Kleinert, A. Pelster "*Recursive Calculation of Effective Potential and Variational Resummation* " J. Math. Phys. **46**, 032101 (2005) (quant-ph/0406206)

   We set up a method for a recursive calculation of the effective potential which is applied to a cubic potential with imaginary coupling. The result is resummed

using variational perturbation theory (VPT), yielding an exponentially fast convergence.

4. O. Zobay, G. Metikas, H. Kleinert "*Nonperturbative Effects on T_c of Interacting Bose Gases in Power Traps* " Phys. Rev. A **71**, 043614 (2005) (cond-mat/0411133)

   The critical temperature T_c of an interacting Bose gas trapped in a general power-law potential $V(x) = \sum_i U_i |x_i|^{p_i}$ is calculated with the help of variational perturbation theory. It is shown that the interaction-induced shift in T_c fulfills the relation $(T_c - T_c^0)/T_c^0 = D_1(eta)a + D'(eta)a^{2eta} + O(a^2)$ with T_c^0 the critical temperature of the trapped ideal gas, a the s-wave scattering length divided by the thermal wavelength at T_c, and $\eta = 1/2 + \sum_i 1/p_i$ the potential-shape parameter. The terms $D_1(\eta)a$ and $D'(\eta)a^{2\eta}$ describe the leading-order perturbative and nonperturbative contributions to the critical temperature, respectively. This result quantitatively shows how an increasingly inhomogeneous potential suppresses the influence of critical fluctuations. The appearance of the $a^{2\eta}$ contribution is qualitatively explained in terms of the Ginzburg criterion.

5. H. Kleinert "*Order of Superconductive Phase Transition* " Berlin Preprint 2005 publ. in Festschrift in honor of R. Folk's 60th birthday (2004).

   On the occasion of Reinhard Folk's 60th birthday, I give a brief review of the theoretical progress in understanding the critical properties of superconductors. I point out the theoretical difficulties in finding a second-order transition in the Ginzburg-Landau Model with O(N)-symmetry in 4-e Dimensions, and the success in predicting the existence and location of a tricritical point with the help of a dual disorder theory.

6. M. Blasone, P. Jizba, and H. Kleinert "*Quantum Behavior of Deterministic Systems with Information Loss. Path Integral Approach* " Phys. Rev. A **71** 052507 (2005) (quant-ph/0409021)

   We present a path-integral formulation of 't Hooft's derivation of quantum from classical physics. The crucial ingredient of this formulation is Gozzi et al.'s supersymmetric path integral of classical mechanics. We quantize explicitly two simple classical systems: the planar mathematical pendulum and the Roessler dynamical system.

7. H. Kleinert and A.J.M. Schakel "*Anomalous Dimension of Dirac's Gauge-Invariant Nonlocal Order Parameter in Ginzburg-Landau Field Theory* " Phys. Lett. B **611**, 182 (2005)

   In a Ginzburg-Landau theory with n fields, the anomalous dimension of the gauge-invariant nonlocal order parameter defined by the long-distance limit of Dirac's gauge-invariant two-point function is calculated. The result is exact for all n to first order in $\epsilon \equiv 4 - d$, and for all $d \in (2, 4)$ to first order in $1/n$,

and coincides with the previously calculated gauge-dependent exponent in the Landau gauge.

8. H. Kleinert, S. Schmidt, and A. Pelster "*Quantum Phase For Homogeneous Bose-Einstein Condensate* " Ann. Phys. **14**, 214 (2005) (cond-mat/0308561)

   We calculate the quantum phase transition for a homogeneous Bose gas in the plane of s-wave scattering length a_s and temperature T. This is done by improving a one-loop result near the interaction-free Bose-Einstein critical temperature $T_c^{(0)}$ with the help of recent high-loop results on the shift of the critical temperature due to a weak atomic repulsion using variational perturbation theory. The quantum phase diagram shows a nose above $T_c^{(0)}$, so that we predict the existence of a reentrant transition above $T_c^{(0)}$, where an increasing repulsion leads to the formation of a condensate.

