The Standard Model in the LHC era

I: Histories and Symmetries

Fawzi BOUDJEMA

LAPTh-Annecy, France
Introduction

EW theory is the combination of two fundamental principles

- Gauge Symmetry Principle
- Hidden symmetry or *Spontaneous symmetry breaking*

This allows
- a correct quantum description
- high degree of precision (LEP, SLC, Babar, ... LHC)
Charge conservation
Charge conservation

Charge can not just disappear like that!
Charge conservation

Total net charge is conserved, global charge conservation
Charge conservation

but for a flicker of a second, as if charge was not conserved

charge can not disappear at one point and reappear *instantaneously* at another point
Charge conservation

Must have Local charge conservation
Charge conservation

A change (in time) in the charge density is accompanied by a current flow.

Local conservation of the electric charge

Charge must be conserved locally

Continuity equation of the electric charge

$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$$

EPAM, Taza, Maroc, March. 2011
Introduction: In the beginning, there was light! Prior to Maxwell

\[ \text{div} \, \vec{E} = \partial_i E_i = \rho \quad \text{(Gauss)} \]

\[ \text{div} \, \vec{B} = 0 \quad \text{(no magnetic charge)} \]

\[ \text{Curl} \, \vec{E} = \epsilon_{ijk} \partial_j E_k = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday)} \]

\[ \text{Curl} \, \vec{B} = \vec{j} \quad \text{(Ampere)} \]
Introduction: In the beginning, there was light! Prior to Maxwell

$$\text{div} \, \vec{E} = \partial_i E_i = \rho \quad \text{(Gauss)}$$

$$\text{div} \, \vec{B} = 0 \quad \text{(no magnetic charge)}$$

$$\text{Curl} \, \vec{E} = \epsilon_{ijk} \partial_j E_k = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday)}$$

$$\text{Curl} \, \vec{B} = \vec{j} \quad \text{(Ampere)}$$

Unfortunately the mathematics implies that

$$\text{div} \, (\text{Curl} \, \vec{B}) = \nabla \cdot (\nabla \times \vec{B}) = 0 \Rightarrow \nabla \cdot \vec{j} = \text{div} \vec{j} = 0$$

$$\nabla \cdot j = \text{div} j = 0$$

in conflict with the continuity equation, local conservation.
Introduction: Electromagnetism as a prototype

Maxwell equations: Unify $\mathbf{E}$ and $\mathbf{B}$

Local conservation of the electric charge

$$\partial j = 0 \quad j^\mu = (\rho, \mathbf{j})$$

$$\text{div} \mathbf{E} = \rho \quad \text{div} \mathbf{B} = 0$$

$$\text{Curl} \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad \text{Curl} \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j}$$

(Gauge Invariance) electrostatic field (force) depends only on difference of potential

The quantum (for photons) is the vector potential $A^\mu(x) = (V, \mathbf{A})$,

$$\mathbf{B} = \text{Curl} \mathbf{A} \quad \mathbf{E} = -\text{Grad} V - \frac{\partial \mathbf{A}}{\partial t}.$$
Different $A_\mu$ lead to the same physical fields $E, B$.  

Gauge invariance: $A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Lambda(x)$. 
Different $A_\mu$ lead to the same physical fields $E, B$. 

Gauge invariance $A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Lambda(x)$.

But Gauge invariance $F^{\mu\nu}(x) \rightarrow F^{\mu\nu}(x)$. 
Introduction: Electromagnetism as a prototype

Different $A_\mu$ lead to the same physical fields $E, B$.

**Gauge invariance**  
$A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Lambda(x)$.

With $A_\mu$ the equations can be written in a manifestly covariant form, with

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \partial^\rho F^{\sigma\sigma}$$

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \partial_\mu \tilde{F}^{\mu\nu} = 0.$$ 

All of this can be derived from the Lagrangian

$$\mathcal{L}_{\text{em}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \equiv \frac{1}{4} \left( (\vec{E} + i \vec{B})^2 + (\vec{E} - i \vec{B})^2 \right).$$

only two Transverse polarisations/helicity states no longitudinal polarisation
Aside, Equation of motion:

The dynamics, the physics, is encoded in the action

\[ S = \int d^4x \ L [\phi_i(x), \partial_\mu \phi_i(x)] . \]

The principle of least action: \( \delta S = 0 \) when varying \( \delta \phi_i \) leads to the Euler–Lagrange equations

\[ \frac{\partial L}{\partial \phi_i} - \partial^\mu \left( \frac{\partial L}{\partial \left( \partial_\mu \phi_i \right)} \right) = 0 \]
Introduction: Electromagnetism as a prototype

\[ F^{\mu \nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix} \]
Introduction: Electromagnetism as a prototype

The equation of motion for the free field, \( j = 0 \), lead to

\[
\partial_{\mu} F^{\mu\nu} = 0 \Rightarrow \Box A^{\nu} - \partial^{\nu} (\partial A) = 0
\]

The freedom from GI allows us to take the gauge fixing

\[
\partial A = 0
\]

Out of the 4 degrees of freedom/components in \( A_{\mu} \), this condition freezes one of them
The equation of motion for the free field, $j = 0$, lead to

$$\partial_{\mu} F^{\mu\nu} = 0 \Rightarrow \Box A^{\nu} - \partial^{\nu}(\partial \cdot A) = 0$$

The freedom from GI allows us to take the gauge fixing

$$\partial A = 0$$

Out of the 4 degrees of freedom/components in $A_{\mu}$, this condition freezes one of them. Moreover there is still a lot freedom in $\partial A = 0$, there is still an invariance, overcounting, due to

$$A^{\mu}(x) + \partial^{\mu} \Lambda(x) \quad \Box \Lambda = 0$$
Introduction: Electromagnetism as a prototype

The equation of motion for the free field, \( j = 0 \), lead to

\[
\partial_\mu F^{\mu\nu} = 0 \Rightarrow \Box A^\nu - \partial^\nu (\partial \cdot A) = 0
\]

The freedom from GI allows us to take the gauge fixing

\[
\partial A = 0
\]

Out of the 4 degrees of freedom/components in \( A_\mu \), this condition freezes one of them

To remember: Working in a particular gauge, breaks gauge invariance, in fact it hides the gauge symmetry, the physics is independent of the gauge fixing
Introduction: Electromagnetism as a prototype

The equation of motion for the free field, \( j = 0 \), lead to

\[
\partial_{\mu} F^{\mu\nu} = 0 \Rightarrow \Box A^\nu - \partial^\nu (\partial \cdot A) = 0
\]

The freedom from GI allows us to take the gauge fixing

\[
\partial A = 0
\]

Out of the 4 degrees of freedom/components in \( A_\mu \), this condition freezes one of them

The free field is then

\[
\Box A^\nu = 0, \quad k^2 A_\nu = 0
\]

Photon is massless, it has 2 transverse polarisations
Introduction: Electromagnetism as a prototype

The equation of motion for the free field, \( j = 0 \), lead to

\[
\partial_\mu F^{\mu\nu} = 0 \Rightarrow \square A^\nu - \partial^\nu (\partial \cdot A) = 0
\]

The freedom from GI allows us to take the gauge fixing

\[
\partial A = 0
\]

Out of the 4 degrees of freedom/components in \( A_\mu \), this condition freezes one of them

The free field is then

\[
\square A^\nu = 0, \quad k^2 A_\nu = 0
\]

Photon is massless, it has 2 transverse polarisations
The equation of motion for the free field, $j = 0$, lead to

$$\partial_\mu F^{\mu\nu} = 0 \Rightarrow \Box A^\nu - \partial^\nu (\partial \cdot A) = 0$$

The freedom from GI allows us to take the gauge fixing

$$\partial A = 0$$

Out of the 4 degrees of freedom/components in $A_\mu$, this condition freezes one of them.

The free field is then

$$\Box A^\nu = 0, \quad k^2 A_\nu = 0$$

Photon is massless, it has 2 transverse polarisations.
\[ \mathcal{L}_{\text{pem}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu. \]

The Lagrangian is not invariant under the gauge transformation \( A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \Lambda(x) \). But the equation of motion requires \( \partial \cdot j = 0 \).

The action is gauge invariant but what is important is the action

\[
\int \mathcal{L}_{\text{pem}} = \int \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu \right) \implies \\
\int \mathcal{L}_{\text{pem}} = \int \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu \right) - \int \Lambda \partial \cdot j
\]
Introduction: Electromagnetism as a prototype, Musings

\[ \mathcal{L}_{\text{pem}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_{\mu} A^\mu - j_{\mu} A^\mu. \]

\[ \square A^\nu + m^2 A^\nu - \partial^\nu (\partial \cdot A) = j^\nu \implies m^2 \partial \cdot A = \partial \cdot j \]

The Lagrangian and the action are not invariant under the gauge transformation
\[ A^\mu (x) \rightarrow A^\mu (x) + \partial^\mu \Lambda (x). \]

But the equation of motion requires \( \partial \cdot A = 0. \)

This is the spin-1 condition, no longer a gauge fixing.

