Integral formulae for finite temperature correlation functions of the XXZ chain

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Introduction

$$H_{XXZ} = J \sum_{j=1}^{L} \left(\sigma^x_{j-1}\sigma^x_j + \sigma^y_{j-1}\sigma^y_j + \Delta(\sigma^z_{j-1}\sigma^z_j - 1) \right)$$

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- Why finite temperatures?
- Algebraic Bethe ansatz approach to finite temperature correlation functions
- The density matrix
- Integral representation
- Numerical results
- Further results and outlook

Work in collaboration with Andreas Klümper, Alexander Seel and Michael Bortz (J. Phys. A, **37** (2004) 7625, Physica B **359-361** (2005) 807, J. Phys. A, **38** (2005) 1833, Eur. Phys. J. B **46** (2005) 399, hep-th/0505091)

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Related work

- Jimbo et al. 1992, Jimbo & Miwa 1996
- Kitanine, Maillet, Slavnov & Terras 2000, 2002, 2004 (‘Lyon group’)
- Boos, Korepin, Smirnov 2001, 2002, 2003
- ‘Takahashi group’ from 2003
- Boos, Jimbo, Miwa, Smirnov & Takeyama 2004, 2005

ABA approach to finite temperature correlation functions

In the framework of Yang-Baxter integrability the XXZ chain is the fundamental model associated with the solution $R(\lambda,\mu) \in \mathrm{End}\big(\mathbb{C}^2 \otimes \mathbb{C}^2\big)$ of the Yang-Baxter equation

$$R^{\alpha\beta}_{\alpha'\beta'}(\lambda,\mu) R^{\alpha'\gamma}_{\alpha''\gamma'}(\lambda,\nu) R^{\beta'\gamma'}_{\beta''\gamma''}(\mu,\nu) = R^{\beta\gamma}_{\beta'\gamma'}(\mu,\nu) R^{\alpha\gamma'}_{\alpha'\gamma''}(\lambda,\nu) R^{\alpha'\beta'}_{\alpha''\beta''}(\lambda,\mu)$$

defined by

$$R(\lambda,\mu) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b(\lambda,\mu) & c(\lambda,\mu) & 0 \\ 0 & c(\lambda,\mu) & b(\lambda,\mu) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$b(\lambda,\mu) = \frac{\mathrm{sh}(\lambda-\mu)}{\mathrm{sh}(\lambda-\mu+\eta)}\,, \quad c(\lambda,\mu) = \frac{\mathrm{sh}(\eta)}{\mathrm{sh}(\lambda-\mu+\eta)}$$

For the exact calculation of thermal averages at temperature T we need to express the statistical operator of the canonical ensemble

$$\rho_L = \mathrm{e}^{-H_{XXZ}/T}$$

in terms of the R-matrix in a way that allows us to utilize the Yang-Baxter algebra efficiently.

ABA approach to finite temperature correlation functions

For this purpose we introduce an auxiliary vertex model defined by a monodromy matrix

$$T_j(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}_j = R_{j\overline{N}}(\lambda, \tfrac{\beta}{N}) R^{t_1}_{\overline{N-1}j}(-\tfrac{\beta}{N}, \lambda) \dots R_{j\overline{2}}(\lambda, \tfrac{\beta}{N}) R^{t_1}_{\overline{1}j}(-\tfrac{\beta}{N}, \lambda)$$

This monodromy matrix acts non-trivially in the tensor product of an auxiliary space $j = 1, \dots, L$ and $N \in 2\mathbb{N}$ 'quantum spaces' $\overline{1}, \dots, \overline{N}$ which are all isomorphic to $\mathbb{C}^2$. By t_1 we mean transposition with respect to the first of the two spaces into which $R(\lambda, \mu)$ is embedded. β is inversely proportional to the temperature T,

$$\beta = \frac{2J\,\mathrm{sh}(\eta)}{T}$$

The monodromy matrix can be used to express the statistical operator as a limit,

$$\rho_L = \lim_{N \to \infty} \mathrm{tr}_{\overline{1} \dots \overline{N}}\, T_1(0) \dots T_L(0)$$

with the implicit identification $\Delta = \mathrm{ch}(\eta)$. The number N is called the Trotter number, and the limit is referred to as the Trotter limit. For large Trotter number the operator

$$\rho_{N,L} = \mathrm{tr}_{\overline{1} \dots \overline{N}}\, T_1(0) \dots T_L(0)$$

is a good approximation to the statistical operator.

