

# *Leptonic BSM,*

## *Magnetic Moments and Majorana Masses*

(things I learned when thinking about reconstructing new physics chez les leptons)

Sacha Davidson

IPN de Lyon

## (ambitious) motivation (= daydreaming)

- we need New Physics to generate  $[m_\nu]$
- Plethora of models — differing at high energy scale  $\gtrsim ? \dots E_{cm}$  of colliders within  $\tau_{sacha}$ ?

⇒ what can we reconstruct from the data? ??

...suppose (!)... $0\nu 2\beta$  but no info on new leptonic physics from the LHC... then:

“if at the weak scale, we know the coupling of every possible interaction involving SM particles, of  $\dim < N$ , what can we learn about the physics at higher energies?”

?? ?? ??

- toy model

- suppose 3 light  $\nu$
- suppose know coefficients of some  $\Delta L = 2$  interactions of  $\dim \leq 7 : [m_\nu], [a]$

- get distracted →
1. run coeffs of effective operators  $up$  in scale ( $SU(2) \times U(1)$  RGEs ... what happens?)
  2. suppose  $[a], [m_\nu]$  from 1-loop New Physics— solve (algebra) for masses and couplings ?
  3. suppose generated at 2-loop, etc...



## ...it “works” in the SUSY seesaw

Suppose usual  $\mathcal{L}$  with 3 added singlets  $N$  (in  $N_K$ , charged lepton mass bases) :

$$\mathcal{W} = \mathcal{W}_{MSSM} + N\mathbf{Y}_\nu L \cdot H_u - \frac{1}{2}N\mathcal{M}N$$

⇒ weak-scale neutrino mass matrix:

$$[m_\nu]_{ij} = [\mathbf{Y}_\nu^T]_{iK} \frac{1}{M_K} [\mathbf{Y}_\nu]_{Kj} \langle H_u^0 \rangle^2$$

flavour violation in sneutrino mass matrix (leading log):

$$[\tilde{m}^2]_{\mu e} \sim \tilde{\nu}_\mu \cdots \overset{N_K}{\underbrace{\begin{matrix} \mathbf{Y}_\nu & \mathbf{Y}_\nu \\ \tilde{h} \end{matrix}}} \cdots \tilde{\nu}_e \sim \frac{3m_0^2 + A_0^2}{8\pi^2} (\mathbf{Y}_\nu^\dagger)_{\mu K} (\mathbf{Y}_\nu)_{Ke} \log \frac{M_{GUT}}{M_K}$$

if universal soft masses, and  $\leq 3$  gen. of  $N$ , then:

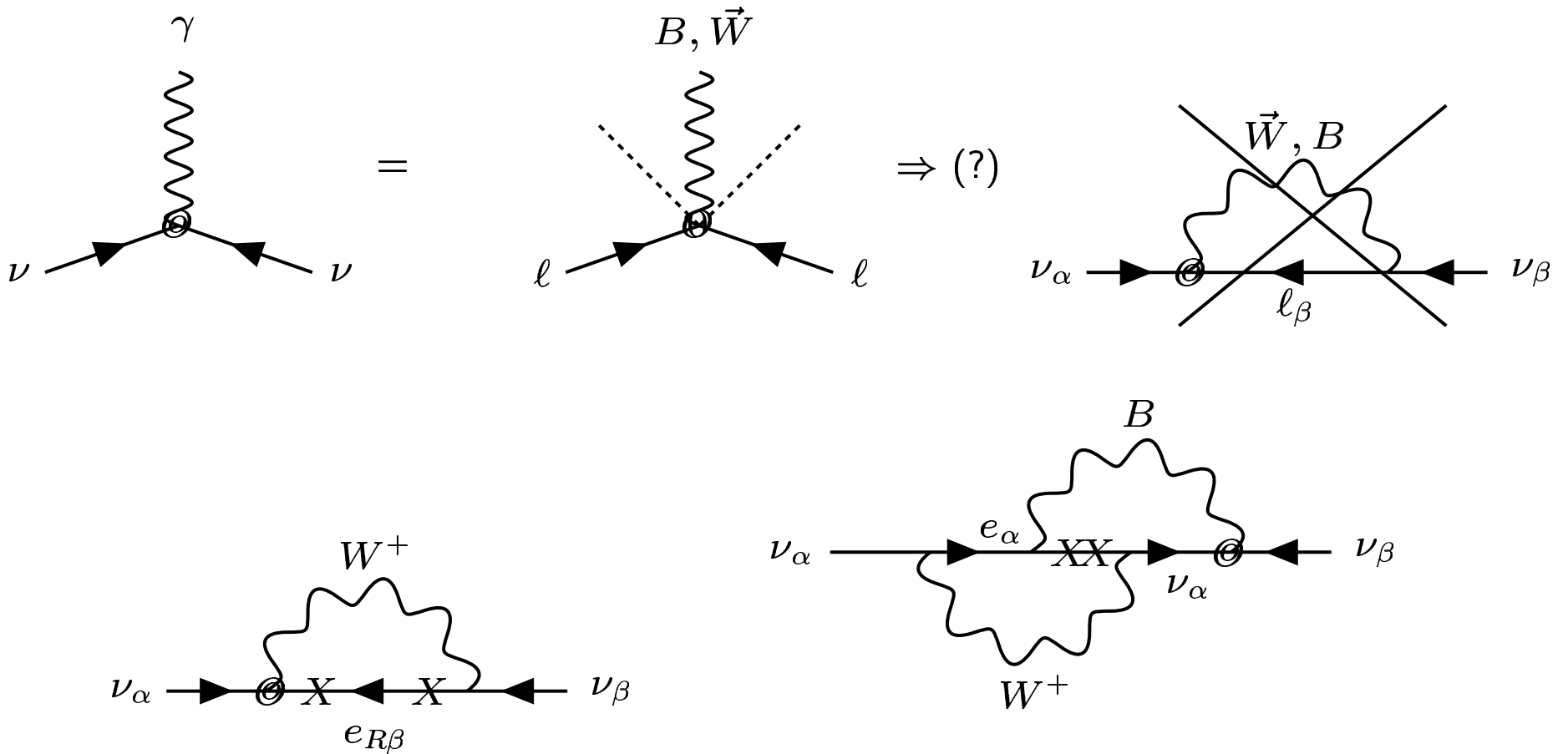
1. extract  $Y^\dagger Y$  from  $[m_{\tilde{\nu}}^2]$ ,  $Y^\dagger Y = V_L^\dagger D_Y^2 V_L$
2. peel  $Y$  off outside of  $[m_\nu]$  :  $D_Y^{-1} V_L^* [m_\nu] V_L^\dagger D_Y^{-1} = V_R^* D_{\mathcal{M}}^{-1} V_R^\dagger$

from masses at  $m_W$ , extract  $D_{\mathcal{M}} = \{M_1, M_2, M_3\}$  and  $\mathbf{Y}_\nu = V_L^\dagger D_Y V_R$  in “ $D_{\mathcal{M}}, D_{m_e}$ ” basis

Constraints on operators responsible for

# Magnetic Moments from Majorana Masses

S D + M Gorbahn + A Santamaria



# Outline

1. (ambitious) motivation...to more manageable project:
2. transition magnetic moments  $a_{\alpha\beta}$ : (from MeV  $\rightarrow$   $\Lambda_{NP}$ )
  - notation
  - experimental bounds
  - SM operators
  - “theoretical expectations”
3. “bottom-up” pheno:
  - SM loops:  $a \rightarrow \delta m_\nu$
  - for certain parameters,  $[\delta m_\nu] > [m_\nu]$
4. detour: what do we know about  $[m_\nu]$ ?
5. back to mag mo calculation: so what?
  - bounds on coefficients of non-renorm operators
6. summary

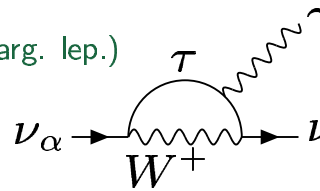
# Transition magnetic moments (for majorana $\nu$ s)