Then 3 degrees of freedom. An extra Longitudinal degree of freedom
Quantization of the EM field

\[ [P, X] = i \]

To quantize, find the conjugate momenta of each degree of freedom and impose equal time commutation

\[ \Pi^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_0 A_\mu)} = F^{\mu 0} \]

\[ \Pi^0 = 0 \]

Fix a gauge for \( \Pi^0 \neq 0 \)

\[ \mathcal{L}_{\text{G pem}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2 \xi} \left( \partial \cdot \mathbf{A} \right)^2 \]
Recall the Lorentz force acting on a moving particle in an electromagnetic field:

\[ \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \]
Gauge Invariance and minimal subtraction, introducing interactions

this is derived from the Hamiltonian

\[ H = \frac{1}{2m} (\vec{p} - q\vec{A})^2 + qV \quad \Leftrightarrow \quad (H - qV) = \frac{1}{2m} (\vec{p} - q\vec{A})^2 \]

whereas for the corresponding free field

\[ H = \frac{p^2}{2m} \]

to introduce the interaction one has made minimum substitution

\[ P_\mu \rightarrow P_\mu - qA_\mu \]
Gauge Invariance and minimal subtraction, introducing interactions

this is derived from the Hamiltonian

\[ H = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + qV \quad \Leftrightarrow \quad (H - qV) = \frac{1}{2m}(\vec{p} - q\vec{A})^2 \]

whereas for the corresponding free field

\[ H = \frac{p^2}{2m} \]

to introduce the interaction one has made minimum substitution

\[ P_\mu \rightarrow P_\mu - qA_\mu \]

in QM

\[ \partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu = \partial_\mu + ieQA_\mu \]

\( D_\mu \) is the covariant derivative.
Take Schrödinger’s equation
\[
\frac{1}{2m}(-i\nabla)^2 \psi = i\frac{\partial \psi}{\partial t}
\]
invariant under a \textit{global} phase transformation
\[
\psi \rightarrow \exp(i\lambda)\psi
\]
what about invariance under \textit{local} phase transformation?
\[
\lambda \rightarrow q\Lambda(x = (t, \vec{x}))
\]
Possible only if one introduces a \textit{compensating} vector field which transforms exactly like $A_\mu$
This prescription gives
\[
(1/2m)\left(-i\nabla + q\vec{A}\right)^2 \psi = \left(i\partial/\partial t + qV\right)\psi.
\]
This equation with interactions is invariant under gauge transformations
Interaction of electrons

consider free Dirac particle whose equation of motion is

\[ \left( i\gamma_\mu \partial^\mu - m \right) \psi = \left( i\partial - m \right) \psi = 0 \]

easily derived from the matter Lagrangian
Interaction of electrons

consider free Dirac particle whose equation of motion is

\[(i\gamma_\mu \partial^\mu - m)\psi = (i\partial - m)\psi = 0\]

easily derived from the matter Lagrangian

\[\mathcal{L}_M = \bar{\psi}(i\partial - m)\psi\]

The interaction is obtained through the covariant derivative, leading to

\[\mathcal{L} = \bar{\psi}(i\partial - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \mathcal{L}_M + \mathcal{L}_G + \mathcal{L}_I\]

\[= \bar{\psi}(i\partial - eQ A - m)\psi + \mathcal{L}_G\]

\[\mathcal{L}_I = -eJ^Q_\mu A^\mu \quad \frac{\partial \mathcal{L}}{\partial A_\mu} = -eJ^Q_\mu \quad \partial_\mu J^\mu = 0\]
Interaction of electrons

\[ \mathcal{L}_M = \bar{\psi} (i \partial \psi - m) \psi \]

The interaction is obtained through the covariant derivative, leading to

\[ \mathcal{L} = \bar{\psi} (i \partial \psi - m) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} = \mathcal{L}_M + \mathcal{L}_G + \mathcal{L}_I \]
\[ = \bar{\psi} (i \partial \psi - eQ A - m) \psi + \mathcal{L}_G \]
\[ \mathcal{L}_I = -e J^Q_\mu A^\mu \frac{\partial \mathcal{L}}{\partial A_\mu} = -e J^Q_\mu \partial_\mu J^\mu = 0 \]

the electromagnetic spin-1/2 current

\[ J^Q_\mu = Q \bar{\psi} \gamma_\mu \psi \]

**UNIVERSAL COUPLING**
Interaction of electrons: Gauge Invariance

\[ \mathcal{L}_M = \bar{\psi} \left( i \partial \psi - m \right) \psi \]

The free Dirac Lagrangian is invariant under a global symmetry: phase transformation

\[ \psi \rightarrow U \psi \quad U = \exp(i\lambda) \quad U^\dagger U = 1 \quad U \text{ is unitary} \]

global means rigid, same for all \( x \)
Interaction of electrons: Gauge Invariance

\[ \mathcal{L}_M = \bar{\psi} \left( i \partial \psi - m \right) \psi \]

The free Dirac Lagrangian is invariant under a global symmetry: phase transformation

\[ \psi \rightarrow U \psi \quad U = \exp(i\lambda) \quad U^\dagger U = 1 \quad U \text{ is unitary} \]

global means rigid, same for all \( x \) Promoting \( U \) to a local symmetry \( \lambda \rightarrow \lambda(x) \) only possible by using covariant derivatives, compensating gauge field.
Interaction of electrons: Gauge Invariance

\[ \mathcal{L}_M \rightarrow \mathcal{L}_{\text{QED}} = \bar{\psi} \left( i \slashed{D} - m \right) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

For \( \lambda = \lambda(x) \)

\[ \bar{\psi} \slashed{D} \psi \rightarrow \bar{\psi} U^\dagger \slashed{D} \left( U \psi \right) \neq \bar{\psi} \slashed{D} \psi \]

must have

\[ \bar{\psi} \slashed{D} \psi \rightarrow \bar{\psi} U^\dagger \slashed{D} \left( U \psi \right) = \bar{\psi} \slashed{D} \psi \]

must require that

\[ D' = U D U^\dagger \quad \left( D \psi \right)' = U \left( D \psi \right) \]

\[ \psi \rightarrow U \psi \quad A_\mu \rightarrow A_\mu + \partial_\mu \Lambda \]

\[ \lambda = -eQ \Lambda(x) \quad \text{Universality} \]

QED U(1) Abelian theory
\[ [D_\mu, D_\nu] \psi = \left( \left( \partial_\mu + i e A_\mu \right) \left( \partial_\nu + i e A_\nu \right) - \mu \leftrightarrow \nu \right) \psi \]

\[ = i e \left( \partial_\mu A_\nu - \partial_\nu A_\mu + i e [A_\mu, A_\nu] \right) \psi \]

\[ = i e \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right) \psi \quad \text{Abelian} \]
$F_{\mu\nu}$ as a covariant derivative

\[
[D_\mu, D_\nu] \psi = \left( \left( \partial_\mu + ieA_\mu \right) \left( \partial_\nu + ieA_\nu \right) - \mu \leftrightarrow \nu \right) \psi
\]

\[
= ie \left( \partial_\mu A_\nu - \partial_\nu A_\mu + ie [A_\mu, A_\nu] \right) \psi
\]

\[
= ie \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right) \psi \quad \text{Abelian}
\]

\[
[D_\mu, D_\nu] \equiv ie F_{\mu\nu}
\]

\[
F_{\mu\nu} \rightarrow UF_{\mu\nu}U^\dagger
\]

\[
\text{Tr} \left( F_{\mu\nu} F_{\mu\nu} \right) \rightarrow \text{Tr} \left( UF_{\mu\nu} F_{\mu\nu} U^\dagger \right) = \text{Tr} F_{\mu\nu} F_{\mu\nu}
\]
charges: $\psi \rightarrow Q$, the antiparticle $\bar{\psi} \rightarrow -Q$ the photon $A_\mu \rightarrow Q = 0$ Lagrangian density has of course no charge whatsoever, is a true scalar in all respects.
charges: $\psi \rightarrow Q$, the antiparticle $\bar{\psi} \rightarrow -Q$ the photon $A_\mu \rightarrow Q = 0$ Lagrangian density has of course no charge whatsoever, is a true scalar in all respects.

The mass term does not break charge (gauge) symmetry
Interaction of electrons: Gauge Invariance, Recap

ψ has 2 chirality states

\[
\psi = \psi_L + \psi_R = P_L \psi + P_R \psi \quad P_{L,R} = \frac{1}{2} (1 \mp \gamma_5)
\]

\[
\bar{\psi}_L = \bar{\psi} P_R \quad \bar{\psi}_R = \bar{\psi} P_L
\]

The em current conserves chirality and from the point of view of the gauge interaction each component \( \psi_{L,R} \) does not talk to each other

\[
J_{\mu}^{e.m} = Q_L \bar{\psi}_L \gamma_\mu \psi_L + Q_R \bar{\psi}_L \gamma_\mu \psi_L \quad Q_L = Q_R = Q
\]
Interaction of electrons: Gauge Invariance, Recap

\( \psi \) has 2 chirality states

\[
\psi = \psi_L + \psi_R = P_L \psi + P_R \psi \quad P_{L,R} = \frac{1}{2} (1 \mp \gamma_5)
\]

\[
\bar{\psi}_L = \bar{\psi} P_R \quad \bar{\psi}_R = \bar{\psi} P_L
\]

The em current conserves chirality and from the point of view of the gauge interaction each component \( \psi_{L,R} \) does not talk to each other

\[
J_{e.m}^\mu = Q_L \bar{\psi}_L \gamma_\mu \psi_L + Q_R \bar{\psi}_L \gamma_\mu \psi_L
\]

\( Q_L = Q_R = Q \)

\( \psi_L \rightleftharpoons \psi_R \) through the mass

\[
m \left( \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \right) = m \bar{\psi}_R \psi_L + h.c \quad m = m^*
\]
Interaction of electrons: Gauge Invariance, Recap

ψ has 2 chirality states

\[ \psi = \psi_L + \psi_R = P_L \psi + P_R \psi \quad P_{L,R} = \frac{1}{2} (1 \mp \gamma_5) \]

\[ \bar{\psi}_L = \bar{\psi} P_R \quad \bar{\psi}_R = \bar{\psi} P_L \]

The em current conserves chirality and from the point of view of the gauge interaction each component \( \psi_{L,R} \) does not talk to each other

\[ J^{e.m.}_\mu = Q_L \bar{\psi}_L \gamma_\mu \psi_L + Q_R \bar{\psi}_L \gamma_\mu \psi_L \quad Q_L = Q_R = Q \]

\( \psi_L \Leftrightarrow \psi_R \) through the mass

\[ m \left( \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \right) = m \bar{\psi}_R \psi_L + h.c \quad m = m^* \]

in general \( \psi_L \) and \( \psi_R \) have different transformation properties. When they transform similarly, one has a vector theory, like in QED
Chiral limit