ABA approach to finite temperature correlation functions

The thermal expectation value of a product of local operators $X^{(n)}$ acting on sites $j,\dots,j+m-1$ of the spin chain is then approximated by

$$\langle X_j^{(1)} \dots X_{j+m-1}^{(m)} \rangle_{N,T} = \frac{\mathrm{tr}_{1\dots L}\, \rho_{N,L} X_j^{(1)} \dots X_{j+m-1}^{(m)}}{\mathrm{tr}_{1\dots L}\, \rho_{N,L}}$$

$$= \frac{\mathrm{tr}_{\bar{1}\dots\bar{N}} \big(\mathrm{tr}\, T(0)\big)^{j-1} \mathrm{tr}\, X^{(1)} T(0) \dots \mathrm{tr}\, X^{(m)} T(0) \big(\mathrm{tr}\, T(0)\big)^{L-m-j+1}}{\mathrm{tr}_{\bar{1}\dots\bar{N}} \big(\mathrm{tr}\, T(0)\big)^{L}}$$

Here the expression

$$Z_{N,L} = \mathrm{tr}_{1\dots L}\, \rho_{N,L} = \mathrm{tr}_{\bar{1}\dots\bar{N}} \big(\mathrm{tr}\, T(0)\big)^{L}$$

in the denominator is the finite Trotter number approximant to the partition function $Z_L = \mathrm{tr}_{1\dots L} \exp(-H_{xxz}/T)$ of the XXZ chain of length L.

The operator occurring on the right hand side

$$t(\lambda) = \mathrm{tr}\, T(\lambda) = A(\lambda) + D(\lambda)$$

is called the quantum transfer matrix.

ABA approach to finite temperature correlation functions

The quantum transfer matrix can be diagonalized by means of the algebraic Bethe ansatz since the monodromy matrix defines a representation of the Yang-Baxter algebra,

$$R_{jk}(\lambda,\mu)T_j(\lambda)T_k(\mu) = T_k(\mu)T_j(\lambda)R_{jk}(\lambda,\mu)$$

and has a pseudo vacuum $|0\rangle = \left[\binom{0}{1} \otimes \binom{1}{0}\right]^{\otimes N/2}$ on which it acts like

$$C(\lambda)|0\rangle = 0\,, \quad A(\lambda)|0\rangle = b^{\frac{N}{2}}\left(-\tfrac{\beta}{N},\lambda\right)|0\rangle = a(\lambda)|0\rangle\,, \quad D(\lambda)|0\rangle = b^{\frac{N}{2}}\left(\lambda,\tfrac{\beta}{N}\right)|0\rangle = d(\lambda)|0\rangle$$

Then the vector

$$|\{\lambda\}\rangle = |\{\lambda_j\}_{j=1}^M\rangle = B(\lambda_1)\dots B(\lambda_M)|0\rangle$$

is an eigenvector of the quantum transfer matrix $t(\lambda)$ if the Bethe roots $\lambda_j,\ j = 1,\dots,M$, satisfy the system

$$\frac{a(\lambda_j)}{d(\lambda_j)} = \prod_{k=1,k\neq j}^{M} \frac{\operatorname{sh}(\lambda_j-\lambda_k+\eta)}{\operatorname{sh}(\lambda_j-\lambda_k-\eta)}$$

of Bethe ansatz equations. The corresponding eigenvalue is

$$\Lambda(\lambda) = a(\lambda)\prod_{j=1}^{M}\frac{\operatorname{sh}(\lambda-\lambda_j-\eta)}{\operatorname{sh}(\lambda-\lambda_j)} + d(\lambda)\prod_{j=1}^{M}\frac{\operatorname{sh}(\lambda-\lambda_j+\eta)}{\operatorname{sh}(\lambda-\lambda_j)}$$

ABA approach to finite temperature correlation functions

The algebraic Bethe ansatz is a means to study the spectrum of the quantum transfer matrix. It has a non-degenerate eigenvalue with largest modulus at $\lambda = 0$ which is characterized by a unique set $\{\lambda\} = \{\lambda_j\}_{j=1}^{N/2}$ of $M = N/2$ Bethe roots that all go to zero as T goes to infinity. We call it the dominant eigenvalue and denote it by $\Lambda_0(\lambda)$. The corresponding eigenvector $|\{\lambda\}\rangle$ will be called the dominant eigenvector. The dominant eigenvalue alone determines the thermodynamic properties of the XXZ chain in the thermodynamic limit $L \to \infty$:

$$f(T) = -T \lim_{L\to\infty} \frac{1}{L} \lim_{N\to\infty} \ln \mathrm{tr}_{\bar{1}\dots\bar{N}}\, t^L(0)$$

