Semilocal Gibbs ensembles

A scent of nonequilibrium symmetry-protected topological order

Lenart Zadnik

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Nonequilibrium symmetry-protected topological order: emergence of semilocal Gibbs ensembles

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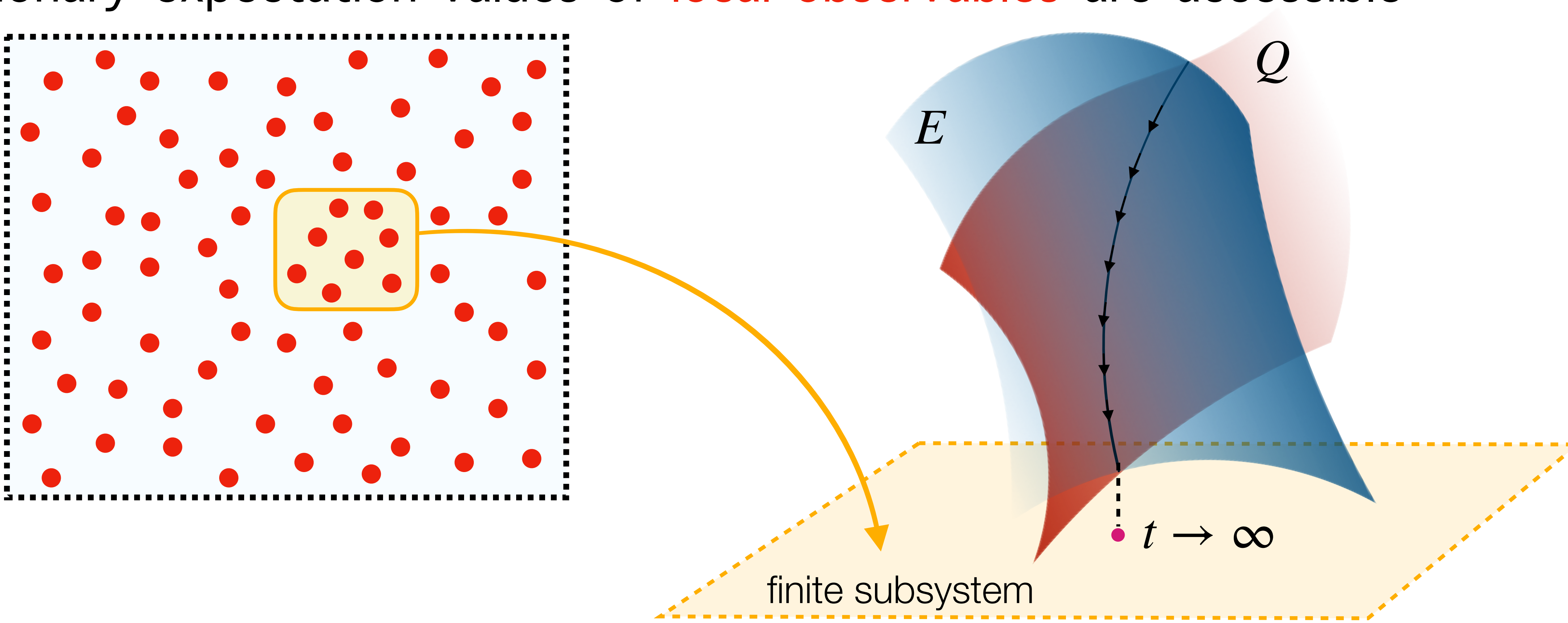
We consider nonequilibrium time evolution in quantum spin chains after a global quench. We show that global symmetries can invalidate the standard picture of local relaxation to a (generalised) Gibbs ensemble and provide a solution to the problem. We introduce in particular a family of statistical ensembles, which we dub "semilocal (generalised) Gibbs ensembles". The issue arises when the Hamiltonian possesses conservation laws that are not (pseudo)local but act as such in the symmetry-restricted space where time evolution occurs. Because of them, the stationary state emerging at infinite time can exhibit exceptional features. We focus on a specific example with a spin-flip symmetry, which is the commonest global symmetry encountered in spin-1/2 chains. Among the exceptional properties, we find that, at late times, the excess of entropy of a spin block triggered by a local perturbation in the initial state grows logarithmically with the subsystem's length. We establish a connection with symmetry-protected topological order in equilibrium at zero temperature and study the melting of the order induced either by a (symmetry-breaking) rotation of the initial state or by an increase of the temperature.

Outline of the talk

1. Local relaxation in isolated quantum many-body systems
2. Beyond locality
3. Semilocal charges
4. Clustering & hidden symmetry breaking
5. Symmetry-protected topological order
6. Summary & outlook

Local relaxation in isolated quantum many-body systems

- Stationary expectation values of local observables are accessible



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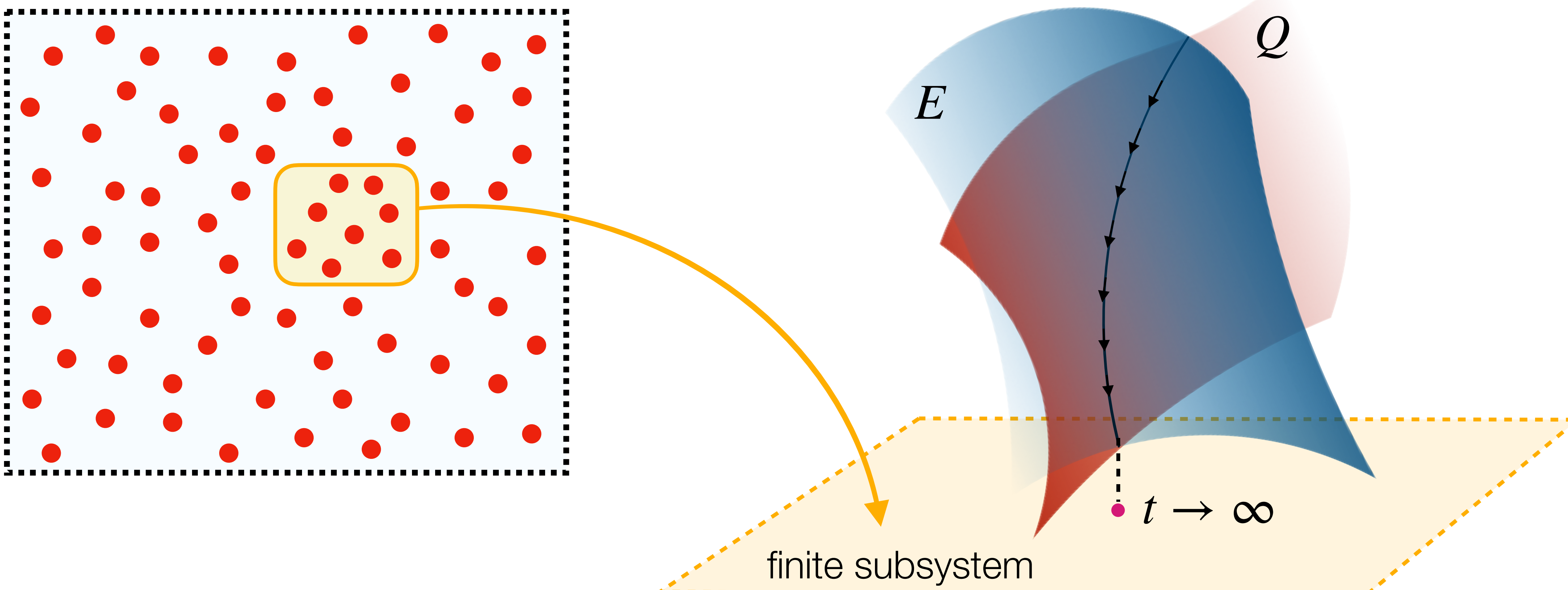
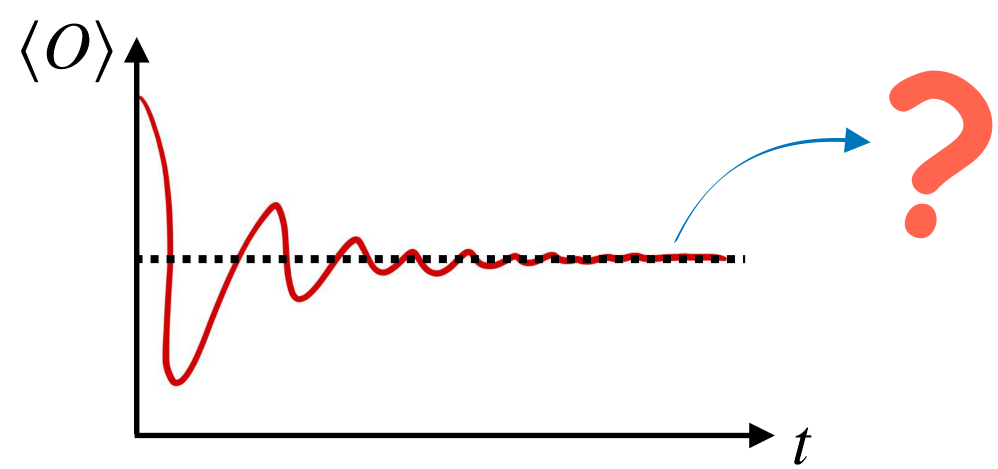