9. H. Kleinert and A. Chervyakov "*Perturbation Theory for Path Integrals of Stiff Polymers*" J. Phys. A: Math. Gen. 39 8231 (2006) (cond-mat/0503199)

   The wormlike chain model of stiff polymers is a nonlinear σ-model in one spacetime dimension in which the ends are fluctuating freely. This causes important differences with respect to the presently available theory which exists only for periodic and Dirichlet boundary conditions. We modify this theory appropriately and show how to perform a systematic large-stiffness expansions for all physically interesting quantities in powers of L/ξ, where L is the length and ξ the persistence length of the polymer. This requires special procedures for regularizing highly divergent Feynman integrals which we have developed in previous work. We show that by adding to the unperturbed action a correction term $\mathcal{A}^{\rm corr}$, we can calculate all Feynman diagrams with Green functions satisfying Neumann boundary conditions. Our expansions yield, order by order, properly normalized end-to-end distribution function in arbitrary dimensions d, its even and odd moments, and the two-point correlation function.

10. H. Kleinert "*Emerging Gravity from Defects in World Crystal*
    Lecture Presented at the 2004 Conference on Emerging Gravity in Piombino Braz. J. Phys. **35**, 359 (2005)

    I show that Einstein Gravity can be thought of as arising from the defects in a world crystal whose lattice spacing is of the order of the Planck length $l_P \sim 10^{-33}$ cm, and whose elastic energy is of the second-gradient type (floppy crystal). No physical experiment so far would be able to detect the lattice structure.

11. H. Kleinert "*Vortex Origin of Tricritical Point in Ginzburg-Landau Theory* "Europhys. Letters **74**, 889 (2006) (cond-mat/0509430) Motivated by recent experimental progress in the critical regime of high-T_c superconductors we show how the tricritical point in a superconductor can be derived from the

Ginzburg-Landau theory as a consequence of vortex fluctuations. Our derivation explains why usual renormalization group arguments always produce a first-order transition, in contrast to experimental evidence and Monte Carlo simulations.

12. F.S. Nogueira and H. Kleinert "*Quantum Electrodynamics in* $2+1$ *Dimensions, Confinement, and the Stability of* $U(1)$ *Spin Liquids* " Phys. Rev. Lett. **95**, 176406 (2005) (cond-mat/0501022)

    Compact quantum electrodynamics in 2+1 dimensions often arises as an effective theory for a Mott insulator, with the Dirac fermions representing the low-energy spinons. An important and controversial issue in this context is whether a deconfinement transition takes place. We perform a renormalization group analysis to show that deconfinement occurs when $N > N_c = 36/\pi^3 \approx 1.161$, where N is the number of fermion replica. For $N < N_c$, however, there are two stable fixed points separated by a line containing a unstable non-trivial fixed point: a fixed point corresponding to the scaling limit of the non-compact theory, and another one governing the scaling behavior of the compact theory. The string tension associated to the confining interspinon potential is shown to exhibit a universal jump as $N \to N_c^-$. Our results imply the stability of a spin liquid at the physical value N=2 for Mott insulators.

13. M. Blasone, P. Jizba, and H. Kleinert "*Quantum Behavior of Deterministic Systems with Information Loss. Path Integral Approach* "Annals Phys. **320**, 468 (2005) (quant-ph/0504200)

    't Hooft's derivation of quantum from classical physics is analyzed by means of the classical path integral of Gozzi et al.. It is shown how the key element of this procedure - the loss of information constraint - can be implemented by means of Faddeev-Jackiw's treatment of constrained systems. It is argued that the emergent quantum systems are identical with systems obtained in [Phys.Rev. A 71 (2005) 052507] through Dirac-Bergmann's analysis. We illustrate our approach with two simple examples - free particle and linear harmonic oscillator. Potential Liouville anomalies are shown to be absent.

