RAQIS' 2020

Quantum exclusion processes

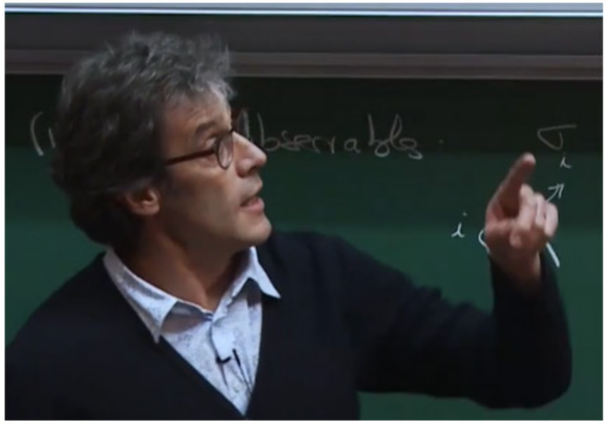
Tony JIN

DQMP, University of Geneva

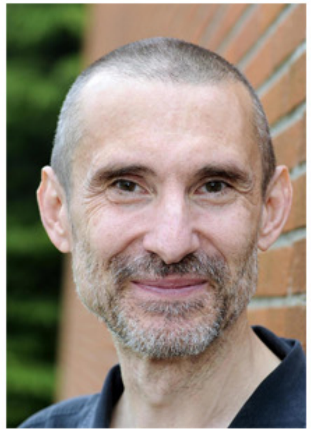
zizhuo.jin@unige.ch

People

- Denis Bernard



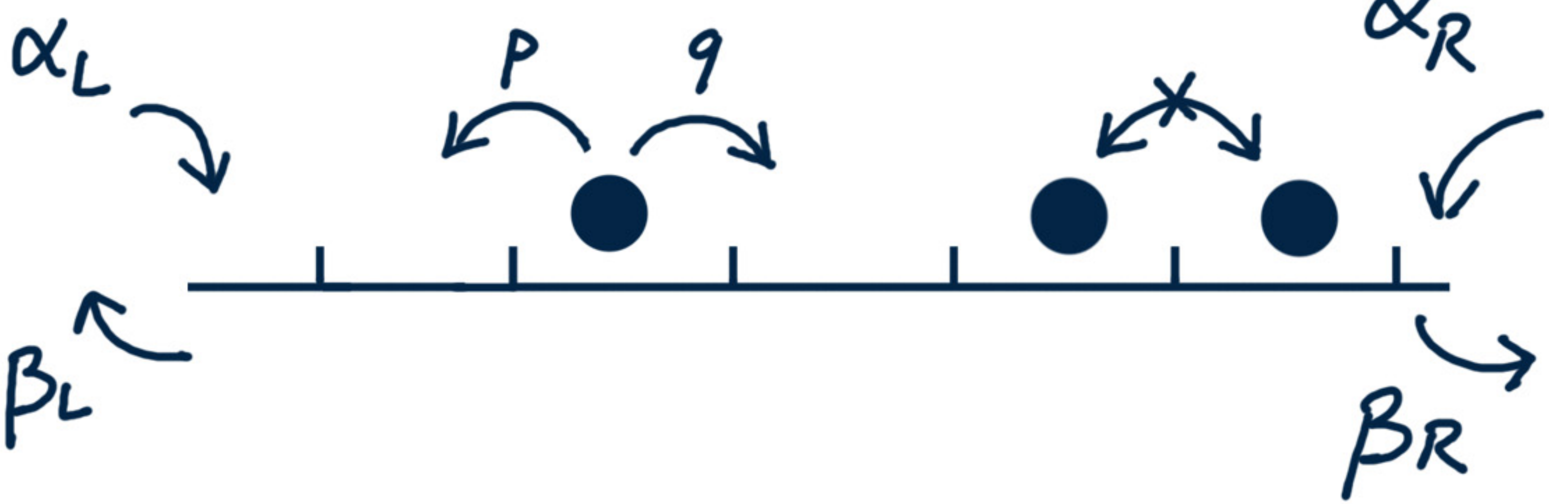
- Michel Bauer



Also:
- Ohad Shpielberg