Nieves  
Kayser  
Shrock  
Li+Wilcek

the magnetic moment interaction :  $\frac{a}{2} \overline{\psi}_R \sigma^{\mu\nu} \psi_L F_{\mu\nu} + h.c.$   $\left\{ \begin{array}{l} \text{chirality flip} \\ \text{dimension 5} \end{array} \right.$

( for  $e^-$ : Gordon decomp. gives  $a = \frac{2e}{2m_e} \equiv 2\mu_B$ )

know: (small) neutrino masses  $\rightarrow$  (smaller) magnetic moments  $a \sim \frac{m_\nu m_\tau^2}{m_W^4}$  ( $\tau = \text{charg. lep.}$ )  
(veery small. forget)



question: mass?  $\leftarrow$  LARGE magnetic moment

suppose neutrinos are majorana, then :  $\frac{a_{\alpha\beta}}{2} \overline{\nu}^c_\alpha \sigma^{\mu\nu} \nu_\beta F_{\mu\nu} + h.c.$   $\left\{ \begin{array}{l} \Delta L = 2 \\ a_{\alpha\beta} = -a_{\beta\alpha} \end{array} \right.$

(so  $a_{\alpha\alpha} = 0$ )

# Constraints

no evidence for  $a$  in  $t \rightarrow 0 \bar{\nu}e$  scattering in the lab:

$$a_{e\beta} \leq 0.9 \times 10^{-10} \mu_B, \quad a_{\mu\beta} \leq 6.8 \times 10^{-10} \mu_B$$

$a$  induces  $\gamma \rightarrow \nu\nu$  cooling of stars. From the lifetime of red giants:

$$a_{\alpha\beta} \leq 3 \times 10^{-12} \mu_B$$

if  $a \neq 0$ , in the solar  $\vec{B}$  field  $\Rightarrow$  time dependence of solar  $\nu_\alpha$  fluxes, also  $\bar{\nu}_\beta$  flux from sun.  
Upcoming experiments could be sensitive to

$$a_{\alpha\beta} \gtrsim 10^{-13} \mu_B$$

$\Rightarrow$  suppose  $a \sim$  upper bound — does it give an “interesting” contribution to  $[m_\nu]$  ?

## in the SM above $m_W \dots$

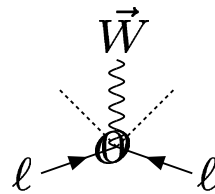
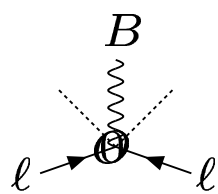
Transition magnetic moment:  $\frac{a_{\alpha\beta}}{2} \bar{\nu}^c \sigma^{\mu\nu} \nu_\beta F_{\mu\nu} + h.c.$

Suppose there is New Physics at  $\Lambda_{NP} \gg m_W$ , that produces  $[a]$ . Then below  $\Lambda_{NP}$ , have possible effective operators in the SM:

$$C_B (\bar{\ell}^c H) \sigma^{\mu\nu} (H \ell) B_{\mu\nu}$$

$$C_W \varepsilon_{abd} (\bar{\ell}^c \tau^a \ell) (H \tau^b H) W_{\mu\nu}^d$$

where  $\ell =$  lepton doublet.  $C_J \sim 1/\Lambda_{NP}^3$



$$\Rightarrow a \sim \frac{v^2}{\Lambda_{NP}^3}$$

*N.B.*  $10^{-12} \mu_B$  is **BIG** :