14. J. Dietel and H. Kleinert, *Triangular lattice model of two-dimensional defect melting* (arXiv:cond-mat/0508780) Phys. Rev. B 73, 024113 (2006)

    We set up a harmonic lattice model for two-dimensional defect melting which, in contrast to earlier simple cubic models, resides on a triangular lattice. Integer-valued plastic defect gauge fields allow for the thermal generation of dislocations and disclinations. The model produces universal formulas for the melting temperature expressed in terms of the elastic constants, which are different from those derived for square lattices. They determine a Lindemann-like parameter for two-dimensional melting. In contrast to the square crystal which underwent a first-order melting transition, the triangular model melts in two

steps. Our results are applied to the melting of Lennard-Jones and electron lattices.

15. V.I. Yukalov and H. Kleinert "*Gapless Hartree-Fock-Bogoliubov Approximation for Bose Gas* " Phys. Rev. A **73**, 063612 (2006) (cond-mat/0606484)

    A dilute Bose system with Bose-Einstein condensate is considered. It is shown that the Hartree-Fock-Bogolubov approximation can be made both conserving as well as gapless. This is achieved by taking into account all physical normalization conditions, that is, the normalization condition for the condensed particles and that for the total number of particles. Two Lagrange multipliers, introduced for preserving these normalization conditions, make the consideration completely self-consistent.

16. J. Dietel and H. Kleinert, *Defect-induced melting of vortices in high- Tc superconductors: A model based on continuum elasticity theory* (arXiv:cond-mat/0511710) Phys. Rev. B 74, 024515 (2006)

    We set up a melting model for vortex lattices in high-temperature superconductors based on the continuum elasticity theory. The model is Gaussian and includes defect fluctuations by means of a discrete-valued vortex gauge field.We derive the melting temperature of the lattice and predict the size of the Lindemann number. Our result agrees well with experiments for $YBa_2Cu_3O_{7-\delta}$, and with modifications also for $Bi_2Sr_2CaCu_2O_8$. We calculate the jumps in the entropy and the magnetic induction at the melting transition.

17. F.S. Nogueira and H. Kleinert "*Thermally Induced Rotons in Two-Dimensional Dilute Bose Gases* "Phys. Rev. **73**, 104515 (2006) (cond-mat/0503523)

    We show that roton-like excitations are thermally induced in a two-dimensional dilute Bose gas as a consequence of the strong phase fluctuations in two dimensions. At low momentum, the roton-like excitations lead for small enough temperatures to an anomalous phonon spectrum with a temperature dependent exponent reminiscent of the Kosterlitz-Thouless transition. Despite the anomalous form of the energy spectrum, it is shown that the corresponding effective theory of vortices describes the usual Kosterlitz-Thouless transition. The possible existence of an anomalous normal state in a small temperature interval is also discussed.

18. H. Kleinert and X.J. Chen "*Boltzmann Distribution and Market Temperature* " Berlin preprint 2006 (physics/0609209)

    The minute fluctuations of of S&P 500 and NASDAQ 100 indices display Boltzmann statistics over a wide range of positive as well as negative returns, thus allowing us to define a *market temperature* for either sign. With increasing time the sharp Boltzmann peak broadens into a Gaussian whose volatility σ measured in $1/\sqrt{\text{min}}$ is related to the temperature T by $T = \sigma/\sqrt{2}$. Plots over the years 1990–2006 show that the arrival of the 2000 crash was preceded by an

increase in market temperature, suggesting that this increase can be used as a warning signal for crashes. A plot of the Dow Jones temperature over 78 years reveals a remarkable stability through many historical turmoils, interrupted only by short heat bursts near the crashes.

19. H. Kleinert "*Stiff Quantum Polymers*" Phys. Rev. B 76, 052202 (2007), Berlin preprint 2006

    At ultralow temperatures, polymers exhibit quantum behavior, which is calculated here for the second and fourth moments of the end-to-end distribution in the large-stiffness regime. The result should be measurable for polymers in wide optical traps.

20. H. Kleinert "*Field Transformations and Multivalued Fields* " 2007 J. Phys.: Conf. Ser. 67 012007 , Berlin preprint 2006

    Changes of field variables may lead to multivalued fields which do not satisfy the Schwarz integrability conditions. Their quantum field theory needs special care as is shown in an application to the superfluid and superconducting phase transitions.