- Alexandre Krajenbrink
- Marko Medenjak

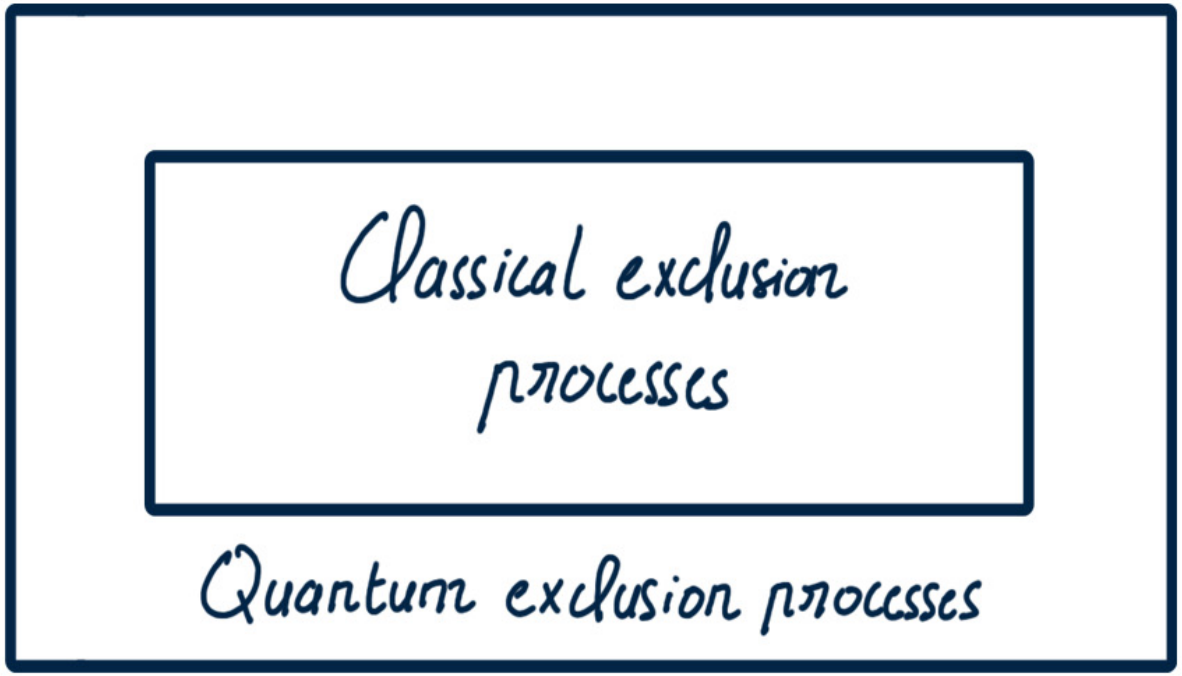
Model

Classical exclusion processes



- $p = q$ SSEP
- $p \neq q$ ASEP
- $p = 0$, $q = 1$ TASEP

Simple but rich model $\rightarrow$ Quantum version ?

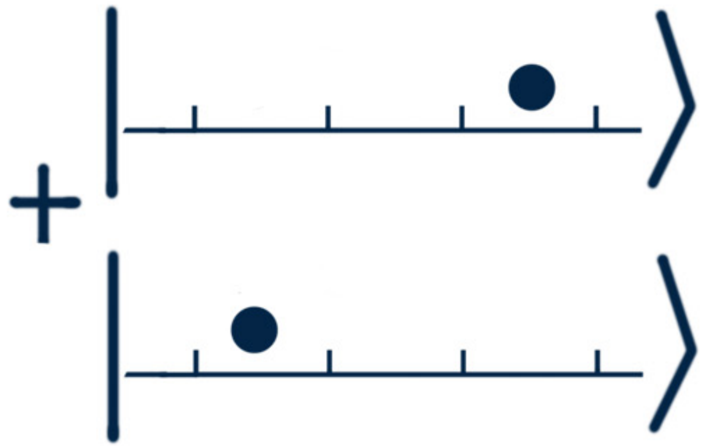


Embedding:

Classical configurations
↓
Quantum basis

Richer structure in the Quantum realm

- Superposition
- Entanglement



Model

$$\boxed{dH_t = \sum_j c_j^\dagger c_{j+1}\, dW_t^j + c_{j+1}^\dagger c_j\, d\overline{W}_t^j}$$

$|\psi_{t+dt}\rangle = e^{-idH_t}|\psi_t\rangle$

$dW_t, d\overline{W}_t$ Quantum noises

Defined by <u>Itô rules</u>

Model

$$\boxed{dH_t = \sum_j c_j^\dagger c_{j+1}\, dW_t^j + c_{j+1}^\dagger c_j\, d\overline{W}_t^j}$$

$|\psi_{t+dt}\rangle = e^{-idH_t}|\psi_t\rangle$ $\quad dW_t, d\overline{W}_t$ Quantum noises

Defined by Itô rules

- Exclusion principle → Fermions
- Quantum master equation restricted to occupation number operators ⟶ Classical master equation
- Itō rules determine the class (QSSEP, QASEP, etc.)

$dW d\overline{W} = d\overline{W} dW \to$ QSSEP $\qquad d\overline{W} dW \neq dW d\overline{W} \to$ QASEP

Model

$$\boxed{dH_t = \sum_j c_j^\dagger c_{j+1}\, dW_t^j + c_{j+1}^\dagger c_j\, d\overline{W}_t^j}$$

- Two sources of randomness : Q uncertainty $\mathrm{tr}(\rho \hat{O})$
  Stochastic process $\mathbb{E}[\hat{O}]$

- Quadratic in fermionic operators
  ↳ Restrict to Gaussian states
  on two-point function $\boxed{G_{ij} := \mathrm{tr}(\rho c_j^\dagger c_i)}$

Simple example : two sites symmetric process

Stationary measure?

SSEP : $p(1=\bullet)=1/2 \quad p(2=\bullet)=1/2$

QSSEP : $\mathbb{E}[G_{11}]=n_1=1/2 \quad \mathbb{E}[G_{22}]=n_2=1/2 \quad \mathbb{E}[G_{12}]=0$

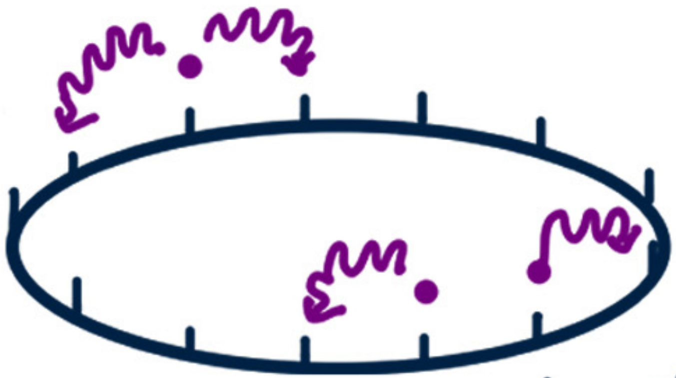
but $\mathbb{E}[G_{12}G_{21}]=1/6 \neq 0$

$\hookrightarrow$ Fluctuating quantum coherences!

Built up dynamically

Results

Stationary measure of QSSEP for PBC



Stationary measure

for $G_{ij} := \mathrm{tr}(\rho c_j^\dagger c_i)$

$$G_{t+dt} = e^{-idh_t} G_t e^{idh_t}$$

$$dH_t = \sum_j c_j^\dagger c_{j+1} \, dW_t^j + c_{j+1}^\dagger c_j \, d\bar{W}_t^j$$

$$\downarrow$$

$$dh_t = \sum_j E_{j+1,j} \, dW_t^j + E_{j,j+1} \, d\bar{W}_t^j$$

$\boxed{\text{the } E_{j,j+1}, E_{j+1,j} \text{ form a simple root system of } su(L)\,!}$

The stationary measure is invariant by the action of elements of $SU(L)$

Compact way of formulating the answer

Generating function $\quad Z(A) = \mathbb{E}_{\infty}\left[e^{\operatorname{tr} AG} \right]$ $\quad A, G$ $L \times L$ matrices

$$\boxed{Z(A) = \int_{U \in (SU(L))} d\eta(U)\, e^{\operatorname{tr}(A U G_0 U^{+})}}$$

