Trilinear and Quartic Gauge Boson Couplings at the LHC

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OUTLINE

- Vector bosons self-couplings and the symmetry breaking connection
- Gauge Invariance and the Hierarchy of couplings: Higgs vs Higgsless description
- Present indirect limits as a yardstick for a meaningful future measurement/constraint
- Important channels. Outlook. Conclusions
Self-couplings: the Higgs and Symmetry Breaking Connection

Without Higgs

If \( g_{VVVV} \neq g_{VVVV}^2 \) \( \implies M_{LLLL} \propto E_W^4 \)

In the SM \( M_{LLLL} \sim \sqrt{2}G_F u \propto E_W^2 \)

Unitarity without Higgs requires \( \sqrt{s_{WW}} \leq 1.2 \text{TeV} \)

Slight departure of the vector bosons self-couplings from SM values is enhanced at high energies
Higgs and Delayed Unitarity

\[ \mathcal{M}_{LLLL} \sim -\sqrt{2} G_F M_H^2 \left( \frac{s}{s-M_H^2} + \frac{t}{t-M_H^2} \right) \]

Unitarity implies \( M_H \leq \frac{4\pi \sqrt{2}}{3G_F} \sim 700 \text{GeV} \)

- expect some collective modes to effectively affect the self-interaction of the gauge bosons
- watch out for the longitudinal modes

Extra-dim with special boundary cdts

Unitarity is delayed up to a few TeV
cancellation from underlying
5-d gauge symmetry.

F. BOUDJEMA, Trilinear and Quartic Gauge Boson couplings at the LHC – p. 3/1
Gauge Invariance: \( g_{f f V} = g_{VVV} \)

- **LEP legacy**: We know that \( WWV \) can not deviate too much (10%) from SM gauge value.
- **But slightest deviations are revealed at higher energies (LHC?)**

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*F. BOUDJEMA, Trilinear and Quartic Gauge Boson couplings at the LHC – p. 4/17*
Gauge Invariance: Indirect Limits

Dittmaier, Schildknecht and Weiglein 1996.
\[ \mathcal{L}_{\text{Gauge}} = -\frac{1}{2} \left[ \text{Tr}(W_{\mu\nu} W^{\mu\nu}) + \text{Tr}(B_{\mu\nu} B^{\mu\nu}) \right] \]

GI kinetic term

\[
W_{\mu\nu} = \frac{1}{2} \left( \partial_\mu W_\nu - \partial_\nu W_\mu + \frac{i}{2} g [W_\mu, W_\nu] \right) = \frac{\tau_i}{2} \left( \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g^{ijk} W^j_\mu W^k_\nu \right)
\]

\[
B_{\mu\nu} = \frac{1}{2} (\partial_\mu B_\nu - \partial_\nu B_\mu) \tau_3 \quad B_\mu = \tau_3 B_\mu
\]

- \( \mathcal{L}_{\text{Gauge}} \) describes **transverse states** (field strength)
- **longitudinal states**, \( Z^L_\mu \propto \partial_\mu \phi_3 \quad \epsilon^L_\mu = \frac{k_\mu}{M_Z} - M_Z \frac{s_\mu}{s.k} \) do not contribute much to \( \mathcal{L}_{\text{Gauge}} \)
- **BUT** to the mass and Symmetry Breaking Lagrangian

\[
\mathcal{L}_M = M_W^2 W^+ \mu W^- \mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \subset \mathcal{L}_{H,M=SB} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - \lambda \left[ \Phi^\dagger \Phi - \frac{\mu^2}{2\lambda} \right]^2
\]

- **Self interacting Goldstone Bosons** \( \equiv \) Self-interacting \( V_L V_L \rightarrow V_L V_L \)

- **Symmetry Breaking has a custodial** \( SU(2) \) symmetry \( \rho = 1 \)