But the two limits commute according to (Suzuki 85, Suzuki and Inoue 87), hence

$$f(T) = -T \lim_{N\to\infty} \ln \Lambda_0(0)$$

Applying this reasoning to the expectation value of the product of local operators in the thermodynamic limit we obtain

$$\langle X_j^{(1)} \dots X_{j+m-1}^{(m)} \rangle_T = \lim_{N\to\infty} \frac{\langle\{\lambda\}|\mathrm{tr} X^{(1)} T(0) \dots \mathrm{tr} X^{(m)} T(0)|\{\lambda\}\rangle}{\langle\{\lambda\}|\{\lambda\}\rangle \Lambda^m(0)}$$

where $|\{\lambda\}\rangle$ denotes the dominant eigenvector. The dominant eigenvector determines the state of thermal equilibrium completely.

It follows that

$$\langle X_j^{(1)} \dots X_{j+m-1}^{(m)} \rangle_T = \lim_{N\to\infty} \lim_{\substack{\xi_j \to 0 \\ j=1,\dots,m}} \frac{\langle \{\lambda\} | \mathrm{tr} X^{(1)} T(\xi_1) \dots \mathrm{tr} X^{(m)} T(\xi_m) | \{\lambda\} \rangle}{\langle \{\lambda\} | \{\lambda\} \rangle \prod_{j=1}^m \Lambda(\xi_j)}$$

where we introduced inhomogeneities $\xi_1, \dots, \xi_m$ that regularize the expression in the numerator and allow us to apply the Yang-Baxter algebra for calculating matrix elements.

Since the Hamiltonian commutes with the z-component

$$S^z = \frac{1}{2} \sum_{j=1}^{L} \sigma_j^z$$

of the total spin, thermodynamics and finite temperature correlation functions can still be treated within the ‘quantum transfer matrix approach’ described above if the system is exposed to an external magnetic field h in z-direction. The external field is properly taken into account by applying a twist to the monodromy matrix,

$$T(\lambda) \to T(\lambda) \begin{pmatrix} \mathrm{e}^{h/2T} & 0 \\ 0 & \mathrm{e}^{-h/2T} \end{pmatrix}$$

ABA approach to finite temperature correlation functions

In (Klümper 92, 93) techniques were developed to treat the Trotter limit in the expression for the free energy analytically: associate with every Trotter number $N \in 2\mathbb{N}$ a meromorphic 'auxiliary function'

$$\mathfrak{a}_N(\lambda) = \frac{d(\lambda)}{a(\lambda)} \prod_{k=1}^{N/2} \frac{\operatorname{sh}(\lambda - \lambda_k + \eta)}{\operatorname{sh}(\lambda - \lambda_k - \eta)}$$

where the λ_j are the Bethe roots that characterize the dominant eigenvalue, and consider the sequence of these functions in the complex plain, rather than sequences of sets of Bethe roots.

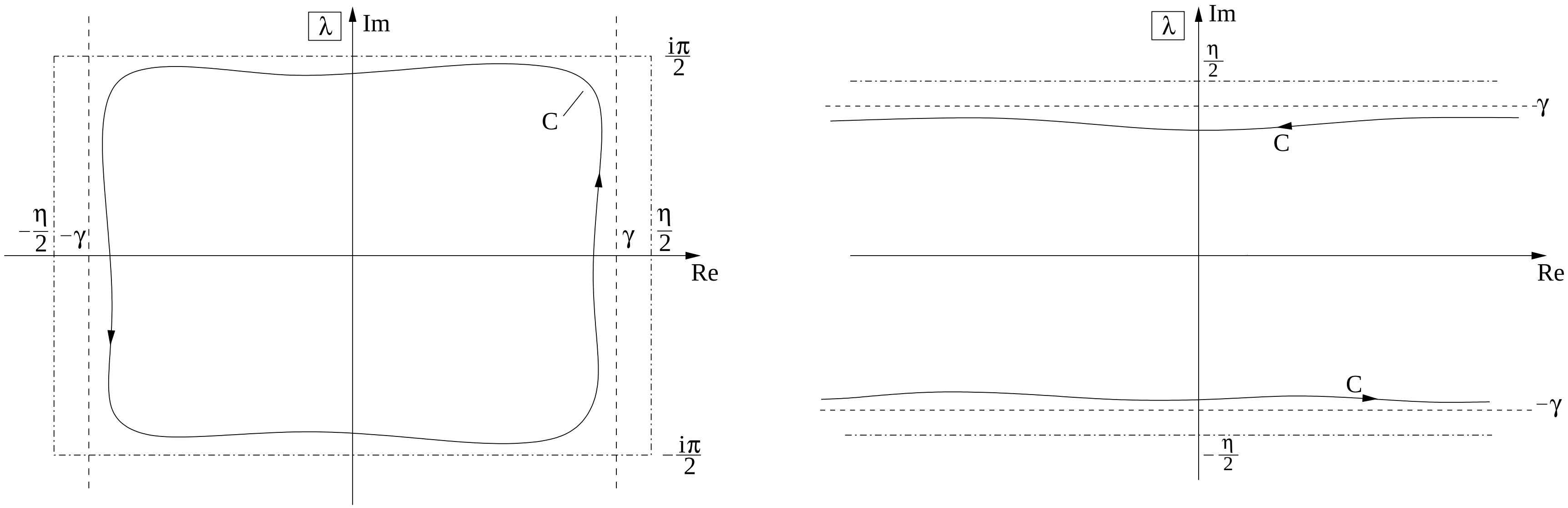
Then express all physical quantities of interest as contour integrals over $\mathfrak{a}_N$ on contours far away from the origin. In particular, $\ln \mathfrak{a}_N$ can be expressed this way, which yields a non-linear integral equation for $\mathfrak{a}_N$,