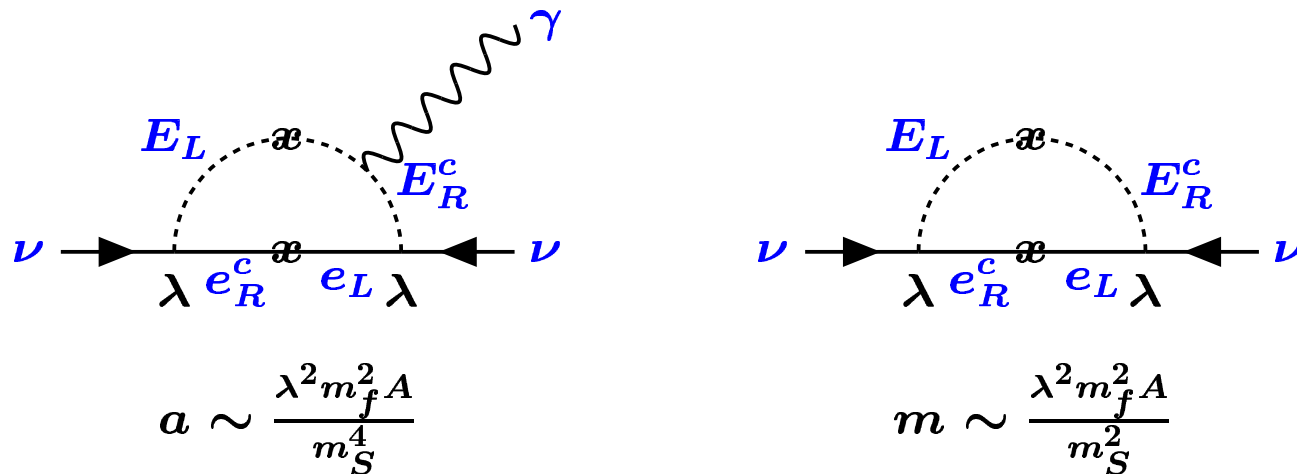
$$\frac{v^2}{\Lambda_{NP}^3} = 10^{-12} \mu_B \Rightarrow \Lambda_{NP} \lesssim 10 TeV$$



# model-building intermission: some possibilities (inside the blob)

Voloshin  
BabuMohapatra  
GeorgiRandall  
Chang+...  
Barr+...

$a \sim 10^{-12} \mu_B$  for  $\nu_{sun}$  (?), but  $m_\nu \lesssim \text{eV} \Rightarrow$  problem for models (e.g. in RPV SUSY):



so  $a \sim 10^{-12} \mu_B \Rightarrow m_\nu \sim m_S^2 a \sim \text{MeV} \left( \frac{m_S}{\text{TeV}} \right)^2$

“natural” solutions:

- Voloshin, ...:  $a_{\alpha\beta}$  is flavour *antisymmetric*, arrange cancellations among the diagrams contributing to the flavour *symmetric* mass matrix
- Barr Freire, Zee: forbid by angular momentum conservation (spin) the magnetic moment diagram with its photon removed.

I am *NOT* building a model ! I do effective theory below  $\Lambda_{NP}$

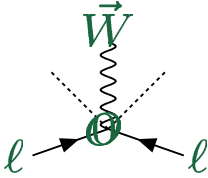
# Outline (...again...where are we?)

1. (ambitious) motivation...to more manageable project:
2. transition magnetic moments  $a_{\alpha\beta}$ : (from MeV  $\rightarrow$   $\Lambda_{NP}$ )
  - notation
  - experimental bounds
  - SM operators
  - “theoretical expectations”
3. “bottom-up” pheno:
  - SM loops:  $a \rightarrow \delta m_\nu$
  - for certain parameters,  $[\delta m_\nu] > [m_\nu]$
4. detour: what do we know about  $[m_\nu]$ ?
5. back to mag mo calculation: so what?
  - bounds on coefficients of non-renorm operators
6. summary

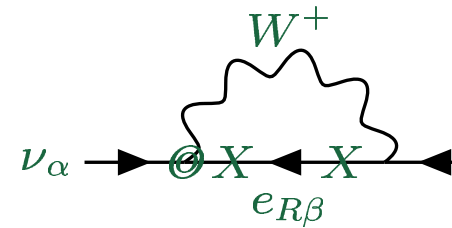


contribution to  $[m_\nu]$  of  $a \sim 10^{-12} \mu_B$  (large)...