Gauge invariance of Mass and SB: Covariant Derivatives

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F. BOUDJEMA, *Trilinear and Quartic Gauge Boson couplings at the LHC* – p. 6/17
kinetic term still there but mass and longitudinals through a system of Goldstones without the Higgs (still gauge invariant): Non-Linear realisation of SB

\[ \Sigma = \exp\left(\frac{i\omega^i \tau^i}{v}\right) \quad (v = 246 \text{ GeV} \text{ is the vev}) \quad \text{and} \quad \mathcal{D}_\mu \Sigma = \partial_\mu \Sigma + \frac{i}{2} \left( g W_\mu \Sigma - g' B_\mu \Sigma \tau_3 \right) \]

\[ \mathcal{L}_M = \frac{v^2}{4} \text{Tr}(\mathcal{D}_\mu \Sigma^\dagger \mathcal{D}_\mu \Sigma) \equiv -\frac{v^2}{4} \text{Tr}(\mathcal{V}_\mu \mathcal{V}_\mu) \quad \text{with} \quad \mathcal{V}_\mu = (\mathcal{D}_\mu \Sigma) \Sigma^\dagger \]
Hierarchy of Operators: Most important effects

POST-LEP 1&2: The SM is a gauge invariant (GI) theory with a custodial SU(2) symm.

as in QED any deviation should be parameterized as a (higher order) GI operators

This approach defines a hierarchy of couplings and order of magnitude for deviations

**QED example**

\[ \mathcal{L}_{\text{eff.}}^{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\beta_1}{m^2} \frac{e^2}{16\pi^2} \left( F_{\mu\nu} \Box F^{\mu\nu} + \frac{e^2}{m^2} F_{\mu\nu} \Box^2 F^{\mu\nu} \right) + \frac{1}{m^4} \frac{e^4}{16\pi^2} \left( \beta_2 (F_{\mu\nu} F^{\mu\nu})^2 + \beta_3 (F_{\mu\nu} F^{\mu\nu})^2 \right) + ...... + \mathcal{L}_{\text{gauge fixing}} \]

**SM** need to describe new physics of longitudinal modes
decide about Linear (with Higgs) or Non-Linear (without Higgs) of Symm. Breaking
### Linear Realisation, Light Higgs

<table>
<thead>
<tr>
<th>Operator</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}_B$</td>
<td>$i g' \frac{e_B}{\Lambda^2} (D_\mu \Phi)^\dagger B^{\mu\nu} D_\nu \Phi$</td>
</tr>
<tr>
<td>$\mathcal{L}_W$</td>
<td>$ig \frac{e_w}{\Lambda^2} (D_\mu \Phi)^\dagger (2 \times W^{\mu\nu})(D_\nu \Phi)$</td>
</tr>
<tr>
<td>$\mathcal{L}_\lambda$</td>
<td>$\frac{2i}{3} \frac{L \lambda}{\Lambda^2} g^3 \text{Tr}(W_{\mu\nu} W^{\nu\rho} W^{\mu}_{\rho})$</td>
</tr>
</tbody>
</table>

### Non Linear-Realisation, No Higgs

<table>
<thead>
<tr>
<th>Operator</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}_{9R}$</td>
<td>$-ig' \frac{L_{9R}}{16\pi^2} \text{Tr}(B^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma)$</td>
</tr>
<tr>
<td>$\mathcal{L}_{9L}$</td>
<td>$-ig \frac{L_{9L}}{16\pi^2} \text{Tr}(W^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger)$</td>
</tr>
<tr>
<td>$\mathcal{L}_1$</td>
<td>$\frac{L_1}{16\pi^2} \left( \text{Tr}(D^{\mu} \Sigma^\dagger D_\mu \Sigma) \right)^2 \equiv \frac{L_1}{16\pi^2} \mathcal{O}_1$</td>
</tr>
<tr>
<td>$\mathcal{L}_2$</td>
<td>$\frac{L_2}{16\pi^2} \left( \text{Tr}(D^{\mu} \Sigma^\dagger D_\nu \Sigma) \right)^2 \equiv \frac{L_2}{16\pi^2} \mathcal{O}_2$</td>
</tr>
</tbody>
</table>
Projecting on the multipole moments, phenomenological parameters