- Quench paradigm:

$$H_0 |\psi\rangle = E_{\mathrm{GS}} |\psi\rangle$$
$$|\psi(t)\rangle = e^{-itH} |\psi\rangle$$

local H, H_0



Local relaxation in isolated quantum many-body systems

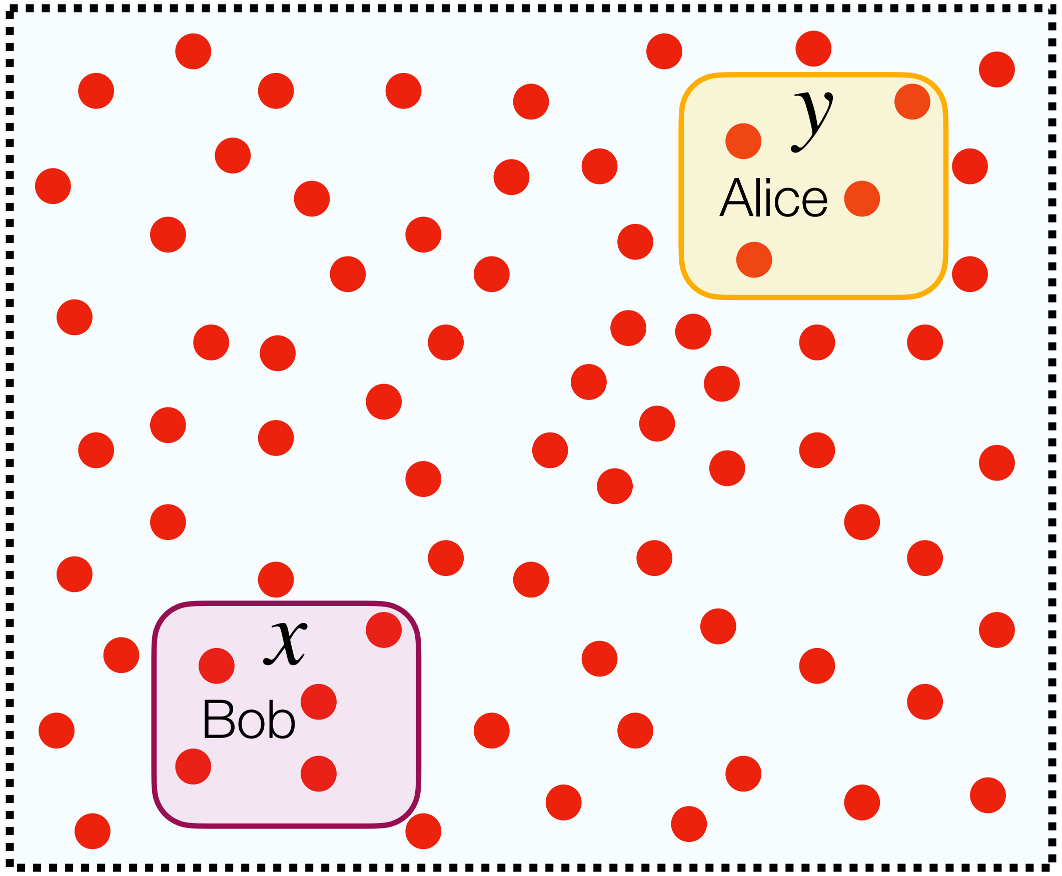
- What are the "few parameters"?

$$\rho \sim e^{-\beta H + \mu Q}, \quad \langle O \rangle_{t\to\infty} = \mathrm{tr}[\rho O]$$

Minimal info lies in the local conserved charges $[Q, H] = 0$.