$$\ln \mathfrak{a}_N(\lambda) = -\frac{h}{T} + \ln\left[\frac{\operatorname{sh}(\lambda - \frac{\beta}{N})\operatorname{sh}(\lambda + \frac{\beta}{N} + \eta)}{\operatorname{sh}(\lambda + \frac{\beta}{N})\operatorname{sh}(\lambda - \frac{\beta}{N} + \eta)}\right]^{\frac{N}{2}} - \int_C \frac{d\omega}{2\pi i} \frac{\operatorname{sh}(2\eta)\ln(1 + \mathfrak{a}_N(\omega))}{\operatorname{sh}(\lambda - \omega + \eta)\operatorname{sh}(\lambda - \omega - \eta)}$$

Notice that we included the magnetic field here.
The contour C of the integral does not depend on the Trotter number. N appears as a mere parameter in the inhomogeneity of the integral equation. Hence, the Trotter limit can be performed,

$$\ln \mathfrak{a}(\lambda) = -\frac{h}{T} - \frac{2J\operatorname{sh}^2(\eta)}{T\operatorname{sh}(\lambda)\operatorname{sh}(\lambda + \eta)} - \int_C \frac{d\omega}{2\pi i} \frac{\operatorname{sh}(2\eta)\ln(1 + \mathfrak{a}(\omega))}{\operatorname{sh}(\lambda - \omega + \eta)\operatorname{sh}(\lambda - \omega - \eta)}$$

The canonical contour $\mathcal{C}$



$1/(1+\mathfrak{a})$ generalizes the Fermi function to the interacting case. Thinking in terms of Fermions it is natural to set $\overline{\mathfrak{a}} = 1/\mathfrak{a}$, and interpret $1/(1+\overline{\mathfrak{a}})$ as the Fermi function for holes satisfying $1/(1+\mathfrak{a}) + 1/(1+\overline{\mathfrak{a}}) = 1$.

Free energy per lattice site as a function of temperature and magnetic field

$$f(h,T) = -\frac{h}{2} - T\int_{\mathcal{C}} \frac{d\omega}{2\pi i} \frac{\mathrm{sh}(\eta)\ln(1+\mathfrak{a}(\omega))}{\mathrm{sh}(\omega)\,\mathrm{sh}(\omega+\eta)} = \frac{h}{2} + T\int_{\mathcal{C}} \frac{d\omega}{2\pi i} \frac{\mathrm{sh}(\eta)\ln(1+\overline{\mathfrak{a}}(\omega))}{\mathrm{sh}(\omega)\,\mathrm{sh}(\omega-\eta)}$$

The density matrix

The density matrix is a means to describe a sub-system as part of a larger system in thermodynamic equilibrium in terms of the degrees of freedom of the sub-system. It is obtained by taking the (normalized) statistical operator and tracing out the 'unwanted' degrees of freedom.