In the limit $m \to 0$ there is another (global) symmetry

$$\psi \to \exp(i\lambda_5 \gamma_5)$$

$$\bar{\psi}_L \gamma_\mu \psi_L \to \bar{\psi}_L \gamma_\mu \psi_L \quad \bar{\psi}_R \gamma_\mu \psi_R \to \bar{\psi}_R \gamma_\mu \psi_R$$

But

$$m\bar{\psi}\psi \not\to m\bar{\psi}\psi$$
Current-Current

\[ J_\mu \rightarrow G_{\mu\nu} \rightarrow J_\nu \]
Current-Current

\[ G_{\mu \nu} \] is the Green's function or propagator.
Current-Current

\[ G_{\mu\nu} \] is the Green's function or propagator

\[ \mathcal{L}_G = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j.A \]

\[ \partial_\mu F^{\mu\nu} = J^\nu \Rightarrow A_\mu = G_{\mu\nu} J^\nu \quad \text{or} \]

\[ G^{-1}_{\mu\nu} A_\mu = J^\nu \Rightarrow G^{-1}_{\mu\nu} = \Box g_{\mu\nu} - \partial_\mu \partial_{\nu} \]
$G_{\mu\nu}$ is the Green's function or propagator

From

$$\mathcal{L}_G = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j.A$$

$$\partial_\mu F^{\mu\nu} = J^\nu \Rightarrow A_\mu = G_{\mu\nu} J^\nu \text{ or }$$

$$G^{-1}_{\mu\nu} A_\mu = J^\nu \Rightarrow G^{-1}_{\mu\nu} = \Box g_{\mu\nu} - \partial_\mu \partial_\nu$$

what is the inverse, $G$?

$$G^{-1}_{\mu\nu} G^{\nu\rho} = g^\rho_\mu \quad G^{\mu\rho} \text{ does not exist !}$$
$G_{\mu\nu}$ is the Green’s function or propagator.

From

$$\mathcal{L}_G = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j.A$$

$$\partial_\mu F^{\mu\nu} = J^\nu \Rightarrow A_\mu = G_{\mu\nu} J^\nu \text{ or}$$

$$G_{\mu\nu}^{-1} A_\mu = J^\nu \Rightarrow G_{\mu\nu}^{-1} = \Box g_{\mu\nu} - \partial_\mu \partial_\nu$$

what is the inverse, $G$?

$$G_{\mu\nu}^{-1} G^{\nu\rho} = g^{\rho}_\mu \text{ G}^{\mu\rho}\text{ does not exist!}$$

must include gauge fixing

$$\mathcal{L}_G = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial A)^2$$

$$G_{\mu\nu} = \frac{-i}{k^2} \left( g_{\mu\nu} - \frac{(1 - \xi) k_\mu k_\nu}{k^2} \right)$$

$k.J = 0 \quad \xi = 1$ Feynman Gauge

unphysical longitudinal part
The amplitude is

\[ \mathcal{M} = e^2 J_\mu^{em} G^{\mu\nu} J_\nu^{em} = -i \frac{e^2}{k^2} J_\mu^{em} J_\mu^{em} \]

\[ \mathcal{L}_{\text{eff.}} = -\frac{e^2}{2} \frac{J_\mu^{em} J_\mu^{em}}{k^2} \]
The amplitude is

\[ \mathcal{M} = e^2 J^e_{\mu} G^{\mu\nu} J^e_{\nu} = -\frac{i e^2}{k^2} J^e_{\mu} J^e_{\mu} \]

\[ \mathcal{L}_{\text{eff.}} = -\frac{e^2}{2} \frac{J^e_{\mu} J^e_{\mu}}{k^2} \]

Contact interaction
The amplitude is

\[ M = e^2 J^e_\mu G^{\mu\nu} J^e_\nu \]

\[ = -i \frac{e^2}{k^2} J^e_\mu J^e_\mu \]

\[ \mathcal{L}_{\text{eff.}} = -e^2 \frac{J^e_\mu J^e_\mu}{2k^2} \]

Fundamental interaction
Massive case

\[ \mathcal{L}_G = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 A^2 \]

\[ G_{\mu\nu} = \frac{-i}{k^2 - M^2} \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2} \right) \]

longitudinal part

\[ \mathcal{L}_{\text{eff.}} = -\frac{e^2}{2} \frac{J_{\mu}^{\text{em}} J_{\mu}^{\text{em}}}{k^2 - M^2} \]

\[ \mathcal{L}_{\text{eff.}} = \frac{e^2}{2} \frac{J_{\mu}^{\text{em}} J_{\mu}^{\text{em}}}{M^2} \equiv G_m J_{\mu}^{\text{em}} J_{\mu}^{\text{em}} \quad k^2 \ll M^2 \]
Weak Interactions and non Abelian theories

First time we became aware of a new type of interaction was at play was through the discovery of

\[ \beta\text{-decay: } n \rightarrow p + e^- + \bar{\nu}_e \] (Fermi 1933)

Difficult to accept and set up theoretically.
Weak Interactions and non Abelian theories

First time we became aware of a new type of interaction was at play was through the discovery of

$$\beta\text{-decay: } n \rightarrow p + e^- + \bar{\nu}_e$$

(Fermi 1933)

Difficult to accept and set up theoretically.

Heisenberg: Thought that $n = (p + e^-)$
Weak Interactions and non Abelian theories

First time we became aware of a new type of interaction was at play was through the discovery of

\[ \beta\text{-decay}: n \rightarrow p + e^- + \bar{\nu}_e \]  

(Fermi 1933)

Difficult to accept and set up theoretically.

Heisenberg: Thought that \( n = (p + e^-) \)

Bohr: Energy not conserved (letter from Gamow to Goudsmit: Well you know that he absolutely does not like this chargeless, massless little things! Thinks that the continuous beta spectrum is compensated by the “emittance” of gravitational waves which play the role of neutrinos but are much more physical things!)
Weak Interactions and non Abelian theories

First time we became aware of a new type of interaction was at play was through the discovery of

$$\beta\text{-decay}: n \rightarrow p + e^- + \bar{\nu}_e$$

(Fermi 1933)

Difficult to accept and set up theoretically.

Heisenberg: Thought that $n = (p + e^-)$

Bohr: Energy not conserved (letter from Gamow to Goudsmit: Well you know that he absolutely does not like this chargeless, massless little things! Thinks that the continuous beta spectrum is compensated by the “emittance” of gravitational waves which play the role of neutrinos but are much more physical things!)

Fermi’s article to Nature rejected: ”contains speculations too remote from reality to be of interest to the reader”…
It was also found that:

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ leptonic decay

- muon capture: $\mu^- + p \rightarrow n + \nu_\mu$

were of the same nature and have the same strength.
Weak Interactions

It was also found that

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ leptonic decay

- muon capture: $\mu^- + p \rightarrow n + \nu_\mu$

were of the same nature and have the same strength.

**Universality, Gauge Interaction?**
Weak Interactions

It was also found that

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ leptonic decay

- muon capture: $\mu^- + p \rightarrow n + \nu_\mu$

were of the same nature and have the same strength.

Universality, Gauge Interaction?

But important differences with QED: they involve

- a change in the identity of the fermion

- only left-handed field/component were found to interact.
Fermi postulated a current-current interaction

\[ \mathcal{L}_{\text{Fermi}} = -\frac{4G_F}{\sqrt{2}} J^+ J^\mu - \frac{G_F}{\sqrt{2}} = 1.03510^{-5} M_P^{-2} \]

\[ J_\mu = L_\mu^{\text{leptons}} + H_\mu^{\text{hadrons}} \]

Structure of the current was purely \((V - A)\): Parity violation

restrict myself to leptonic current

\[
J^-_\mu = \bar{e}\gamma_\mu \left( \frac{1 - \gamma_5}{2} \right) e + \bar{\mu}\gamma_\mu \left( \frac{1 - \gamma_5}{2} \right) \mu + \bar{\tau}\gamma_\mu \left( \frac{1 - \gamma_5}{2} \right) \tau
\]

\[ = \bar{e}\gamma_\mu \nu_e + \cdots \]

\[
J^+_\mu = (J^+_\mu)^\dagger = \bar{\nu}_e L\gamma_\mu e_L + \cdots
\]

analogy

\[ J^\text{em}_\mu = \bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R \]

for em same entity \(e \leftrightarrow e\) here \(e_L \leftrightarrow \nu_e\) looks like it is not the same entity, some charge not conserved. Nope.

Make it \(E_L \leftrightarrow E_L\)
Weak Current

\[ J_\mu = \bar{e} \gamma_\mu \nu_e L + \cdots \]

\[ J_\mu = \bar{E}_L \gamma_\mu \ ? \ E_L \]
Doublets and Isospin:

Define the doublet

\[ E_L = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \bar{E}_L = \begin{pmatrix} \bar{\nu}_{e_L} \\ \bar{e}_L \end{pmatrix} \]
Doublets and Isospin:

Define the doublet

\[ E_L = \begin{pmatrix} \nu e_L \\ e_L \end{pmatrix} \quad \bar{E}_L = \begin{pmatrix} \bar{\nu} e_L \\ \bar{e}_L \end{pmatrix} \]

\[ J^+_{\mu} = \bar{\nu} e_L \gamma^\mu e_L = \left( \begin{array}{c} \bar{\nu} e_L \\ \bar{e}_L \end{array} \right) \gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu e_L \\ e_L \end{pmatrix} = \frac{1}{\sqrt{2}} \bar{E}_L \gamma^\mu \tau^+ E_L \]

= \sqrt{2} \bar{E}_L \gamma^\mu T^+_L E_L
Doublets and Isospin:

This is the same maths are your spin/ 2-level system in QM

\[
\begin{pmatrix}
\uparrow \\
\downarrow
\end{pmatrix}
\]

QM of rotations:

\[O(3) \sim SU(2)\]
Doublets and Isospin:

Pauli matrices, fundamental representation
Pauli matrices, 3 generators

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau^\pm = \frac{1}{\sqrt{2}} \left( \tau_1 \pm i\tau_2 \right)$$

$$\left[ \frac{\tau_i}{2}, \frac{\tau_j}{2} \right] = i f_{ijk} \frac{\tau_k}{2} = i \epsilon_{ijk} \frac{\tau_k}{2}$$

$$f_{ijk}$$ structure constants

$$\sum_i (t^i t^i)_{ab} = C_F \delta_{ab} = \frac{N^2 - 1}{2N} \delta_{ab} = \frac{3}{4} \delta_{ab}$$

$$\sum_{ij} f_{ijk} f_{ijl} = C_A \delta_{kl} = N \delta_{kl} = 2 \delta_{kl}$$
Neutral weak current

where is $\tau_3$??
Neutral weak current

We are forced to consider the group $SU(2)_W$:

there is no group of continuous transformation that has only 2 generators.
Neutral weak current

We are forced to consider the group $SU(2)_W$:

there is no group of continuous transformation that has only 2 generators

\[
J^+ \mu = \bar{E}_L \gamma_\mu \frac{\tau^+}{\sqrt{2}} E_L \rightarrow
\]

\[
J^3 \mu = \bar{E}_L \gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2} \left( \bar{\nu}_e L \gamma_\mu \nu_e L - \bar{e}_L \gamma_\mu e_L \right)
\]
Neutral weak current

We are forced to consider the group $SU(2)_W$:

there is no group of continuous transformation that has only 2 generators

\[ J^+_\mu = \bar{E}_L \gamma_\mu \frac{\tau^+}{\sqrt{2}} E_L \rightarrow \]

\[ J^3_\mu = \bar{E}_L \gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2} \left( \bar{\nu}_e \gamma_\mu \nu_e - \bar{e} \gamma_\mu e \right) \]

This is a neutral current but it is not the em current, this would have been too good

- neutrinos have no em charge
- no $e_R$, P violation
- strength G! This current is much weaker at lower energies than the em current and is therefore very difficult to detect at those earlier energies
Neutral weak current

We are forced to consider the group $SU(2)_W$:

there is no group of continuous transformation that has only 2 generators

$$J^+_{\mu} = \bar{E}_L \gamma_\mu \frac{\tau^+}{\sqrt{2}} E_L \rightarrow$$

$$J^3_{\mu} = \bar{E}_L \gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2} \left( \bar{\nu}_e L \gamma_\mu \nu_e L - \bar{e}_L \gamma_\mu e_L \right)$$

This is a neutral current but it is not the em current, this would have been too good

- neutrinos have no em charge
- no $e_R$, P violation
- strength G! This current is much weaker at lower energies than the em current and is therefore very difficult to detect at those earlier energies

However part of $J^{em}_{\mu}$ is contained in $J^3_{\mu}$
The rest of the current and the hypercharge current

\[ J^3_\mu = \bar{E}_L \gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2} \left( \bar{\nu}_L \gamma_\mu \nu_L - \bar{e}_L \gamma_\mu e_L \right) \]

\[ J^{em}_\mu(Q) = Q \left( \bar{e}_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R \right) = - \left( \bar{e}_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R \right) \]

\[ = - \left( \bar{e}_R \gamma_\mu e_R \right) + J^3_\mu - \frac{1}{2} \left( \bar{\nu}_L \gamma_\mu \nu_L + \bar{e}_L \gamma_\mu e_L \right) \]

\[ = J^3_\mu - \frac{1}{2} \bar{E}_L \gamma_\mu \cancel{E}_L - \left( \bar{e}_R \gamma_\mu e_R \right) = J^3_\mu + Y_\mu \]
The rest of the current and the hypercharge current

$$J_{\mu}^3 = \bar{E}_L \gamma_{\mu} \frac{\tau^3}{2} E_L = \frac{1}{2} \left( \bar{\nu}_e L \gamma_{\mu} \nu_e L - \bar{e}_L \gamma_{\mu} e_L \right)$$

$$J_{\mu}^{em}(Q) = Q \left( \bar{e}_L \gamma_{\mu} e_L + \bar{e}_R \gamma_{\mu} e_R \right) = - \left( \bar{e}_L \gamma_{\mu} e_L + \bar{e}_R \gamma_{\mu} e_R \right)$$

$$= - \left( \bar{e}_R \gamma_{\mu} e_R \right) + J_{\mu}^3 - \frac{1}{2} \left( \bar{\nu}_e L \gamma_{\mu} \nu_e L + \bar{e}_L \gamma_{\mu} e_L \right)$$

$$= J_{\mu}^3 - \frac{1}{2} \bar{E}_L \gamma_{\mu} \tau^3 E_L - \left( \bar{e}_R \gamma_{\mu} e_R \right) = J_{\mu}^3 + Y_{\mu}$$

$Y_{\mu}$ is the hypercharge current
The rest of the current and the hypercharge current

\[
J^3_\mu = \bar{E}_L \gamma_\mu \frac{\tau^3}{2} E_L = \frac{1}{2} (\bar{\nu}_e L \gamma_\mu \nu_e L - \bar{e}_L \gamma_\mu e_L)
\]

\[
J^\text{em}_\mu (Q) = Q \left( e_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R \right) = - \left( e_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R \right)
\]

\[
= \left( e_R \gamma_\mu e_R \right) + J^3_\mu - \frac{1}{2} \left( \bar{\nu}_e L \gamma_\mu \nu_e L + \bar{e}_L \gamma_\mu e_L \right)
\]

\[
= J^3_\mu - \frac{1}{2} \bar{E}_L \gamma_\mu E_L - \left( e_R \gamma_\mu e_R \right) = J^3_\mu + Y_\mu
\]

\[
Y_\mu = \gamma_{e_R} (\bar{e}_R \gamma_\mu e_R) + \gamma_{E_L} (\bar{E}_L \gamma_\mu E_L)
\]
The rest of the current and the hypercharge current

\[
J^3_{\mu} = \bar{E}_L \gamma_{\mu} \frac{\tau^3}{2} E_L = \frac{1}{2} \left( \bar{\nu}_L \gamma_{\mu} \nu_L - \bar{e}_L \gamma_{\mu} e_L \right)
\]

\[
J^e_{\mu} (Q) = Q \left( \bar{e}_L \gamma_{\mu} e_L + \bar{e}_R \gamma_{\mu} e_R \right) = - \left( \bar{e}_L \gamma_{\mu} e_L + \bar{e}_R \gamma_{\mu} e_R \right)
\]

\[
= - \left( \bar{e}_R \gamma_{\mu} e_R \right) + J^3_{\mu} - \frac{1}{2} \left( \bar{\nu}_L \gamma_{\mu} \nu_L + \bar{e}_L \gamma_{\mu} e_L \right)
\]

\[
= J^3_{\mu} - \frac{1}{2} \bar{E}_L \gamma_{\mu} \gamma_1 E_L - \left( \bar{e}_R \gamma_{\mu} e_R \right) = J^3_{\mu} + Y_{\mu}
\]

\[
Y_{\mu} = y_{e_R} (\bar{e}_R \gamma_{\mu} e_R) + y_{E_L} (\bar{E}_L \gamma_{\mu} \gamma_1 E_L)
\]

This is indeed a $U(1)$ current

$E_L$ is a doublet and there is a separate entity $e_R$ which is a singlet (under $SU(2)$) each entity has its own hypercharge

\[
y_{e_R} = -1 \\
y_{e_L} = -1/2
\]

\[
Q = T_3 + \frac{Y}{2} = \frac{\tau^3}{2} + y
\]

\[
Q_{e_R} = 0 - 1 = -1 \\
Q_{e_L} = -\frac{1}{2} - \frac{1}{2} = -1
\]

\[
Q_{\nu_L} = +\frac{1}{2} - \frac{1}{2} = -1
\]
For the effective QED operator and with \( \alpha = \frac{e^2}{4\pi} \)

\[
\mathcal{L}_{\text{eff.}} = -\frac{e^2}{2} \frac{J_{\mu}^{em}(e^+, e^-) J_{\mu}^{em}(\mu^+, \mu^-)}{k^2}
\]

The cross section \( e^+e^- \rightarrow \mu^+\mu^- \) behaves as

\[
\sigma \propto \alpha^2 / s
\]

decreases as the energy decreases. Less probability, \( \mathcal{P} \), to produce muons.
Unitarity

For the effective QED operator and with \( \alpha = e^2/4\pi \)

\[
\mathcal{L}_{\text{eff.}} = -\frac{e^2}{2} \frac{J^e_{\mu} (e^+, e^-) J^\mu_{\mu} (\mu^+, \mu^-)}{k^2}
\]

The cross section \( e^+ e^- \rightarrow \mu^+ \mu^- \) behaves as

\[
\sigma \propto \alpha^2 / s
\]

decreases as the energy decreases. Less probability, \( \mathcal{P} \), to produce muons.

For the Fermi interaction

\[
\mathcal{L}_{\text{Fermi}} = -\frac{4G_F}{\sqrt{2}} J^+_{\mu} (e\nu_e) J^{\mu} (\mu, \nu_\mu)
\]

\[
\frac{G_F}{\sqrt{2}} = 1.03510^{-5} M^{-2}_P
\]

The cross section \( e^- \bar{\nu}_e \rightarrow \mu^- \bar{\nu}_\mu \) behaves as

\[
\sigma \propto G^2_F \times s
\]

The probability \( \mathcal{P} \) increases indefinitely.

But \( \mathcal{P} < 1 \), unitarity must be preserved.