Rigol et al. 07/08, and others

(G)GE — (generalized) Gibbs ensemble



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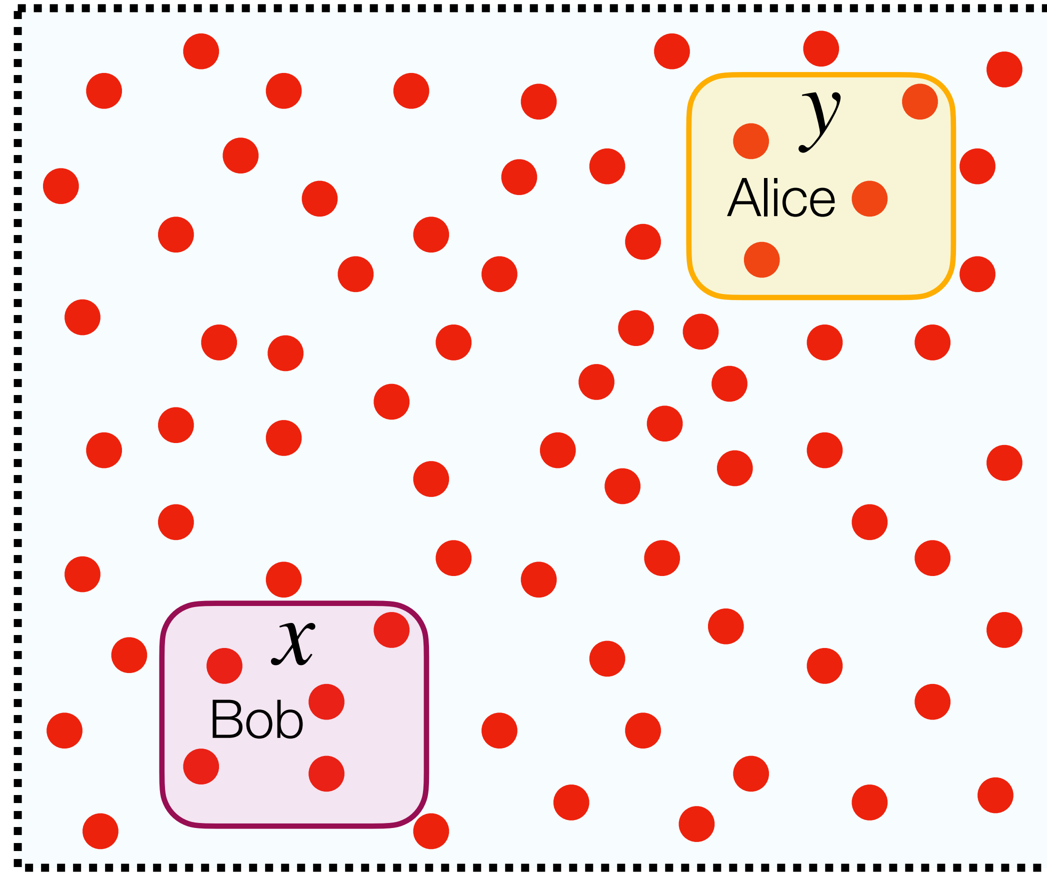


$$\left[O(x), O(y) \right] \to 0$$

- Typically we assume also clustering of the (initial) state:

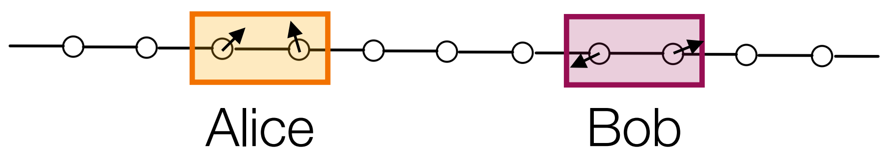
$$\langle O(x) O(y) \rangle \to \langle O(x) \rangle \langle O(y) \rangle$$

$\Rightarrow$ "locality" of the theory!

Murthy & Srednicki 19, Gluza et al. 19

Local relaxation in isolated quantum many-body systems

- Intuition: "local" observables → "local" operators



1D spin chain

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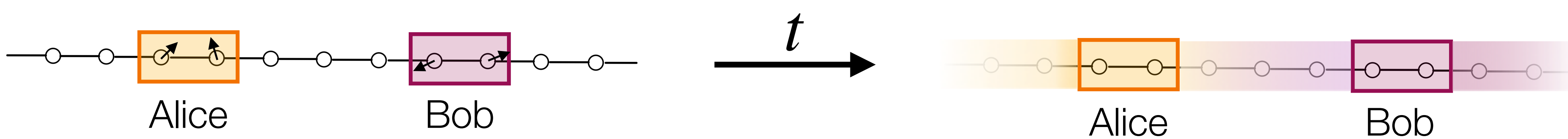

1D spin chain

“locality” → quasilocality (exponential tails)

Lieb & Robinson 72, Bratteli & Robinson 79, Prosen 14, Ilievski et al. 16, Doyon 17, …

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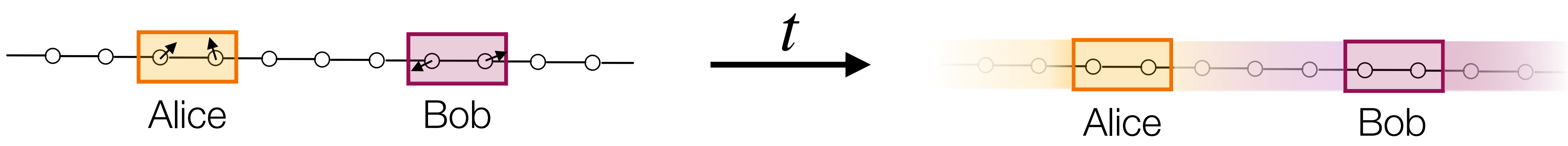

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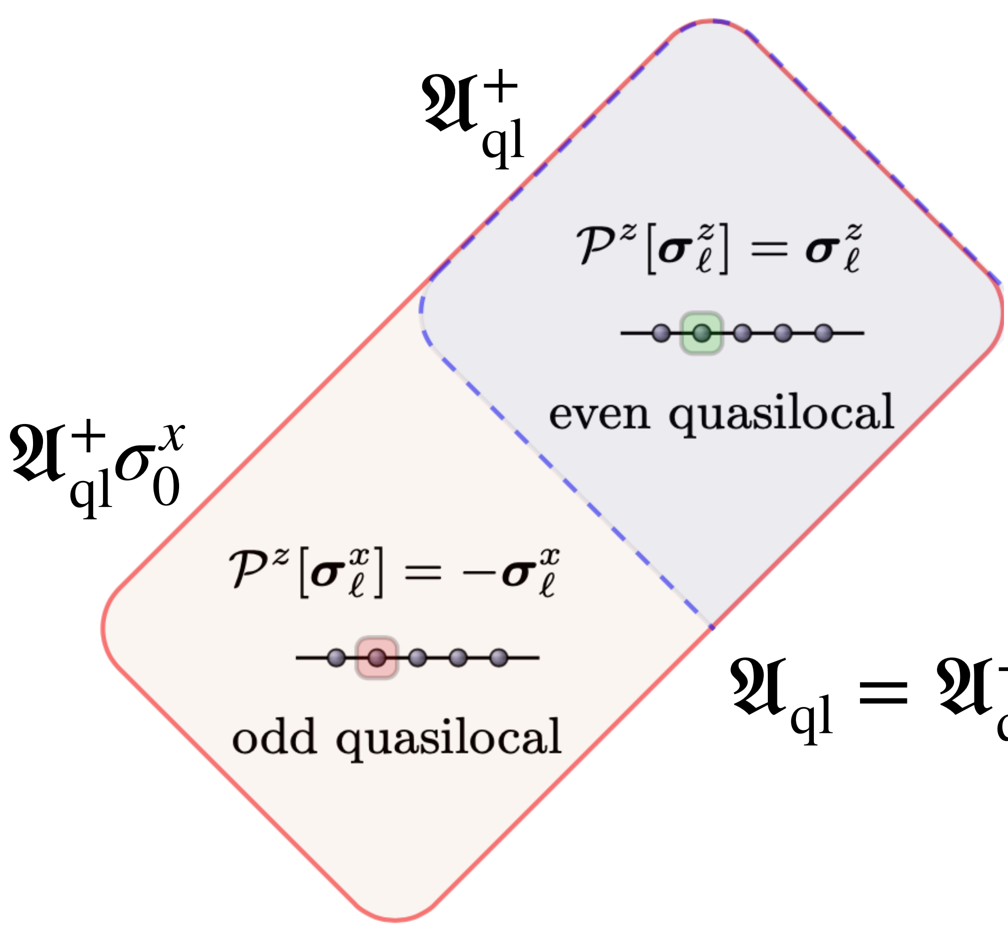
- (Quasi)local rep. of local observables in $\mathbb{Z}_2$-symmetric systems