\[ W^+ W^- Z, W^+ W^- \gamma \text{ couplings} \]

\[
\Delta \kappa_\gamma = \frac{e^2}{s_w^2} \frac{v^2}{4 \Lambda^2} (\epsilon_W + \epsilon_B) = \frac{e^2}{s_w^2} \frac{1}{32 \pi^2} (L_{9L} + L_{9R})
\]

\[
\Delta \kappa_Z = \frac{e^2}{s_w^2} \frac{v^2}{4 \Lambda^2} (\epsilon_W - \frac{s_w^2}{c_w^2} \epsilon_B) = \frac{e^2}{s_w^2} \frac{1}{32 \pi^2} \left( L_{9L} - \frac{s_w^2}{c_w^2} L_{9R} \right)
\]

\[
\Delta g_1^Z = \frac{e^2}{s_w^2} \frac{v^2}{4 \Lambda^2} \left( \frac{\epsilon_W}{c_w^2} \right) = \frac{e^2}{s_w^2} \frac{1}{32 \pi^2} \left( \frac{L_{9L}}{c_w^2} \right)
\]

\[
\lambda_\gamma = \lambda_Z = \left( \frac{e^2}{s_w^2} \right) L \lambda \frac{M_W^2}{\Lambda^2}
\]

- At this order no \(C\) violating tri-linear couplings in particular no \(ZZZ, ZZ\gamma, Z\gamma\gamma\) couplings. These appear at higher orders. Should not receive high priority.

- For LHC fits and measurements better to switch to the \(L_i\) parameters
Pheno of tri-linear couplings

\[ \mathcal{L}_{WWV} = -ie \left\{ A_\mu \left( W^{-\mu\nu} W_{\nu}^+ - W^{+\mu} W_{\nu}^- \right) + \frac{\kappa_{\gamma\gamma}}{1 + \Delta \kappa_{\gamma\gamma}} F_{\mu\nu} W^{+\mu} W^{-\nu} \right\} \]

\[ + \frac{c_W}{s_W} g_1^Z \left[ (1 + \Delta g_1^Z) Z_\mu \left( W^{-\mu\nu} W_{\nu}^+ - W^{+\mu} W_{\nu}^- \right) + \frac{\kappa_{Z\gamma}}{1 + \Delta \kappa_{Z\gamma}} Z_{\mu\nu} W^{+\mu} W^{-\nu} \right] \]

\[ + \frac{1}{M_W^2} \left( \lambda_\gamma F^{\nu\lambda} + \lambda_Z \frac{c_W}{s_W} Z^{\nu\lambda} \right) W_\lambda^+ W^{-\mu}_{\nu} \}

No \ ZZZ, \ ZZ\gamma, \ Z\gamma\gamma(\text{higher order})
Current direct limits as a guide

\[ \lambda_\gamma \quad 0.2 \]
\[ 0.15 \]
\[ 0.1 \]
\[ 0.05 \]
\[ 0 \]
\[ -0.05 \]
\[ -0.1 \]
\[ -0.15 \]
\[ -0.2 \]
\[ g_1^Z \]

\[ \kappa_\gamma \quad 1.25 \]
\[ 1.2 \]
\[ 1.15 \]
\[ 1.1 \]
\[ 1.05 \]
\[ 1.0 \]
\[ 0.95 \]
\[ 0.9 \]
\[ 0.85 \]
\[ 0.8 \]
\[ 0.75 \]
\[ g_1^Z \]