Harish-Chandra, Itzykson, Zuber integral $\qquad \eta$ Haar measure

Interesting fact : respects a large deviation principle

$$\boxed{\mathbb{E}_\infty\left[e^{L\,\mathrm{tr}(AG)}\right] \underset{L\to\infty}{\asymp} e^{L\mathcal{F}(A)}}$$

$$\boxed{\mathcal{P}rob_\infty(G=g) \underset{L\to\infty}{\asymp} e^{-L\,I(g)}}$$

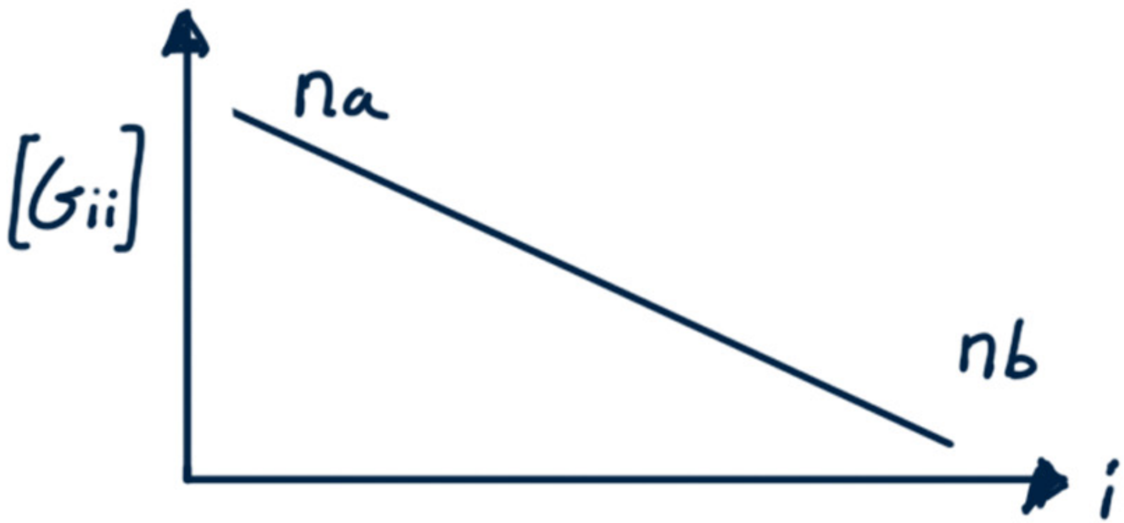
Open case

We can still use fermionic Gaussian states technology

Steady-state distribution of $G_{ij} = \mathrm{tr}(\rho_t \, c_j^\dagger c_i)$

In mean

$$[G_{ij}] = 0, \quad i \neq j$$

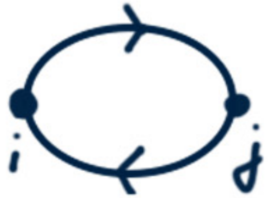


Fluctuations ?

Diagrammatic

$G_{ij} =$ 

$G_{ii} =$ 

$G_{ij}G_{ji} =$ 

Steady-state

- only "closed" diagrams survive

e.g. $\mathbb{E}_\infty\Big[$ 

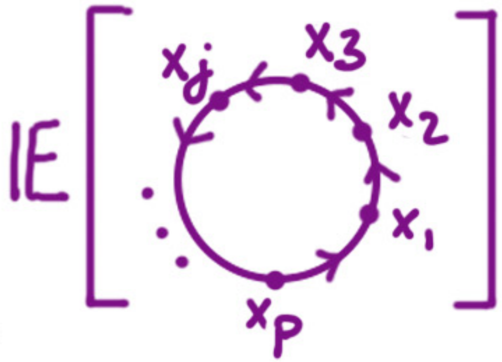
$\Big] = 0$

- Large L limit dominant contribution : loops

N-point diagram $\mathbb{E}\Bigg[$ 

$\Bigg] \sim L^{-(N-1)}$

Continuous limit



$:= [\sigma]$

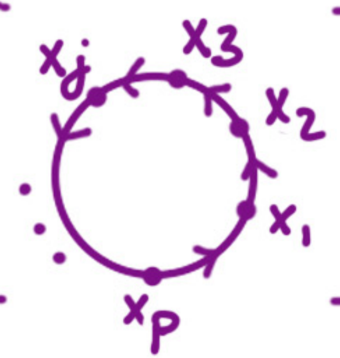
i) Bulk $\sum_j \Delta_{x_j}[\sigma](\vec{x}) = 0$