21. F. Nogueira and H. Kleinert "*Compact quantum electrodynamics in* $2+1$ *dimensions and spinon deconfinement: a renormalization group analysis* " Berlin preprint 2007 (arXiv:0705.3541)

    We discuss compact (2+1)-dimensional Maxwell electrodynamics coupled to fermionic matter with N replica. For large enough N, the latter corresponds to an effective theory for the nearest neighbor SU(N) Heisenberg antiferromagnet, in which the fermions represent solitonic excitations known as spinons. Here we show that the spinons are deconfined for $N > N_c = 36$, thus leading to an insulating state known as spin liquid. A previous analysis considerably underestimated the value of N_c. We show further that for $20 < N \leq 36$ there can be either a confined or a deconfined phase, depending on the instanton density. For $N \leq 20$ only the confined phase exist. For the physically relevant value N=2 we argue that no paramagnetic phase can emerge, since chiral symmetry breaking would disrupt it. In such a case a spin liquid or any other nontrivial paramagnetic state (for instance, a valence-bond solid) is only possible if doping or frustrating interactions are included.

22. J.W. Zhang, Y. Zhang, and H. Kleinert "*Power tails of Index Distributions in Chinese Stock Market* " Physica A 377, 166 (2007)

    The power a of the Lvy tails of stock market fluctuations discovered in recent years are generally believed to be universal. We show that for the Chinese stock market this is not true, the powers depending strongly on anomalous daily index changes short before market closure, and weakly on the opening data.

23. K. Glaum, A. Pelster, H. Kleinert, and T. Pfau "*Critical Temperature of Weakly Interacting Dipolar Condensates* " Phys. Rev. Lett. **98**, 080407/1-4 (2007) (cond-mat/0606569)

    We calculate perturbatively the effect of a dipolar interaction upon the Bose-Einstein condensation temperature. This dipolar shift depends on the angle between the symmetry axes of the trap and the aligned atomic dipole moments, and is extremal for parallel or orthogonal orientations, respectively. The difference of both critical temperatures exhibits most clearly the dipole-dipole interaction and can be enhanced by increasing both the number of atoms and the anisotropy of the trap. Applying our results to chromium atoms, which have a large magnetic dipole moment, shows that this dipolar shift of the critical temperature could be measured in the ongoing Stuttgart experiment.

24. J. Dietel and H. Kleinert, *Phase diagram of vortices in high-Tc superconductors from lattice defect model with pinning* (arXiv:cond-mat/061204) Phys. Rev. B 75, 144513 (2007) Erratum: Phys. Rev. B 78, 059901 (2008) The theory presented is based on a simple Hamiltonian for a vortex lattice in a weak impurity background which includes linear elasticity and plasticity, the latter in the form of integer valued fields accounting for defects. Within a quadratic approximation in the impurity potential, we find a first-order Bragg-glass, vortexglass transition line showing a reentrant behavior for superconductors with a melting line near H_{c2}. Going beyond the quadratic approximation by using the variational approach of Mzard and Parisi established for random manifolds, we obtain a phase diagram containing either a first-order or a third-order glass transition line depending on the form of the disorder potential. Disorder potentials resulting in a unified glass transition line of a third-order part ahigh magnetic fieldsb and a first-order part alow magnetic fieldsb are possible. The glass transition line separates the vortex glass and the vortex liquid. Furthermore, we find a unified first-order line consisting of the melting transition between the Bragg glass and the vortex liquid phase as well as a disorder induced first-order line between the Bragg glass and the vortex glass phase. The reentrant behavior of this line within the quadratic approach mentioned above vanished. We calculate the entropy and magnetic induction jumps over the first-order line.

25. H. Kleinert and S.-S. Xue Photoproduction in *Semiconductors by Onset of Magnetic Field Eur. Phys. Letters* 81, 57001 (2008).

    The energy bands of a semiconductor are lowered by an external magnetic field. When a field is switched on, the straight-line trajectories near the top of the occupied valence band are curved into Landau orbits and Bremsstrahlung is emitted until the electrons have settled in their final Fermi distribution. We calculate the radiated energy, which should be experimentally detectable, and suggest that a semiconductor can be cooled by an oscillating magnetic field.