$$\mathfrak{A}_{\mathrm{ql}} = \mathfrak{A}^{+}_{\mathrm{ql}} \oplus \mathfrak{A}^{+}_{\mathrm{ql}}\sigma^x_0$$

$$\boxed{\mathscr{P}^z \leftrightarrow \prod_\ell \sigma^z_\ell}$$

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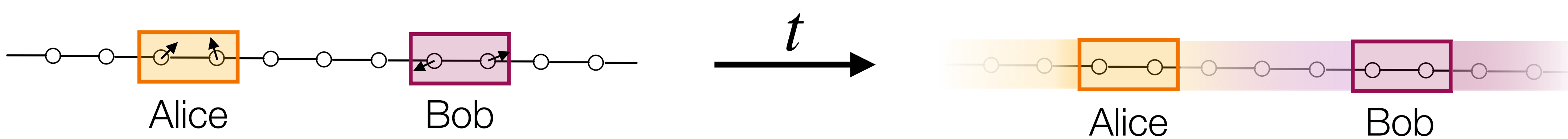

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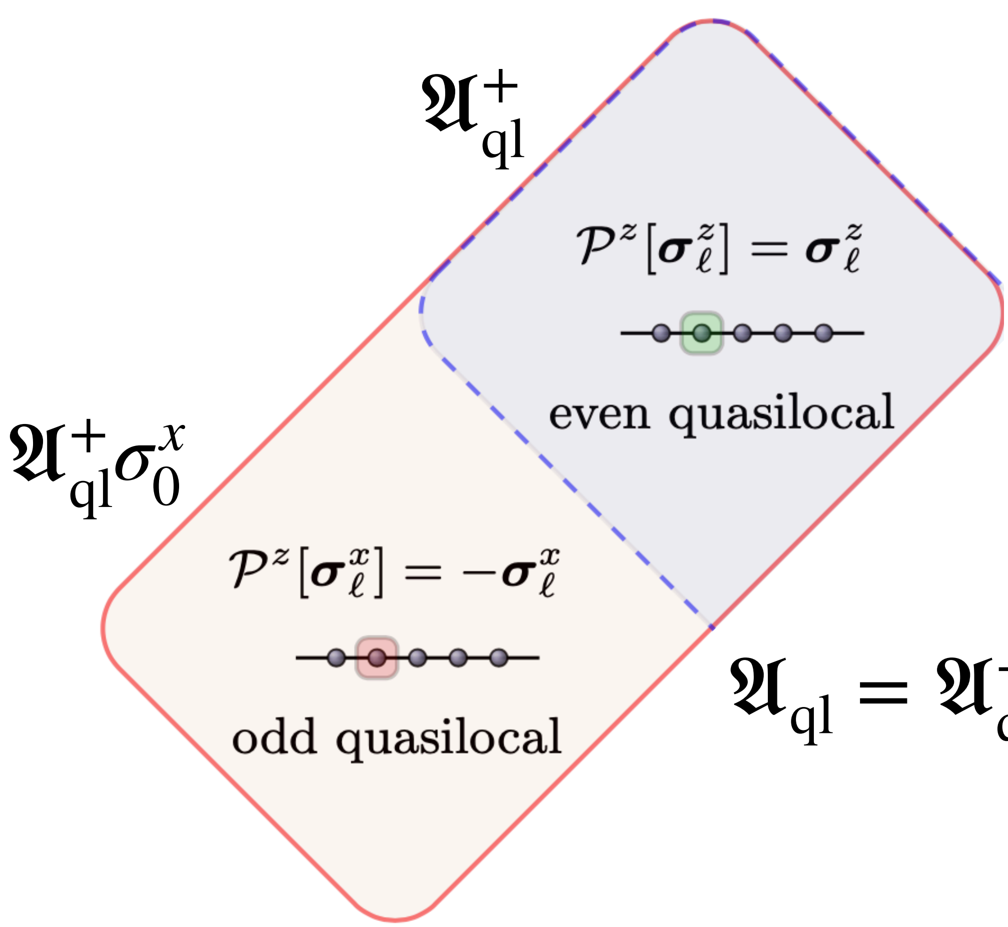

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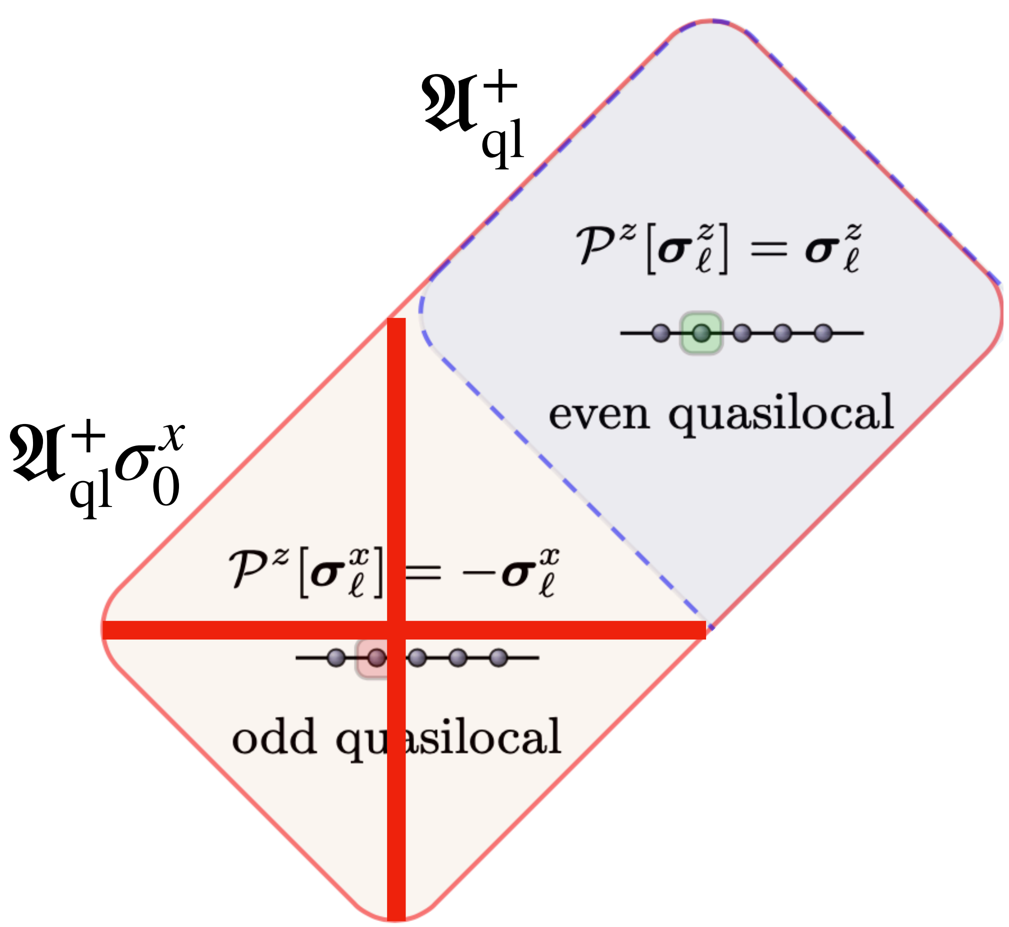
What lies beyond?