1. Suppose  $a_{\alpha\beta}$  is generated by  $O^W = \varepsilon_{abd}(\bar{\ell}^c \tau^a \ell)(H \tau^b H) W_{\mu\nu}^d$ :



2. leading log contribution of  $O_W$  to the dim 7 mass operator  $O^M = (\bar{\ell}^c H)(H \ell)(H^\dagger H)$  from:



3. approximate solution:

$$[\delta m]_{\alpha\beta} \simeq \frac{6g}{16\pi^2 s_W} (m_\alpha^2 - m_\beta^2) a_{\alpha\beta} \ln \left( \frac{\Lambda_{NP}}{m_W} \right)$$

$$\sim 0.1 \text{eV} \quad (\text{for } a_{\alpha\tau} = 10^{-12} \mu_B)$$

## results

$O_W$  contributes to mag mo:

$$[a] = 2v^2 s_W [C_W]$$

and to the neutrino mass matrix ( $a_{\alpha\beta} = \tilde{a}_{\alpha\beta} \cdot 10^{-12} \mu_B$ ):

$$[\delta m] = \begin{bmatrix} 0 & 0.004\tilde{a}_{e\mu} & \tilde{a}_{e\tau} \\ 0.004\tilde{a}_{e\mu} & 0 & \tilde{a}_{\mu\tau} \\ \tilde{a}_{e\tau} & \tilde{a}_{\mu\tau} & 0 \end{bmatrix} \times .1\text{eV} \quad \begin{array}{l} \text{large atm mixing!} \\ \text{...but not pheno. viable} \\ \text{: (} \end{array}$$

$$\Delta m_{atm}^2 = (0.049\text{eV})^2, \Delta m_{sol}^2 = (0.0089\text{eV})^2, \theta_{23} \simeq \pi/4, \theta_{12} \simeq \pi/6, s \equiv \sin \theta_{13} \leq .2$$

- if  $a_{\alpha\tau} \gtrsim 10^{-12} \mu_B$  and non- degen  $m_\nu$ , then mild cancellation  $\delta m \leftrightarrow [m_\nu]$  required.
- if  $a_{e\tau} \sim 10^{-12} \mu_B$  and hierarchical  $m_\nu$ ,

$$[m_\nu]^{hier} \sim \begin{bmatrix} .06 & .06 + .35s & -.06 + .35s \\ .06 + .35s & .06 + .25 & -.06 + .25 \\ -.06 + .35s & -.06 + .25 & .06 + .25 \end{bmatrix} \times .1\text{eV} .$$

$$\Rightarrow \frac{[\delta m_\nu]_{e\tau}}{[m_\nu]_{e\tau}} \sim 10$$

$\Rightarrow a_{e\tau} < 10^{-13} \mu_B$ , or cancellation (peculiar, unexpected, not impossible)

- $[\delta m]_{\mu e} \ll m_{\mu e}$

## detour: what do we know about $[m_\nu]$ ?

not enough  $\nu_\mu$  arrive from the atmosphere, deficit consistent with KEK to SK deficit (K2K)

$$\begin{aligned} (\pi \rightarrow \mu \bar{\nu}_\mu & \quad (\nu_\mu \text{ beam}) \\ & \rightarrow e \nu_\mu \bar{\nu}_e) \end{aligned}$$

not enough  $\nu_e$  arrive from the sun (MSW) , *and* deficit of  $\bar{\nu}_e$  at KamLAND (vacuum osc)

$$(\nu_e \text{ from nuclear fusion}) \quad (\bar{\nu}_e \text{ from fission reactors})$$

$\Rightarrow \nu$  have small masses and “oscillate”:

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta_{\alpha\beta} \sin^2 \left( \frac{(m_\alpha^2 - m_\beta^2)L}{2E} \right) \quad (2 \text{ flavour})$$

(*NOT* seeing decays, precession due to mag mos, non-standard interactions...)