Since we want to include a magnetic field in z-direction we have to modify the statistical operator

$$\rho_L = \exp\bigl(-(H_{XXZ} - hS^z)/T\bigr)$$

Then the density matrix of the sub-system consisting of the first m lattice sites is defined as

$$D_L(T,h) = \frac{\mathrm{tr}_{m+1\dots L}\, \rho_L}{\mathrm{tr}_{1\dots L}\, \rho_L}$$

By construction, the thermal average of every operator A acting non-trivially only on sites 1 to m can now be written as

$$\langle A\rangle_{T,h} = \frac{\mathrm{tr}_{1\dots L}\, A\rho_L}{\mathrm{tr}_{1\dots L}\, \rho_L} = \frac{\mathrm{tr}_{1\dots m}\, A_{1\dots m}\, \mathrm{tr}_{m+1\dots L}\, \rho_L}{\mathrm{tr}_{1\dots L}\, \rho_L} = \mathrm{tr}_{1\dots m}\, A_{1\dots m} D_L(T,h)$$

where $A_{1\dots m}$ is the restriction of A to a chain consisting of sites 1 to m. In particular, every two-point function of local operators in the segment 1 to m of the XXZ chain can be brought into the above form.

The density matrix

Denote the gl(2) standard basis of 2×2 matrices with a single non-zero entry at the intersection of row β and column α by e^α_β, $\alpha, \beta = 1, 2$. The canonical embedding into $(\mathbb{C}^2)^{\otimes L}$ will be denoted by ${e_j}^\alpha_\beta$, $j = 1, \dots, L$. For the restriction to the first m sites we use the same symbols. Then

$$D_L{}^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m}(T,h) = \mathrm{tr}_{1 \dots m}\, {e_1}^{\alpha_1}_{\beta_1} \dots {e_m}^{\alpha_m}_{\beta_m} D_L(T,h) = \langle {e_1}^{\alpha_1}_{\beta_1} \dots {e_m}^{\alpha_m}_{\beta_m} \rangle_{T,h}$$

which holds for all L and is still valid in the thermodynamic limit.

For $L \longrightarrow \infty$ we can apply our general formula and obtain

$$\left\langle {e_1}^{\alpha_1}_{\beta_1} \dots {e_m}^{\alpha_m}_{\beta_m} \right\rangle_{T,h} = \lim_{N \to \infty} \lim_{\xi_1, \dots, \xi_m \to 0} D^{(N)}{}^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m}(\xi_1, \dots, \xi_m)$$

where

$$D^{(N)}{}^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m}(\xi_1, \dots, \xi_m) = \frac{\langle \{\lambda\} | T^{\alpha_1}_{\beta_1}(\xi_1) \dots T^{\alpha_m}_{\beta_m}(\xi_m) | \{\lambda\} \rangle}{\langle \{\lambda\} | \prod_{j=1}^m t(\xi_j) | \{\lambda\} \rangle}$$

is the 'inhomogeneous finite Trotter number approximant' to the density matrix element in the thermodynamic limit. In order to simplify the notation we have suppressed the dependence on T and h on the left hand side.

Integral representation

The density matrix elements

$$\langle e_1{}^{\alpha_1}_{\beta_1} \dots e_m{}^{\alpha_m}_{\beta_m} \rangle_{T,h} = \left[\prod_{j=1}^{|\alpha^+|} \int_C \frac{d\omega_j}{2\pi i} \frac{\operatorname{sh}^{\tilde{\alpha}_j^+ - 1}(\omega_j - \eta) \operatorname{sh}^{m - \tilde{\alpha}_j^+}(\omega_j)}{1 + \mathfrak{a}(\omega_j)} \right]$$

$$\left[\prod_{j=|\alpha^+|+1}^{m} \int_C \frac{d\omega_j}{2\pi i} \frac{\operatorname{sh}^{\tilde{\beta}_j^- - 1}(\omega_j + \eta) \operatorname{sh}^{m - \tilde{\beta}_j^-}(\omega_j)}{1 + \overline{\mathfrak{a}}(\omega_j)} \right] \det \left[- \frac{\partial_\xi^{(k-1)} G(\omega_j, \xi)|_{\xi=0}}{(k-1)!} \right] \frac{1}{\prod_{1 \le j < k \le m} \operatorname{sh}(\omega_j - \omega_k - \eta)}$$

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Notation exemplified: here $m = 8$, $|\alpha^+| = 3$,

j	1	2	3	4	5	6	7	8
α_j	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
β_j	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\tilde{\alpha}_j^+$	6	4	3					
$\tilde{\beta}_j^-$				1	3	4	5	8

$$G(\lambda, \xi) = \frac{\mathrm{sh}(\eta)}{\mathrm{sh}(\xi - \lambda)\, \mathrm{sh}(\xi - \lambda + \eta)} + \int_C \frac{d\omega}{2\pi i (1 + \mathfrak{a}(\omega))} \frac{\mathrm{sh}(2\eta) G(\omega, \xi)}{\mathrm{sh}(\lambda - \omega + \eta)\, \mathrm{sh}(\lambda - \omega - \eta)}$$