ii) Boundaries $[\sigma](\vec{x})\Big|_{x_1=0} = [\sigma](\vec{x})\Big|_{x_p=1} = 0$

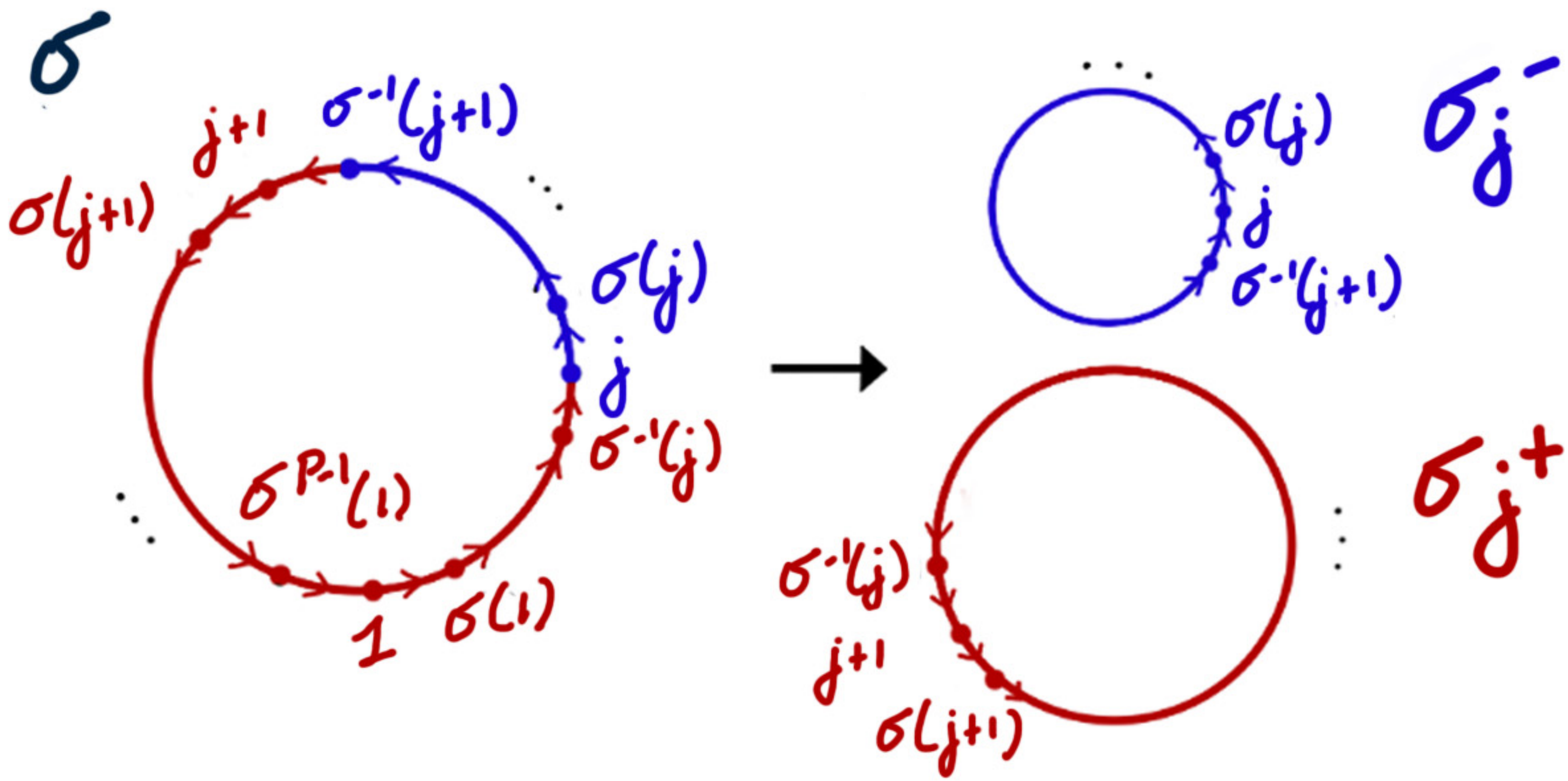
iii) Exchange $[\sigma](\vec{x})\Big|_{x_j=x_{j+1}} = [\tau_j \circ \sigma](\vec{x})\Big|_{x_j=x_{j+1}}$

