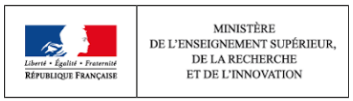
Recent Advances in Quantum Integrable Systems 2022

On the occasion of Nikita Slavnov's 60th Birthday



The Slavnov determinant formula

- Scalar product formula **'89** (Slavnov)

$$\left(\psi\left(\{\lambda_a\}_1^N\right), \psi\left(\{\nu_a\}_1^N\right)\right) = \frac{\det_N\left[\mathtt{M}(\lambda_a, \nu_b)\right]}{\det_N\left[\mathtt{C}(\lambda_a, \nu_b)\right]}$$

- Det. Rep. for Form factors **'98** (Kitanine, Maillet, Terras) : $\left(\psi\left(\{\lambda_a\}_1^N\right), \sigma_1^\alpha \psi\left(\{\nu_a\}_1^N\right)\right)$

  ⇝ High precision numerics (DRF) **'05** (Caux, Maillet), **'06** (Caux, Calabrese, Slavnov)

- Elementary Blocks **'99** (Kitanine, Maillet, Terras) : $\left(\psi\left(\{\lambda_a\}_1^N\right), \mathtt{E}_1^{\epsilon_1 \epsilon_1'} \cdots \mathtt{E}_m^{\epsilon_m \epsilon_m'} \psi\left(\{\nu_a\}_1^N\right)\right)$

- Explicit Reps for correlation functions **'02-'05** (Kitanine, Maillet, Slavnov, Terras)

- Finite T Reps for correlation functions **'05** (Klümper, Göhmann, Seel)

...

- Form factor expansions in Free Fermionic models **'90** (Korepin, Slavnov)

$$\Big(\Omega, \mathcal{O}_1\, \mathcal{O}_{m+1}(t)\, \Omega\Big) \;=\; \sum_{\Upsilon} \Big|\big(\Omega, \mathcal{O}_1\, \Upsilon\big)\Big|^2 \cdot \mathrm{e}^{\mathrm{i}m\mathcal{P}_{\Omega,\Upsilon} - \mathrm{i}t\mathcal{E}_{\Omega,\Upsilon}}$$
$$= \det_{L^2([-q;q])}\Big[\mathrm{id} \,+\, \mathbb{V}\Big]$$

  - $\mathbb{V}$ integrable integral operator

$$V(\lambda,\mu) \;=\; \frac{1}{\lambda-\mu} \cdot \sum_{a=1}^{N} f_a(\lambda) g_a(\mu)$$

    - ⇝ Connection with Riemann–Hilbert Problems **'91** (Its, Izergin, Korepin, Slavnov)

- $\chi \in \mathcal{M}_N\big(\mathcal{O}(\mathbb{C} \setminus [-q\,;q]\,)\big)$;
- $\chi(\lambda) \;=\; I_N \,+\, \mathrm{O}(\lambda^{-1})\;, \qquad \lambda \to \infty\,;$
- $\chi_-(\lambda) = \mathsf{G}_\chi(\lambda) \cdot \chi_+(\lambda)\;, \qquad \lambda \in \,]-q\,;q\,[\,;$

Fully Systematic approach

i) Representation for multi-point fcts

ii) Asymptotics

- ⊛ **'87** (Korepin) $\hookrightarrow$ Dual Fields
  $\rightsquigarrow$ simplification of interacting algebra
- ⊛ **'97** (Kojima, Korepin, Slavnov)

$$\hat{\rho}(x,t) = \frac{\mathrm{Tr}\left[\Psi(0,0)\Psi^{\dagger}(x,t)\mathrm{e}^{-\frac{1}{T}\mathrm{H}_{\mathrm{NLS}}}\right]}{\mathrm{Tr}\left[\mathrm{e}^{-\frac{1}{T}\mathrm{H}_{\mathrm{NLS}}}\right]} \underset{L\to+\infty}{\longrightarrow} \left(0\right|\det\left[\mathrm{id} + \boldsymbol{V}\right]\left|0\right)$$

- ♦ $\boldsymbol{V}$: dual field valued infinite dimensional $\int$ op.

- ♦ $(0|\cdot|0)$: dual field average

- ⊛ Large-x behaviour of $\rho(x,t)$
  - Operator Valued RHP **'98** (Its, Slavnov)
  - Dual field average $\rightsquigarrow$ non-commutativity of $(0|\cdot|0)$ and $x\to+\infty$ **'98** (Slavnov)
- ♦ Ingenious resummation techniques

$$\ln[\rho(x,t)] \simeq \frac{t}{\xi(x/t,T)} + \cdots$$

- ⊛ **'02-'05** (Kitanine, Maillet, Slavnov, Terras)
  Constructions of series of multiple $\int$ reps. for correlation fcts.

- ⊛ **'06-'08** (Kitanine, K, Maillet, Slavnov, Terras)
  RHP & Combinatorial resummations & Asymptotic resummations

$$\left(\Omega, \sigma_1^z \, \sigma_{m+1}^z \Omega\right) = < \sigma_1^z >^2 - \frac{2\mathcal{Z}^2}{\pi^2 m^2} + |\mathcal{F}|^2 \frac{2\cos[2p_F m]}{m^{2\mathcal{Z}^2}} + \cdots$$

- $|\mathcal{F}|^2 = \lim_{L\to+\infty} \left\{ L^{2\mathcal{Z}^2} \cdot \left|\left(\Omega, \sigma_1^z \Upsilon\right)\right|^2 \right\}$

- **'12-...** (Hutsalyuk, Liashyk, Pakuliak, Ragoucy, Slavnov)
  ⇝ Towards correlation functions of higher rank models

  - Scalar products for rank 2 case ;
  - Norms and combinatorics of scalar products for general rank
  - Better understanding of Bethe vectors

Happy 60^{th} Nikita