Integral representation

Remarks:

(i) For $T \to 0$ the formulae for the density matrix elements reduce to known results (Kitanine et al. 00, Jimbo et al. 92, 96)

(ii) The formulae for the density matrix elements are obtained as the homogeneous limit ($\xi_j \to 0$, $j = 1, \dots, m$) of the more general inhomogeneous functions

$$D^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m}(\xi_1, \dots, \xi_m) = \left[\prod_{j=1}^{|\alpha^+|} \int_C \frac{d\omega_j}{2\pi i (1 + \mathfrak{a}(\omega_j))} \prod_{k=1}^{\tilde{\alpha}_j^+ - 1} \mathrm{sh}(\omega_j - \xi_k - \eta) \prod_{k=\tilde{\alpha}_j^+ + 1}^{m} \mathrm{sh}(\omega_j - \xi_k) \right]$$
$$\left[\prod_{j=|\alpha^+|+1}^{m} \int_C \frac{d\omega_j}{2\pi i (1 + \overline{\mathfrak{a}}(\omega_j))} \prod_{k=1}^{\tilde{\beta}_j^- - 1} \mathrm{sh}(\omega_j - \xi_k + \eta) \prod_{k=\tilde{\beta}_j^- + 1}^{m} \mathrm{sh}(\omega_j - \xi_k) \right]$$
$$\frac{\det(-G(\omega_j, \xi_k))}{\prod_{1 \le j < k \le m} \mathrm{sh}(\xi_k - \xi_j)\, \mathrm{sh}(\omega_j - \omega_k - \eta)}$$

These functions in the limit $T \to 0$ satisfy a set of functional equations in the parameters ξ_j, the reduced q-Knizhnik-Zamolodchikov equation (Boos et al. 04).

Lemma 1. The general left action. For $j=1,\dots,m$ and $\ell_j \in \{1,\dots,M+m\}$ define