Beyond locality

Consider a $\mathbb{Z}_2$-symmetric system

$$|\psi\rangle = |\uparrow \cdots \uparrow\rangle \quad \Big(H_0 = -\sum_\ell \sigma_\ell^z\Big), \qquad \mathscr{P}^z[H] = H \quad \Big(\mathscr{P}^z \leftrightarrow \prod_\ell \sigma_\ell^z\Big)$$

By symmetry odd operators vanish: $\langle\psi\,|\,O\,|\,\psi\rangle = -\langle\psi\,|\,\mathscr{P}^z[O]\,|\,\psi\rangle = -\langle\psi\,|\,O\,|\,\psi\rangle$



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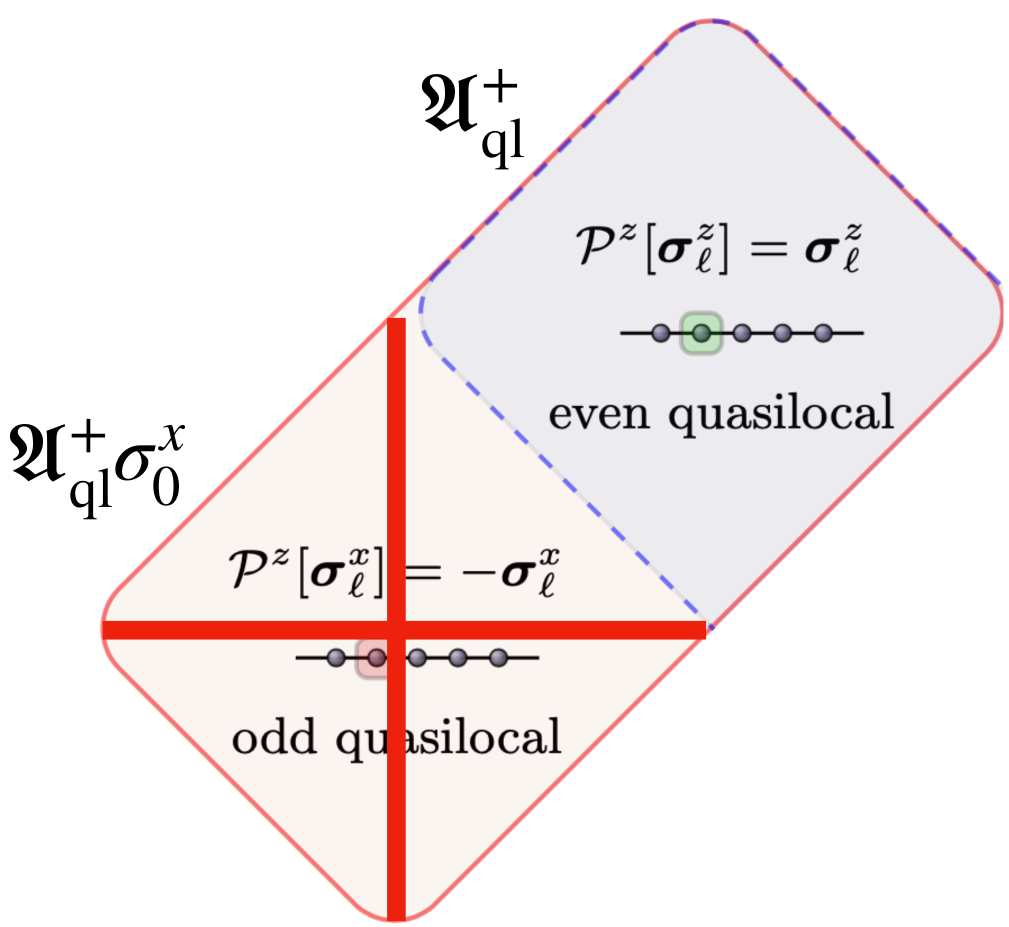
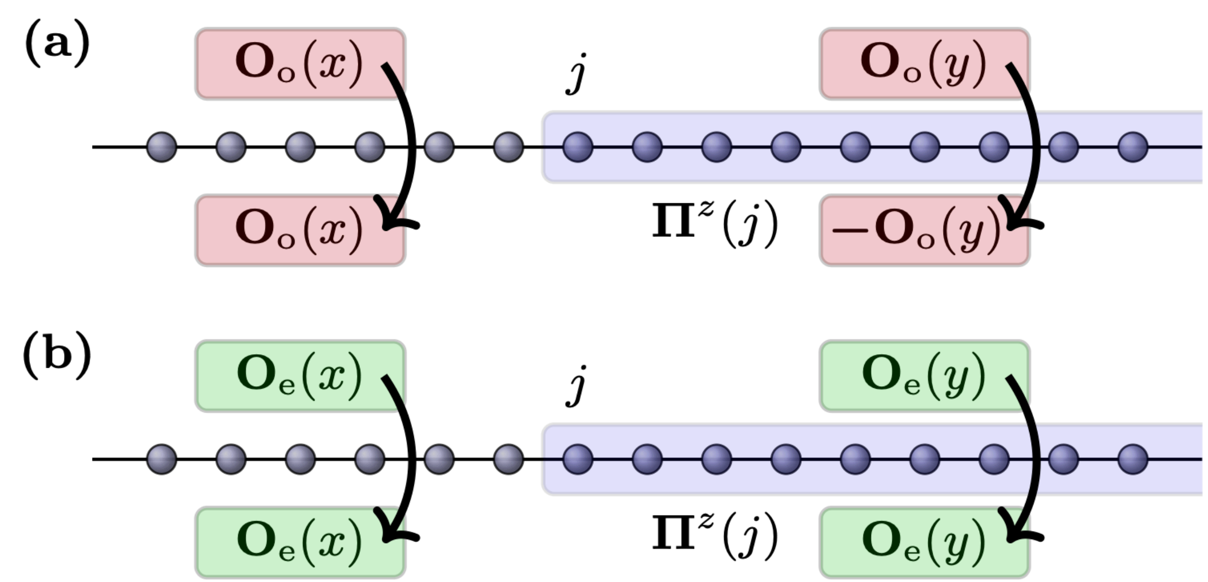