## combined fit numbers

in “flavour” basis (charged lepton mass basis):

$$[m_\nu^2]_{\alpha\beta} = V D_{m_\nu}^2 V^\dagger \quad D_{m_\nu}^2 = \text{diag}\{m_1^2, m_2^2, m_3^2\}$$

$$V_{fm} = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix}.$$

energies/baselines for  $\Delta m_{31}^2 = m_3^2 - m_1^2$  (atm, K2K, CHOOZ):

$$\Delta m_{31}^2 = (0.049\text{eV})^2 \pm 0.6 \times 10^{-3} \text{eV}^2$$

$$\sin^2 \theta_{23} = 0.45 \pm \sim .15 \quad (\text{incident } \nu_\mu)$$

$$\theta_{13} \lesssim 0.18 \quad (\text{incident } \bar{\nu}_e)$$

energies/baselines for  $\Delta m_{21}^2 = m_2^2 - m_1^2$  (sun, KamLAND):

$$\Delta m_{21}^2 \simeq (0.0089\text{eV})^2 \pm 0.6 \times 10^{-5} \text{eV}^2$$

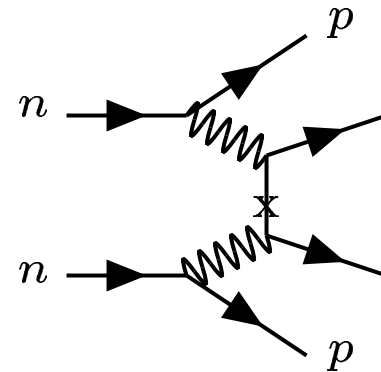
$$\sin^2 \theta_{12} \simeq 0.31 \pm + 0.05$$



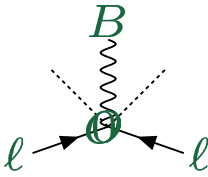
## what we don't know...

1. how many  $\Delta m^2$ ? — are there sterile mass eigenstates?  
 test LSND ( $\Delta m^2 \sim eV^2$ ) with MiniBoone— results  $\nu_\mu \rightarrow \nu_e$  late 2005?
2. are  $\nu$  masses majorana ( $\Delta L = 2$ ) or Dirac (6 light 2-comp  $\nu$ )  
 $\Delta L$  process...e.g.  $0\nu 2\beta$
3. mass pattern:
 

● hierarchical: $m_3 > m_2 > m_1$	3 ———	—————	2
● inverted : $m_1, m_2 > m_3$		—————	1
$0\nu 2\beta$ , $\nu$ beams under discussion			
	2 ———	—————	
	1 ———	—————	3
4. absolute mass scale
  - spectrum endpoint in  $\beta$  decay  $|U_{ei}|^2 |m_i|^2, \lesssim 1 \text{ eV}$
  - $0\nu 2\beta$  :  $\mathcal{M}_{nucl} |U_{ei}^2 m_i|, !! .2 \rightarrow 1 \text{ eV}$
  - cosmology  $\sum_i m_i \lesssim 0.46 \text{ eV} (\rightarrow \sim 2 \text{ eV})$
5.  $\theta_{13}, \delta \dots$



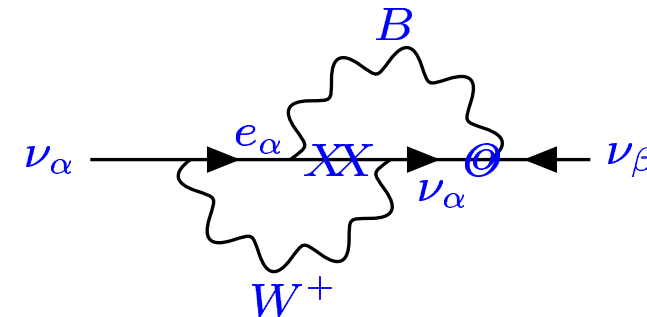
## what about $O_B$



1. Suppose  $a_{\alpha\beta}$  is generated by  $O^B = (\bar{\ell}^c H)\sigma^{\mu\nu}(H\ell)B_{\mu\nu}$ :

2. *could* calculate contribution to  $[m_\nu]$  by operator mixing during RG running  $\Lambda_{NP} \rightarrow m_W$  :