This means something must happen at some energy to restore \( \mathcal{P} < 1 \)
or theory not good!
Unitarity

Fermi Contact interaction
Unitarity

Fermi Contact interaction

Where is the underlying fundamental interaction?
Gauging, the vector bosons

To each current associate a vector particle, spin-1, a gauge particle

QED recap

\[ J_{\mu}^{em} \rightarrow e J_\mu (Q) A_\mu \]

\[ \mathcal{L} = -e J_\mu (Q) A_\mu \]

Turn the derivatives of the free Lagrangian into covariant derivatives to get the interaction with the gauge field

\[ \mathcal{L}_{int, QED} = i \bar{\psi} e \gamma^\mu \left( \partial_\mu + i e Q A_\mu \right) \psi_e \]

Universality

\[ e \leftrightarrow A_\mu \]
To each current associate a vector particle, spin-1, a gauge particle

\[ J^i_{\mu} = \pm,3 \quad \rightarrow \quad W^i_{\mu} = \pm,3 \quad \rightarrow \quad g \]

\[ Y_{\mu} \quad \rightarrow \quad B_{\mu} \quad \rightarrow \quad g' \]

\[ g \neq g' \text{ (partial unification)} \]
Gauging, the vector bosons

To each current associate a vector particle, spin-1, a gauge particle

\[
\mathcal{L} = i\bar{E}_L \gamma_\mu \left( \partial_\mu \mathbb{1} + ig \left( \frac{\tau^i}{2} \right) W^i_\mu + ig' (y_{E_L}) B_\mu \right) E_L \\
+ i\bar{e}_R \gamma_\mu \left( \partial_\mu + ig' (y_{e_R}) B_\mu \right) e_R
\]
Gauging, the vector bosons

To each current associate a vector particle, spin-1, a gauge particle

\[ \mathcal{L} = i\bar{E}_L \gamma_\mu \left( \partial_\mu \mathbb{1} + ig \left( \frac{\tau^i}{2} \right) W^i_\mu + ig' (y_{EL}) B_\mu \right) E_L \]

\[ + \quad i\bar{e}_R \gamma_\mu \left( \partial_\mu + ig' (y_{eR}) B_\mu \right) e_R \]

\[ \tau^i W^i = \tau^3 W^3 + \tau^1 W^1 + \tau^2 W^2 = \tau^3 W^3 + W^+ + W^- \]

\[ W^\pm_\mu = \frac{W^1_\mu \pm iW^2_\mu}{\sqrt{2}} \]
Gauging, the vector bosons

To each current associate a vector particle, spin-1, a gauge particle

\[ \mathcal{L} = i \bar{E}_L \gamma_\mu \left( \partial_\mu \mathbb{I} + ig \left( \frac{\tau^i}{2} \right) W^i_\mu \right) + ig' (y_{E_L}) \mathbb{I} B_\mu \right) E_L + i \bar{e}_R \gamma_\mu \left( \partial_\mu + ig' (y_{e_R}) B_\mu \right) e_R \]

\[ \tau^i W^i = \tau^3 W^3 + \tau^1 W^1 + \tau^2 W^2 = \tau^3 W^3 + \tau^+ W^+ + \tau^- W^- \]

\[ W^\pm_\mu = \frac{W^1_\mu \pm i W^2_\mu}{\sqrt{2}} \]

\[ \mathcal{L}_{cc} = \frac{-g}{\sqrt{2}} \left( J^+_\mu W^+ + J^-_\mu W^- \right) \]

\( W^\pm \) have electric charge, they should couple to the photon
The weak mixing angle (unorthodox way)

Counting the gauge fields one has $4/3$ of the triplet $W$ and $B$. The photon must emerge as the physical field $A_\mu$ of $W^3 - B$. The other orthogonal physical field is the $Z_\mu$ boson. Two requirements, i) $A_\mu$ couples to the em current ii) with strength $e$.

\[ \mathcal{L}_{NC} = - \left( g J_\mu^3 W^3 + g' Y_\mu B^\mu \right) = -eJ_\mu (Q) A^\mu - g Z J_\mu^Z Z^\mu \]

\[
\begin{pmatrix}
  W_\mu^3 \\
  B_\mu
\end{pmatrix}
= 
\begin{pmatrix}
  \cos \theta_W & \sin \theta_W \\
  -\sin \theta_W & \cos \theta_W
\end{pmatrix}
\begin{pmatrix}
  Z_\mu \\
  A_\mu
\end{pmatrix}.
\]

using $J_\mu^Q = J_\mu^3 + Y_\mu$

\[ g \sin \theta_W = g' \cos \theta_W = e \]

\[ J_\mu^Z = J_\mu^3 - \sin^2 \theta_W J_\mu^Q \quad ; \quad g Z = \frac{g}{\cos \theta_W} = \frac{g}{\cos \theta_W \sin \theta_W} \]
Feynman Rules, coupling $g, s_W$ unspecified as yet

\[ \gamma \quad f \quad f \quad f \quad f \quad l^- \quad v_1 \]

\[ e \quad e Q_f \quad f \quad f \quad f \quad f \quad v_1 \]

\[ \frac{e}{2 s_\theta c_\theta} (v_f - a_f \gamma_5) \quad \frac{g}{2^{3/2}} (1 - \gamma_5) \]
Cross sections

\[ e^- + f^+ \rightarrow \gamma, Z \]

\[ f^- + \gamma, Z \]

\[ e^- + f^- \rightarrow \mu^-, \mu^+ \]

\[ e^+ + \mu^+ \rightarrow \gamma, Z \]

\[ e^- + e^+ \rightarrow f^, f^- \]

\[ \theta \]

\[ e^- + e^+ \rightarrow \nu, \bar{\nu} \]
Weak Interactions

It was also found that

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ leptonic decay

- muon capture: $\mu^- + p \rightarrow n + \nu_\mu$

were of the same nature and have the same strength.
It was also found that

- $\beta$-decay: $n \to p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ leptonic decay

- muon capture: $\mu^- + p \to n + \nu_\mu$

were of the same nature and have the same strength.

Universality, Gauge Interaction?
Weak Interactions

It was also found that

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ leptonic decay

- muon capture: $\mu^- + p \rightarrow n + \nu_\mu$

were of the same nature and have the same strength.

Universality, Gauge Interaction?

But important differences with QED: they involve

- a change in the identity of the fermion

- only left-handed field/component were found to interact.
Weak Interactions

It was also found that

- $\beta$-decay: \( n \rightarrow p \) + \( e^- + \bar{\nu}_e \) semi-leptonic decay,

- muon decay: \( \mu^- \rightarrow \nu_\mu \) + \( e^- + \bar{\nu}_e \) leptonic decay

were of the same nature and have the same strength.
It was also found that

- $\beta$-decay: \( n \rightarrow p \) + \( e^- + \bar{\nu}_e \) semi-leptonic decay,

-_muon decay: \( \mu^- \rightarrow \nu_\mu \) + \( e^- + \bar{\nu}_e \) leptonic decay

Old writing of the (isospin) doublet \( \begin{pmatrix} p \\ n \end{pmatrix} \)
Weak Interactions

It was also found that

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ leptonic decay

Old writing of the (isospin) doublet

\[ \begin{pmatrix} p \\ n \end{pmatrix} \]

Modern writing of the (isospin) doublet

\[ \begin{pmatrix} p = (uud) \\ n = (udd) \end{pmatrix} \]
Weak Interactions

It was also found that

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ leptonic decay

Old writing of the (isospin) doublet

\[
\begin{pmatrix}
  p \\
  n
\end{pmatrix}
\]

Modern writing of the (isospin) doublet

\[
\begin{pmatrix}
  p = (uud) \\
  n = (udd)
\end{pmatrix} \xrightarrow{EW} \begin{pmatrix}
  u \\
  d
\end{pmatrix}
\]
Weak Interactions

It was also found that

- $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ leptonic decay

Old writing of the (isospin) doublet $\begin{pmatrix} p \\ n \end{pmatrix}$

Modern writing of the (isospin) doublet $\begin{pmatrix} p = (uud) \\ n = (udd) \end{pmatrix} \xrightarrow{EW} \begin{pmatrix} u \\ d \end{pmatrix}$

$Q_u = 2/3 \quad Q_d = -1/3 \quad \begin{pmatrix} u \\ d \end{pmatrix}_L$ and singlets $u_R; d_R$ (both $R$ have hypercharge)
Feynman Rules, coupling $g$, $s_W$ unspecified as yet

\[ \mathcal{L}_{NC}^Z = -\frac{e}{2 \sin \theta_W \cos \theta_W} Z^\mu \sum_f \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f, \quad a_f = T_3^f \quad v_f = T_3^f (1 - 4|Q_f| \sin^2 \theta_W) \]

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$d$</th>
<th>$\nu_e$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2v_f$</td>
<td>$1 - \frac{8}{3} \sin^2 \theta_W$</td>
<td>$-1 + \frac{4}{3} \sin^2 \theta_W$</td>
<td>$1$</td>
<td>$-1 + 4 \sin^2 \theta_W$</td>
</tr>
<tr>
<td>$2a_f$</td>
<td>$1$</td>
<td>$-1$</td>
<td>$1$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>
Inclusion of hadrons

When the precision got better (1960) it was in fact found that the Fermi constant in $\beta$ decay was **3% smaller** than that measured in muon decay!

- $\beta$-decay: $d \rightarrow u + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ leptonic decay
Inclusion of hadrons

When the precision got better (1960) it was in fact found that the Fermi constant in $\beta$ decay was \textit{3\% smaller} than that measured in muon decay!

- $\beta$-decay: $d \rightarrow u + e^- + \bar{\nu}_e$ semi-leptonic decay,

- muon decay: $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ leptonic decay

Moreover at that time we knew of strange particles, $\Lambda \equiv (uds)$ for example. $\Lambda$ seemed to decay like the neutron but the associated effective coupling was measured much smaller: strangeness suppression!

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e \equiv s \rightarrow u + e^- + \bar{\nu}_e$$
Inclusion of hadrons

When the precision got better (1960) it was in fact found that the Fermi constant in $\beta$ decay was 3% smaller than that measured in muon decay!

$\beta$-decay: $d \rightarrow u + e^- + \bar{\nu}_e$ semi-leptonic decay,

muon decay: $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ leptonic decay

Moreover at that time we knew of strange particles, $\Lambda \equiv (uds)$ for example. $\Lambda$ seemed to decay like the neutron but the associated effective coupling was measured much smaller: strangeness suppression!