$$\Pi^z(j) \sim \prod_{n\geq j} \sigma_n^z$$



semilocal representation
of local observables

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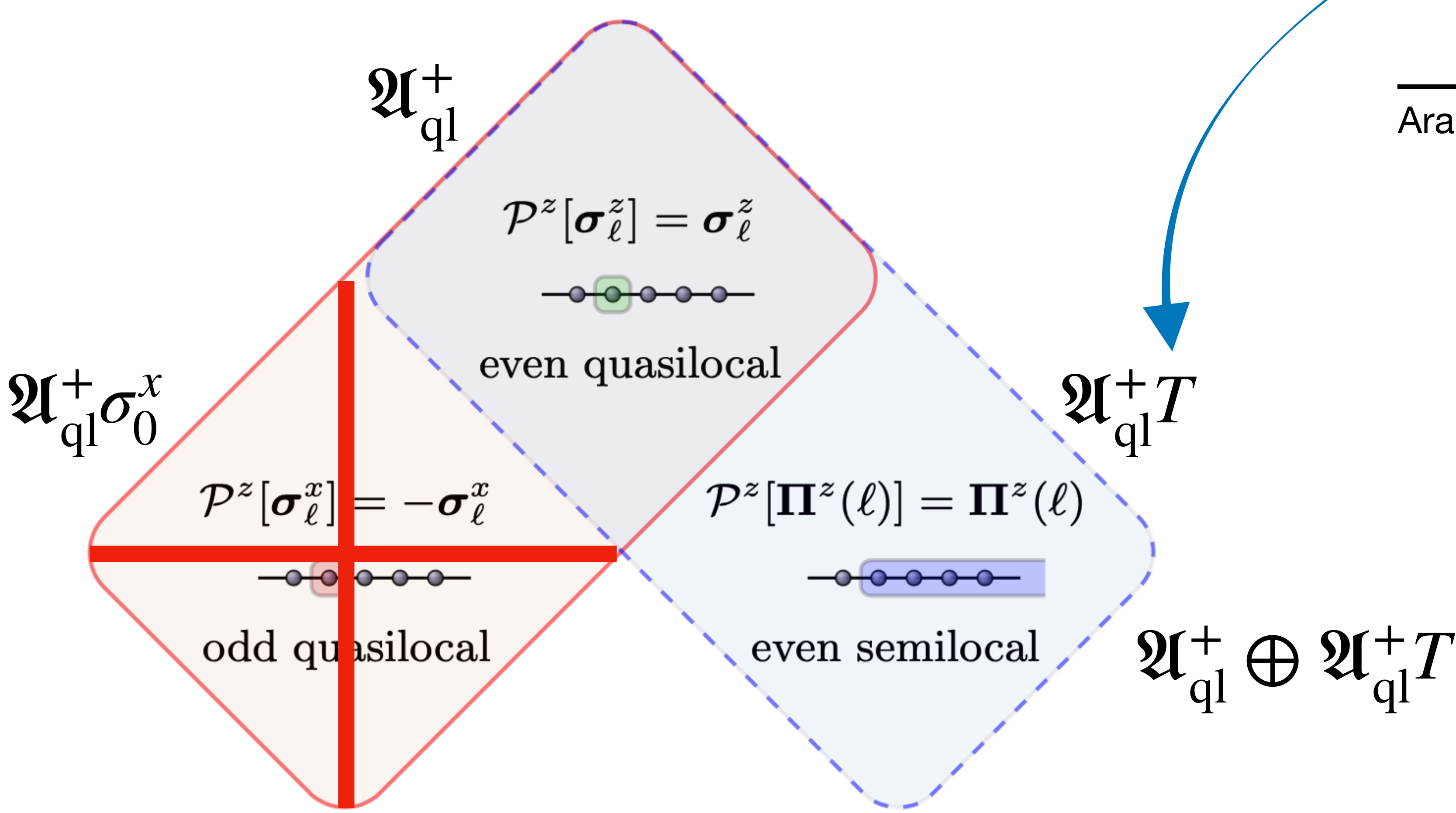
Araki 84 & 90

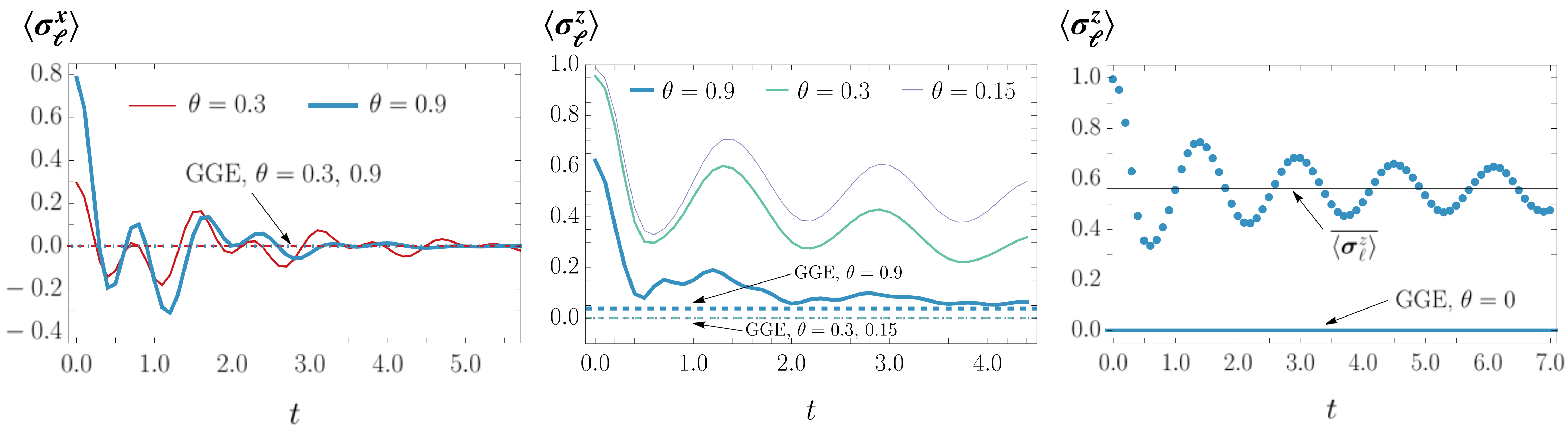


Should we care?