- match  $[C_B]$  onto  $[a]$  at  $\mu \sim m_W$
- Run  $[C_B]$ :  $m_W \rightarrow \Lambda_{NP}$  (order 1 change, so forget)
- run back to  $m_W$ :  $O^B$  can mix with  $O_W$  and other dim 7 operators...
- calculate anomalous dim matrix  $[\gamma]$  for  $\{O_B, \text{ops it mixes into at one loop which also mix into } O^M \text{ at one loop, } O^M\}$



3. but... estimate:

$$\begin{aligned}
 [\delta m]_{\alpha\beta} &\sim \frac{6g'\alpha_W |m_\alpha^{e2} - m_\beta^{e2}|}{64\pi^3} a_{\alpha\beta} \log^2 \left( \frac{\Lambda_{NP}}{m_W} \right) \\
 &\sim \frac{\alpha_{em}}{\pi} \cdot (1 - \text{loop estimate}) \\
 \Rightarrow &[\delta m]_{\alpha\beta} \leq [m]_{\alpha\beta}
 \end{aligned}$$

better bound on  $[C_B]_{\alpha\beta}$  from mag mos than masses



## so what? Back to motivation...

...umm...no progress, going backwards:

another source of confusion in reconstruction:  $[m_\nu]/v^2$  is the coefficient of

$$(H\ell)(H\ell) \left( C^0 + C^1 \frac{v^2}{\Lambda_{NP}^2} + \dots \right)$$

I want to set  $C^0 = [m_\nu]/v^2$ . But is it?? If  $C^0 = 0$  by some symmetry, how could I tell??

## Summary

current bounds on neutrino magnetic moments are  $a \lesssim 10^{-10} \mu_B$  (lab),  $3 \cdot 10^{-12} \mu_B$  (astro).  
Upcoming expts and solar physics could be sensitive to  $a \gtrsim 10^{-13} \mu_B$ . [ $\mu_B = e/(2m_e)$ ].

neutrino mass differences and mixing angles we know from oscillation expts:

$$\Delta m_{atm}^2 = (0.049 eV)^2, \Delta m_{sol}^2 = (0.0089 eV)^2, \theta_{23} \simeq \pi/4, \theta_{12} \simeq \pi/6, \sin \theta_{13} \leq .2$$

It is well known that a majorana mass induces a (small) transition magnetic moment  
but...small  $m_\nu$  as observed  $\Rightarrow$  undetectable  $a$

Converse is also true: operators generating mag. mo.  $a_{\alpha\beta}$  contribute via loops to  $[m_\nu]_{\alpha\beta}$ .  
large  $a$  near current bound  $\Rightarrow$  contribution  $[\delta m_\nu] \sim [m_\nu]$

two  $SU(2) \times U(1)$  invariant, dimension 7 operators that could induce the transition mag moment.

Operator coefficients are bounded by  $[a], [\delta m_\nu] < \text{expt}$ :

$$C_W \varepsilon_{abd} (\bar{\ell}^c \tau^a \ell) (H \tau^b H) W_{\mu\nu}^d \text{ (so } e\nu W^+ \text{ exists)}$$

- if non-degen  $m_\nu$ , then  $[\delta m_\nu] \lesssim [m_\nu] \Rightarrow a_{\alpha\tau} \lesssim 3 \times 10^{-13} \mu_B$
- if hierarchical  $m_\nu$ , then  $[\delta m_\nu] \lesssim [m_\nu] \Rightarrow a_{e\tau} \lesssim 10^{-13} \mu_B$
- strongest bounds on  $[C_W]_{e\mu}$  are from  $a_{e\mu}$

$$C_B (\bar{\ell}^c H) \sigma^{\mu\nu} (H \ell) B_{\mu\nu} \text{ (only } \nu\nu Z/\gamma \text{ mag mo interaction),}$$

- best limit on  $[C_B]_{\alpha\beta}$  is astro mag mo