26. H. Kleinert, R. Ruffini, and S.-S. Xue "*Electron-Positron Pair Production in Space- or Time-Dependent Electric Fields* Phys. Rev. D 78, 025011 (2008).

    Treating the production of electron and positron pairs by a strong electric field from the vacuum as a quantum tunneling process we derive, in semiclassical approximation, a general expression for the pairproduction rate in a z-dependent electric field pointing in the z direction. We also allow for a smoothly varying magnetic field parallel to z. The result is applied to a confined field that is nonzero for $|z| > \ell$, a semiconfined field that is nonzero for $z > 0$, and a linearly increasing field. The boundary effects of the confined fields on pair-production rates are exhibited. A simple variable change in all formulas leads to results for electric fields depending on time rather than space. In addition, we discuss tunneling processes in which empty atomic bound states are spontaneously filled by negative-energy electrons from the vacuum under positron emission. In particular, we calculate the rate at which the atomic levels of a bare nucleus of finite-size r_n and large $Z \gg 1$ are filled by spontaneous pair creation.

27. H. Kleinert, *From Landau's Order Parameter to Modern Disorder Field Theory*, in *L.D. Landau and his Impact on Contemporary Theoretical Physics*, Horizons in World Physics 264 (2008), A. Sakaji and I. Licata (ed.). (`http://www.physik.fu-berlin.de/~kleinert/373`)

    Landau's work was crucial for the development of the modern theory of phase transitions. He showed that such transitions can be classified by an order parameter, which in the low-temperature phase becomes nonzero. Together with Ginzburg he made this order parameter a spacetimedependent order field and introduced a local energy functional whose extrema yield field equations and whose fluctuations determine the universal critical behavior of second-oder transitions. In the same spirit, but from a dual point of view, I have developed in the last twenty years a disorder field theory that describes phase transitions via the statistical mechanics of grand-canonical ensembles of vortex lines in superfluids and superconductors, or of defect lines in crystals. The Feynman diagrams of the disorder fields are pictures of the vortex or defect lines. A nonzero ground state expectation value of the disorder field at high temperature signalizes the proliferation of line like excitations in the ordered phase. It was this description of the superconductor that led in 1982 to a first understanding of the order of the superconducting phase transition. Recent experimental progress in the critical regime of high-Tc superconductors will be able to verify the predicted tricritical point of the Ginzburg parameter $\kappa \approx 0.8/\sqrt{2}$ where the second-order transition becomes first-order.

28. H. Kleinert and P. Kienle, *Neutrino Mass Differences from Interfering Recoil Ions*, Lecture presented at the 3rd Stueckelberg Workshop on Relativistic Field Theories ICRANET Stueckelberg July 8-18, 2008 - ICRANet Center, Pescara (Italy), and EJTP 6, 107 (2009)

We show that the recent observation of the time modulation of two-body weak decays of heavy ions reveals the mass content of the electron neutrinos via interference patterns in the recoiling ion wave function. From the modulation period we derive the difference of the square masses $m^2 \approx 22.5 \times 10^{-5}$ eV^2, which is about 2.8 times larger than that derived from a combined analysis of KamLAND and solar neutrino oscillation experiments. It is, however, compatible with a data regime to which the KamLAND analysis attributes a smaller probability. The experimental results displayed in Fig. 1 imply that the neutrino mixing matrix violates unitarity by about 10%.

29. A. Chervyakov and H. Kleinert, *Exact Pair Production Rate for a Smooth Potential Step* Phys. Rev. D 80, 065010 (2009) We present an explicit formula for the exact rate of pair production of oppositely charged scalar particles by a smooth potential step $V \propto tanhkz$ in three dimensions. As a check we recover from this the known results for an infinitely sharp step as well as for a uniform electric field.