LEP Preliminary

- 95% c.l.
- 68% c.l.
- 2d fit result

5 – 10% accuracy \( \Rightarrow |L_9| \sim 50 \)
\[
\mathcal{L}_{WWV_1V_2} \\
- e^2 \left\{ (A_\mu A_\nu W_\mu^+ W_\nu^- - A_\nu A_\mu W_\mu^+ W_\nu^-) \right. \\
+ 2 \frac{c_w}{s_w} \left( 1 + \frac{l_9}{c_w^2} \right) (A_\mu Z_\mu W_\mu^+ W_\nu^- - \frac{1}{2} A_\mu Z_\nu (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-)) \\
+ \frac{c_w^2}{s_w^2} \left( 1 + \frac{2l_9}{c_w^2} - \frac{l_-}{c_w^4} \right) (Z_\mu Z_\mu W_\mu^+ W_\nu^- - Z_\mu Z_\nu W_\mu^+ W_\nu^-) \\
+ \frac{1}{2s_w^2} \left( 1 + 2l_9 - l_- \right) (W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- - W_\nu^+ W_\nu^- W_\mu^+ W_\mu^-) \\
- \frac{l_+}{2s_w^2} \left( (3W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + W_\nu^+ W_\mu^+ W_\mu^- W_\nu^-) \right) \\
+ \frac{2}{c_w^2} \left( Z_\mu Z_\mu W_\mu^+ W_\nu^- + Z_\mu Z_\nu W_\mu^+ W_\nu^- \right) \\
\left. + \frac{1}{c_w^4} (Z_\mu Z_\mu Z_\nu Z_\nu) \right\} \\
\text{with} \quad l_{9l} = \frac{e^2}{32\pi^2 s_w^2} L_9 L \quad ; \quad l_{\pm} = \frac{e^2}{32\pi^2 s_w^2} (L_1 \pm L_2) 
\]

No Anomalous $WW\gamma\gamma$, No $V_1V_2\gamma\gamma$

New $ZZZZ$, New $WWWW$ and $WWZZ$ structures.

Best probed in $WW$ scattering $(L_1,2) \implies \implies$

(higher order structures see Bélanger et al.)
I only listed operators giving contribution to $VVV$ and $VVVV$ but...

A general approach would also include contributions to 2-point function (see QED example)

\[
\mathcal{L}_{WB} = gg' \frac{\epsilon_{WB}}{\Lambda^2} (\Phi^\dagger \times W_{\mu\nu} \Phi) B_{\mu\nu}
\]

\[
\mathcal{L}_{10} = gg' \frac{L_{10}}{16\pi^2} \text{Tr}(B_{\mu\nu}^\dagger W_{\mu\nu}^\dagger) \rightarrow L_{10} = -\pi S_{\text{New}} \simeq \frac{4\pi s_W}{\alpha} \epsilon_3
\]

LEP-Tevatron precision EW physics implies strong constraint on $S_{\text{New}}(L_{10})$ and consequently on the other operators

This constraint could be considered as a yardstick for future measurements
For $M_H = 1\text{TeV}$ need $S_{\text{New}} = -0.07$ and $T_{\text{New}} = +0.3 \implies \text{Constraint } L_{10} < 1$

future $L_i$ measurements of this order! $L_i \sim 1$

unless some global symmetry makes $L_{10} = 0 (SU(2)_A ?)$
Future measurements I.

The $S, T, U$ Post-LEP

From $WW$ scattering $L_{1,2} \sim 1@\text{LHC}$
Future measurements and Conclusions.

- Especially if Higgs not found precision measurements of (longitudinal) vector bosons self-couplings is crucial
- Update simulations in the channels $pp \rightarrow WZ, W\gamma(WW?)$ including NLO (MCFM)
- $pp \rightarrow (W, Z)X$ (VV fusion to single V): worth it?
- Can more be done to optimize $WW \rightarrow WW$?
- Quartic through $pp \rightarrow WWZ, ZZZ, \ldots$ though falls like $1/\hat{s}$ could it help? combined with $WW$ fusion?
- work out in detail consequences of some extra-dim models, (Little Higgs?)...

...could constitute a nice subgroup at Les Houches 2005....

Official deadline for registration TOMORROW
http://lappweb.in2p3.fr/conferences/LesHouches/Houches2005/application.html