$\Lambda \rightarrow p + e^- + \bar{\nu}_e \equiv s \rightarrow u + e^- + \bar{\nu}_e$

breakdown of universality? gauge principle ??
Inclusion of hadrons

When the precision got better (1960) it was in fact found that the Fermi constant in $\beta$ decay was 3\% smaller than that measured in muon decay!

$\beta$-decay: $d \to u + e^- + \bar{\nu}_e$ semi-leptonic decay,

muon decay: $\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$ leptonic decay

Moreover at that time we knew of strange particles, $\Lambda \equiv (uds)$ for example. $\Lambda$ seemed to decay like the neutron but the associated effective coupling was measured much smaller: strangeness suppression!

$\Lambda \to p + e^- + \bar{\nu}_e \equiv s \to u + e^- + \bar{\nu}_e$

breakdown of universality? gauge principle ???

Universal coupling but apparent non universality due to mixing, cf $Z, W$
Inclusion of hadrons

\[ J_\mu^- = \cos \theta_c \bar{u} \gamma_\mu P_L d + \sin \theta_c \bar{u} \gamma_\mu P_L s \]

\[ \Delta s \neq 0 \]

\[ = \bar{u} \gamma_\mu P_L \left( \cos \theta_c d + \sin \theta_c s \right) \]
Inclusion of hadrons

\[
J_{\mu}^- = \cos \theta_c \bar{u}\gamma_\mu P_L d + \sin \theta_c \bar{u}\gamma_\mu P_L s
\]

\[
= \bar{u}\gamma_\mu P_L \left( \cos \theta_c d + \sin \theta_c s \right)
\]

\[
\Delta s \neq 0
\]

\[
\cos \theta_c = V_{ud} = 0.97
\]

\[
\sin \theta_c = V_{us} = 0.24
\]
Inclusion of hadrons

\[ J_\mu^- = \left( \cos \theta_c \bar{u}\gamma_\mu P_L d + \sin \theta_c \bar{u}\gamma_\mu P_L s \right) \]

\[ = \bar{u}\gamma_\mu P_L \begin{pmatrix} \cos \theta_c d + \sin \theta_c s \end{pmatrix} \]

with \( d' \) universality reinstated

use weak current eigenstate

\[ U_L = \begin{pmatrix} u \\ d' \end{pmatrix}_L \]

what to do with the orthogonal state to \( d' \)?

\[ s'_L = -\sin \theta_c d + \cos \theta_c s \]
Flavour changing neutral currents

\[ J_{\mu}^\pm = \bar{U}_L \gamma_\mu \tau^+ U_L \]

\[ \Downarrow \]

\[ J_\mu^3 = \frac{1}{2} \left( \bar{u}_L \gamma_\mu u_L - \bar{d}'_L \gamma_\mu d'_L \right) \]

\[ = \frac{1}{2} \left( \bar{u}_L \gamma_\mu u_L - \cos^2 \theta_c \bar{d}_L \gamma_\mu d_L - \sin^2 \theta_c \bar{s}_L \gamma_\mu s_L - \sin \theta_c \cos \theta_c \left( \bar{d}_L \gamma_\mu s_L \bar{s}_L \gamma_\mu d_L \right) \right) \]

\[ \Delta S = 1, \text{ FCNC} \]

of course one requires the em current to be diagonal

\[ J_\mu (Q) = \frac{2}{3} \bar{u}_\gamma_\mu u - \frac{1}{3} \left( \bar{d}_\gamma_\mu d + \bar{s}_\gamma_\mu s \right) \]

\[ = J_\mu^3 + Y_\mu \]

ex. find all quantum numbers of \( s \) including ...\( s_R \)
\( \Delta S = 1 \) lead to FCNC \( K_0(d\bar{s}) \rightarrow \mu^+\mu^- \) occurs at tree-level and leads to a large rate for this decay! But experimentally

\[
B_{K\mu} = \frac{\Gamma(K^0 \rightarrow \mu^+\mu^-)}{K^+ \rightarrow \mu^+\nu_\mu} \sim 10^{-8}
\]
$\Delta S = 1$ lead to FCNC $K_0(d\bar{s}) \rightarrow \mu^+\mu^-$ occurs at tree-level and leads to a large rate for this decay! But experimentally

$$B_{K\mu} = \frac{\Gamma(K^0 \rightarrow \mu^+\mu^-)}{K^+ \rightarrow \mu^+\nu\mu} \sim 10^{-8}$$

The $sdZ$ coupling must be eliminated!
\[ \Delta S = 1 \] lead to FCNC \( K_0(d\bar{s}) \rightarrow \mu^+\mu^- \) occurs at tree-level and leads to a large rate for this decay! But experimentally

\[
B_{K\mu} = \frac{\Gamma(K^0 \rightarrow \mu^+\mu^-)}{K^+ \rightarrow \mu^+\nu_\mu} \sim 10^{-8}
\]

GIM=Glashow Illiopoulos Maiani postulate that a cousin of the \( u \) exists, the \( c \) quark that should form a doublet with \( s'_L \)

\[
C_L = \begin{pmatrix} u \\ s'_L \end{pmatrix}
\]

with this new entry \( J_\mu \) is diagonal and no FCNC occur!
\( \Delta S = 1 \) lead to FCNC \( K_0(d\bar{s}) \rightarrow \mu^+\mu^- \) occurs at tree-level and leads to a large rate for this decay! But experimentally

\[
B_{K\mu} = \frac{\Gamma(K^0 \rightarrow \mu^+\mu^-)}{K^+ \rightarrow \mu^+\nu_\mu} \sim 10^{-8}
\]

GIM=Glashow Illiopoulos Maiani postulate that a cousin of the \( u \) exists, the \( c \) quark that should form a doublet with \( s'_L \)

\[
C_L = \begin{pmatrix} u \\ s' \end{pmatrix}_L
\]

with this new entry \( J^3_\mu \) is diagonal and no FCNC occur!

\[
J^+_\mu = \begin{pmatrix} u, c \end{pmatrix}_L \gamma_\mu \tau^+ V_{\text{Cabbibo}} \begin{pmatrix} d \\ s \end{pmatrix} \text{family space}
\]
GIM mechanism

$\Delta S = 1$ lead to FCNC $K_0(d\bar{s}) \rightarrow \mu^+\mu^-$ occurs at tree-level and leads to a large rate for this decay! But experimentally

$$B_{K\mu} = \frac{\Gamma(K^0 \rightarrow \mu^+\mu^-)}{K^+ \rightarrow \mu^+\nu_\mu} \sim 10^{-8}$$

GIM=Glashow Illiopoulos Maiani postulate that a cousin of the $u$ exists, the $c$ quark that should form a doublet with $s_L'$

$$C_L = \begin{pmatrix} u \\ s' \end{pmatrix}_L$$

with this new entry $J^3_{\mu}$ is diagonal and no FCNC occur!

$$J^+_{\mu} = \overbrace{\begin{pmatrix} u, c \end{pmatrix}_L}^{\text{family space}} \gamma_\mu \tau^+ V_{\text{Cabbibo}} \begin{pmatrix} d \\ s \end{pmatrix} \} \text{family space}$$
With a bonus!

\[
W^- \rightarrow W^+ \nu \mu^-
\]

\[
K_0 \rightarrow W^- W^+ \nu \mu^-
\]

\[
K_0 \rightarrow W^- W^+ \nu \mu^-
\]

if \( m_u = m_c \) total cancellation, GIM suppression even at one-loop.

if \( m_c \gg m_u \) rate still too large. J. Ellis and M.K. Gaillard, predicted \( m_c \) in the range 1 – 3GeV to account for the experimental rate.

Loop calculations and masses!
Generalisation, third family

\[ J^+ = \left( \begin{array}{c} u \\ c \\ t \end{array} \right) L \gamma_\mu \tau^+ V_{CKM} \left( \begin{array}{c} d \\ s \\ b \end{array} \right) \] family space

gauge eigenstates

\[ V_{ij} \]

\[ d_j \rightarrow u_i \]

\[ W \]

\[ u \rightarrow c \rightarrow t \]

\[ d \rightarrow s \rightarrow b \]
Gauge invariance, non-Abelian theory

\[ G = SU(2)_L \otimes U(1)_Y \] theory

\[ \psi_1(x) = \begin{pmatrix} u \\ d \end{pmatrix}_L \], \quad \psi_2(x) = u_R, \quad \psi_3(x) = d_R. \]

\[ \psi_1(x) = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \], \quad \psi_2(x) = \nu_{eR}, \quad \psi_3(x) = e_{R}. \]

As in the QED start from the free Lagrangian (no masses)

\[ \mathcal{L}_0 = \sum_{j=1}^{3} i \overline{\psi}_j(x) \gamma^\mu \partial_\mu \psi_j(x) \]

\( \mathcal{L}_0 \) is invariant under global \( G \) transformations in flavour space:

\[ \psi_1(x) \xrightarrow{G} \psi'_1(x) \equiv \exp \{ iy_1 \beta \} \ U_L \ \psi_1(x), \quad U_L \equiv \exp \left\{ i \frac{\tau_i}{2} \ \alpha^i \right\} \ (i = 1, 2, 3) \]

\[ \psi_2(x) \xrightarrow{G} \psi'_2(x) \equiv \exp \{ iy_2 \beta \} \ \psi_2(x) \]

\[ \psi_3(x) \xrightarrow{G} \psi'_3(x) \equiv \exp \{ iy_3 \beta \} \ \psi_3(x), \]
Requiring local gauge transformation, $\alpha^i = \alpha^i(x)$ and $\beta = \beta(x)$ we must make $\partial_\mu \rightarrow D_\mu$

$$D_\mu \psi_1(x) \equiv \left[ \partial_\mu - i g \tilde{W}_\mu(x) - i g' y_1 B_\mu(x) \right] \psi_1(x), \quad \tilde{W}_\mu(x) \equiv \frac{\tau_i}{2} W^i_\mu(x)$$

$$D_\mu \psi_2(x) \equiv \left[ \partial_\mu - i g' y_2 B_\mu(x) \right] \psi_2(x),$$

$$D_\mu \psi_3(x) \equiv \left[ \partial_\mu - i g' y_3 B_\mu(x) \right] \psi_3(x),$$