$$\left(\nabla_{x_j} - \nabla_{x_{j+1}}\right)\left([\sigma](\vec{x}) + [\tau_j \circ \sigma](\vec{x})\right)\Big|_{x_j=x_{j+1}} = 2\nabla_{x_j}[\sigma_j^-](\vec{x})\nabla_{x_{j+1}}[\sigma_j^+](\vec{x})$$

Continuous limit

$\mathbb{E}\Big[$  $\Big] := [\sigma]$

$$\left(\nabla_{x_j} - \nabla_{x_{j+1}}\right)\left([\sigma](\vec{x}) + [\tau_j \circ \sigma](\vec{x})\right)\Big|_{x_j = x_{j+1}} = 2\nabla_{x_j}[\sigma_j^-](\vec{x})\,\nabla_{x_{j+1}}[\sigma_j^+](\vec{x})$$

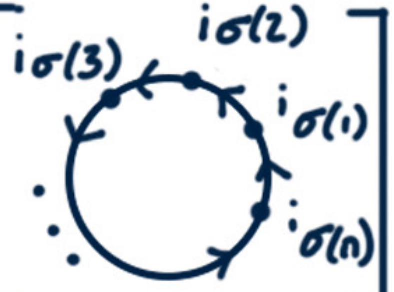


Large deviations

$$\boxed{Z(A) = \mathbb{E}_{\infty}\left[e^{\mathrm{tr}(AG)}\right] \underset{L\to\infty}{\asymp} e^{L\,\mathcal{Z}(A)}}$$

$\mathcal{Z}(A)$ large-deviation function

$$\mathcal{Z}(A) = \sum_{N} \frac{1}{N!} F_N(A)$$

$$F_N(A) = \sum_{i_1 < i_2 \cdots < i_N} \sum_{\sigma} \mathbb{E}\left[\right.$$ 

$$\left.\right] A_{i_{\sigma(1)} i_{\sigma(2)}} \cdots A_{i_{\sigma(N-1)} i_{\sigma(N)}}$$

QASEP and QKPZ

Clasically, we know: ASEP $\xrightarrow[\text{asymmetric limit}]{\text{weakly}}$ KPZ

Same holds for quantum! QASEP $\xrightarrow[\text{asymmetric limit}]{\text{weakly}}$ QKPZ

$\hat{h}_k = \sum_{-\infty}^{k} \hat{c}_j^{\dagger} \hat{c}_j$ height field

$$\boxed{d\hat{h}_k = \left[(\alpha+\tfrac{1}{2})\Delta \hat{h}_k - (\nabla \hat{h}_k)^2 + \tfrac{1}{4}\right]dt + i\left[c_{k+1}^{\dagger} c_k \, dW_t^k - c_k^{\dagger} c_{k+1} \, d\bar{W}_t^k\right]}$$

$d\bar{W} dW = \alpha \quad dW d\bar{W} = 1+\alpha \qquad (1;2;3)$ scaling $\rightarrow$ Continuous KPZ equation

Conclusion

- Generalisation of exclusion processes to the quantum realm.
- The structure is not fully deciphered yet for the open case
- The approach for the closed case can be generalized to give a new point-of-view on quantum thermodynamics.

Thank you!

References

- Closed case

  SciPost Phys 6, 45

- Open case

  Phys. Rev. Lett. 123, 080601
  arXiv:2006.12222

- QASEP and QKPZ

  Phys. Rev. Lett. 125, 040603