Semilocal charges

- Example: $\mathscr{P}^z$-symmetric integrable Hamiltonian and "local" GGE

$$H = \sum_\ell \sigma^x_{\ell-1}(J_x - J_y\sigma^z_\ell)\sigma^x_{\ell+1}, \qquad |\psi\rangle = |\nearrow \cdots \nearrow_\theta\rangle$$



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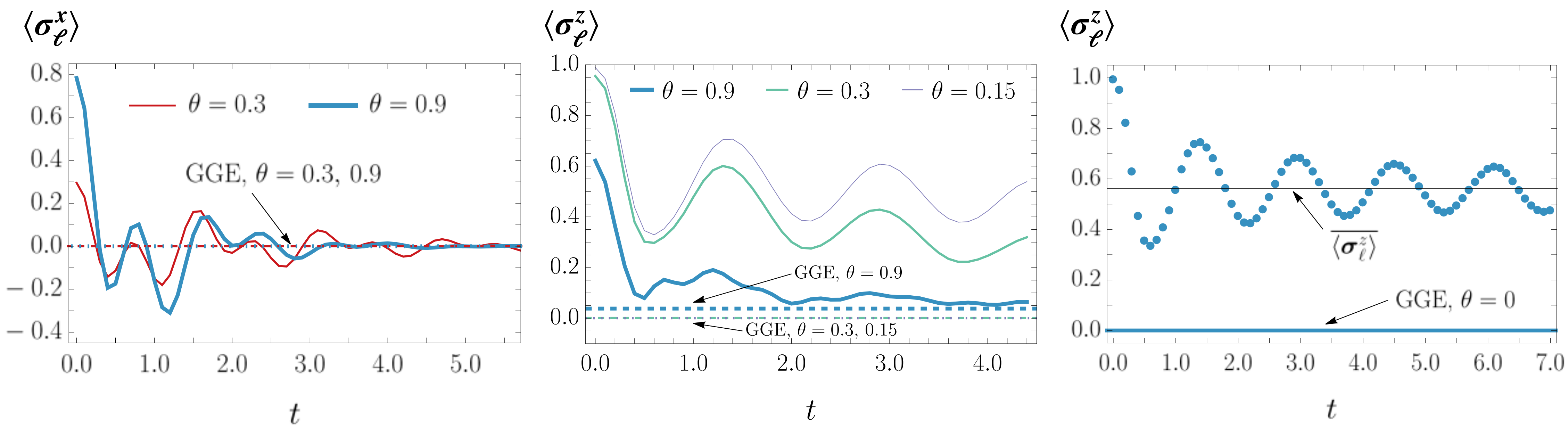

Duality map $\sigma^x_{j-1}\sigma^x_j \mapsto \sigma^x_j, \quad \sigma^z_j \mapsto \sigma^z_j\sigma^z_{j+1} \quad \left(\Pi^z(j) \mapsto \sigma^z_j\right)$

$H = \sum_\ell \sigma^x_{\ell-1}(J_x - J_y\sigma^z_\ell)\sigma^x_{\ell+1}$	$\longrightarrow$	$\tilde{H} = \sum_\ell J_x\sigma^x_\ell\sigma^x_{\ell+1} + J_y\sigma^y_\ell\sigma^y_{\ell+1}$
local charges	$\longrightarrow$	local $\mathscr{P}^x$-even charges
?	$\longrightarrow$	local $\mathscr{P}^x$-odd charges

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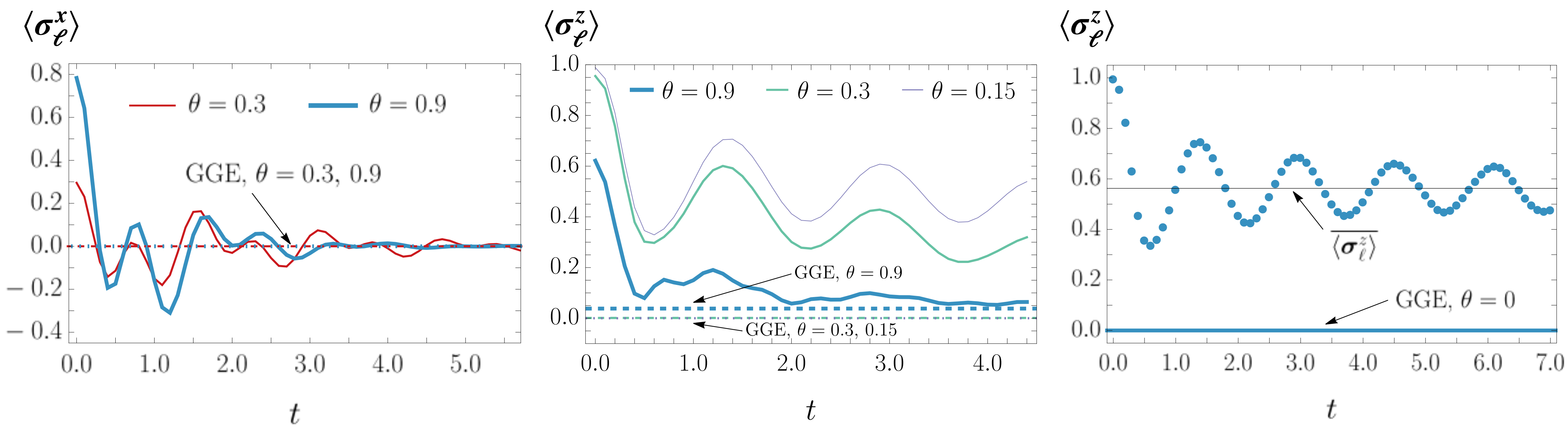

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Semilocal charges

- Example of a semilocal charge:

$$H = \sum_{\ell} \sigma^x_{\ell-1}(J_x - J_y\sigma^z_{\ell})\sigma^x_{\ell+1}, \quad Q = \sum_{\ell} \left[\sigma^x_{\ell-1}\left(J_x\sigma^x_{\ell}\sigma^x_{\ell+1} + J_y\sigma^y_{\ell}\sigma^y_{\ell+1}\right)\sigma^x_{\ell+2} - (J_x + J_y)I\right]\Pi^z(\ell+1)$$

duality map

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the second charge from the boost procedure