$$|\ell^+| = \mathrm{card}\{1,\dots,M\} \cap \{\ell_j\}_{j=1}^m$$

and $\lambda_{M+j} = \xi_j$. Then

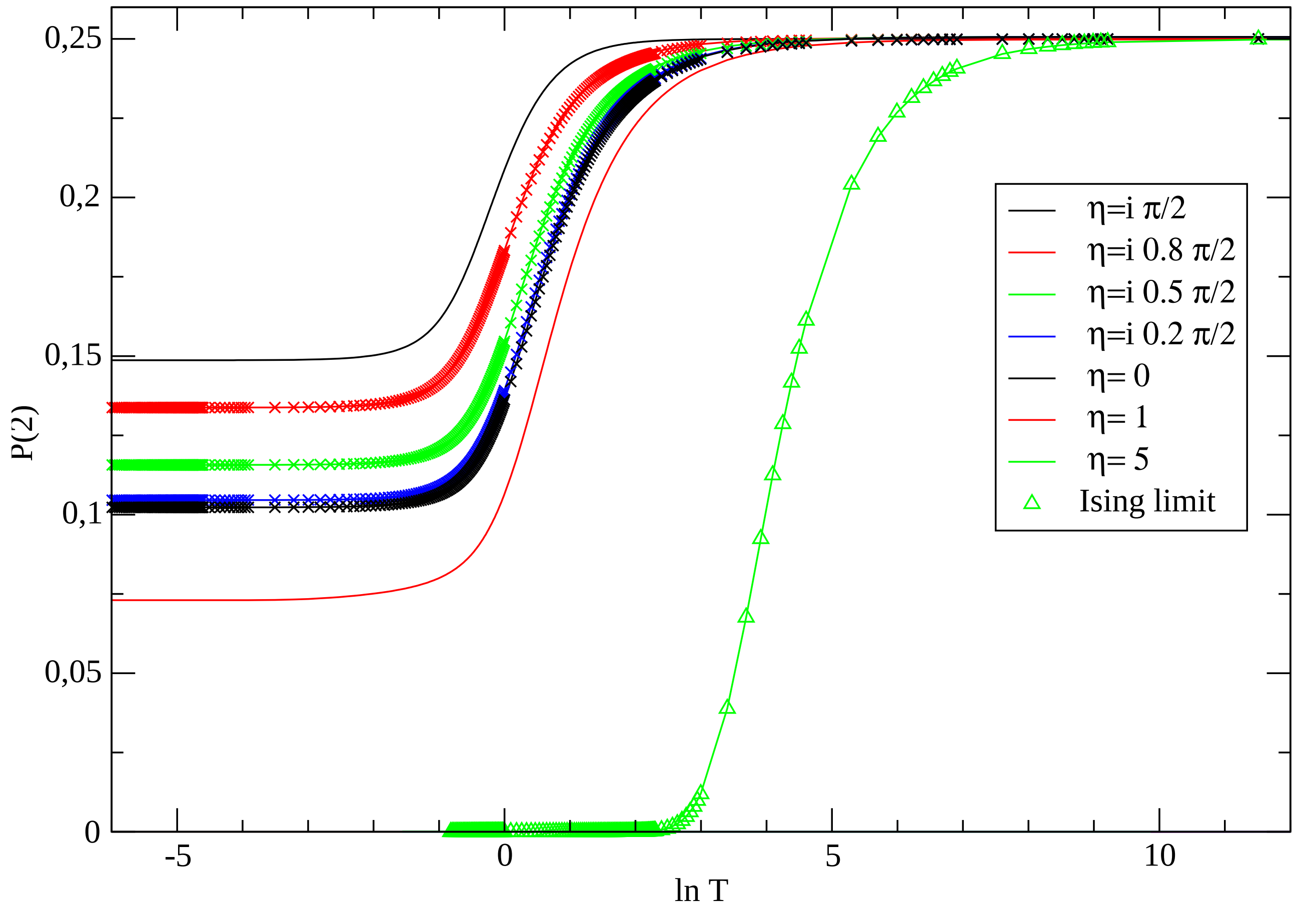
$$\langle 0| \prod_{j=1}^{M} C(\lambda_j) T^{\alpha_1}_{\beta_1}(\xi_1)\dots T^{\alpha_m}_{\beta_m}(\xi_m) = \sum_{\substack{\ell_1,\dots,\ell_m=1 \\ \ell_j\neq\ell_k \text{ for } j\neq k}}^{M+m} \big(-\mathrm{sh}(\eta)\big)^{|\ell^+|} \langle 0| \prod_{\substack{k=1 \\ k\neq\ell_1\dots\ell_m}}^{M+m} C(\lambda_k)$$
$$\left[\prod_{j=|\alpha^+|+1}^{m} \mathfrak{a}_N(\lambda_{\ell_j})\right] \left[\prod_{j=1}^{m} a(\lambda_{\ell_j}) \prod_{\substack{k=1 \\ k\neq\ell_j}}^{M} \frac{1}{b(\lambda_k,\lambda_{\ell_j})}\right] \left[\prod_{j=1}^{m} \prod_{\substack{k=1 \\ k+M\neq\ell_j}}^{m} \frac{1}{\mathrm{sh}(\lambda_{\ell_j}-\xi_k)}\right] \left[\prod_{1\le j<k\le m} b(\lambda_{\ell_k},\lambda_{\ell_j})\right]$$
$$\left[\prod_{j=1}^{|\alpha^+|} \prod_{k=1}^{\tilde\alpha_j^+-1} \mathrm{sh}(\omega_j-\xi_k-\eta) \prod_{k=\tilde\alpha_j^+ +1}^{m} \mathrm{sh}(\omega_j-\xi_k)\right] \left[\prod_{j=|\alpha^+|+1}^{m} \prod_{k=1}^{\tilde\beta_j^- -1} \mathrm{sh}(\omega_j-\xi_k+\eta) \prod_{k=\tilde\beta_j^- +1}^{m} \mathrm{sh}(\omega_j-\xi_k)\right]$$

Lemma 2. Matrix element formula.

$$\frac{\langle\{\xi^+\}\cup\{\lambda^-\}|\{\lambda\}\rangle}{\langle\{\lambda\}|\{\lambda\}\rangle \prod_{j=1}^m \Lambda_0(\xi_j)} = \left[\prod_{j=1}^{|\xi^-|} \frac{\prod_{k=1}^{N/2} b(\lambda_k,\xi_j^-)}{a(\xi_j^-)(1+\mathfrak{a}_N(\xi_j^-))}\right] \left[\prod_{j=1}^{|\lambda^+|} \frac{\prod_{k=1,\lambda_k\neq\lambda_j^+}^{N/2} b(\lambda_k,\lambda_j^+)}{\mathrm{sh}(\eta) a(\lambda_j^+) \mathfrak{a}'_N(\lambda_j^+)}\right] \frac{\det G(\lambda_j^+,\xi_k^+)}{\det\left[\frac{1}{\mathrm{sh}(\lambda_j^+-\xi_k^+)}\right]}$$