$D_\mu \psi_j(x)$ transforms (covariantly) like $\psi_j(x) \Rightarrow$

$$B_\mu(x) \rightarrow G \quad B'_\mu(x) \equiv B_\mu(x) + \frac{1}{g'} \partial_\mu \beta(x),$$

$$\tilde{W}_\mu \rightarrow G \quad \tilde{W}'_\mu \equiv U_L(x) \tilde{W}_\mu U_L^\dagger(x) - \frac{i}{g} U_L(x) \partial_\mu U_L^\dagger(x).$$

$$W^i_\mu \rightarrow G \quad W^i'_\mu \equiv W^i_\mu - \frac{1}{g} \alpha^i - (\vec{\alpha} \times \vec{W})^i \quad \text{(infinitesimal)}$$

This fixes the gauge-matter interaction

$$\mathcal{L} = \sum_{j=1}^{3} i \bar{\psi}_j(x) \gamma^\mu D_\mu \psi_j(x)$$
for the $U(1)_Y$ field do the same as with the electromagnetic field. The field strength

$$B_{\mu \nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \quad B_{\mu \nu} \overset{G}{\rightarrow} B_{\mu \nu},$$

$$\mathcal{L}_{\text{Kin,B}} = -\frac{1}{4} B_{\mu \nu} B^{\mu \nu}$$
• for the $U(1)_Y$ field do the same as with the electromagnetic field. The field strength

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \quad B_{\mu\nu} \xrightarrow{G} B_{\mu\nu},$$

$$\mathcal{L}_{\text{Kin},B} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

We can not just use $W^{i}_{\mu\nu} = \partial_\mu W^{i}_{\nu} - \partial_\nu W^{i}_{\mu}$ Note the right gauge transformation. $W^{i}$ is isospin charged and the normal derivative should be turned into a covariant derivative

$$W^{i}_{\mu\nu} = \partial_\mu W^{i}_{\nu} - \partial_\nu W^{i}_{\mu} - g \epsilon^{ijk} W^{j}_{\mu} W^{k}_{\nu}$$

$$\mathcal{L}_{\text{Kin}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr} \left[ \widetilde{W}_{\mu\nu} \widetilde{W}^{\mu\nu} \right] = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^{i}_{\mu\nu} W^{i}_{\mu\nu}.$$
Trilinear and Quartic Gauge Boson Couplings
Gauge Invariance: \( g_{ffV} = g_{VVV} \)

\[
\sigma(e^+e^- \rightarrow W^+W^-) = \sigma(e^+e^- \rightarrow W^+W^-) \quad (\gamma + Z) \text{ s-channel exchange only}
\]

\[
\nu_e t\text{-channel exchange only}
\]

\[
\nu_e \text{ exchange}
\]

\[
\text{no ZWW vertex}
\]

\[
\text{only } \nu_e\text{ exchange}
\]

- **LEP legacy**: We know that \(WWV\) cannot deviate too much (10%) from SM gauge value.
- **But slightest deviations are revealed at higher energies (LHC?)**
Origin of VVVV and VVV: Gauge Invariance

\[ \mathcal{L}_{\text{Gauge}} = -\frac{1}{2} \left[ \text{Tr}(W_{\mu \nu} W^{\mu \nu}) + \text{Tr}(B_{\mu \nu} B^{\mu \nu}) \right] \] GI kinetic term

\[ W_{\mu \nu} = \frac{1}{2} \left( \partial_\mu W_\nu - \partial_\nu W_\mu + \frac{i}{2} g [W_\mu, W_\nu] \right) = \frac{\tau^i}{2} \left( \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g \epsilon^{ijk} W^j_\mu W^k_\nu \right) \]

\[ B_{\mu \nu} = \frac{1}{2} (\partial_\mu B_\nu - \partial_\nu B_\mu) \quad B_\mu = B_\mu \]

\[ \mathcal{L}_{WWV} = -ie \left\{ A_\mu \left( W^{\mu \nu} W^{\nu}_\nu - W_{\mu \nu} W^{\nu}_\nu \right) + F_{\mu \nu} W^{\mu \nu} - F_{\mu \nu} \right\} \]

\[ + \frac{c_W}{s_W} \left[ Z_\mu \left( W^{\mu \nu} W^{\nu}_\nu - W_{\mu \nu} W^{\nu}_\nu \right) + Z_{\mu \nu} W^{\mu \nu} - Z_{\mu \nu} \right] \} \]

No \( \text{ZZZ}, \text{ZZ}_{\gamma}, \text{Z}_{\gamma\gamma} \)
tri-linear couplings

\[ \mathcal{L}_{W W V} = -i e \left\{ A_\mu \left( W^{-\mu \nu} W^\nu_\nu - W^{+\mu \nu} W^-_\nu \right) + \left( 1 + \Delta \kappa_\gamma \right) F_{\mu \nu} W^{+\mu} W^{-\nu} \right\} \]
\[ + \frac{c_W}{s_W} \left( 1 + \Delta g_1^Z \right) Z_\mu \left( W^{-\mu \nu} W^\nu_\nu - W^{+\mu \nu} W^-_\nu \right) + \left( 1 + \Delta \kappa_Z \right) Z_{\mu \nu} W^{+\mu} W^{-\nu} \]
\[ + \frac{1}{M_W^2} \left( \lambda_\gamma F^{\nu \lambda} + \lambda_Z \frac{c_W}{s_W} Z^{\nu \lambda} \right) W^+_\lambda W^{-\mu}_\nu \} \]

No $ZZZ, ZZ\gamma, Z\gamma\gamma$. 
\[
\mathcal{L}_{WWV_1V_2} = \\
- e^2 \left\{ (A_\mu A_\nu W_\nu^+ W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) \right. \\
+ 2 \frac{c_W}{s_W} \left( A_\mu Z_\mu W_\nu^+ W_\nu^- - \frac{1}{2} A_\mu Z_\nu (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) \right) \\
+ \frac{c_W^2}{s_W^2} \left( Z_\mu Z_\nu W_\mu^+ W_\nu^- - Z_\mu Z_\nu W_\mu^+ W_\nu^- \right) \\
+ \frac{1}{2 s_W^2} \left( W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- - W_\mu^+ W_\mu^+ W_\nu^- W_\nu^- \right) \right\}
\]
Aspects of Colour
QCD
Make sense of the hadron spectrum economically

The Particle Data Book lists a plethora, that might seem as a zoo, of particles in the hadron family, between the mesons and the baryons. But! everything fits neatly with much fewer fundamental constituents of spin-1/2

**Quarks that come in 6 flavours**

The whole spectrum of hadrons can be constructed out of

- **Mesons** $M = q\bar{q}$
- **Baryons** $B = qqq$

This picture faces a major problem

$$\Delta^{++}_{JZ=+3/2,J=3/2} = u^\uparrow u^\uparrow u^\uparrow$$

The wave function is symmetric for a fermion! and therefore does not obey spin-statistics.

For such a state to exist, the three $u$ can not be the same, they must carry (at least) different quantum numbers.

$$\Delta^{++} = u^\uparrow u^\uparrow u^\uparrow$$
Simplicity and minimality suggests number of colours is $N_c = 3$

then each quark is $q^\alpha, \alpha = 1, 2, 3 =$red, green blue.

Then $\Delta^{++} \sim \epsilon^{\alpha\beta\gamma} |u^\alpha u^\beta u^\gamma\rangle$.  

Baryons and mesons are described by colour singlet construct

$$ B = \frac{1}{\sqrt{6}} \epsilon^{\alpha\beta\gamma} |q_\alpha q_\beta q_\gamma\rangle, \quad M = \frac{1}{\sqrt{3}} \delta^{\alpha\beta} |q_\alpha \bar{q}_\beta\rangle $$

We observe no free quarks and moreover we do not see any combination of states/particles that carry colour: confinement
Colour, more evidence

\[ R_{e^+e^-} \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}. \]

Probability for quarks to hadronize is one and for \( S \ll M_Z^2 \)

\[ R_{e^+e^-} \approx N_C \sum_{f=1}^{N_f} Q_f^2 = \begin{cases} 
\frac{2}{3} N_C = 2, & (N_f = 3 : u, d, s) \\
\frac{10}{9} N_C = \frac{10}{3}, & (N_f = 4 : u, d, s, c) \\
\frac{11}{9} N_C = \frac{11}{3}, & (N_f = 5 : u, d, s, c, b) \end{cases} \]
Colour, more evidence

\[ R_{e^+e^-} \approx N_C \sum_{f=1}^{N_f} Q_f^2 = \begin{cases} 
\frac{2}{3} N_C = 2, & (N_f = 3 : u, d, s) \\
\frac{10}{9} N_C = \frac{10}{3}, & (N_f = 4 : u, d, s, c) \\
\frac{11}{9} N_C = \frac{11}{3}, & (N_f = 5 : u, d, s, c, b) 
\end{cases} \]
Colour, more evidence

\[ R_{e^+e^-} \approx N_C \sum_{f=1}^{N_f} Q_f^2 = \begin{cases} \frac{2}{3} N_C = 2, & (N_f = 3 : u, d, s) \\ \frac{10}{9} N_C = \frac{10}{3}, & (N_f = 4 : u, d, s, c) \\ \frac{11}{9} N_C = \frac{11}{3}, & (N_f = 5 : u, d, s, c, b) \end{cases} \]

\[ R_{had}^Z = N_C \frac{\sum_q v_q^2 + a_q^2}{v_\mu^2 + a_\mu^2} = 20.095. \]
Exact colour symmetry $\text{Symm}_3$

Colour is what binds hadrons together.