30. P. Jizba, H. Kleinert, and P. Haener *Perturbation Expansion for Option Pricing with Stochastic Volatility* Physica A 388 (2009) 3503-3520

    We fit the volatility fluctuations of the S&P 500 index well by a Chi distribution, and the distribution of log-returns by a corresponding superposition of Gaussian distributions. The Fourier transform of this is, remarkably, of the Tsallis type. An option pricing formula is derived from the same superposition of BlackuScholes expressions. An explicit analytic formula is deduced from a perturbation expansion around a BlackuScholes formula with the mean volatility. The expansion has two parts. The first takes into account the non-Gaussian character of the stock-fluctuations and is organized by powers of the excess kurtosis, the second is contract based, and is organized by the moments of moneyness of the option. With this expansion we show that for the Dow Jones Euro Stoxx 50 option data, a Δ-hedging strategy is close to being optimal.

31. J. Dietel and H. Kleinert, *Modeling two-dimensional crystals and nanotubes with defects under stress* (arXiv:0812.0226) Phys. Rev. B 79, 245415 (2009)

    We calculate analytically the phase diagram of a two-dimensional planar crystal and its wrapped version with defects under external homogeneous stress as a function of temperature using a simple elastic square lattice model that allows for defect formation. The temperature dependence turns out to be very weak. The results are relevant for recent stress experiments on carbon nanotubes at high temperatures. Under increasing stress, we find a crossover regime which we identify with a cracking transition that is almost independent of temperature. Furthermore, we find an almost stress-independent melting point. In addition, we derive an enhanced ductility with relative strains before

cracking between 200% and 400%, in agreement with carbon nanotube experiments. The specific values depend on the Poisson ratio and the angle between the external force and the crystal axes. We give arguments that the results for carbon nanotubes should be not much different from these results in spite of the different lattice structures.

32. J. Dietel and H. Kleinert, *Lindemann parameters for solid membranes focused on carbon nanotubes* (arXiv:0806.1656) Phys. Rev. B 79, 075412 (2009)

    Temperature fluctuations in the normal direction of planar crystals such as graphene are quite violent and may be expected to strongly influence their melting properties. In particular, they will modify the Lindemann melting criterion. We calculate this modification in a self-consistent Born approximation. The result is applied to graphene and its wrapped version represented by single-walled carbon nanotubes. It is found that the out-of-plane fluctuations dominate over the in-plane fluctuations. This makes strong restrictions to possible Lindemann parameters. Astonishingly we find that these large out-of-plane fluctuations have only a small influence on the melting temperature.

33. J. Dietel and H. Kleinert, Phase diagram of vortices in high-Tc superconductors with a melting line in the deep H_{c2} region (arXiv:0807.2757) Phys. Rev. B 79, 014512 (2009)

    We use a simple elastic Hamiltonian for the vortex lattice in a weak impurity background, which includes defects in the form of integer-valued fields to calculate the free energy of a vortex lattice in the deep H_{c2} region. The phase diagram in this regime is obtained by applying the variational approach of Mzard and Parisi developed for random manifolds. We find a first-order line between the Bragg-glass and vortex-glass phases as a continuation of the melting line. In the liquid phase, we obtain an almost vertical third-order glass transition line near the critical temperature in the H-T plane. Furthermore, we find an almost vertical second-order phase transition line in the Bragg-glass as well as the vortex-glass phases, which crosses the first-order Bragg-glass-vortex-glass transition line. We calculate the jump of the temperature derivate of the induction field across this second-order line as well as the entropy and magnetic-field jumps across the first-order line.

34. H. Kleinert, *Equivalence Principle and Field Quantization in Curved Spacetime* (arxiv.org:0910.4034) EJTP 6, 1 (2009)

    To comply with the equivalence principle, fields in curved spacetime can be quantized only in the neighborhood of each point, where one can construct a freely falling *Minkowski* frame with *zero* curvature. In each such frame, the geometric forces of gravity can be replaced by a selfinteracting spin-2 field, as proposed by Feynman in 1962. At *any fixed* distance R from a black hole, the vacuum in each freely falling volume element acts like a thermal bath of all parti-

cles with Unruh temperature $T_U = \hbar GM/2\pi cR^2$. At the horizon $R = 2GM/c^2$, the falling vacua show the Hawking temperature $T_H = \hbar c^3/8\pi GMk_B$.

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