Numerical results

- Numerical results for small *m* (M. Bortz)



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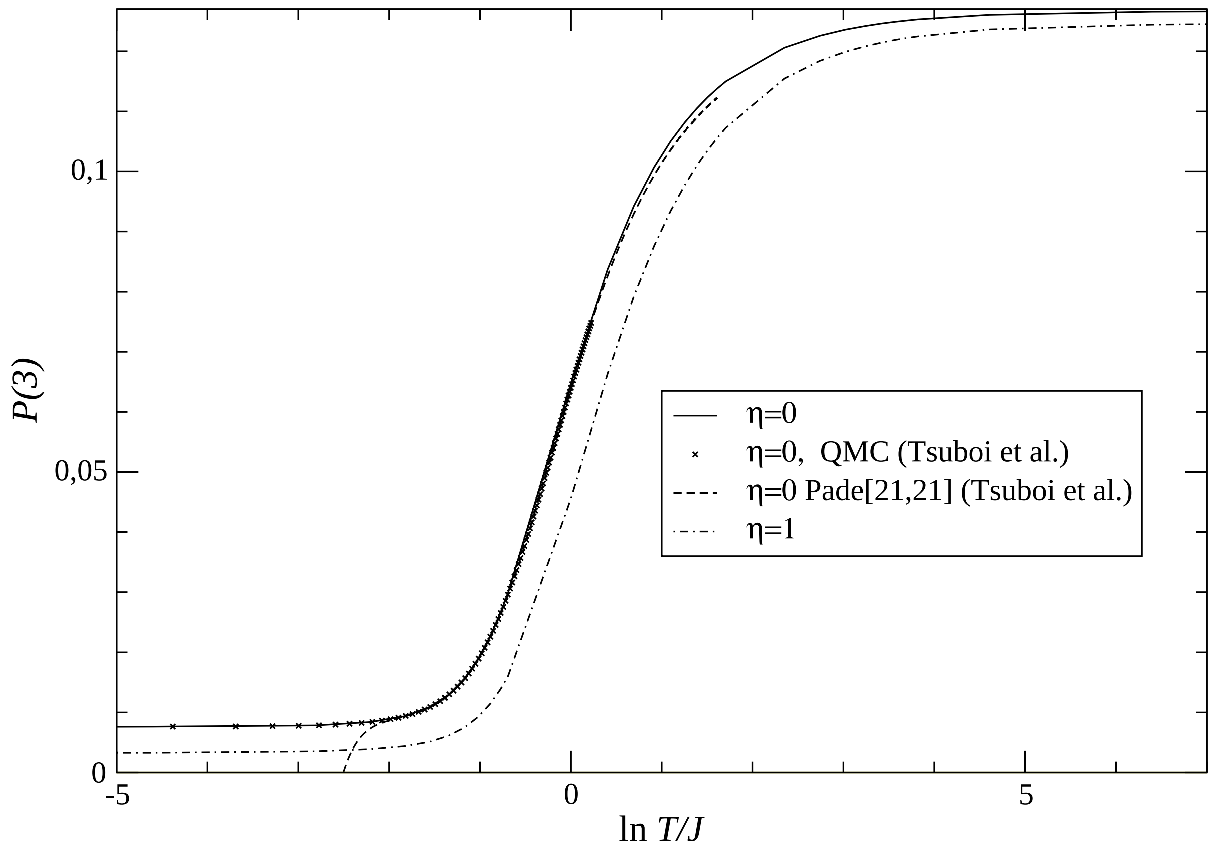

Further results and outlook

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  (i) Integral formulae for a one-parameter generating function of the σ^z-σ^z correlation functions (FG, A. Klümper and A.Seel 04, FG and A. Seel hep-th/0505091)
  (ii) Summation of density matrix elements (FG unpublished)
  (iii) Open XXZ chain with boundary fields (FG, M. Bortz and H. Frahm cond-mat/0508377)
  (iv) High order high temperature expansions (Z. Tsuboi and M. Shiroishi 05)
  (v) Ansatz solution (no integrals!) of reduced q-Knizhnik-Zamolodchikov equation for the inhomogeneous model (H. Boos, M. Jimbo, T. Miwa, F. Smirnov and Y. Takeyama 04, 05)

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- Outlook
  (i) Large distance asymptotics for emptiness formation probability?
  (ii) Density matrix elements – reduction of integrals (cf Boos, Korepin)?
  (iii) Density matrix elements – reduced q-Knizhnik-Zamolodchikov equation at finite temperature?
  (iv) Time dependent correlation functions?
  (v) Other models (higher rank)?