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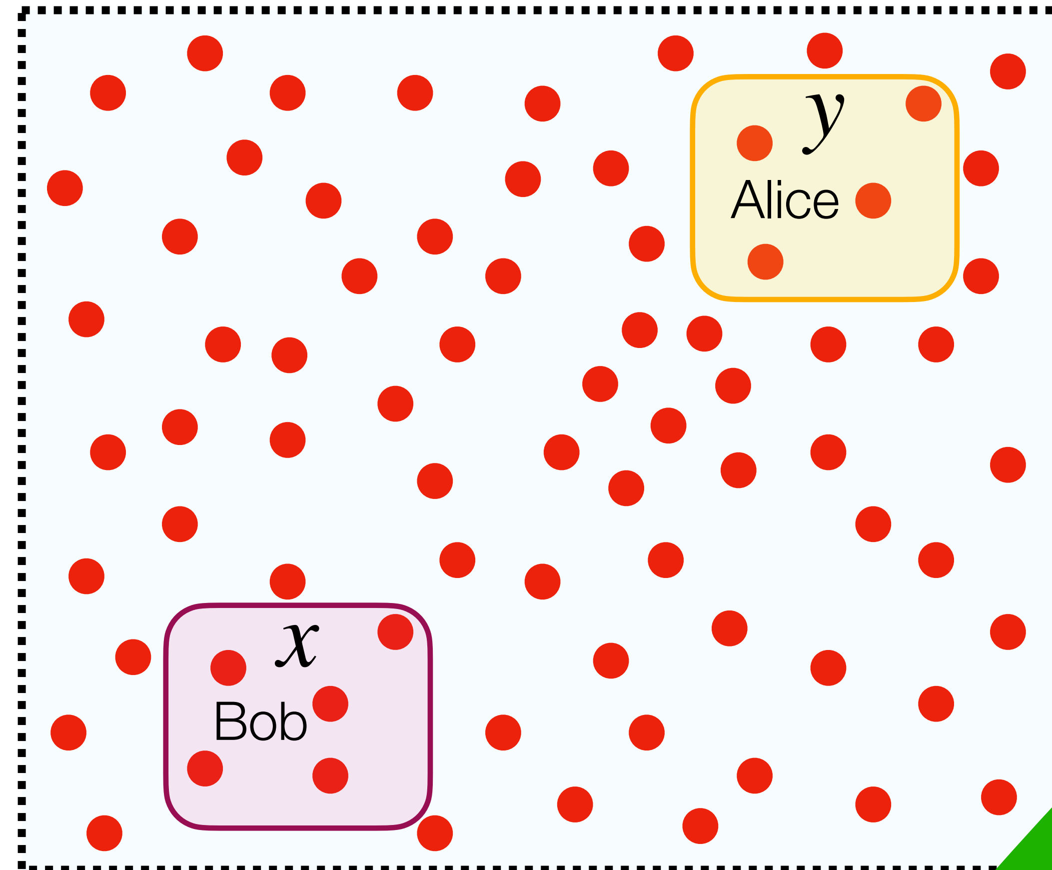

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$$|\mathrm{GS}\rangle = |\uparrow\cdots\uparrow\rangle$$

$$\tilde{H}_0 = -\sum_\ell \sigma^z_\ell\sigma^z_{\ell+1}, \quad \mathscr{P}^x[\tilde{H}_0] = \tilde{H}_0$$

$$|\mathrm{GS}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\cdots\uparrow\rangle + |\downarrow\cdots\downarrow\rangle\big)$$

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hidden symmetry breaking

Hidden symmetry breaking enables:

1. Nonzero semilocal operators $\Rightarrow$ string order
2. Clustering $\Rightarrow$ relaxation to a canonical GGE

Else et al. 13, Kennedy & Tasaki 92

Symmetry-protected topological order

- Landau phases of matter (standard)

$$|\leftarrow \cdots \leftarrow \rangle \qquad\qquad |\rightarrow \cdots \rightarrow \rangle$$

local operator σ^x suffices to determine the symmetry-broken GS of a ferromagnet

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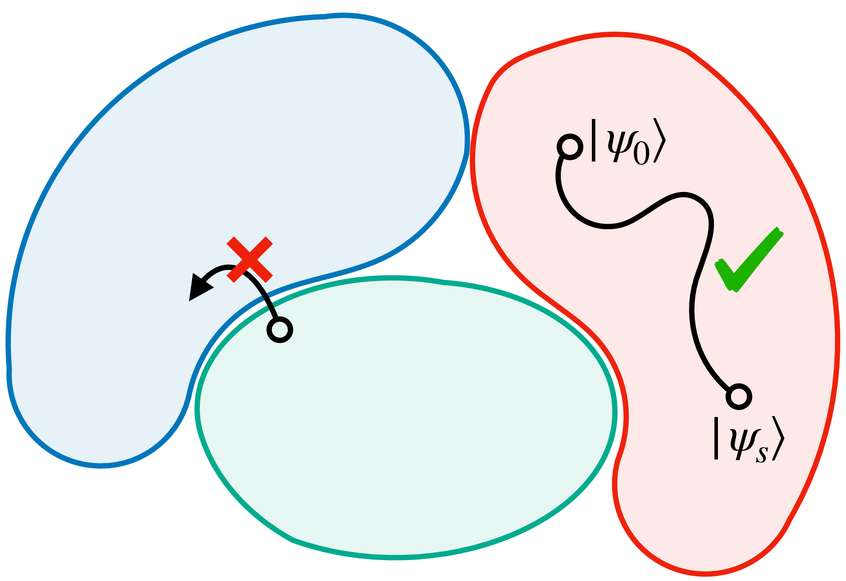

$$|\psi_s\rangle = e^{isW}|\psi_0\rangle, \quad \mathscr{P}^z[W] = W$$

W translationally invariant local

knowledge of H and its symmetries is required to determine the phase

A. string order
B. edge modes
C. topological entanglement entropy

Wen et al. 12, Pollmann et al. 12, Fendley et al. 16, Kitaev 06 & many others

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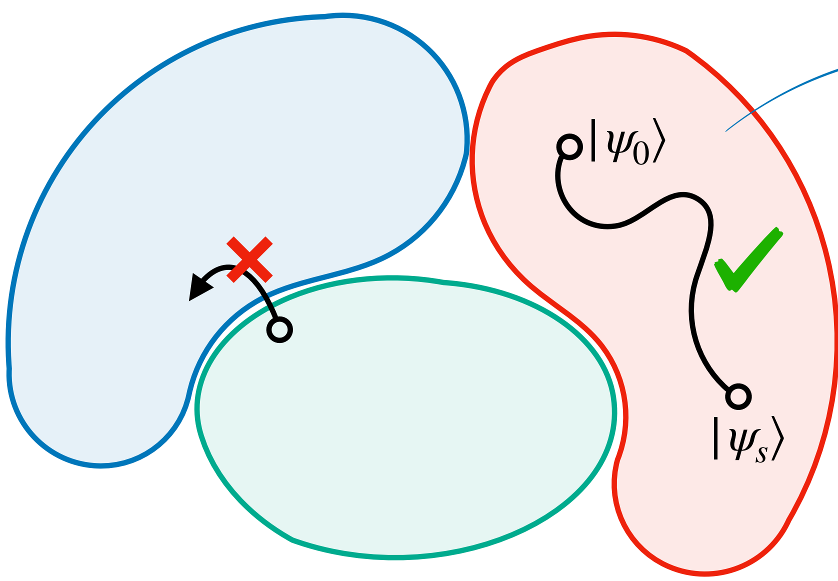

we can play with semilocal theory

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information from the edge of the system is carried by the strings extending to ∞

arXiv:2205.02221

Wen et al. 12, Pollmann et al. 12, Fendley et al. 16, Kitaev 06 & many others

Message & outlook

1. Nonlocal objects can be relevant for a complete picture of local relaxation

2. Symmetry protected order: new representations of local observables

A. Relevance of semilocal charges in inhomogeneous states:
(−) we break transl. invariance
(+) string order can still be present $\langle \mathbf{\Pi}^z(x)\mathbf{\Pi}^z(y)\rangle \neq 0$ (scaling!)

generalized hydrodynamics

Fagotti 22

B. Can we obtain semilocal charges from the transfer matrix?

Gombor & Pozsgay 21

C. Edge modes — solving the finite-L open boundary chain

Fendley et al. 16/17

D. Interacting models with $\mathbb{Z}_2$-breaking charges

Prosen 14, Pasquier et al. 14