$N_C = 3$. Quarks belong to the triplet representation $\mathbf{3}$ of $\text{Symm}_3$.

Quarks and antiquarks are different states. Therefore, $\mathbf{3}^* \neq \mathbf{3}$.

Confinement hypothesis: hadronic states are colour singlets.

The interaction must be quite strong $\rho \rightarrow 2\pi \quad \tau \sim 10^{-22} \text{s}$ compared to $\mu \rightarrow e\bar{\nu}_e\nu_\mu \quad 10^{-6} \text{s}$

$$\text{Symm}_3 = SU(3)_C$$

\[
\begin{align*}
q\bar{q} & : \quad \mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}, \\
qqq & : \quad \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}, \\
qq & : \quad \mathbf{3} \otimes \mathbf{3} = \mathbf{3}^* \oplus \mathbf{6}, \\
qqqq & : \quad \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{6}^* \oplus \mathbf{6}^* \oplus \mathbf{15} \oplus \mathbf{15} \oplus \mathbf{15} \oplus \mathbf{15}',
\end{align*}
\]
Recap $SU(2)$ set up $SU(3)$, $SU(N)$

$SU(2)_{\text{weak isospin}}$

- Matter fields in doublets, $L_a$, a=1,2
- Generators of the group, $\tau^A$ Pauli matrices, $A = 1 \ldots (N^2 - 1) = 3$
- $N^2 - 1 = 3$ gauge bosons
- Invariance under $U = exp(i\tau^A \alpha^A)$
Recap $SU(2)$ set up $SU(3)$, $SU(N)$

$SU(2)_{\text{weak isospin}}$
- Matter fields in doublets, $L_a$, $a=1,2$
- Generators of the group, $\tau^A$ Pauli matrices, $A = 1... (N^2 - 1) = 3$
- $N^2 - 1 = 3$ gauge bosons
- Invariance under $U = exp(i\tau^A \alpha^A)$

$SU(3)_C$
- Matter fields, $q_a$, $a=1,2,3$
- Generators of the group, $t^A$, $A = 1... (N^2 - 1) = 8$
- $N^2 - 1 = 8$ gauge bosons
- $U = exp(i\tau^A \theta^A)$
Recap $SU(2)$ set up $SU(3), SU(N)$

$SU(2)_{\text{weak isospin}}$
- Matter fields in doublets, $L_a, a=1,2$
- Generators of the group, $\tau^A$ Pauli matrices, $A = 1...(N^2 - 1) = 3$
- $N^2 - 1 = 3$ gauge bosons
- Invariance under $U = e^{i\tau^A A^A}$

$SU(3)_C$
- Matter fields, $q_a, a=1,2,3$
- Generators of the group, $t^A, A = 1...(N^2 - 1) = 8$
- $N^2 - 1 = 8$ gauge bosons
- $U = e^{i\tau^A A^A}$

\[
\begin{align*}
T_R &= \frac{1}{2}, \\
C_F &= \frac{4}{3}, \\
C_A &= 3,
\end{align*}
\]
The fundamental representation $T^a = \lambda^a/2$ is $N$–dimensional. For $N = 2$, $\lambda^a$ are the usual Pauli matrices, while for $N = 3$, they correspond to the eight Gell-Mann matrices:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$
Fundamental representation 3:
\[ i \rightarrow j = \delta_{ij} \]

Adjoint representation 8:
\[ a \rightarrow b = \delta_{ab} \]

Trace identities:
\[ a \rightarrow b = 0 \]
\[ \text{Tr}(t^a) = 0 \]
\[ a \rightarrow b = T_R \]
\[ \text{Tr}(t^a t^b) = T_R \delta^{ab} \]
Fierz identity:

\[
(t^a)_k^i (t^a)_j^l = \frac{1}{2} \delta^i_j \delta^k_l - \frac{1}{2N_c} \delta^i_k \delta^j_l
\]

Fundamental representation 3:

\[
\sum_a (t^a_{ij})(t^a_{kj}) = C_F \delta_{ij} \quad C_F = \frac{N_c^2 - 1}{2N_c}
\]

Adjoint representation 8:

\[
\sum_{cd} f^{acd} f^{bde} = C_A \delta^{ab} \quad C_A = N_c
\]
QCD Lagrangian

From the free Lagrangian

\[ \mathcal{L}_{\text{quarks}} = \sum_{i}^{n_f} \bar{q}_i^a (i\slashed{D} - m_i)_{ab} q_i^b, \]

turn to the covariant derivative

\[ D_{\mu, ab} = \partial_{\mu} 1_{ab} + ig_s (t \cdot A_\mu)_{ab}, \]

\( A^a_\mu \) are coloured vector fields, gluons

\[ \mathcal{L}_{\text{kin}} = -\frac{1}{4} F^{A}_{\mu\nu} F_{\mu\nu}^A, \]

\[ F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A - g_s f^{ABC} A_B^\mu A_C^\nu, \]

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^A F_{\mu\nu}^A + \sum_i^n \bar{q}_i^a (i\slashed{D} - m_i)_{ab} q_i^b - \frac{1}{2\lambda} (\partial^\mu A_\mu^A)^2 + \mathcal{L}_{\text{ghost}}. \]
QCD Feynman rules

\[ q_\alpha \rightarrow G^a_\mu q_\beta \]

\[ g_s \frac{\lambda^{a}_{\alpha\beta}}{2} \gamma^\mu \]

\[ G^a_\mu, G^b_\nu, G^c_\sigma, G^d_\rho \]

\[ f_{abc}, f_{abc}f_{ade} \]
A gluon emission **repaints** the quark colour.
A gluon itself carries colour and anti-colour.
QCD Feynman rules

\[ -g_s f^{ABC} [(p - q)^\rho g_{\mu\nu} \]
\[ + (q - r)^\mu g_{\nu\rho} \]
\[ + (r - p)^\nu g^{\rho\mu} \]

A gluon emission also repaints the gluon colours.
Because a gluon carries colour + anti-colour, it emits \(~\) twice as strongly as a quark (just has colour)
Quantum numbers

<table>
<thead>
<tr>
<th></th>
<th>SU(3)</th>
<th>SU(2)$_L$</th>
<th>U(1)$_Y$</th>
<th>$Q = T_3 + Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q = (u_L, d_L)$</td>
<td>3</td>
<td>2</td>
<td>$\frac{1}{6}$</td>
<td>$(\frac{2}{3}, -\frac{1}{3})$</td>
</tr>
<tr>
<td>$u_R$</td>
<td>3</td>
<td>1</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>$d_R$</td>
<td>3</td>
<td>1</td>
<td>$-\frac{1}{3}$</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>$L = (\nu_L, e_L)$</td>
<td>1</td>
<td>2</td>
<td>$-\frac{1}{2}$</td>
<td>$(0, -1)$</td>
</tr>
<tr>
<td>$e_R$</td>
<td>1</td>
<td>1</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\nu_R$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
De quoi sommes nous faits

- Matter
- Atom
- Electron
- Nucleus
- Neutron
- Proton
- Quarks
De quoi sommes nous faits
De quoi sommes nous faits

interaction électromagnétique (QED)
interaction forte (QCD)
vers 1880, D’ou vient l’énergie du soleil?
Kelvin, Helmholtz, etc...: ”contraction gravitationnelle du nuage solaire...”

age de l’Univers (soleil): qq millions d’annees
Darwin (evolution, erosion,...): milliards d’annees pour la terre!
vers 1880, D’ou vient l’énergie du soleil?
Kelvin, Helmholtz, etc…: ”contraction gravitationnelle 
du nuage solaire…”
age de l’Univers (soleil): qq millions d’années
Darwin (evolution, erosion,...): milliards d’années pour 
la terre!

Interaction faible

desintegration

\[ n \rightarrow p e^- \nu_e \]
vers 1880, D’ou vient l’énergie du soleil?
Kelvin, Helmholtz, etc...: ”contraction gravitationnelle
du nuage solaire…”
age de l’Univers (soleil): qq millions d’annees
Darwin (evolution, erosion,...): milliards d’annees pour
la terre!

reaction nucleaire: fusion
bruler de l’hydrogene:
\[ 4^1H + 2e \rightarrow ^4He + 2\nu + 6\gamma + \text{Energie} \]
\[ (CH_4 + 2O_2 \rightarrow 2H_2O + \text{chaleur}) \]
La première famille

Spin-1/2

Quarks

u
d
ν_e

Leptons

Matière

Les Forces: Spin-1

γ Photon Lumière, QED

W±, Z Interaction faible

Théorie électrofaible

Gluons, g

Interaction forte
Le ciel encore...

Rayons Cosmiques

\( \mu: \) muon a part la masse, en tout point comme l’électron
accelérateurs pour produire ces nouvelles particules ou d’autres pour mieux les étudier
Spin-0

Higgs, $H$

Mass
Le Modèle Standard

<table>
<thead>
<tr>
<th>Spinning</th>
<th>Leptons</th>
<th>Quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin-1/2</td>
<td>( u )</td>
<td>( d )</td>
</tr>
<tr>
<td></td>
<td>( \nu_e )</td>
<td>( \nu_e )</td>
</tr>
<tr>
<td></td>
<td>( c )</td>
<td>( s )</td>
</tr>
<tr>
<td></td>
<td>( \nu_\mu )</td>
<td>( \nu_\mu )</td>
</tr>
<tr>
<td></td>
<td>( t )</td>
<td>( b )</td>
</tr>
<tr>
<td></td>
<td>( \nu_\tau )</td>
<td>( \nu_\tau )</td>
</tr>
</tbody>
</table>

- Matière
- Génération 1
- Génération 2
- Génération 3

Masse

- Interaction
- \( \gamma \) : Photons
- \( W^\pm, Z \) : Bosons
- Théorie électrofaible
- Gluons

- Spin-1
- Masse

- Spin-0
- Higgs, \( H \), ??